REPUTATION AND PERFORMANCE FEE EFFECTS
ON PORTFOLIO CHOICE BY INVESTMENT ADVISERS

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Reputation and Performance Fee Effects on Portfolio Choice by Investment Advisers†

by

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This paper examines a two-period model of investment management. Investors reallocate their wealth between two mutual funds managed by different investment advisers after observing the performance of each adviser in the first period. A reputation effect causes one adviser to use his private information about investment returns too aggressively in the first period. This is costly for risk-averse advisers and investors because mutual funds are riskier than in one-period or single-adviser settings. A commitment by investors not to reallocate their investment among advisers or the adoption of a performance fee mitigate undesirable reputation effects and results in superior ex ante payoffs to investors.

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This paper explores distortions in the portfolio choices of investment advisers caused by reputation concerns. Because investors can easily compare the past performance of advisers and appoint a new adviser to manage their portfolios at any time, advisers face enormous pressure to produce a history of investment choices that compares favorably with those of their peers. When advisers differ in ability, past portfolio choices shed some light on the ability of the adviser. In this setting, the investment choice that maximizes investor perceptions of ability differs from the investment choice that optimally trades off risk and return. Thus reputation concerns adversely affect investor welfare. This paper shows either performance fees or closing a fund can redress the distortions caused by reputation concerns.

The investment advisory relationship is important because a large and increasing fraction of wealth is invested through this vehicle. Total assets in mutual funds in the U.S. today exceed $1 trillion. Mutual funds account for about one-third of NYSE trading volume.\footnote{Steven E. Levingston, “Mutual Funds Could Smooth Stock Market’s Swings,” \textit{Wall Street Journal} (June 28, 1993) p. C1.} Investors who own mutual fund shares delegate some of their investment decisions to the advisers who manage these funds, though they retain the right to switch advisers at any time. The compensation of investment advisers consists almost entirely of fees that are a percentage of net assets under management (an “asset fee”), and fees that depend on the realized performance of the adviser’s portfolio relative to a benchmark portfolio (a “performance fee”). An investment adviser can increase his income in two ways: by choosing a portfolio that yields a high return, and by inducing investors to purchase additional units of the mutual fund. Many argue that garnering new funds is more important than earning high returns on invested funds, although the two objectives are related.\footnote{See, for example, Jonathan Clements, “Pilgrim Group, No Puritan, Irks More-Staid Funds,” \textit{Wall Street Journal} (October 29, 1993) p. C1.}

Statistics about mutual funds are collected and distributed in a manner consistent with the hypothesis that investors make inferences about ability from the history of past portfolio choices and return realizations. Portfolio composition and cumulative returns for mutual funds are prominently reported and carefully scrutinized by investors and rating agencies. Cumulative returns over various time periods are the principal statistics quoted in advertisements for mutual funds.

Moreover, investing in funds with good track records appears to be a good strategy for investors. Hendricks et al. (1993) document that “hot hands” in investment manage-
ment are persistent.\textsuperscript{3} Selecting every quarter the top performers based on the last four quarters can significantly outperform the average mutual fund. One explanation for this phenomenon is that some especially able advisers have a superior ability to receive or interpret information. If this is so, then it is indeed sensible for investors in mutual funds to attempt to identify the informed advisers and to invest with them.

This paper models the interaction between investors and advisers in a two-period setting. It is common knowledge that one adviser is informed while the other is not, but investors do not know which adviser is informed. The informed adviser receives a signal about asset payoffs each period before making his portfolio choice. The uninformed adviser does not receive a signal. Each period, each adviser receives a fee from the investors whose assets he manages. The fee is a fraction of the assets under management. At the end of the first period, investors reallocate their wealth to the adviser they believe is most likely to be informed.\textsuperscript{4} Because asset fees are a fraction of assets under management, there is a benefit to investment advisers, realized in the second period, of appearing to be informed at the end of the first period. The portfolio choice that most enhances reputation is not necessarily the portfolio choice that best trades off return and risk in the current period. Thus, attempts to create a good reputation may distort first-period portfolio choices by advisers relative to the single-period optimum.\textsuperscript{5} When fees are based solely on assets under management, competition for clients leads investment advisers to engage in overly risky investment strategies.

What kind of investment track record constitutes evidence of superior information?

\textsuperscript{3} The question of whether mutual funds earn superior risk-adjusted returns has been the subject of extensive empirical investigation. Grinblatt and Titman (1989b) find evidence that risk-adjusted returns of some growth funds were significantly abnormally positive over the years 1975 to 1984. Brown et al. (1992) survey the evidence and consider the impact of survivorship bias on previous studies. Grinblatt and Titman (1992), Brown and Goetzmann (1994), and Goetzmann and Ibbotson (1994) also document persistence.

\textsuperscript{4} The opportunity to reallocate funds among advisers exists in settings other than the one described here. Many large institutions such as pension funds employ several investment advisers. Pension fund managers have discretion over the amount allocated to each adviser. Even some mutual funds have multiple investment advisers. In the Vanguard Explorer Fund, each adviser is responsible for a fraction of the funds invested: "The proportion of the net assets of the Fund managed by each adviser was established by the Board of Directors effective with the acquisition of Explorer II, and may be changed in the future by the Board of Directors as circumstances warrant. Presently, WMC is responsible for approximately 70% of the assets of the Fund, and Granahan for the remaining 30%." (Vanguard Explorer Fund Prospectus, May 24, 1993, p. 8)

\textsuperscript{5} The offering documents for some mutual funds discuss the possibility that advisers' actions may not conform to the fund's objectives: "The Fund is subject to manager risk[.] The investment adviser manages the Fund according to the traditional methods of "active" investment management, which involves the buying and selling of securities based upon economic, financial and market analysis and investment judgment. Manager risk refers to the possibility that the Fund's investment advisers may fail to execute the Fund's investment strategy effectively. As a result, the Fund may fail to achieve its stated objective." (Vanguard Explorer Fund Prospectus, May 24, 1993, p. 5)
A decision maker who has private information that a given stock is likely to perform very well in the coming period will choose to hold more of it in his portfolio than he would in the absence of such information. How much more is a function of the strength of his information and his risk tolerance. This implies informed advisers tend to take more extreme positions in assets than equally risk-tolerant uninformed advisers. Absent reputation effects, it is not optimal for an adviser to select a portfolio that is inconsistent with his private information. I show that reputation concerns can cause either the informed or uninformed adviser to choose a portfolio that puts a more extreme weight on a risky asset than his information justifies.

Most previous work on the incentives of investment advisers assumes a one-period setting (e.g., Barry and Starks, 1984; Bhattacharya and Pfleiderer, 1985; Starks, 1987; Khilstrom, 1988; and, Grinblatt and Titman, 1989a). Thus, these models give no consideration to the tradeoff an adviser makes between selecting the portfolio that offers the best possible combination of risk and return in the current period and the portfolio that will create the best possible track record in the next period.

This paper extends earlier research on multiperiod considerations in investment advisory relationships by Trueman (1988) and Heinkel and Stoughton (1994) in four ways. First, the analysis adopts the more realistic assumption that investors and advisers are risk averse, rather than risk neutral. Assuming risk neutrality, Trueman identifies one pooling equilibrium in which uninformed investment advisers engage in noise trading. The second contribution of this paper is a demonstration that a separating equilibrium can exist in a model where all market participants are risk averse. In the equilibrium, the informed adviser chooses a portfolio that deters the uninformed adviser from attempting to mimic him. The deterrent portfolio is riskier than the one the informed adviser would choose in the absence of reputation concerns. The incremental cost of the risk imposed on the informed adviser is less than the incremental cost imposed on the uninformed adviser who attempts to mimic because of the difference in the information held by the advisers. This difference permits the informed adviser to separate himself from the uninformed adviser. The separating equilibrium cannot arise when market participants are risk neutral. Third, this paper considers a setting with two advisers rather than one. Consequently, investors’ perceptions of whether an adviser is informed are conditioned on portfolio choice (as in Trueman and Heinkel and Stoughton) and performance relative to a peer. A second adviser allows asset shifts between advisers following the release of performance results to be
studied. Fourth, I document the impact of performance fees on the behavior of investment advisers. In some situations, imposing a performance fee increases the welfare of investors without altering the welfare of advisers. This last result is important because it provides a new rationale for performance fees, namely: performance fees can serve to mitigate excessive risk-taking by informed and uninformed investment advisers alike. I also show that closing a fund can have a similar effect.

Section 1 presents the model. This section establishes the existence of a separating and a pooling equilibrium when only asset fees are charged. Section 2 considers the effects of performance fees. Section 3 discusses some extensions to the model. Section 4 concludes the paper.

1. Model

A simple model will serve to illustrate how the desire to signal superior information or obscure lack of information can color investment decisions when advisers are endowed with different, privately known, but unobservable information.

1.1 Assumptions

(A1) The compensation contract is exogenous.

Throughout the paper, assume that fund advisers receive periodic compensation that is a fraction of the assets under management. I do not argue that this contract is optimal. Rather, I treat the contractual form as an institutional fact in order to explain the behavior of investment advisers. Many details of the contracts between investors and investment advisers are regulated by the Investment Advisers Act of 1940.

(A2) Mutual funds are price takers.

In the rational expectations literature [e.g., Admati and Pfleiderer (1990)], prices partly reveal information when trades based on private information are executed. This paper assumes the funds run by investment advisers are price takers regardless of the trade they propose to undertake.\(^6\)

(A3) Moves are simultaneous.

In contrast to work by Scharfstein and Stein (1990) on herding and Gul and Lundholm (1993) on clustering in financial markets, I study a game in which players (the advisers)

\(^6\) Price taking is consistent with a limiting case of rational expectations models. As the uncertainty associated with the supply of assets becomes infinite, then all investors are price takers.
move simultaneously. Thus, advisers’ strategies are not conditioned on the actions of advisers who have already acted, or on the knowledge that to this point no one has acted.

\textit{(A4) There is no collusion.}

Fund advisers do not coordinate their investment decisions or pool their information.

\textit{(A5) One agent is strictly better informed.}

Employing more than one investment adviser may be desirable if advisers have different pieces of information. I suppress such considerations by assuming that one adviser has information and the other is uninformed.\footnote{This is consistent with Bhattacharya and Pfleiderer (1985), who assume that advisers can be ranked by the criterion of Blackwell sufficiency.}

\textit{(A6) Neither adviser can make personal investments outside the fund he manages.}

Grinblatt and Titman (1989) explore performance fees in a one-period setting when the adviser can hedge the fee he receives under the management contract with the return on his personal portfolio. I assume advisers cannot make investments on their own account. The payoff to the adviser consists solely of the fee he receives under the management contract.

\subsection{1.2 Setup}

Suppose there are two assets, two investment advisers, and many identical small investors. The investors cannot invest in the assets directly. Instead, they invest in a mutual fund run by one or the other of the investment advisers. To simplify the analysis, I assume that each investor must place all her wealth in a single fund.\footnote{If investors could divide their wealth between the two mutual funds, they would do so. In the first period, the decision would be to allocate one half of their wealth to each portfolio. In the second period, investors would reallocate their wealth between the two portfolios based on the realized returns and investment decisions. If both advisers choose portfolios that signal private information, then investors cannot conclude for certain which adviser is informed. They do know that the adviser whose portfolio produced the best return is more likely to be informed. Accordingly, investors would place more (but not all) of their wealth with the adviser who had the best first-period return. Allowing investors this freedom makes it more difficult to derive first-period portfolio choices, though the same intuitions apply. Often, mutual funds require a minimum contribution of $5,000 or more. For many individual investors, this investment floor may preclude them from dividing their money among the funds with similar objectives over which comparisons of managers’ abilities are drawn.} Initially suppose that half of the small investors invest with one adviser, and half with the other.

The investment advisers and the investors exhibit the same preferences, described by the constant relative risk aversion utility function for wealth $U(W) = W^\alpha / \alpha$ for some $\alpha < 1$. The investors and advisers have no wealth other than the amounts they will receive from the mutual fund at dissolution. Thus, in a one-period or one-adviser setting, paying
the adviser a fraction of the assets under management (computed at the end of the period) induces the adviser to make the portfolio choice that is optimal for investors conditional on the adviser's information. That is, given the assumptions about utility and wealth, paying the investment adviser a fraction of the ending value of the mutual fund is an optimal contract in a one-shot game. The optimality of this contract in the one-shot game highlights the distortions introduced by reputation effects in the two-stage game.

Suppose there are two assets, \( \hat{A} \) and \( M \). Per dollar invested, risky asset \( \hat{A} \) returns 2 with probability 1/2 and 0 with probability 1/2 each period. The other asset \( M \) always returns the amount invested (i.e., the riskless return is normalized to zero). Interpret asset \( M \) as the market portfolio in an economy without systematic risk. Given these asset returns, a risk averse uninformed investor would choose to invest all his wealth in the market portfolio, \( M \), since it offers the same expected return as \( \hat{A} \).

Prior to investing, one adviser (the informed adviser) receives a private signal about the payoff to asset \( \hat{A} \). Half the time the adviser receives the signal \( G \) and half the time he receives the signal \( B \). If he receives \( G \), then the revised probability that \( \hat{A} \) will pay 2 is \( p \in (1/2, 1) \). Of course, with complementary probability \( \hat{A} \) will pay 0. If he receives \( B \), then the revised probability that \( \hat{A} \) will pay 2 is \( 1 - p \).

### 1.3 One-period Analysis

When an adviser invests solely on his own account for one period, he buys the portfolio that maximizes expected returns conditional on his information.\(^9\) If his information indicates the probability that the risky asset will pay 2 is \( q \), his choice of portfolio weight, \( \omega \), maximizes:

\[
EU(\omega \hat{A} + (1 - \omega)M) = qU(1 + \omega) + (1 - q)U(1 - \omega).
\]  

(1)

The first order condition implies that the optimal weight is\(^10\)

\[
\omega^* = \frac{(H - 1)}{(H + 1)} \quad \text{where} \quad H = \left(\frac{q}{1-q}\right)^{\frac{1}{\alpha}}.
\]

(2)

In the case of an uninformed adviser, \( q = 1/2 \); so \( \omega^* = 0 \). The uninformed adviser invests his entire portfolio in asset \( M \). The parameter \( p \) is a measure of the precision of the

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\(^9\) This paper presumes advisers cannot invest their own wealth (or the wealth of their clients) in the fund managed by the other adviser. Such an investment would never be attractive to the informed adviser, though it would appeal to the uninformed adviser.

\(^10\) If \( U(W) = \log W \), then \( \omega^* = 2q - 1 \).
information conveyed by the signal. As the precision of the signal increases, the optimal portfolio choice involves taking a more extreme position in the risky asset; however, for any value of \( p < 1, \omega^* \in (-1,1) \). This follows because the marginal utility for wealth becomes infinite as wealth approaches zero. A portfolio choice of \( \omega = 1 \) or \( \omega = -1 \) exposes the investors and the adviser to the possibility of zero wealth in some state. Let \( \hat{\omega} \) and \( \tilde{\omega} \) be the solutions to (1) when the informed adviser receives signals \( G \) and \( B \), respectively. By symmetry, \( \tilde{\omega} = -\hat{\omega} \).

1.4 Two-period Analysis

The investors know exactly one of the advisers is informed. Only the informed adviser receives a signal at the beginning of each period. Signals and outcomes are serially independent. Initially, the investors believe it is equally likely that either adviser is the informed one. At the end of the first period, each investor may stay put or switch her wealth from one fund to the other. The investors observe the composition of the portfolios chosen by the advisers. The performance and portfolio weights of the two funds help investors divine which adviser is informed. In particular, if the investors observe that an adviser chooses a portfolio weight of \( \omega = 0 \), then they conclude the adviser is uninformed. After the investors reallocate their wealth between advisers, the informed adviser receives a signal. Both advisers then pick portfolios for period 2. Consumption takes place after period 2 returns are realized. The precision of the signal received by the informed adviser, \( p \), is assumed to be common knowledge. Table 1 summarizes these facts.

[Table 1]

Since the advisers receive a fee for managing the portfolio that is a fraction of the assets under management, they care about investors’ perceptions of ability. As soon as the investors conclude that an adviser is uninformed, they cease to allocate any wealth to him, and in the succeeding period, the adviser’s fee income is zero because the fund he manages has no investors. To forestall this possibility, the uninformed adviser has incentives to appear informed.

The uninformed adviser has an incentive to pretend he is acting upon a signal. Assuming the informed adviser chooses either portfolio weight \( \hat{\omega} \) or \( \tilde{\omega} \) (depending on whether he receives signal \( G \) or \( B \)), the uninformed adviser can mimic the informed adviser at least some of the time by randomly choosing either portfolio weight \( \hat{\omega} \) or \( \tilde{\omega} \). Half the time, the uninformed adviser will choose the same weight as the informed adviser. The rest of the
time, the choices of the two advisers will differ and one adviser will outperform the other. Table 2 lists the possible outcomes.

Let \( q \) be the probability that the uninformed adviser selects portfolio weight \( \hat{\omega} \). It is straightforward to show that for any \( q \in [p, 1 - p] \) the best response for an investor is to place her wealth in the second period with the adviser who performs best in the first period. In the event of a tie, the investor is indifferent to whether she invests her money in period 2 with one adviser or the other. It is also straightforward to show that strategies in which the uninformed adviser plays \( \hat{\omega} \) with a probability less than \( p \) or more than \( 1 - p \) are strictly dominated.

[Table 2]

Table 2 shows there are three possible outcomes from the strategy of faking receipt of a signal. Only in the third outcome do the investors revise downward their perceptions of the likelihood the uninformed adviser is informed.

**OUTCOME 1:** Sometimes the uninformed adviser is lucky because his action turns out to be different from that of the informed adviser and the informed adviser's action turns out to be incorrect, *ex post*. This is the best outcome for the uninformed adviser because investors' posterior probability that he is informed increases.

Given the structure of uncertainty noted above, this happens \((1 - p)/2\) of the time.

**OUTCOME 2:** Sometimes the uninformed adviser is lucky because his action turns out to be identical to that of the informed adviser. In this case, investors have no evidence on which to conclude that one adviser is superior to the other, so the uninformed adviser retains his share of investors' wealth.

This happens half of the time. Finally, the uninformed adviser can do worse than the informed adviser.

**OUTCOME 3:** Sometimes the uninformed adviser is unlucky because his action turns out to be different from that of the informed adviser and the informed adviser's action turns out to be correct, *ex post*. This is the worst outcome for the uninformed adviser because the investors' posterior probability that the uninformed adviser is informed decreases.

This happens \(p/2\) of the time.
Note that the effect of outcome 3 on the adviser's reputation is less severe than choosing \( \omega = 0 \) would have been. If the adviser picks \( \omega = 0 \), the investors can conclude with certainty that the adviser is uninformed. If one adviser chooses \( \omega \) while the other chooses \( \tilde{\omega} \), the investors can only conclude that, with probability \( p \), it is the uninformed adviser who performed worst.

This analysis suggests the informed adviser may want to adopt a portfolio position that is consistent with being informed since the reputation effect of doing so strictly dominates acting on the basis of no information. What could cause the uninformed adviser not to mimic the informed adviser? The answer is that the uninformed adviser bears more risk when he attempts to mimic the informed adviser.

1.5 Equilibrium

To develop this intuition more carefully, let \( \omega_{it} \) be the fraction of the assets under management that adviser \( i \) places in asset \( \tilde{A} \) in period \( t \). Then one dollar invested in adviser \( i \)'s portfolio returns

\[
\tilde{R}_{it} = \omega_{it}\tilde{A} + (1 - \omega_{it})M.
\]

Suppose that both advisers levy a management fee of \( f \) per dollar of assets invested with the adviser at the end of each period. Restrict \( f \) to lie in \([0, 1]\). Typically, \( f \) is about 1% in an equity mutual fund. Then in period \( t \) an investor receives either \((1 - f)\tilde{R}_{1t}\) or \((1 - f)\tilde{R}_{2t}\) per dollar invested. The investor's objective is to maximize her expected utility at the end of two periods assuming that she remains fully invested in one of the mutual funds offered by adviser 1 and adviser 2.

Let \( \gamma_{it} \) be the proportion of total funds invested with adviser \( i \) at the beginning of period \( t \). Fix \( \gamma_{11} = 1/2 \). The proportion \( \gamma_{12} \) depends on the information available at the end of period 1, namely the portfolio choices of the two advisers and the realized outcomes, as well as the strategies of the investors. If the advisers choose the same portfolio, then the investors' posterior beliefs about the abilities of the advisers are the same as their prior beliefs because the investors have received no information. An inference about the identity of the informed adviser can only be drawn when the actions of the advisers differ.

Each adviser's objective is to maximize his expected utility at the end of two periods. Let

\[
\tilde{\Gamma} = \gamma_{11}\tilde{R}_{11} + \gamma_{21}\tilde{R}_{21}
\]

be the aggregate wealth of investors and advisers at the end of period 1. In each period
the fee earned by adviser 1 is the fee per dollar of assets, $f$, times the fraction of investors’ wealth allocated to the adviser, $\gamma_{it}$, times investors’ total wealth at the beginning of the period (1 in the first period and $(1 - f)\bar{\Gamma}$ in the second period), times the return on those funds, $\bar{R}_{i1}$.\footnote{Aggregate initial wealth has been normalized to unity since the problem is independent the level of wealth.} For each adviser, expected utility is calculated with respect to the fee earned in the first period times the return earned on that fee income in the second period, $\bar{R}_{i2}$, plus the fee earned in the second period. For adviser 1, this is

$$EU\left(f\gamma_{i1}\bar{R}_{i1}\bar{R}_{i2} + f\gamma_{i2}(1 - f)\bar{\Gamma}\bar{R}_{i2}\right) = EU\left(f\bar{R}_{i2}(\gamma_{i1}\bar{R}_{i1} + \gamma_{i2}(1 - f)\bar{\Gamma})\right).$$

Similarly, the expected utility of adviser 2 is

$$EU\left(f\bar{R}_{22}(\gamma_{21}\bar{R}_{21} + \gamma_{22}(1 - f)\bar{\Gamma})\right).$$

In this formulation, the adviser has utility for wealth at the end of two periods. There is no intermediate consumption.

\textit{1.5.1 Optimal Investment Policy in Period 2}

In the last period, there is no benefit to the investment adviser from improving the investors’ assessments of his ability since there is no subsequent time at which investors may reallocate their wealth among advisers. Once each investor has chosen who will invest her wealth in the second period, the expected utility of the adviser depends only on the performance of the portfolio he buys for the final period. With a power utility function, the portfolio weights chosen by the adviser are independent of (i.e., multiplicatively separable from) the adviser’s wealth and the size of the fee.\footnote{The utility function $U(W) = \log W$ corresponds to risk aversion parameter $\alpha = 0$. In this case, portfolio weights chosen by the adviser are additively separable from the adviser’s wealth.} Also, the adviser chooses to invest both the fee he earned in the first period and money he manages on behalf of the investor identically. Moreover, since the investor is assumed to have preferences identical to those of the adviser, the portfolio selected by the adviser is the one that would be chosen by the investor if she were endowed with the same information as the adviser. Thus in the second period, the uninformed adviser chooses portfolio weight $\omega = 0$. The informed adviser chooses $\hat{\omega}$ on receipt of signal $G$ and $\tilde{\omega}$ on receipt of signal $B.$
1.5.2 Optimal Investment Policy in Period 1

Having established the optimal action choice and payoffs in period 2, now consider the optimal strategies of the advisers in period 1. A strategy profile consists of strategies for the informed and uninformed advisers and the investors. A strategy for the uninformed adviser is $s_U = (\Omega; \Pi)$ where $\Omega = (\omega_1, \ldots, \omega_n)$ is a vector of first period portfolio choices and $\Pi = (\pi_1, \ldots, \pi_n)$ is a vector of probabilities. The probability of playing $\omega_i$ is $\pi_i$ and $\sum_i \pi_i = 1$. A strategy for the informed adviser is $s_I = (\Omega; \Pi^B, \Pi^G)$ where $\Omega$ is a vector of portfolios as above and $\Pi^I$ is a vector of probabilities. The probability of playing $\omega_i$ after receiving signal $j$ is $\pi_i^j$ and $\sum_i \pi_i^j = 1$ for $j \in \{G, B\}$.

A fund investor’s pure strategies are $s_F(\omega_1, \omega_2; R_{11}, R_{21}) \rightarrow \{\text{stayput, switch}\}$. The investor decides whether to stay invested in the mutual fund she held in the first period or switch to the other fund on observing the portfolio choices and returns of the two advisers. Assume investors choose to stay in the event both advisers choose the same portfolio. Then, the fraction of aggregate investor wealth managed by adviser $i$ in the second period can take one of only three values. That is, $\gamma_{i2} \in \{0, 1/2, 1\}$.

There are many sequential equilibria to this game because there are no restrictions on the beliefs of investors when they observe a portfolio choice by an adviser that is off the equilibrium path of the game. Portfolio choices for the informed and uninformed advisers can be supported in equilibrium by supposing investors conclude that an adviser who deviates from the equilibrium play is surely uninformed. An adviser who investors believe is uninformed receives no assets to manage in the second period. Such beliefs can be used to enforce a wide range of equilibria in which advisers play mixed strategies.

Nevertheless, it is possible to provide some characterization of equilibrium play. Suppose the uninformed adviser chooses $\omega \neq 0$ with positive probability and the informed adviser never plays $\omega$ in some equilibrium. Then the investors conclude that any adviser who chooses $\omega$ is uninformed, so an adviser who plays $\omega$ is allocated no funds by the investors in period 2. That is, the period 2 outcome is the worst one possible when an adviser plays $\omega$. Also, $\omega$ is strictly dominated by $\omega_0 = 0$ for the uninformed adviser in period 1. Since $\omega$ is strictly dominated by $\omega_0$, $\omega$ cannot be chosen by the uninformed adviser in equilibrium.

Remark 1: If the uninformed adviser chooses portfolio $\omega \neq 0$ with positive probability in some equilibrium, then the informed adviser must choose $\omega$ with positive probability as well.
In any equilibrium where the uninformed adviser chooses $\omega \neq 0$, investors bear avoidable risk and are not compensated by superior expected returns. This is a cost borne by investors due to reputation concerns on the part of the uninformed adviser.

Reputation concerns can also distort the investment choices of the informed adviser. To illustrate and contrast these two effects, the remainder of this paper treats only two of the sequential equilibria to this game. These equilibria are simple and appealing because the informed adviser plays a pure strategy. They serve to illustrate the fundamental tensions that all equilibria share. The first one is a pooling equilibrium,

$$\left\{ s_U((\hat{\omega}, \tilde{\omega}); (q, 1-q)), s_I((\hat{\omega}, \tilde{\omega}); (1, 0), (0, 1)) \right\} \quad \text{where } q \in (p, 1-p);$$

the second is a separating equilibrium,

$$\left\{ s_U(0; 1), s_I((\hat{\omega}, \tilde{\omega}); (1, 0), (0, 1)) \right\}.$$ 

In the first equilibrium, $\hat{\omega}$ and $\tilde{\omega}$ are the optimal one-period portfolio choices of the informed adviser conditional on signals $G$ and $B$, respectively. In the second equilibrium, $\hat{\omega}$ and $\tilde{\omega}$ are portfolio choices that are more extreme (i.e., further from zero) than $\hat{\omega}$ and $\tilde{\omega}$.

1.5.3 Pooling Equilibrium

In the pooling equilibrium, the informed adviser chooses the portfolio that is optimal (in a one-period setting ignoring reputation effects) given his information. The uninformed investment adviser mimics the informed adviser by randomly choosing either $\hat{\omega}$ or $\tilde{\omega}$. Half of the time, the two advisers’ portfolio choices are identical. Then the uninformed adviser will manage half of the investors’ aggregate wealth in the second period. The rest of the time, the uninformed adviser’s portfolio either outperforms or underperforms the informed adviser’s portfolio. In the former case, the uninformed adviser manages all investor wealth in the second period; in the latter case, he manages none of it. This pooling strategy is more costly to the uninformed adviser in the first period than the strategy of choosing a portfolio that is optimal in a one-period setting given his information, namely $\omega_0 = 0$.

Two conditions need to be satisfied for the pooling equilibrium to exist. First, the pooling equilibrium will only exist when the contribution to expected utility associated with the lottery in which the uninformed adviser gets some, all, or none of the investors’ funds to manage in the second period more than offsets the loss in expected utility from making a risky portfolio choice in the first period. Second, the informed adviser must find
the pooling equilibrium more attractive than the separating equilibrium described below. The first proposition establishes conditions under which some pooling must occur.

**Proposition 1:** Suppose \( \alpha > \frac{1}{2} \) and

\[
f < 2 - 2^{\frac{1-\alpha}{\alpha}}.
\]  

Then no separating equilibrium exists.

The proof of this proposition is in the appendix. As \( \alpha \) approaches 1, investors and advisers become risk neutral. At this point, the inequality holds regardless of the asset fee, \( f \). Separation is impossible when advisers are risk neutral because the uninformed adviser assesses the same one-period expected return on every portfolio. Since mimicry offers higher expected returns in the second period, the informed adviser will mimic. As \( \alpha \) decreases, the informed adviser becomes more risk averse, so mimicking the informed adviser becomes more costly. Thus it is intuitive that the right-hand side of (3) is decreasing in \( \alpha \).

The performance fee, \( f \), also affects the nature of the equilibrium. As \( f \) increases, more of the initial wealth of investors is owned by advisers at the end of the first period. Consequently, the second-period fee income of advisers, expressed as a fraction of total income over both periods, decreases. Thus as \( f \) grows larger, the second-period fee income of the uninformed adviser becomes less important to his overall utility. Since the first-period contribution to the overall utility of the uninformed adviser is maximal when portfolio \( \omega_0 = 0 \) is chosen, a high asset fee facilitates separation. Thus, the inequality is harder to satisfy as \( f \) increases.

**1.5.4 Separating Equilibrium**

Suppose the uninformed adviser prefers mimicking the informed adviser to choosing the market portfolio \( \omega_0 = 0 \) when the informed adviser selects the portfolio that maximizes his one-period expected utility conditional on his information, \( \hat{\omega} \) following signal \( G \) and \( \hat{\omega} \) following signal \( B \). To discourage mimicry, the informed adviser could use his information more aggressively (i.e., the adviser chooses a riskier portfolio than he would choose in the absence of reputation concerns).

If his information suggests that \( \hat{A} \) is more likely to pay 2 than 0, then the informed adviser may choose \( \hat{\omega} > \hat{\omega} \), thereby over-investing in \( \hat{A} \). If the information suggests that \( \hat{A} \) will have lower returns in the coming period, then the adviser will choose \( \hat{\omega} < \hat{\omega} \), thereby
over-investing in $M$. The informed adviser will choose $\hat{\omega}$ and $\bar{\omega}$ at levels that just discourage the uninformed adviser from mimicking him. The portfolio choices of $\hat{\omega}$ following $G$ and $\bar{\omega}$ following $B$ impose a cost on the informed adviser in the first period relative to $\hat{\omega}$ and $\bar{\omega}$, but this cost is offset in the second period if it discourages the uninformed adviser from mimicking. Then in period 2, the informed adviser manages all investors’ wealth for sure. Under the separating equilibrium, an adviser whose portfolio performs badly but whose actions indicate that he had private information is assigned the task of managing all wealth in period 2. The next proposition establishes the existence of a separating equilibrium for one case where advisers are more risk averse than in Proposition 1. The interactions among the precision of the informed adviser’s information, the size of the asset fee, and risk tolerance are complex. The case of logarithmic utility, corresponding to $\alpha = 0$, is the easiest to analyze so it is presented in the proposition. More generally, increases in risk aversion, the asset fee, and the precision of information make separation easier to support.

**Proposition 2:** For $U(W) = \log W$, the unique sequential equilibrium satisfying the intuitive criterion is defined by strategies:

$$s_U = (w_0; 1) \quad \text{and,}$$

$$s_I = (\hat{\omega}, \bar{\omega}; (1, 0), (0, 1))$$

where $\hat{\omega} = \max(\hat{\omega}, \bar{\omega})$ and $\bar{\omega}$ is the unique solution to

$$1 = (2 - f)(1 - \omega^2)^p \left(3 - 2f - 2(1 - f)\omega - \omega^2\right)^{1-p} \quad (4)$$

for $\omega \in (0, 1)$.

The proof is in the appendix. The intuitive criterion is described in Cho and Kreps (1987). In the equilibrium, the uninformed adviser invests solely in the market portfolio, $M$. If the informed adviser receives signal $G$, he chooses portfolio weight $\hat{\omega}$. If the informed adviser receives signal $B$, he chooses portfolio weight $\bar{\omega}$. In the second period, the investors invest with the adviser who chooses $\hat{\omega}$ or $\bar{\omega}$. If both advisers or neither adviser plays $\hat{\omega}$ (an event off the equilibrium path), the investors stay put.

Choosing portfolio weights $\hat{\omega} > \hat{\omega}$ and $\bar{\omega} < \bar{\omega}$ is strictly less costly in the first period for the informed adviser than for the uninformed adviser. To see this, consider a small increase, $\epsilon$, in the portfolio weight on asset $A$ following signal $G$. Since $\hat{\omega}$ is the optimal one-period weight for the informed adviser, the change in expected utility in moving from
\* to \* + \* is approximately zero. For the uninformed adviser, the slope on first-period expected utility at \* is strictly negative. The slope on expected utility of the uninformed adviser is more negative than the slope on expected utility of the informed adviser at every \* > \*. Thus the first-period costs of using information too aggressively are higher for the uninformed adviser than for the informed adviser. This allows the informed adviser to use his first-period portfolio choice to signal his superior information to investors.

In either the pooling or separating equilibrium, one adviser is using his information too aggressively. This increases the variance of mutual fund returns.

**Proposition 3:** The variance of mutual fund returns is higher in the presence of a reputation effect.

**Proof:** Define the risk of a mutual fund as the variance in portfolio returns given portfolio choice \* with respect to the probability distribution of asset returns conditional on the signal, \( \text{Var}_p(\cdot) \),

\[
\text{Var}_p(\omega \bar{A} + (1 - \omega)M) = 4p(1 - p)\omega^2.
\]

The unconditional variance has \( p = 1/2 \). Variance increases as \* increases in absolute value. Since the reputation effect leads to a higher (in absolute value) choice of \* for one adviser in either the pooling equilibrium or the separating equilibrium, the risk of a mutual fund to an investor is greater in the presence of reputation effects.

An investor cannot hedge this risk by holding some quantity of the risky asset directly because she does not know whether she is invested with the informed or uninformed adviser. Moreover, the investor does not have the informed adviser's information and the uninformed adviser is playing a mixed strategy, so the investor does not know whether to go long or short in the risky asset to hedge the exposure due to the reputation effect. The next section shows how performance fees reduce the variance of mutual fund returns and enhance investor welfare.

2. **Performance Fees**

Performance fees reward (punish) advisers when the return on the portfolio they manage is greater (less) than the return on a benchmark portfolio. This section examines the impact of linear performance fees on the portfolio choices made by advisers.\(^{13}\) It

\(^{13}\) A linear performance fee is a constant \( g \) times the difference between the return on the adviser's portfolio and the return on the benchmark portfolio.
also considers the consequent effects on investors. A linear performance fee induces a Pareto improvement when advisers make portfolio choices consistent with the separating equilibrium described in the previous section. A linear performance fee also increases \textit{ex ante} investor wealth under the pooling equilibrium.

Contracts for investment advisory services are regulated in the United States under the Investment Advisers Act of 1940. Since 1970, §205(a)1 of the Act prohibits a registered investment adviser from receiving compensation "on the basis of a share of capital gains upon or capital appreciation of the funds or any portion of the funds of the client," including clients who are registered investment companies (i.e., mutual funds). The reason for this prohibition is Congress' concern that such fees would induce advisers to take inappropriate risks that would harm investors [Division of Investment Management (1992), pp. 237–238]. One exception to this rule is a "fulcrum fee." With a fulcrum fee, an adviser's compensation increases or decreases symmetrically depending on how the fund's return compares to a benchmark.

One reason the SEC insists on fulcrum fees is the fear that other types of fees cause advisers to make speculative investments. In such cases, the adviser wins a great deal if the investment turns out well because he participates in the upside of the transaction. If the investment turns out badly, the adviser is not greatly harmed because his downside risk is limited. Also, if fees were not symmetric then advisers would have incentives to set up multiple funds (Kritzman, 1987). In one fund they could buy, and in the other sell, the same asset. One of these strategies is bound to be profitable while the other will not be. If fees are not symmetric, then the adviser is rewarded with a performance bonus in the fund that performed well, but there is no offsetting penalty in the fund with the opposite position. The linear fees considered below do not suffer from this criticism.

\textbf{Remark 2:} Relative to the case of no performance fees, linear performance fees induce advisers to use their information less aggressively.

Suppose that a performance fee of \( g \) is imposed and the benchmark portfolio is \( M \). The payoff to the adviser who assesses a probability \( q \) that the risky asset \( \tilde{A} \) will be worth 2 next period and chooses portfolio \( \omega \) is

\[
EU \left[ f(\omega \tilde{A} + (1 - \omega)M) + g(\omega \tilde{A} + (1 - \omega)M - M) \right].
\]

The effect of imposing a performance fee in a one-period setting is shown in figure 1. For the uninformed adviser, all portfolios offer the same expected return. Since the portfolio
\( \omega_0 = 0 \) is riskless, it is preferred by the uninformed adviser. While the uninformed adviser’s optimal portfolio choice does not change with the imposition of a performance fee, the difference between the expected one-period payoff from holding the optimal portfolio, \( \omega_0 = 0 \), and the payoff from holding any other portfolio, \( \omega \), increases with the imposition of a performance fee. The larger the performance fee \( g \), the greater the difference. The performance fee penalizes an adviser for choosing a portfolio other than the benchmark portfolio by increasing risk as the chosen portfolio departs from the benchmark portfolio.

[Figure 1]

A performance fee \( g \) imposes risk on the adviser that can be avoided entirely by holding the benchmark portfolio.\(^{14}\) For the informed adviser, both returns and risk differ across portfolios. If the informed adviser holds the benchmark portfolio, he forgoes the superior payoff he could earn by betting on his private information. As the performance fee increases, the manager is willing to bet less, because the risk imposed by the performance fee increases. The optimal one-period portfolio when performance fees are imposed lies closer to the benchmark portfolio than when no performance fee is imposed. Thus, when the performance fee is positive the informed adviser holds a portfolio that is an average of the benchmark portfolio and the optimal portfolio in the absence of a performance fee.

It is easy to envision the effect of varying the sign and magnitude of the performance fee, \( g \), on the portfolio chosen by the informed adviser. Recall there are two assets in the economy, \( \bar{A} \) and \( M \). Feasible portfolio weights are ordered pairs \( (\omega, 1-\omega) \). Call the portfolio weight on asset \( \bar{A} \) chosen by the investment adviser in the absence of a performance fee \( \omega^*(0) \). Call \( \lambda \) the weight on asset \( \bar{A} \) in the benchmark portfolio whose return is subtracted from the return on the adviser’s portfolio in computing the performance fee. Each point on the line through \( \omega^*(0) \) and \( \lambda \) corresponds to the optimal portfolio choice for a particular performance fee, \( g \). As \( g \) increases from zero, the portfolio chosen by the investment adviser moves along the line from \( \omega^*(0) \) toward the benchmark portfolio, \( \lambda \).\(^{15}\)

**Proposition 4:** Define

\[
\omega(g) = \frac{f \omega(0) + g \lambda}{f + g}.
\]  

\(^{14}\) The utilities of the uninformed and informed advisers are equal at \( \omega_0 = 0 \).

\(^{15}\) As \( g \) decreases in the range \(-f < g < 0\), the portfolio chosen by the manager moves along the line away from \( \lambda \) beyond \( \omega^*(0) \). As \( g \) decreases in the range \( g < -f \), the portfolio chosen by the manager approaches \( \lambda \) from the side opposite \( \omega^*(0) \).
The one-period payoff to an investment adviser in the absence of performance fees when portfolio \( \omega(0) \) is chosen is identical to the payoff when performance fee \( g \) is imposed and portfolio \( \omega(g) \) is chosen.\(^\text{16}\)

Proof:

\[
EU \left[ f \left( \omega(g) \bar{A} + (1 - \omega(g)) M \right) + g \left( \omega(g) \bar{A} + (1 - \omega(g)) M - \lambda \bar{A} - (1 - \lambda) M \right) \right] \\
= EU \left[ f \left( \omega(0) \bar{A} + (1 - \omega(0)) M \right) \right]
\]

for all \( g \) is immediate. Hence an optimal solution to one objective function is feasible for the other. In fact, the payoff to the adviser is identical state by state. \( \blacksquare \)

Proposition 4 shows that the compensation received by an adviser is independent of the choice of benchmark or the magnitude of the performance fee. For any portfolio chosen by the adviser in a regime where there is no performance fee, there corresponds a related portfolio defined by (5) that provides the same compensation state-by-state when performance fee \( g \) and benchmark \( \lambda \) are imposed. In particular, the utility-maximizing compensation is the same regardless of the benchmark and performance fee imposed.

Why are performance fees imposed at all? One answer to this question is that the portfolio selected by the adviser changes as a function of the benchmark and performance fee. Thus a benchmark and performance fee can serve as tools to modify the portfolio choices of advisers. In particular, the performance fee and benchmark can be chosen to counteract the undesirable distortions in portfolio choice induced by reputation concerns.

2.1 Separating Equilibrium

Suppose the adviser's choices are given by the separating equilibrium identified in section 1.5.4. Also, suppose that \( \hat{\omega} < \hat{\omega} \). If performance fees are adopted, the informed adviser chooses a portfolio that is closer to \( \hat{\omega} \) than \( \hat{\omega} \). Separation is maintained because it is as costly for the uninformed adviser to mimic \( \hat{\omega} \) in the absence of a performance fee as it is to mimic \( \hat{\omega}(g) \) in the presence of performance fee \( g \).

Proposition 5: There is a performance fee imposed in the first period that induces a Pareto superior outcome under the separating equilibrium.

\(^{16}\) Proposition 4 generalizes naturally to an economy where there are many assets. If there are \( n \) assets, the result holds when \( \omega \) and \( \lambda \) are interpreted as \( n \)-dimensional vectors. The \( i^{\text{th}} \) component of each vector is the weight on asset \( i \).
Proof: Suppose the uninformed adviser plays \( s_U(0; 1) \), while the informed adviser plays \( s_I((\hat{\omega}, \hat{\omega}); (1, 0), (0, 1)) \). Choose \( M \) as the benchmark portfolio. The uninformed adviser's utility from strategy \( s_U(0; 1) \) does not change when a performance fee is imposed. The informed adviser's expected utility is lower with strategy \( s_I((\hat{\omega}, \hat{\omega}); (1, 0), (0, 1)) \) when \( g \neq 0 \) than when \( g = 0 \). However, the informed adviser can receive the same utility as when \( g = 0 \) by using strategy \( s_I((\frac{f}{f+g}\hat{\omega}, \frac{f}{f+g}\hat{\omega}); (1, 0), (0, 1)) \). The uninformed adviser will not find it worthwhile to mimic the informed adviser, even though the informed adviser's portfolio choice is less aggressive, because the expected utility from strategy \( s_U((\frac{f}{f+g}\hat{\omega}, \frac{f}{f+g}\hat{\omega}); (\frac{1}{2}, \frac{1}{2})) \) is the same for all \( g \) by Proposition 4.

Recall the signals received by the informed adviser are \( G \) and \( B \). With the performance fee, investors who invest with the informed adviser in the first period receive

\[
\frac{1}{2} E \left\{ U \left[ (1 - f)(f + g)\hat{\omega}A + (1 - \frac{f}{f + g}\hat{\omega})M \ight] - g\left(\frac{f}{f + g}\hat{\omega}A + (1 - \frac{f}{f + g}\hat{\omega})M - 1\right) \right\} | G \\
+ \frac{1}{2} E \left\{ U \left[ (1 - f)(f + g)\hat{\omega}A + (1 - \frac{f}{f + g}\hat{\omega})M \ight] - g\left(\frac{f}{f + g}\hat{\omega}A + (1 - \frac{f}{f + g}\hat{\omega})M - 1\right) \right\} | B. 
\]

Since \( \hat{\omega} = -\hat{\omega} \) and \( \Pr(\hat{A} = 2 | G) = \Pr(\hat{A} = 0 | B) = p \), this expression reduces to

\[
= pU \left[ (1 - f)(1 + \hat{\omega}) - \frac{g}{f + g}\hat{\omega} \right] + (1 - p)U \left[ (1 - f)(1 - \hat{\omega}) + \frac{g}{f + g}\hat{\omega} \right].
\]

Increasing \( g \) shifts consumption, unit for unit, from states where the adviser’s private information turns out to be correct to states where the adviser’s private information turns out to be incorrect. Choosing \( g \) to satisfy

\[
(1 - f)(1 + \hat{\omega}) - \frac{g}{f + g}\hat{\omega} = (1 - f)(1 + \hat{\omega})
\]

provides the investors with the first-period utility they would obtain in the absence of reputation concerns. This is a strict improvement whenever \( \hat{\omega} < \hat{\omega} \). \( \blacksquare \)

Imposing the optimal performance fee \( g \) does not require investors to know which adviser is informed or the information of the informed adviser. Thus, a performance fee alters the equilibrium portfolio choices made by the advisers without altering the separating
nature of the equilibrium. In the case of the separating equilibrium, the imposition of performance fees results in a Pareto improvement in advisers’ and investors’ welfare.

2.2 Pooling Equilibrium

Both types of advisers find it optimal to choose a portfolio closer to the market portfolio when a performance fee applies. If advisers choose portfolios consistent with the pooling equilibrium identified in section 1.5.3, then performance fees increase the welfare of investors who invested with the uninformed adviser, but decrease the welfare of investors who invested with the informed adviser. The imposition of a performance fee moves the uninformed adviser’s portfolio choice closer to his one-period optimum, but moves the informed adviser’s portfolio choice further from the one-period optimum. The effect of a judiciously chosen performance fee on the ex ante utility of investors under the pooling equilibrium is positive.

**Proposition 6:** There is a performance fee imposed in the first period that improves ex ante welfare under the pooling equilibrium.

**Proof:** Suppose the uninformed adviser plays \( s_U((\hat{\omega}, \hat{\omega}); (q, 1 - q)) \), while the informed adviser plays \( s_I((\omega, \omega); (1, 0), (0, 1)) \). Let \( r \in (0, 1) \) be the probability that an investor places his money with the informed adviser in period 1. Choose \( M \) as the benchmark portfolio. The ex ante first-period utility of an investor in the absence of a performance fee is

\[
r \left( pU((1 - f)(1 + \hat{\omega})) + (1 - p)U((1 - f)(1 - \hat{\omega})) \right) \\
+ (1 - r) \left( \frac{1}{2} U((1 - f)(1 + \hat{\omega})) + \frac{1}{2} U((1 - f)(1 - \hat{\omega})) \right) \\
= \frac{1 - r + 2rq}{2} U((1 - f)(1 + \hat{\omega})) + \frac{1 + r - 2rq}{2} U((1 - f)(1 - \hat{\omega}))
\]

(6)

since it is equally likely the investor chooses the informed or uninformed adviser in the first period. Expression (6) has the same form as (1). The weight depends on the precision of information about the payoff on the risky asset \( \hat{A} \). Since the decisions of the informed and uninformed advisers are mixed together from the perspective of the investor, the optimal portfolio weight \( \omega^* \) is calculated using \( q = \frac{1 - r + 2rq}{2} \) in (2). Since \( \hat{\omega} \) is based on precision \( p > \frac{1 - r + 2rq}{2} \), \( \hat{\omega} \) is too large from the perspective of the investors. Similar to Proposition 5,
choosing $g$ to satisfy

$$(1 - f)(1 + \omega) - \frac{g}{f + g} \omega = (1 - f)(1 + \omega^*)$$

improves welfare.

In section 1 of the paper it was assumed that an investor was as likely to invest with the informed adviser as with the uninformed adviser in period 1. That assumption simplifies the analysis section 1 but is not required here. Propositions 5 and 6 hold as long as there is some positive probability that investors place their money with the uninformed adviser in period 1.

3. Extensions

Section 2 showed that undesirable portfolio distortions caused by reputation concerns can be mitigated by imposing performance fees. The example below shows that closing a fund can have a similar effect.

3.1 Closed Funds

It seems plausible that investors who are unhappy with the performance of their adviser in the first period should be allowed to invest their money elsewhere in the second period. Investors will switch their money to the adviser they believe is most likely to be informed. Given the information about ability released in the first period, investors are more likely to be invested with the informed manager in the second period if they are allowed to switch. This suggests investors are better off when they are free to vote with their feet. This reasoning is incomplete because it neglects the costs imposed on investors in the first period by the uninformed manager who attempts to build a reputation for ability. When investors cannot switch their money, the uninformed adviser has no reason to attempt to appear informed. Since he does not distort his portfolio choice, the reputation costs are avoided. Investors may be worse off if they can vote with their feet.

**Example:** An *ex ante* Pareto superior outcome is possible if investors commit not to switch funds once first-period performance is realized.

Suppose an investor randomly places her wealth with one of the investment advisers at the beginning of period. If she cannot reallocate her wealth at the end of the first period,
her expected utility at the end of two periods is:

\[
\frac{1}{2} EU((1 - f)^2 \tilde{R}_{11} \tilde{R}_{12}) + \frac{1}{2} EU((1 - f)^2 \tilde{R}_{21} \tilde{R}_{22}).
\]

Since neither adviser has an incentive to build a reputation, each invests optimally in both periods conditional on his private information. In each period, the uninformed adviser chooses \( \omega_0 = 0 \) while the informed adviser picks \( \omega^* \in \{ \tilde{\omega}, \tilde{\omega} \} \), depending on his private information. Thus, the payoff to the investor (suppressing a factor of \( (1 - f)^2 \) from the argument of the utility function) is:

\[
\frac{1}{2} U(1) + \frac{1}{2} \left( (1-p)^2 U((1 - \tilde{\omega})^2) + 2(1-p)p U((1 - \tilde{\omega})(1 + \tilde{\omega})) + p^2 U((1 + \tilde{\omega})^2) \right).
\]

Half of the time, the investor places her money with the uninformed adviser, who holds the market portfolio which returns \( 1 \). Half of the time the investor places her money with the informed adviser who goes long or short in the risky asset each period depending on his information. Each period, the return on the portfolio is \( 1 + \tilde{\omega} \) with probability \( p \) or \( 1 - \tilde{\omega} \) with probability \( 1 - p \).

If switching is permitted, the analysis is more complicated. The associated payoffs and probabilities are laid out in figure 2. The figure shows there are nine cases to consider, depending on the return on the risky asset, whether the investor places her money with the informed or uninformed adviser, and whether the portfolios of the advisers are the same or different. The calculation of expected utility in the figure presumes the investor switches advisers whenever the portfolio of her adviser underperforms the portfolio of the other adviser. In each period the informed adviser chooses \( \omega^* \in \{ \tilde{\omega}, \tilde{\omega} \} \), depending on his private information. In the first period, the uninformed adviser attempts to mimic the informed adviser by randomly choosing either \( \tilde{\omega} \) or \( \tilde{\omega} \). The uninformed adviser chooses \( \omega^* = 0 \) in the second period.

The payoff to the investor (again suppressing a factor of \( (1 - f)^2 \)) reduces to:

\[
\frac{p^2 U((1 + \tilde{\omega})^2)}{2} + \frac{(3p - 2p^2) U((1 + \tilde{\omega})(1 - \tilde{\omega}))}{4} + \frac{(1-p) U(1 - \tilde{\omega})}{2} + \frac{(1-p) U((1 - \tilde{\omega})^2)}{4} + \frac{U(1 + \tilde{\omega})}{4}.
\]

Suppose \( f \) is chosen to satisfy (3) so that no separating equilibrium is possible. Then the uninformed adviser prefers to mimic the informed adviser rather than to signal that he
is uninformed by choosing $\omega_0 = 0$. If $p = 7/8$ and $\alpha = 3/4$, then the payoff to an investor in the regime without switching (2.11064) exceeds the payoff in the regime with switching (2.00661).

[Figure 2]

This example suggests transactions costs that make switching funds costly could improve shareholder welfare to the extent they alleviate advisers' reputation concerns. Such costs include loads paid to enter or exit a mutual fund, and any capital gains taxes triggered on the sale of appreciated mutual fund shares. Another implication of this example is that investors would prefer investing in closed-end rather than open-end mutual funds. Since the investment adviser need not worry about the flight of assets from his fund in response to perceived bad performance and cannot attract new assets into the fund, reputation effects are reduced. Moreover, investors are free to enter and leave the fund at any time by selling their units to other investors. However, the possibilities that an adviser could start a new fund, be hired away to manage another fund, or be dismissed as manager of the current fund suggest that closing a fund does not eliminate reputation effects.

3.2 New Funds

The basic tension arises in this paper because a prize available tomorrow (i.e., additional funds to manage) depends on whether a risky bet (i.e., a portfolio choice that signals ability) is taken today. Making no bet means no probability of winning the prize. Making a bet imposes risk that reduces utility but raises expected payout. If the prize is big in relation to the risk, then the uninformed adviser will take the bet. In this model, the prize comes from money leaving one fund to enter another. This structure is appealing because it creates a closed system. Patel et al. (1992) document that old money is slow to leave established funds with poor track records but new money flows to funds with good track records.\(^\text{17}\) The intuitions in this paper apply to this setting as well. Suppose investors leave their period 1 investments with the same adviser in period 2 regardless of performance. Also suppose some new money will be invested with the advisers at the start of period 2. If the allocation of new money between the advisers is sensitive to first-period performance,

\(^{17}\) Goetzmann and Peles (1994) argue that cognitive dissonance can explain why money is slow to leave funds with poor track records. Taxes offer another possible explanation. Remaining invested in a fund that has performed poorly may be better than switching to another fund if shifting money to another fund would trigger capital gains taxes that otherwise could be deferred. Early payment of these taxes could lower returns more than the anticipated gain from investing with an informed adviser over the remaining time horizon. Loads have a similar effect.
then the uninformed adviser will wish to mimic the informed adviser while the informed adviser will wish to separate himself from the uninformed adviser. Whether pooling or separation obtains depends on the amount of new money invested in period 2 relative to the amount of money invested in period 1 as well as the precision of the informed adviser's information and the risk tolerance of the advisers.

4. Conclusions

This paper suggests that informed and uninformed advisers are rewarded for taking bold actions, but for different reasons. It is rational for the uninformed adviser to undertake a bold action in order to appear informed. This gives rise to a pooling equilibrium. Given this possibility, it is rational for the informed adviser to take an even bolder action in order to increase the cost to the uninformed adviser of mimicry. Making mimicry prohibitively costly for the uninformed adviser serves to distinguish the informed adviser. This gives rise to a separating equilibrium. Attempts at both mimicry and separation cause investors to bear additional risk. Mimicry increases risk without increasing expected payoff, while separation increases both risk and expected payoff. An implication of attempts by uninformed advisers to mimic informed advisers, and attempts by informed advisers to distinguish themselves from uninformed advisers, is that portfolio risk is higher than it would otherwise be. Linear performance fees mitigate the undesirable effects of reputation on portfolio choice.

Whether a pooling equilibrium or a separating equilibrium is played depends on the parameters of the model. As the precision of the informed adviser's information increases, separation becomes cheaper for the informed adviser and mimicry becomes more expensive for the uninformed adviser. As advisers become less risk averse, the cost of choosing a risky portfolio is reduced so it is harder for an informed adviser to separate himself from an uninformed adviser.

Aside from the economic significance of mutual funds to investors, three features of the investment advisory relationship make it an attractive object of study. First, many of the terms of the compensation contract between advisers and their investors are publicly available. Second, mutual funds must file quarterly listings of asset holdings with the Securities Exchange Commission, so it is possible to document the actions taken by the fund adviser. Third, the universe of possible actions is the set of possible portfolios. Thus it is possible for the researcher to observe the action taken (and those not taken) by the adviser and the contractual incentives that gave rise to that action. Previous research
on project selection and compensation has chiefly considered the relationship between
top executive pay and performance in non-financial public corporations. This work, of
necessity, is conducted in the absence of exact knowledge of the menu of possible actions
from which the executive chooses a course of action, the action taken, or the outcome of the
action. A promising avenue for research in compensation and project choice is the study
of investment companies. Because investment advisers are numerous and homogenous, it
should be possible to examine empirically the intuitions about compensation policy and
investment choice developed here.

There are many issues in portfolio management that remain to be addressed. This
paper treats the compensation contracts imposed on managers as exogenous and asks what
behavior emerges when advisers act as rational agents who care about their reputations.
While a strong case can be made that the form of the contract is imposed by the Securities
and Exchange Commission, it is less plausible that the level of asset and performance
fees is exogenous. It would be interesting to develop a model in which fees are selected
endogenously by advisers.
Appendix I. Proofs

Proof of Proposition 1: The payoff from choosing \( \omega = 0 \) with probability 1 is \( \frac{1}{2} U \).

From table 2, the payoff to the uninformed adviser from mimicking the informed adviser at \( \omega \) is

\[
\frac{pq}{2} U \left[ f(\frac{1}{2}(1 + \omega) + \frac{1}{2}(1 - f)(1 + \omega)) \right] \\
+ \frac{p(1-q)}{2} U \left[ f(\frac{1}{2}(1 - \omega) + 0) \right] \\
+ \frac{(1-p)q}{2} U \left[ f(\frac{1}{2}(1 - \omega) + \frac{1}{2}(1 - f)(1 - \omega)) \right] \\
+ \frac{(1-p)(1-q)}{2} U \left[ f(\frac{1}{2}(1 + \omega) + (1 - f)(\frac{1}{2}(1 + \omega) + \frac{1}{2}(1 - \omega))) \right] \\
+ \frac{pq}{2} U \left[ f(\frac{1}{2}(1 - \omega) + 0) \right] \\
+ \frac{p(1-q)}{2} U \left[ f(\frac{1}{2}(1 + \omega) + \frac{1}{2}(1 - f)(1 + \omega)) \right] \\
+ \frac{(1-p)q}{2} U \left[ f(\frac{1}{2}(1 + \omega) + (1 - f)(\frac{1}{2}(1 + \omega) + \frac{1}{2}(1 - \omega))) \right] \\
+ \frac{(1-p)(1-q)}{2} U \left[ f(\frac{1}{2}(1 - \omega) + \frac{1}{2}(1 - f)(1 - \omega)) \right] \\
= \frac{p}{2} U \left( f(\frac{2-f}{2}(1+\omega) \right) + \frac{1-p}{2} U \left( f(\frac{2-f}{2}(1-\omega) \right) + \frac{p}{2} U \left( f(\frac{2}{2}(1-\omega) \right) \\
+ \frac{1-p}{2} U \left( f(\frac{2-f}{2}(1+\omega) + (1 - f)(1-\omega)) \right).
\]

The most costly strategy for the uninformed adviser to mimic is the one in which the informed adviser chooses \( \hat{\omega} = 1 \) following signal \( G \) and \( \hat{\omega} = -1 \) following signal \( B \). Then, for \( \alpha > 0 \) the payoff from mimicry is:

\[
= \frac{p}{2} U (f(2-f)) + \frac{1-p}{2} U (f(2-f)) \\
= \frac{1}{2} U (f(2-f)).
\]

The payoff from mimicry exceeds the payoff from choosing \( \omega = 0 \) whenever (3) is satisfied. In equality (3) holds for some positive \( f \) when \( \alpha > \frac{1}{2} \).

Lemma 1 is needed to prove Lemma 2. Lemma 2 is needed in the proof of Proposition 2.

Lemma 1: Let \( K(\omega) \) be the expected utility of the uninformed adviser in the event both advisers play \( \omega \) in period 1. Let \( L(\omega, \omega) \) be the expected utility of the uninformed adviser
in the event (i) the uninformed adviser chooses $\tilde{\omega}$, (ii) the informed adviser chooses $\omega$, and (iii) with probability one the uninformed adviser captures all of the investors’ funds in period 2. If $U(W) = \log W$, then there exists a $\tilde{\omega}$ such that $K(\omega) = L(\tilde{\omega},\omega)$ where $|\omega| < |\tilde{\omega}|$.

**Proof of Lemma 1:** Take $\omega > 0$. The argument for $\omega < 0$ is symmetric.

$$K(\omega) = EU\left(f\tilde{R}_{u2}(\gamma_{u1}\tilde{R}_{u1} + \gamma_{u2}(1-f)\tilde{R})\right)$$
$$= EU\left(\frac{f}{2}(2-f)\tilde{R}_{u1}\right)$$

since $\tilde{R}_{u2} = 1$, $\tilde{R}_{u1} = \tilde{R}$, and $\gamma_{u1} = \gamma_{u2} = \frac{1}{2}$ when both advisers choose $\omega$,

$$= \frac{1}{2}U\left(\frac{f}{2}(2-f)(1+\omega)\right) + \frac{1}{2}U\left(\frac{f}{2}(2-f)(1-\omega)\right)$$
$$= U\left(\frac{f}{2}\right) + U(2-f) + \frac{1}{2}U(1-\omega^2). \tag{I.1}$$

$$L(\tilde{\omega},\omega) = EU\left(f\tilde{R}_{u2}(\gamma_{u1}\tilde{R}_{u1} + \gamma_{u2}(1-f)\tilde{R})\right)$$
$$= EU\left(f\frac{1}{2}\tilde{R}_{u1} + (1-f)\tilde{R}\right)$$

since $\tilde{R}_{u2} = 1$, $\gamma_{u1} = \frac{1}{2}$, and (by assumption) $\gamma_{u2} = 1$,

$$= \frac{1}{2}U\left(f\frac{1}{2}(1-\tilde{\omega}) + (1-f)(\frac{1}{2}(1-\tilde{\omega}) + \frac{1}{2}(1-\omega))\right)$$
$$+ \frac{1}{2}U\left(f\frac{1}{2}(1+\tilde{\omega}) + (1-f)(\frac{1}{2}(1+\tilde{\omega}) + \frac{1}{2}(1+\omega))\right)$$

$$= U\left(\frac{f}{2}\right) + \frac{1}{2}\left[U\left((1-\tilde{\omega})(2-f) + (1-\omega)(1-f)\right) + U\left((1+\tilde{\omega})(2-f) + (1+\omega)(1-f)\right)\right]$$
$$= U\left(\frac{f}{2}\right) + U(2-f) + \frac{1}{2}\left[U\left((1-\tilde{\omega} + (1-\omega)\left(\frac{1-f}{2-f}\right)(1+\tilde{\omega} + (1+\omega)\left(\frac{1-f}{2-f}\right))\right)\right]$$
$$= U\left(\frac{f}{2}\right) + U(2-f) + \frac{1}{2}(1-\omega^2) + \frac{1}{2}U\left[\frac{1-\tilde{\omega}}{1-\omega} + k\left(\frac{1+\tilde{\omega}}{1+\omega} + k\right)\right] \tag{I.2}$$

where $k = \frac{1-f}{2-f}$. Recall that $U(W) = \log W$. Comparing (I.1) and (I.2), it is clear that $K(\omega) = L(\tilde{\omega},\omega)$ if and only if

$$\left(\frac{1-\tilde{\omega}}{1-\omega} + k\right)\left(\frac{1+\tilde{\omega}}{1+\omega} + k\right) = 1. \tag{I.3}$$

Rearranging this expression yields a quadratic equation in $\tilde{\omega}$ with roots $\tilde{\omega} = -k\omega \pm \sqrt{k^2 + 2k + \omega^2}$. Choose

$$\tilde{\omega} = -k\omega + \sqrt{k^2 + 2k + \omega^2}. \tag{I.4}$$

27
Noting that $\omega < 1$, $\omega < \hat{\omega}$ follows easily from (I.4).

Lemma 2: If $U(W) = \log W$, then there is no sequential equilibrium that satisfies the intuitive criterion in which the informed and uninformed advisers both choose portfolio $\omega$ with positive probability.

Proof of Lemma 2: Suppose both advisers choose $\omega$ with positive probability after the informed adviser receives signal $G$. By Lemma 1, there exists $\hat{\omega} > \omega$ such that the uninformed adviser prefers (i) choosing $\omega$ in period 1 and capturing half of the investors' funds in period 2 to (ii) choosing $\hat{\omega}$ in period 1 and capturing all of the investors' funds in period 2. Thus, there do not exist out of equilibrium beliefs for the investors that rationalize a choice of $\hat{\omega}$ by the uninformed adviser. If $\hat{\omega}$ is observed, the investors must conclude that the adviser who selected $\hat{\omega}$ is informed. Hence, in period 2, the investors allocate all their wealth to the adviser who selected $\hat{\omega}$ (i.e., the informed adviser) in period 1.

It remains to show that the informed adviser prefers $\hat{\omega}$ to $\omega$, given that selecting $\hat{\omega}$ results in investors allocating all their wealth to the informed adviser.

The expected utility to the informed adviser from $\hat{\omega}$ is

$$EU \left( f \hat{\omega} \left( \gamma_{t1} \hat{\omega} + \gamma_{t2} (1 - f) \hat{\omega} \right) \right)$$

$$= EU \left( f \hat{\omega} \left( \frac{1}{2} \hat{\omega} + (1 - f) \hat{\omega} \right) \right)$$

since $\gamma_{t1} = \frac{1}{2}$ and (by the argument above) $\gamma_{t2} = 1$.

$$= p^2 U \left\{ f(1 + \omega^*) \left( \frac{1}{2} (1 + \hat{\omega}) + (1 - f) \left[ \frac{1}{2} (1 + \hat{\omega}) + \frac{1}{2} (1 + \omega) \right] \right) \right\}$$

$$+ p(1 - p) U \left\{ f(1 + \omega^*) \left( \frac{1}{2} (1 - \hat{\omega}) + (1 - f) \left[ \frac{1}{2} (1 - \hat{\omega}) + \frac{1}{2} (1 + \omega) \right] \right) \right\}$$

$$+ (1 - p)p U \left\{ f(1 - \omega^*) \left( \frac{1}{2} (1 + \hat{\omega}) + (1 - f) \left[ \frac{1}{2} (1 + \hat{\omega}) + \frac{1}{2} (1 + \omega) \right] \right) \right\}$$

$$+ (1 - p)^2 U \left\{ f(1 - \omega^*) \left( \frac{1}{2} (1 - \hat{\omega}) + (1 - f) \left[ \frac{1}{2} (1 - \hat{\omega}) + \frac{1}{2} (1 - \omega) \right] \right) \right\},$$

where $\omega^* = \hat{\omega}$ if signal $G$ is received in period 2 and $\omega^* = \hat{\omega}$ if signal $B$ is received in period 2. Recall that $\hat{\omega} = -\omega$.

$$= U \left( \frac{f}{2} \right) + p^2 U \left\{ (1 + \omega^*) \left[ (2 - f)(1 + \hat{\omega}) + (1 - f)(1 + \omega) \right] \right\}$$

$$+ p(1 - p) U \left\{ (1 + \omega^*) \left[ (2 - f)(1 - \hat{\omega}) + (1 - f)(1 - \omega) \right] \right\}$$

$$+ (1 - p)p U \left\{ (1 - \omega^*) \left[ (2 - f)(1 + \hat{\omega}) + (1 - f)(1 + \omega) \right] \right\}$$

28
\[ + (1 - p)^2 U \left\{ (1 - \omega^*) (2 - f) (1 - \omega) + (1 - f)(1 - \omega) \right\} \]
\[ = U \left( \frac{f}{2} \right) + p U (1 + \omega^*) + (1 - p) U (1 - \omega^*) \]
\[ + p U \left[ (2 - f)(1 + \bar{\omega}) + (1 - f)(1 + \omega) \right] \]
\[ + (1 - p) U \left[ (2 - f)(1 - \bar{\omega}) + (1 - f)(1 - \omega) \right]. \]  
\( \text{(I.5)} \)

The expected utility to the informed adviser from choosing \( \omega \) in period 1 is

\[ EU \left( f \tilde{R}_{t2} (\gamma_{t1} \tilde{R}_{t1} + \gamma_{t2} (1 - f) \tilde{R}) \right) \]
\[ = EU \left( f \tilde{R}_{t2} \left( \frac{1}{2} \tilde{R}_{t1} + \frac{1}{2} (1 - f) \tilde{R} \right) \right) \]

since \( \gamma_{t1} = \gamma_{t2} = \frac{1}{2} \),

\[ = p^2 U \left\{ f(1 + \omega^*) \left( \frac{1}{2} (1 + \omega) + \frac{1}{2} (1 - f)(1 + \omega) \right) \right\} \]
\[ + p(1 - p) U \left\{ f(1 + \omega^*) \left( \frac{1}{2} (1 - \omega) + \frac{1}{2} (1 - f)(1 - \omega) \right) \right\} \]
\[ + (1 - p)^2 U \left\{ f(1 - \omega^*) \left( \frac{1}{2} (1 - \omega) + \frac{1}{2} (1 - f)(1 - \omega) \right) \right\} \]
\[ = U \left( \frac{f}{2} \right) + p U (1 + \omega^*) + (1 - p) U (1 - \omega^*) \]
\[ + p U \left[ (2 - f)(1 + \omega) \right] + (1 - p) U \left[ (2 - f)(1 - \omega) \right]. \]  
\( \text{(I.6)} \)

It must be shown that \( \text{(I.6)} < \text{(I.5)} \). This reduces to showing

\[ p U ((2 - f)(1 + \omega)) + (1 - p) U ((2 - f)(1 - \omega)) \]
\[ < p U ((2 - f)(1 + \tilde{\omega}) + (1 - f)(1 + \omega)) + (1 - p) U ((2 - f)(1 - \tilde{\omega}) + (1 - f)(1 - \omega)). \]

This is equivalent to

\[ 0 < p U \left( \frac{1 + \tilde{\omega}}{1 + \omega} + \frac{1 - f}{2 - f} \right) + (1 - p) U \left( \frac{1 - \tilde{\omega}}{1 - \omega} + \frac{1 - f}{2 - f} \right). \]

By construction (see \( \text{(I.3)} \) in the proof to Lemma 2),

\[ x \equiv \left( \frac{1 + \tilde{\omega}}{1 + \omega} + \frac{1 - f}{2 - f} \right) = \left( \frac{1 - \tilde{\omega}}{1 - \omega} + \frac{1 - f}{2 - f} \right)^{-1}. \]

29
Thus, it must be shown that

\[ 0 < pU(x) + (1 - p)U\left(\frac{1}{x}\right). \]

This last inequality follows since \( p > \frac{1}{2} \) and \( x > 1 \).

Therefore, informed advisers prefer \( \bar{\omega} \) to \( \omega \) and the investors must conclude the selection of \( \bar{\omega} \) could only be made by the informed adviser. This breaks the equilibrium. By symmetry, an identical argument applies if the informed adviser receives signal \( B \) in period 1. \( \square \)
Proof of Proposition 2: From Remark 1, the uninformed adviser will only play $\omega \neq 0$ with positive probability if the informed adviser plays $\omega$ with positive probability. From Lemma 2, this cannot happen; so the uninformed adviser must play $\omega = 0$ with probability 1 in equilibrium. The informed adviser will choose a first period portfolio equal to the single-period optimum ($\hat{\omega}$ following signal $G$, $\tilde{\omega}$ following signal $B$), provided the uninformed adviser does not find it worthwhile to mimic this strategy. From table 1, the payoff to the uninformed adviser from mimicking the informed adviser at $\omega$ is

$$
\frac{pq}{2}U\left[f\left(\frac{1}{2}(1 + \omega) + \frac{1}{2}(1 - f)(1 + \omega)\right)\right]
+ \frac{p(1 - q)}{2}U\left[f\left(\frac{1}{2}(1 - \omega) + 0\right)\right]
+ \frac{(1 - p)q}{2}U\left[f\left(\frac{1}{2}(1 - \omega) + \frac{1}{2}(1 - f)(1 - \omega)\right)\right]
+ \frac{(1 - p)(1 - q)}{2}U\left[f\left(\frac{1}{2}(1 + \omega) + (1 - f)\left[\frac{1}{2}(1 + \omega) + \frac{1}{2}(1 - \omega)\right]\right)\right]
\frac{pq}{2}U\left[f\left(\frac{1}{2}(1 - \omega) + 0\right)\right]
+ \frac{p(1 - q)}{2}U\left[f\left(\frac{1}{2}(1 + \omega) + \frac{1}{2}(1 - f)(1 + \omega)\right)\right]
+ \frac{(1 - p)q}{2}U\left[f\left(\frac{1}{2}(1 + \omega) + (1 - f)\left[\frac{1}{2}(1 + \omega) + \frac{1}{2}(1 - \omega)\right]\right)\right]
+ \frac{(1 - p)(1 - q)}{2}U\left[f\left(\frac{1}{2}(1 - \omega) + \frac{1}{2}(1 - f)(1 - \omega)\right)\right]
= \frac{p}{2}U\left(\frac{f(2 - f)}{2}(1 + \omega)\right) + \frac{1 - p}{2}U\left(\frac{f(2 - f)}{2}(1 - \omega)\right) + \frac{p}{2}U\left(\frac{f}{2}(1 - \omega)\right)
+ \frac{1 - p}{2}U\left(\frac{f}{2}((2 - f)(1 + \omega) + (1 - f)(1 - \omega))\right)
= U\left(\frac{f}{2}\right) + \frac{1}{2}pU\left[(2 - f)(1 + \omega) + (2 - f)(1 - \omega)\right] + pU(1 - \omega)
+ (1 - p)U\left[(2 - f)(1 + \omega) + (1 - f)(1 - \omega)\right]
= U\left(\frac{f}{2}\right) + \frac{1}{2}U\left[(2 - f)(1 - \omega^2)p(3 - 2f - 2(1 - f)\omega - \omega^2)^{1-p}\right].
$$

The payoff from choosing $\omega = 0$ with probability 1 is $U\left(\frac{f}{2}\right)$. Therefore, to discourage mimicking, the informed adviser must choose $\hat{\omega}$ and $\tilde{\omega}$ far enough away from 0 so that (4) is satisfied.

Nothing essential to the argument in the proof requires that the number of strategies played by either of the advisers with positive probability be finite. If there were a countable
number of strategies played with positive probability, the argument applies by redefining $\Omega$ and $\Pi$ to be sequences. If a continuum of strategies is played, the argument proceeds by considering the supports of the distributions over strategies. ■
References


Figure 1. One-period payoff to informed and uninformed investment advisers as a function of portfolio choice. This figure plots the one-period payoffs to the uninformed and informed advisers under two different scenarios. In the first scenario, an asset fee is paid to each adviser. No performance fee is paid. In the second scenario, an asset fee and a symmetric performance fee are paid to each adviser. Performance is measured relative to the market portfolio.

Parameter values are: coefficient of risk aversion, $\alpha = 1/2$; probability asset $A$ will be worth 2 at the end of next period, $p = 3/4$; asset fee, $f = 1\%$; performance fee, $g = 0.5\%$. The figure assumes the informed adviser receives the signal $G$. If the informed adviser receives the signal $B$, the curves are reflected in the vertical axis.
<table>
<thead>
<tr>
<th>Event</th>
<th>Probability</th>
<th>Payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>$\frac{p^2}{2}$</td>
<td>$U[(1 - f)^2(1 + \hat{\omega})^2]$</td>
</tr>
<tr>
<td>(b)</td>
<td>$\frac{p(1-p)}{2}$</td>
<td>$U[(1 - f)^2(1 + \hat{\omega})(1 - \hat{\omega})]$</td>
</tr>
<tr>
<td>(c)</td>
<td>$\frac{1-p}{4}$</td>
<td>$U[(1 - f)^2(1 - \hat{\omega})]$</td>
</tr>
<tr>
<td>(d)</td>
<td>$\frac{p(1-p)}{4} \quad U[(1 - f)^2(1 - \hat{\omega})(1 + \hat{\omega})]$</td>
<td></td>
</tr>
<tr>
<td>(e)</td>
<td>$\frac{(1-p)^2}{4} \quad U[(1 - f)^2(1 - \hat{\omega})^2]$</td>
<td></td>
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<tr>
<td>(f)</td>
<td>$\frac{1}{4} \quad U[(1 - f)^2(1 + \hat{\omega})]$</td>
<td></td>
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<tr>
<td>(g)</td>
<td>$\frac{p^2}{4} \quad U[(1 - f)^2(1 - \hat{\omega})(1 + \hat{\omega})]$</td>
<td></td>
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<tr>
<td>(h)</td>
<td>$\frac{p(1-p)}{4} \quad U[(1 - f)^2(1 - \hat{\omega})^2]$</td>
<td></td>
</tr>
<tr>
<td>(i)</td>
<td>$\frac{1-p}{4} \quad U[(1 - f)^2(1 - \hat{\omega})]$</td>
<td></td>
</tr>
</tbody>
</table>

Figure 2. This figure depicts the possible outcomes for an investor who is free to move her wealth to the other adviser at the end of the first period. The outcomes are laid out at the leaves of the tree. The figure assumes the informed adviser plays a pure strategy (\( \hat{\omega} \) if signal G is received and \( \hat{\omega} \) if signal B is received) that the uninformed adviser attempts to mimic. The investor switches her funds at the beginning of the second period to the adviser with the best performance in the first period. If both advisers choose the same portfolio, investors stay put.
Table 1. Summary of model. There are two periods and two investment advisers. At the beginning of both periods, each investment adviser divides the funds he has under management between the risky asset and the riskless asset. One investment adviser receives an informative signal about the return on the risky asset; the other does not. The returns on the risky asset and both portfolios are realized and known to all parties. Based on the realized asset and fund returns at the end of the first period, investors choose to reallocate their wealth between the funds managed by the investment advisers.

Investment advisers are concerned about the reputation they develop in the first period because they are paid according to the assets they have under management. If an adviser’s reputation at the end of the first period is adverse, he may have no assets to manage in the second period. If his reputation is favorable he manages all investors’ wealth in the second period. Both advisers and all investors seek to maximize their expected utility from consuming all their wealth at the end of period 2.
<table>
<thead>
<tr>
<th>Signal</th>
<th>$s_G$</th>
<th>$s_G$</th>
<th>$s_G$</th>
<th>$s_G$</th>
<th>$s_B$</th>
<th>$s_B$</th>
<th>$s_B$</th>
<th>$s_B$</th>
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<tbody>
<tr>
<td>Payoff on asset $A$</td>
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<td>0</td>
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<td>0</td>
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<td>2</td>
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<tr>
<td>Informed adviser's portfolio choice</td>
<td>$\hat{\omega}$</td>
<td>$\hat{\omega}$</td>
<td>$\hat{\omega}$</td>
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<tr>
<td>Uninformed investor's portfolio choice</td>
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<tr>
<td>Outcome:</td>
<td></td>
<td></td>
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<tr>
<td>informed adviser outperforms uninformed adviser tie</td>
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<tr>
<td>uninformed adviser outperforms informed adviser</td>
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<tr>
<td>Probability of outcome assuming the uninformed adviser chooses portfolio weights $\hat{\omega}$ and $\hat{\omega}$ with probabilities $q$ and $1-q$, respectively</td>
<td>$pq$</td>
<td>$p(1-q)$</td>
<td>$(1-p)q$</td>
<td>$(1-p)(1-q)$</td>
<td>$pq$</td>
<td>$p(1-q)$</td>
<td>$(1-p)q$</td>
<td>$(1-p)(1-q)$</td>
</tr>
</tbody>
</table>

Table 2. Summary of signals, actions, and outcomes in the pooling equilibrium. The signal seen by the informed adviser is either $G$ or $B$. With probability $p$, asset $A$ is worth 2 (0) following signal $G$ ($B$). The informed adviser chooses portfolio weight $\hat{\omega}$ following $G$ and $\hat{\omega}$ following $B$. The uninformed adviser randomly chooses either $\hat{\omega}$ or $\hat{\omega}$. 