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September 1977

A THEORY OF RISK-AVERSE BANK BEHAVIOR

Working Paper 151

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FOR DISCUSSION PURPOSES ONLY

The purpose of this paper and of its companion paper, "Effects of Market Structure on Bank Performance" (No. 152 in this series), is to set forth a theory of the banking firm and to demonstrate how variations in market structure affect the behavior of banks. Any microeconomic analysis of the banking firm should seek to explain at least the following behavioral characteristics: (1) the composition of the bank's asset portfolio, (2) the composition of the bank's liabilities, (3) the rate of interest on bank loans, and (4) the yield the bank offers on its time and demand deposits. The models presented below provided solutions for the above characteristics, and provide possible paths for future empirical research.

Although the great majority of previous studies on the relationship between bank behavior and market structure have been empirical rather than theoretical, two major studies, one by Michael Klein [1] and another by Bernard Shull [2], have provided theoretical models of bank behavior. Shull, however, is concerned only with the effects of loan market structure on the allocation of assets within the bank's portfolio and does not discuss the effects of deposit market structure. Klein, on the other hand, does investigate the relationship between input (deposit) market characteristics and performance in the loan market. He concludes that bank asset selection is not functionally dependent on deposit supply characteristics, but his results depend in part on his assumption that bankers are risk neutral.1/ The purpose of this paper is

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1 John Pringle [3] points out that Klein's results also depend on the existence of an asset with perfect price elasticity. See footnote 5.

Broaddus's [4] and Langohr [5] have shown that if net disbursements are functionally dependent on the mix or volume of deposits then bank asset selection is joint dependent with liability management. This result is shown in part II of this paper to incorporate risk aversion into a model of bank behavior. It will be shown that by including risk in the model, deposit market characteristics will affect bank asset selection and the rate of interest charged on bank loans.

The paper is divided into two parts and an appendix. In Part I a risk-neutral model of bank behavior will be presented, solved, and the results analyzed. The analysis in Part I draws heavily on the work of Klein and is intended to provide a basic structure of the model to be presented later in the paper. In Part II the model constructed in Part I will be modified to incorporate risk and the results compared to Klein's. The Appendix provides mathematical proofs of some of the results derived in Part II.

The implications of incorporating risk into the model of the banking firm and how this change affects the relationship between market structure and performance is analyzed in the second paper.

Part I

In the case of risk neutrality, banks can be viewed as attempting to maximize the single period return on stockholder's equity. They are risk neutral in the sense that their sole objective is the maximization of expected returns on owner's equity. The bank has three sources of funds: stockholder's equity, denoted as w; time deposits, denoted as TD; and demand deposits, denoted as DD. Therefore, total funds available to the bank, F, is equal to the sum of time deposits, demand deposits, and owner's equity. To simplify things somewhat and to isolate the effects of deposit market characteristics, it will be assumed that owner's equity is fixed, i.e., $w = \bar{w}$. Further, we will assume that there exists only one type of time deposit and only one class of demand deposit. Both types of deposits will be assumed to be available in less than perfectly competitive markets. In other words, both time and demand deposits are characterized by upward sloping supply functions, and the volume of each supplied is a function of their explicit and implicit yields and the yield on alternative deposits. We have, therefore,

$$TD = TD(r_{TD}, r_D) \text{ and } \frac{\partial TD}{\partial r_{TD}} > 0, \qquad \frac{\partial TD}{\partial r_D} < 0$$

$$DD = DD(r_D, r_{TD}) \text{ and } \frac{\partial DD}{\partial r_D} > 0, \qquad \frac{\partial DD}{\partial r_{TD}} < 0.$$

On the asset side, banks will be assumed to be able to choose from a universe of only three assets: loans, government securities, and cash (reserves). The loan market is assumed to be imperfectly competitive so that, to increase their portfolio holdings of loans, banks must accept a lower expected rate of return. Government securities, on the other hand, can be bought or sold in a perfectly competitive market. Cash is held by banks both as required reserves and to meet unexpected cash withdrawals.

If we denote the proportion of loans, government securities, and cash held in the bank's portfolio as X_L, X_g, and X_c respectively and the expected return on the ith asset as E_i, the demand function for government securities and loans can be written as follows

$$E_g = \bar{E}_g$$

$$E_L = E_L(X_L) \text{ and } \frac{\partial E_L}{\partial X_L} < 0,$$

where E_g is the expected return on government securities and E_L is the expected return on loans.[2/]

Expected returns on loans, as opposed to the actual explicit interest rate charged on loans, will be used here because we want to account for the fact that loans, unlike government securities, are risky. Under somewhat restrictive assumptions as to the functional form of the utility function investors possess, the risk of a loan can be summarized by the standard deviation of returns. In other words, the risk associated with loans can be denoted as σ_L. In the risk-neutrality case it will be assumed that σ_L is exogenously given. Later, when risk aversion is added to the model, this assumption will be dropped.

Finally, cash is assumed to be held to cover both required reserves and to meet expected cash disbursements. If there exists a penalty rate at which banks can borrow to meet cash withdrawals in excess of holdings, then cash can be viewed as yielding an implicit return determined by the penalty rate and the distribution of withdrawal likelihoods. Let us assume that net disbursements (cash outflows in excess of inflows) are seen by the bank as having a probability distribution characterized by

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[2]Klein assumes, as we have here, that the demand function for loans is a function of X_L, the proportion of the portfolio held in private securities, and not L the total volume of loans granted in the market. John Broadduss develops a model of the banking firm similar to Klein using as endogenous variables the volume of loans, government securities and resources. This latter specification provides a simple approach to solving for the optimum size of the banking firm. In this paper proportions, as opposed to absolute quantities, will be used since the main purpose of this paper is to show how risk aversion affects portfolio decisions, and not how risk aversion affects the absolute size of the firm.

the density function k(Z). Further, assume that net disbursements have some upper and lower bound during any planning period, so that withdrawls in excess of the upper bound or less than the lower bound can be assumed to occur with probability equal to zero. If n denotes the constant penalty cost per dollar on borrowed funds, and if Z denotes the ratio of net disbursements to total assets, then the expected cost due to net disbursements can be written as

$$N = n \int_{X_c}^{c} (Z - X_c)\, k(Z)\, dZ,$$

or in functional form

$$N = N(X_c)$$

where N equals the expected losses as a percentage of total funds, c equals the upper limit on withdrawals, and X_c equals the proportion of the portfolio held in cash. In Part II, for simplicity, it will be assumed that k(Z) is a uniform distribution and that the upper and lower limits are a function of the percentage of the deposit portfolio held in demand and time deposits. In the risk-neutral case it is sufficient to leave the density function unspecified.

As mentioned above, the bank attempts to maximize the expected return to stockholder's equity. Given the above definitions, the expected returns on all bank funds can be written as

$$E_F = X_L E_L + X_g \bar{E}_g - N - r_{TD} \frac{TD}{F} - r_D \frac{DD}{F},$$

where F equals the total funds and all other terms are as defined above.

Define E_W as the expected return on owner's equity. Then, E_W

can be written as[3/]

$$E_w = E_F \frac{F}{w}$$

and this is the function the bank seeks to maximize by choosing X_L, X_g, X_c, r_D, and r_{TD}. The constraint is that $X_L + X_g + X_c = 1$, i.e., the proportions of the portfolio held in various assets sums to one. The first order conditions for a constrained maximum are:

$$(1) \qquad \frac{\partial E_w}{\partial r_D} = \frac{1}{w}\left(\frac{\partial DD}{\partial r_D} + \frac{\partial TD}{\partial r_D}\right)E_a - \frac{1}{w}\left[r_d \frac{\partial DD}{\partial r_D} + DD + r_{TD} \frac{\partial TD}{\partial r_D}\right] = 0$$

$$(2) \qquad \frac{\partial E_w}{\partial r_{TD}} = \frac{1}{w}\left(\frac{\partial DD}{\partial r_{TD}} + \frac{\partial TD}{\partial r_{TD}}\right)E_a - \frac{1}{w}\left[r_{TD} \frac{\partial TD}{\partial r_{TD}} + TD + r_D \frac{\partial DD}{\partial r_{TD}}\right] = 0$$

$$(3) \qquad \frac{\partial E_w}{\partial X_L} = \frac{F}{w}\left(\frac{\partial E_L}{\partial X_L} X_L + E_L\right) + \lambda = 0$$

$$(4) \qquad \frac{\partial E}{\partial X_g} = \frac{F}{w} \bar{E}_g + \lambda = 0$$

$$(5) \qquad \frac{\partial E_w}{\partial X_c} = - \frac{F}{w} \frac{\partial N}{\partial X_c} + \lambda = 0$$

$$(6) \qquad \frac{\partial E_w}{\partial \lambda} = X_L + X_g + X_c - 1 = 0$$

where E_a equals the return on assets and is identical to $X_L E_L + X_g E_g - N$. Equations (3), (4), and (5) can be used to solve for the optimum allocation of funds within the portfolio. Since

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[3]The expected return on owners equity may also be written as

$$E_w = \left(1 + \frac{DD + TD}{w}\right) (X_L E_L + X_g E_g - N) - \frac{1}{w}(r_D\, DD + r_{TD}\, TD).$$

This will be the form used in the maximization that follows.

$$\frac{F}{w} E_g = - \lambda$$

and

$$\frac{F}{w}\left(\frac{\partial E_L}{\partial X_L} X_L + E_L\right) = - \lambda$$

and

$$- \frac{F}{w} \frac{\partial N}{\partial X_c} = - \lambda$$

then

$$\frac{\partial E_L}{\partial X_L} X_L + E_L = E_g = - \frac{\partial N}{\partial X_c}. \tag{7}$$

Equation (7) is a significant result of the risk-neutral model. The optimum proportion of loans in the portfolio will be found by setting the marginal returns on loans equal to the marginal return on government securities. But, since government securities have a perfectly elastic supply, their marginal return is equal to average returns. In a similar fashion, cash will be held in the portfolio until its marginal implicit return equals the return on government securities and the marginal return on loans.[4]

Figure 1 illustrates this result; total credit output will be allocated so that the marginal return associated with each credit product is equal to the average revenue on government securities. If E_g equals the expected return on government securities, and if DD_L equals the demand

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[4]John Pringle [3] points out that if there exists at least one controlable source of funds with a rate exogenously determined then it will be the "peg" for the system if that rate exceeds the return on government securities. In Figure 1 this modification would imply that the marginal cost curve is horizonal and lies above E_g. This modification would, of course, imply an interdependance between asset and liability management.

curve for loans, and MR_L equals the expected marginal revenue earned on loans, then total credit allocated to loans, and therefore X_L, is determined by the intersection of MR_L with E_g. MC refers to the marginal cost of funds, and is assumed to be upward sloping.

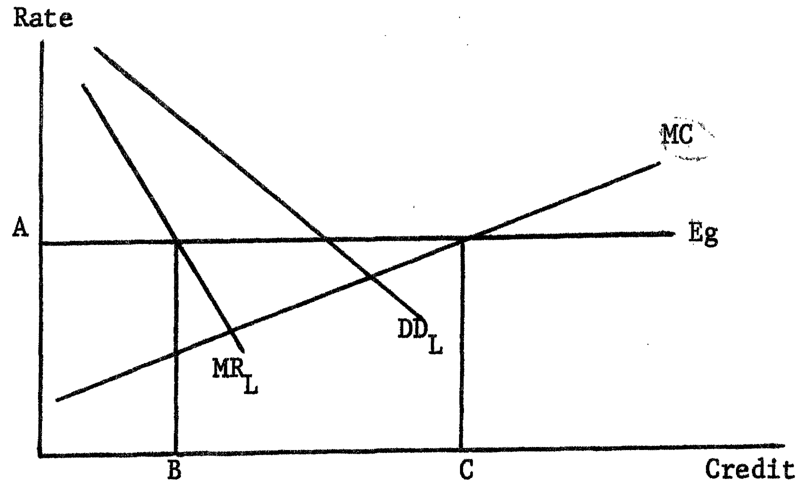

Fig. 1 Illustration of allocation of total credit output.

More important than determining the allocation of assets in the portfolio, equation (7) indicates that the allocation of funds among the securities and, therefore, the interest rate charged on loans, is independent of either of the deposit supply functions. This result is significant in that the loan market can be viewed as insulated from changes in the competitive structure of the deposit market. An increase in competition in the deposit market can be viewed as a leftward shift and downward pivot in the supply curves of either time or demand deposits. Since the supply functions for deposits do not enter into the equation for the determination of loan rates, the risk neutrality analysis indicates that deposit market characteristics will not, ceteris paribus, affect performance in the loan markets.

Equations (1) and (2) can be used to solve simultaneously for both the return on demand deposits and on time deposits. Given the optimal returns on demand and time deposits, using the respective supply functions, the optimum scale of the bank can be solved for. If equations (1) and (2) are rewritten to express r_D and r_{TD} explicitly in terms of the supply functions, and if we define

(8) $$(\frac{\partial DD}{\partial r_D} + \frac{\partial TD}{\partial r_D} = \frac{\partial F}{\partial r_D} > 0$$

and

(9) $$(\frac{\partial DD}{\partial r_{TD}} + \frac{\partial TD}{\partial r_{TD}}) = \frac{\partial F}{\partial r_{TD}} > 0$$

then[5/]

(10) $$(E_a - r_{TD})\frac{\partial F}{\partial r_D} = \frac{\partial DD}{\partial r_D}(r_D - r_{TD}) + DD$$

and

(11) $$(E_a - r_D)\frac{\partial F}{\partial r_{TD}} = \frac{\partial DD}{\partial r_{TD}}(r_{TD} - r_D) + TD$$

and therefore

(12) $$r_{TD} = E_a - (E_a - r_D)\frac{\partial TD}{\partial r_{TD}}/\frac{\partial DD}{\partial r_{TD}} + \frac{TD}{\frac{\partial DD}{\partial r_{TD}}}$$

and

(13) $$r_D = E_a - (E_a - r_{TD})\frac{\partial TD}{\partial r_D}/\frac{\partial DD}{\partial r_D} + \frac{DD}{\frac{\partial DD}{\partial r_D}}.$$

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[5]Equation 10 follows from the fact that

$$F = DD + TD + w$$

and therefore

10.1 $$\frac{\partial DD}{\partial rD} = \frac{\partial F}{\partial r_D} - \frac{\partial TD}{\partial r_D}$$

and

10.2 $$\frac{\partial TD}{\partial r_{TD}} = \frac{\partial F}{\partial r_{TD}} - \frac{\partial D}{\partial r_{TD}}$$

Substituting 10.2 into equation 1

10.3 $$\frac{\partial F}{\partial r_D}E_a = rd\frac{\partial D}{\partial r_D} + r_{TD}(\frac{\partial F}{\partial r_D} - \frac{\partial D}{\partial r_D}) + DD,$$

and therefore

10 $$\frac{\partial F}{\partial r_D}(E_a - r_{TD}) = (r_D - r_{TD})\frac{\partial D}{\partial rD} + DD,$$

and following the same procedure,

11 $$(E_a - r_D)\frac{\partial F}{\partial r_{TD}} = \frac{\partial TD}{\partial r_{TD}}(r_{TD} - r_D) + TD.$$

Equations (12) and (13) are significant in that they indicate that the optimum rates to be paid on time and demand deposits, and therefore the optimum scale of the bank, is a function of three factors:

1) The supply function and, therefore, the competitive characteristics of the demand deposit market
2) The supply function and, therefore, the competitive characteristics of the time deposit market
3) The return on earning assets (E_a), which is a function of asset market characteristics

Equation (7), together with the results of equations (12) and (13), are the basis of the argument, first put forth by George Benston [6], that the regulation of interest rates on deposits is unnecessary. If, as the risk-neutral model suggests, it is the return on earning assets that determines the rate paid on deposits, and not vice versa, then the concern of regulators that the absence of interest rate ceilings will lead to banks' investing in high yield but risky assets, is unfounded. In other words, the model indicates the causal relationship runs from returns on assets to deposit rates and not, as proponents of interest rate ceilings claim, from deposit rates to earning asset returns. As will be shown in Part II, this result does not hold when risk aversion is entered into the model.

Finally, equations (12) and (13) give a clue to the relationship between the rate paid on time deposits and the implicit rate paid on demand deposits. By solving for r_{TD} and r_D in equations (12) and (13) we find that

$$r_D = E_a + \frac{TD \frac{\partial TD}{\partial r_D} + DD \frac{\partial DD}{\partial r_{TD}}}{\frac{\partial DD}{\partial r_{TD}} \frac{\partial DD}{\partial r_D} - \frac{\partial TD}{\partial r_{TD}} \frac{\partial TD}{\partial r_D}} \tag{14}$$

$$r_{TD} = E_a + \frac{DD \frac{\partial TD}{\partial r_{TD}} + TD \frac{\partial DD}{\partial r_D}}{\frac{\partial DD}{\partial r_{TD}} \frac{\partial DD}{\partial r_D} - \frac{\partial TD}{\partial r_{TD}} \frac{\partial TD}{\partial r_D}} . \tag{15}$$

Now if we assume that the levels of time and demand deposits are more sensitive to changes in their own prices than to changes in prices of substitute deposits, and if time deposits are more sensitive to changes in interest rates than demand deposits, then equations (14) and (15) indicate that $r_{TD} > r_D$, as we would expect.[6]

The analysis provided above ignores, however, several dimensions of the bank's environment which must be included in the analysis. First notice that no account has been taken of the risky characteristics of loans other than expressing returns as expected values rather than as actual contracted rates. Second, expected losses due to cash withdrawals are assumed to be independent of the proportion of funds held in time and demand deposits. As the work of John Broadduss [4] and others suggests, cash

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[6]Equations (14) and (15) illustrate the well-known results that a discriminating monopolist will equate the marginal costs of perfect substitutes in the production function. If we assume that demand deposits have a lower supply elasticity then time deposits, then lower rates will be paid on demand as opposed to time deposits.

withdrawals increase as the ratio of demand deposits to total funds increases. This suggests that the upper and lower limit on the probability function should be a variable dependent on how banks hold their deposits. Finally, only the expected value of losses due to withdrawals is accounted for in the model. As with loan returns, the standard deviation of losses must also be considered once the assumption of risk neutrality is dropped. In the next section these criticisms will be taken into account, and a model that incorporates risk aversion will be constructed.

Part II

There are essentially two approaches to incorporating risk into the decision-making process. The first deals explicitly with the management's utility function. The second approach is based on the capital asset pricing model and incorporates risk into the analysis by making very general assumptions as to the distribution of returns or the functional form of investors' preference mappings.

If the first approach to risk aversion is followed, then one would specify a utility function and assume that utility is a function of profits. If profit flows are assumed to have random fluctuations, then, following Friedman and Savage [7], bankers will attempt to maximize expected utility. The objective function for banks, therefore, could be written as:

$$\max_{r_D,\ r_{TD},\ X_L,\ X_g} E[U(\Pi)]$$

A person is risk averse, if "starting from a position of certainty, [he] is unwilling to take a bet which is actuarially fair" [8]; mathematically this can be expressed as

$$U''(\Pi) < 0.$$

As Arrow and others have noted, for any utility function $U = U(\Pi)$, $U''(\Pi)$ is not invariant under positive linear transformations of $U(\Pi)$, and therefore risk aversion can be measured either in terms of absolute risk

$$R_A(\Pi) \equiv -U''(\Pi)/U'(\Pi)$$

or relative risk aversion

$$R_R(\Pi) \equiv -\Pi U''(\Pi)/U'(\Pi).$$

Having defined risk aversion, we can analyze the effects of changes in deposit market characteristics on loan markets. General results can be derived. The problem with this approach is that it relies on an analysis of management's utility function and neglects the belief that it is stockholder's preferences and capital markets in general that shape management's decisions.

An alternative, and more flexible, means of dealing with risk aversion is to use the capital asset pricing model and its implications for the value of the firm. Under quite general conditions, it has been shown that the equilibrium market value of the firm under certainty can be expressed as

(16) $$V = 1/\rho\ [E(w) - R\ \beta\ \sigma_w]$$

where ρ is the riskless rate of interest, R is the market value of a unit of risk, β is the correlation coefficient relating the firm's return on equity, w, to overall returns.[7] R is the reciprocal of the slope of the market line, and

$$\beta = \frac{\text{cov }(R_w, M)}{\sigma_m \sigma_w}$$

where R_w = return on stockholder's equity

M = return on market portfolio

σ_w = standard deviation of returns to the firm

σ_m = standard deviation on market portfolio.

The use of this type of objective function in decision making under risk has several desirable properties: (1) it is formally derivable from the equilibrium conditions for capital asset prices; (2) it includes the risk-neutral expected profit criterion as a special case; (3) it explicitly introduces the influence of both interest rates and the equilibrium market price of risk; and (4) the capital asset pricing model incorporates the same definition of risk aversion as given above but relies on the utility function of investors instead of focusing just on the utility function of management.

If banks are risk averse, they can be viewed as attempting to maximize the risk-adjusted value of stockholder's equity. Define the expected returns on stockholder equity as before, i.e.:

$$E_w = (1 + \frac{DD + TD}{w})[X_L E_L + X_g E_g - N] - \frac{1}{w}[r_{TD}TD + r_D DD].$$

Again, let N equal the expected loss due to cash withdrawals during any given planning period. For simplicity, assume that the density function of possible cash withdrawals is uniform, running from a lower limit, b,

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[7] See Robert Meyer [9] for a discussion of this model.

to an upper limit, c, but assume the upper limit c and therefore the density function, k(Z), is a function of the ratio of demand deposits to total deposits. In particular, let c be an increasing function of the ratio of demand deposits to total deposits, to capture the more volatile nature of demand deposits. Expected losses, therefore, can be defined in terms of the following equation

$$N = n \int_{X_c}^{c} (Z - X_c)\, k(Z)\, dZ \equiv \frac{n(c - X_c)^2}{2(c - b)}, \tag{17}$$

and given the specification of c above[8]

$$\frac{\partial N}{\partial r_D} > 0 \qquad \frac{\partial N}{\partial r_{TD}} < 0 \qquad \frac{\partial N}{\partial X_c} < 0 .$$

The standard deviation of returns on equity (σ_W) is a function of the standard deviation on loans (σ_L) and the standard deviation of losses due to cash withdrawals (σ_n); therefore

$$\sigma_W = (E(R_W - E_W)^2)^{1/2}$$

where R_W equals the return on owner's equity.

σ_W can be written, therefore, as

$$\sigma_W = \frac{F}{W}(X_L^2\sigma_L^2 + \sigma_n^2)^{1/2} . \tag{18}$$

Now,

$$X_L^2\sigma_L^2 + \sigma_n^2 \equiv \sigma_\Pi^2,$$

or the variance of profits.

The variance of losses due to cash withdrawals is a function of X_c and both r_D and r_{TD}. It can be shown (see appendix) that under the

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[8] As shown in the appendix both the lower limit b and the upper limit c may change with an increase in time or demand deposits. While this specification is probably more realistic, it does not change the results derived in Part II. For simplicity only the upper bound c will be assumed to change with a change in deposits.

assumption of a uniform distribution and the specification of c,

$\frac{\partial \sigma_n^2}{\partial r_D} > 0$, $\frac{\partial \sigma_n^2}{\partial r_{TD}} < 0$, and $\frac{\partial \sigma_n^2}{\partial X_c} < 0$ and, therefore

$$\frac{\partial \sigma_w}{\partial r_D} > 0 \qquad \frac{\partial \sigma_w}{\partial r_{TD}} < 0 \qquad \frac{\partial \sigma_w}{\partial X_c} < 0 \ .$$

The variance of returns on loans is a function of the proportion of loans in the portfolio. In particular, $\frac{\partial \sigma_L^2}{\partial X_L} > 0$. Following the Jaffee and Modigliani theory of credit rationing [10], banks can be viewed as dividing their loan customers into distinct credit classes. Within each class, an explicit interest rate is set and credit is rationed on the basis of credit risk. As banks expand loans in any given class they are forced to accept riskier borrowers. If the risk associated with a borrower is defined in terms of capital gain or loss, then the risk of any customer can be summarized by the standard deviation of actual returns. Given this view of bank credit rationing behavior, it is clear that

$$\frac{\partial \sigma_L^2}{\partial X_L} > 0$$

and therefore

$$\frac{\partial \sigma_w}{\partial X_L} > 0.$$

Further, since the distribution of returns on a loan are likely to be skewed to the left (toward capital loss) as σ_L increases, E_L will decline.

Finally, before solving the model and analyzing the results, the relationship between $R\beta$ and the endogenous variables must be specified. To simplify the analysis, it will be assumed that changes in the endogenous variables do not affect the correlation of the bank's returns with the market. $R\beta$ can therefore be viewed as exogenously given and constant.[10]

Given the above definitions, the objective of the bank becomes

$$\max_{r_D, r_{TD}, X_L, X_g, X_c} V = \frac{1}{\rho}\left(E(w) - R\beta\sigma_w\right),$$

or

$$\max \frac{1}{\rho}\left\{\left[1 + \frac{DD(r_D, r_{TD}) + TD(r_D, r_{TD})}{w}\right] X_L E_L(X_L) + X_g E_g - N(r_D, r_{TD}, X_c) - \frac{1}{w}\left[TDr_{TD} + DDr_D\right] - R\beta\sigma_w(r_{TD}, r_D, X_c, X_L)\right\},$$

subject to the constraint

$$X_L + X_g + X_c = 1.$$

The first order conditions for a maximum are, therefore:

$$\text{(20)} \qquad \frac{\partial V}{\partial X_L} = \frac{F}{w}\frac{\partial E_L}{\partial X_L} X_L + X_L - R\beta\frac{\partial \sigma_w}{\partial X_L} + \lambda = 0$$

$$\text{(21)} \qquad \frac{\partial V}{\partial X_g} = \frac{F}{w} E_g + \lambda = 0$$

$$\text{(22)} \qquad \frac{\partial V}{\partial X_c} = -\frac{F}{w}\frac{\partial N}{\partial X_c} - R\beta\frac{\partial \sigma_w}{\partial X_c} + \lambda = 0$$

$$\text{(23)} \qquad \frac{\partial V}{\partial r_{TD}} = \frac{1}{w}\frac{\partial F}{\partial r_{TD}} E_g - \frac{F}{w}\frac{\partial w}{\partial r_{TD}} - \frac{1}{w}\frac{\partial TD}{\partial r_{TD}} r_{TD} + TD + \frac{\partial DD}{\partial r_{TD}} r_D - R\beta\frac{\partial \sigma_w}{\partial r_{TD}} = 0$$

$$\text{(24)} \qquad \frac{\partial V}{\partial r_D} = \frac{1}{w}\frac{\partial F}{\partial r_D} E_g - \frac{F}{w}\frac{\partial N}{\partial r_D} - \frac{1}{w}\frac{\partial TD}{\partial r_D} r_{TD} + \frac{\partial DD}{\partial r_D} r_D + r_D - R\beta\frac{\partial \sigma_w}{\partial r_D} = 0$$

$$\text{(25)} \qquad \frac{\partial V}{\partial X} = X_L + X_g + X_c - 1 = 0 \ .$$

The first order conditions indicate that the allocation of loans in the asset portfolio, and therefore E_L, is not, as was demonstrated in the

[10] The appendix provides a justification for this assumption.

certainty case, functionally independent of the deposit supply function. Equations (20), (21), and (22) together with the fact that

$$\sigma_w = \frac{F}{w}(\sigma_\Pi^2)^{1/2}$$

indicate that to maximize stock value the following must hold

(26) $$\left(\frac{\partial E_L}{\partial X_L} X_L + X_L\right) - R\beta \frac{\partial \sigma_\Pi}{\partial X_L} = E_g$$

(27) $$-\frac{\partial N}{\partial X_c} - R\beta \frac{\partial \sigma_\Pi}{\partial X_c} = E_g \quad .$$

If $R\beta > 0$, then equation (26) indicates that a lower proportion of total assets will be placed in loans than in the certainty case. Further,

(28) $$\frac{\partial \sigma_\Pi}{\partial X_L} = (\sigma_\Pi^2)^{-1/2} \left(X_L \sigma_L^2 + X_L \sigma_L \frac{\partial \sigma_L}{\partial X_L}\right)$$

where

(29) $$(\sigma_\Pi^2)^{-1/2} = (X_L^2 \sigma_L^2 + \sigma_n^2)^{-1/2} \quad .$$

Then X_L is dependent on deposit supply functions, since σ_n^2 is dependent on both the level of time and demand deposits. Intuitively, the left-hand side can be viewed as the risk adjusted marginal returns on a bank loan, while equation (27) can be interpreted as the marginal savings or implicit return on cash holdings. Again, the proportion held in cash also is dependent upon σ_Π, which in turn depends upon the deposit supply characteristics. Equations (26) and (27) also give an indication of the relationship between the proportion of assets held in loans and the ratio of time to demand deposits. As mentioned above, σ_n^2 is defined as an increasing function of the ratio of demand to time deposits. Thus, the higher the proportion of demand to time deposits, the larger will be

σ_{Π}^{2} and, therefore, the smaller will be $\frac{\partial \sigma_{\Pi}}{\partial X_L}$. In other words, the relationship between the asset and liability portfolios is, in the risk-averse case, joint dependent.

This last result is significant since previous models of the banking firm have indicated that there is no relationship between the ratio of demand deposits to total deposits and the asset portfolio decision (see Broadduss [4]). This result seems peculiar, given the empirical work of Jacobs and Edwards [11], which found a strong and negative relationship between interest rates charged on bank loans and the ratio of demand deposits to time deposits. This result will hold in the above model even if we follow a less restrictive set of assumptions and assume only that

$$\frac{\partial \sigma_n}{\partial DD} > \frac{\partial \sigma_n}{\partial TD}$$

instead of the more restrictive assumption that

$$\frac{\partial \sigma_n}{\partial DD} > 0 \text{ and } \frac{\partial \sigma_n}{\partial TD} < 0.$$

Equations (20) through (25) can also be used to solve for the optimum scale of bank operations and the rate to be paid on time and demand deposits. As in the risk-neutral case, both r_{TD} and r_D will be functions of the return on earning assets and the rate paid on other types of deposits. Given the assumption that time deposits are more sensitive to interest rate changes, and the assumption that all types of deposits are more sensitive to own, as opposed to substitute prices, equations (23) and (25) can be used to show that the rate paid on time deposits will be greater than the rate paid on demand deposits. Now, however, the higher rate is paid on time deposits not only because of their supply characteristics but, in addition, because time deposits, at best, reduce the risk of the firm and at worse contribute less to the variation in profits than do demand deposits.

The above model of bank behavior conflicts with the risk-neutral model in that it indicates a link between the input market and the bank's output market. As equations (26) and (27) indicate, the link is through the net disbursement loss function. By recognizing that different types of deposits subject the bank to different amounts of risk, and further recognizing that risk can be managed on both the asset and liability sides of the balance sheet, one reaches the conclusion that asset portfolio decisions cannot be made independently of liability portfolio decisions.

The results of the risk averse-model of bank behavior can be used to analyze the effects of entry on the performance of banks. This analysis is pursued in the second, or companion, paper, which follows.

Similarly for an increase in time deposits, if $\left|\frac{\partial b}{\partial r_{TD}}\right| < \frac{\partial c}{\partial r_{TD}}$ then

$$\frac{\partial N}{\partial r_{TD}} < 0.$$

The effect of an increase in demand deposits on the variance of losses can be clearly seen in the simple case in which X_c equals zero. The distribution of losses is shown in Figure 2.

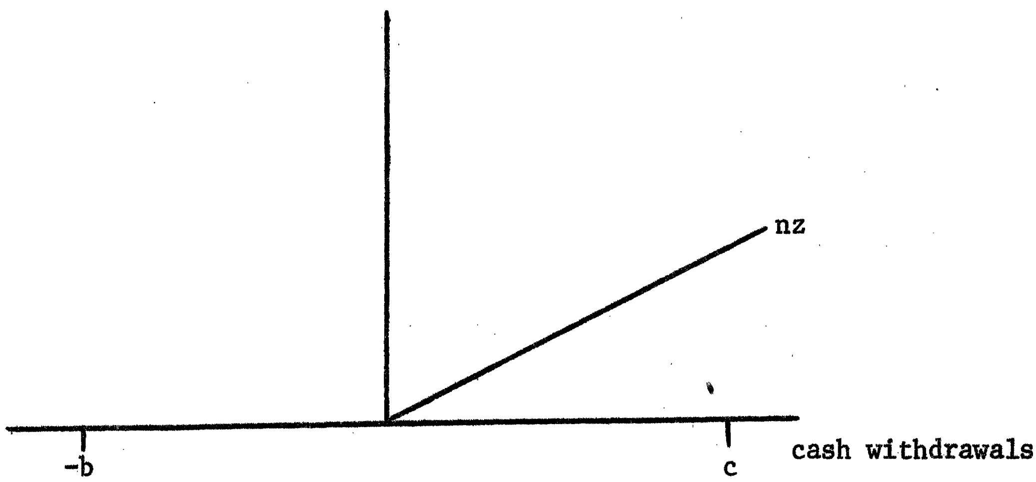

Fig. 2 Distribution of losses.

where

$$Z \sim U(-b,\ c)$$

then

(i) $$N = E(L) = \int_{-b}^{c} L(z)f(z)dz = \int_{0}^{c} nz\ f(z)dz = \frac{n}{c+b}\int_{0}^{c} zdz = \frac{nc^2}{2(c+b)}\ ;$$

and

(ii) $$E(L^2) = \int_{0}^{c} n^2z^2f(z)dz = \frac{n^2}{c+b}\frac{z^3}{3} = \frac{n^2c^3}{3(c+b)}$$

$$\sigma_n^{\ 2} = E(L^2) - E(L)^2 = \frac{n^2c^3}{3(c+b)} - \frac{n^2c^4}{4(c+b)^2}$$

$$= \frac{n^2c^3}{12(c+b)^2}(4b+c)$$

$$\frac{\partial \sigma_n}{\partial r_D} = \frac{\partial c}{\partial r_D} \frac{\partial}{\partial c} \left\{ \frac{n^2 c^3 (4b+c)}{12(c+b)^2} \right\}$$

$$= \left(\frac{2c^2 + 12b^2 + 8bc}{c + b} \right) \frac{n^2}{12} > 0 \quad ,$$

and therefore

$$\frac{\partial \sigma_n}{\partial r_D} > 0 \quad .$$

Similarly it can be shown that

$$\frac{\partial \sigma_n}{\partial r_{TD}} < 0 \text{ if } \frac{\partial c}{\partial r_{TD}} < 0.$$

Following Mossin [12] the value of the ith firm is given by

$$V_i = \frac{1}{1+R_f} [E(\Pi_i) - R \text{ Corr } (\Pi_i, M) \, \sigma_{\Pi_i}] \quad ,$$

where

R_f = risk free rate

σ_{Π_i} = standard deviation of profits for the ith firm

Corr (Π_i, M) = correlation of the ith firms returns with the market portfolio returns.

Corr (Π_i, M) may be written as

$$\frac{\sum_{j=1}^{n} x_{jm} \sigma_{ji}}{\sigma_i \sigma_m} \quad ,$$

where x_{jm} = proportion of the jth asset held in the market portfolio

and σ_{ji} = covariance of the returns of the ith firm with the jth firm.

Now if we assume

$$\frac{\partial \sigma_{ij}}{\partial r_{TD}} = 0 \qquad \frac{\partial \sigma_{ij}}{\partial r_D} = 0$$

the the first order conditions appearing in equations (22)-(24) are correct. For changes in the asset portfolio the results are more complex. If we assume that banks allocate loans to the best credit risk first and expand loans by allocating credit to riskier customers, then $\frac{\partial\beta_i}{\partial X_L} > 0$ and β_i will converge to one.

In other words, since

$$\beta_i = \frac{X_{im}\sigma_i^{\,2} + j \sum_{\substack{j=1 \\ j \neq i}}^{n} X_{jm}\sigma ji}{\sigma_i\sigma_m}$$

then

$$\frac{\partial\beta_i}{\partial X_L} = \frac{X_{im}}{\sigma_m} + \left(\frac{\sum_{\substack{j=1 \\ j \neq i}}^{n} X_{jm} \frac{\partial\sigma_{ji}}{\partial X_L}}{\sigma_i\sigma_m} + \frac{\sum_{j=i}^{n} x_{jm}\sigma_{ij}}{\sigma_i^{\,2}\sigma_m} \frac{\partial\sigma_i}{\partial X_L} \right) > 0.$$

Since $\frac{\partial\beta_i}{\partial X_L} > 0$ this does not change qualitatively the results in equations (20)-(22). In addition for any given increase in the proportion of loans held by the bank, the effect on the correlation coefficient is likely to be small.

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