Evidence Based Decision Process: Some Perspectives On Investor Strategies

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ABSTRACT

In this paper we develop a framework for an Evidence Based Decision Process (EBDP) for investors who allocate money into the financial sector. In particular, we analyze the structure of such a process for those investing in mutual funds under the assumption they consider the measure of performance, the modeling approach and the selection of the pricing model as their decision criteria. Using seven groups of equity mutual funds, classified by investment objective, we exemplify the impact of the EBDP on investor strategies.

We find that the selection of the performance measure may have a critical impact on the resulting decision an investor makes. Additionally, the choice of risk adjusting pricing model as well as the econometric tool used could lead to very different decisions using the same data. We demonstrate the advantages of using a predictive approach to quantifying the uncertainty accompanying each EBDP. A key finding is, at the present time, quantifying mangerial skill in selecting funds is a very difficult challenge; diametrically opposite conclusions can be drawn in this context as exemplified by the EBDP analysis.

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I. Introduction

It is necessary to formalize a procedure that carefully accounts for various levels of uncertainty in an investor's world; we state such a procedure as an Evidence Based Decision Process (EBDP). It is true that any attempt at constructing such a process will *always* be incomplete and imprecise because of the complex uncertainties an investor encounters in practice. But it is also true that *all* attempts to formalize this process should help an investor make calculated decisions. The EBDP developed in this paper tackles, simultaneously, three critical aspects that investors typically consider: (a) the measure of performance; (b) the pricing model; and (c) the modeling approach.

Throughout, "inference" and "decision" are used interchangeably. As Smith (1984) notes:

> The supposed dichotomy between inference and decision is illusory, since any report or communication of beliefs following the receipt of information inevitably itself constitutes a form of action.

That is, the problem of reporting inferences is essentially a special case of a decision problem. This fact is important in light of (c) above because it automatically suggests the need for statistical methods that recognize an investor's strategy as actions based on the ubiquitous notion of "learning from experience." Formally, of course, this is what we are calling EBDP. However, the explicit recognition of an investor's decision will necessarily involve considerations of utility functions which is outside the scope of this paper.

Why (a), (b) and (c)? Because the plethora of papers that have appeared in the literature recognize, among others, these three aspects as most critical. The point of departure here, however, is the simultaneous consideration of the three. The resulting inference for an investor's strategy, it turns out, is substantially different from previous work reported in this context.

Although the idea of EBDP is general and, in principle, applies to many problems in finance, here focus is on its use in the equity mutual funds sector simply because there has been considerable interest in this sector in recent years from a theoretical and practical perspective; see for example, Hendricks, Patel, Zeckhauser (1993); Elton, Gruber, Das, and Hlavka(1993); Goetzmann and Ibbotson (1994); Brown and Goetzmann (1995); Malkiel(1995); Gruber (1996); Wermers (1996); Nanda, Narayanan and Warther (1998); Zheng (1999); Wermers (2000).

In order to employ an EBDP:

under (a), we consider two measures: (i) raw returns; (ii) performance evaluation, typically referred to as a firm's *alpha*;

under (b), three pricing models are entertained: (i) Capital Asset Pricing Model (CAPM); (ii) Fama & French (1993) 3-factor model; and (iii) Carhart's (1997) 4-factor model;

under (c), two parametric hierarchical models are developed: (i) correlated random effects; and (ii) error components.

The rationale for the selections under (a) and (b) is based on related research discussed below. The rationale for the choice of the hierarchical models under (c) will be discussed in several parts of the paper starting with the following.

A central idea in the paper is the notion of persistence of returns. Theoretically, investing money into funds with high positive persistence should result in a simpler decision criteria than investing in funds which exhibit no clear patterns. In practice, however, the performance of the mutual funds rarely exhibits clear patterns, and the differences among the funds or groups of funds are too small to give conclusive investment strategies. Consequently, investors following advice based only on the persistence phenomenon may very often be mistaken. Recognizing this *uncertainty* in the persistence of returns is to recognize it has to be studied via a formal mathematical model. The hierarchical models under (c) are designed to accomodate this feature.

But modeling persistence also depends on factors such as the performance measure used (see (a)); the pricing model (see (b)); statistical assumptions (see (c)); and the horizon of the analysis.

Recognizing and modeling this interdependency between (a) (b) and (c) is at the heart of the EBDP developed in this paper. From an investor's perspective failure to understand and model this interdependency could lead to substantially different investment strategies. That this is so will be exemplified via illustrative analyses.

The problem of performance evaluation and performance persistence of the equity mutual funds has been well documented in the literature. Carhart (1997) provides an excellent summary of most of the previous findings in this context. Christopherson, Ferson, and Glassman (1998) consider conditional measures of persistence. Carhart (1997) and Christopherson et al. (1998) focus on factors which contribute to persistence without giving any suggestions how those measures could be utilized by investors facing potential portfolio allocation problem. Baks, Metrick, and Wachter (2000, henceforth BMW) provide more intuition about the decision making process and the conditions under which investors allocate their money into equity funds. But they do not relate performance to persistence. Pástor and Stambaugh (2000(a), henceforth PS) consider the adequacy of the pricing model and quanitfy the stock picking ability of the managers. Additionally, both BMW and PS use Bayesian methods in their analysis. But neither consider persistence in such framework.

The research described in this paper complements and extends the work summarized above. First, we mathematically relate the factors that influence persistence to different performance measures. This is important because investors are typically divided on the choice of performance measures that they deem ideal to develop their personal investment strategies. Thus, if raw returns were the focus for an investor, persistence may play a critical role in their assessment of the risk underlying future investements. On the other hand, an investor may feel that a firm's *alpha* (performance evaluation) is a better indicator of risk; in this case as well, persistence of performance may be of considerable importance to the investor. In both cases, the influence of the pricing model(s) could substantially influence the investor's strategy.

How should one encapsulate all these dynamic elements together so that each investor can rely on a *process* that appeals to them?

This question is answered in this paper. But to answer this query satisfactorily, a key feature is the statistical models required in the analysis. Since investor strategies are hardly, if ever, static, the natural choice is to develop mathematical forms that explicitly recognize conditioning on all relevant information available to the investor. Hence a Bayesian approach is used under (c).

Two nice consequences of the approach result. First, it naturally provides "what if" alternatives to the investor. Smith (1984) writes:

> ...one of the most attractive features of the Bayesian approach is its recognition of the legitimacy of a plurality of (coherently constrained) responses to data. Any approach to scientific inference which seeks to legitimize an answer in response to complex uncertainty is, for me, a totalitarian parody of a would-be rational learning process.

To which we may add, attempting to legitimize an answer in the equity mutual fund sector could prove costly. From a practical perspective, the investor is presented with easy-to-understand summaries of the uncertainty via complete probability descriptions of the random quantities of interest; for example, the probability distribution of the persistence parameter. It is not sufficient to know whether or not there is persistence; it is necessary to quantify the uncertainty associated with it.

Second, for obvious reasons, investors, typically, are interested in potential *future* outcomes. As an example, investors may be anxious to discern – with some reasonable degree of confidence – the future performance of a fund; the future may be a week, month, 2-months and so on. *Conditional* on expected behavior of certain market variables, the EBDP developed in this paper can provide a complete probability description of the uncertainties involved in future performance of equity funds that an investor is keen on; this is obtained via so-called *predictive distributions.*

The Bayesian models developed in this paper differ substantially from those used in BMW and PS. This is because we are posing a different question *and* because our approach models time-series *and* cross-sectional data; more details are provided later.

Section II discusses the construction and properties of the data used. In particular, we distinguish between performance measures with respect to simple raw returns without controlling for any fund-specific characteristics, such as loads, expenses, turnover, etc., as well as performance adjusted for all those factors. Under the latter, we construct the sample given the standard Fama-MacBeth (1973) type regression. In Section III, the methodology is developed. Section IV provides a comprehensive empirical analysis under various combinations of (a) (b) and (c) described earlier. Comparative analysis, where appropriate, to previous work is also provided. Some conclusions are discussed in Section V.

II. Data

In this study, we construct data based on the CRSP 99' Survivor-Bias Free US Mutual Fund Database. In particular, the CRSP 99' data is consistent with the dataset used by Carhart (1995). This database reports monthly returns of the open-end mutual funds from January 1^{st} 1962 to December 31^{st} 1999 for funds of all objectives, including defunct funds.[1]

CRSP dataset provides the cumulative Total Returns per share, calculated as follows:

$$R_{t-1,t} = (\frac{NAV_t}{NAV_{t-1}}) (\prod_{k=1}^{J} (1 + \frac{X\_AMT_j^D}{RE\_NAV_j^D})) (\prod_{k=1}^{K} (\frac{X\_AMT_k^S}{RE\_NAV_k^S})) - 1 \tag{1}$$

where NAV_{t-1} is the Net Asset Value(NAV) at the end of the previous period; NAV_t is the NAV at the end of the current period; J is the number of dividend or capital gains distributions during the period; K is the number of NAV splits during the period; $X\_AMT_j^D$ is the j^{th} dividend or capital gains distribution during the period; $RE\_NAV_j^D$ is the NAV at which the j^{th} dividend or capital gains distribution was reinvested; $X\_AMT_k^S$ is the number of new shares per RE_NAV of old shares investors received in the k^{th} NAV split. That is, $\frac{X\_AMT_k^S}{RE\_NAV_k^S}$ is the split ratio for the k^{th} NAV split.

In order to obtain comparable results, in this paper we constrain our analysis to the period from January 1^{st} 1989 to December 31^{st} 1998. In particular, we include only those equity funds which survived until the end of 1998, consistent with the methodology used by BMW(2000). However, our dataset includes only those funds which fall under one of the seven broad investment objectives, according to the classifications provided by Wiesenberger, ICDI, and Strategic Insight. Following PS (2000a), we additionally exclude multiple share classes for the same fund as well as funds with less than 10 years of available returns.[2]

For the purpose of this study we construct two sets of initial data. The motivation for this separation comes from the supposition that the evidence base for investors might very well differ. One set – henceforward Set A – may prefer raw returns while the other set –henceforward Set B – may be more comfortable with performance measure of a fund (*alpha*); see (a) in the Introduction.

Set A consists of residuals obtained from the time-series regression of the excess returns of the funds on the respective pool of factors coming from the following pricing models: CAPM, 3-factor model of Fama and French (1993), and 4-factor model of Carhart (1997); see (b) in the Introduction.[3]

These models were chosen because they are the ones most widely considered in the literature cited in this paper. Note *each* model is itself part of the evidence in the EBDP; stated differently, our analysis will provide the investor a form of *robust* inference – robustness of the decision to the choice of the pricing model assumed.

The excess returns have been calculated based on one month Treasury bill. For the CAPM model the only regressor is the excess return on the broad market index (MKT); for the 3-factor model we additionally have SMB defined as payoffs on long-short spreads constructed by sorting stocks according to market capitalization and HML defined as the book-to-market ratio; for Carhart's 4-factor model we also include the so called momentum factor, denoted as PR1YR. PR1YR is constructed as the equal-weight average of firms with the lowest 30% eleven-month returns lagged one month. The portfolios include all NYSE, AMEX, and NASDAQ stocks and are re-formed monthly.

Set B is constructed in two steps and is mostly consistent with the approach used by Carhart (1997). In the first step we estimate monthly *alphas*, each month on every fund according to one of the following models, CAPM, 3-factor model, and 4-factor model, respectively:

$$\alpha_{i,t} \equiv R_{i,t} - R_{F,t} - \hat{b}_{i,t-1} MKT_t \tag{2}$$

$$\alpha_{i,t} \equiv R_{i,t} - R_{F,t} - \hat{b}_{i,t-1} MKT_t - \hat{h}_{i,t-1} HML_t - \hat{s}_{i,t-1} SMB_t \tag{3}$$

$$\alpha_{i,t} \equiv R_{i,t} - R_{F,t} - \hat{b}_{i,t-1} MKT_t - \hat{h}_{i,t-1} HML_t - \hat{s}_{i,t-1} SMB_t - \hat{p}_{i,t-1} PR1YR_t \tag{4}$$

The loadings on the factors are estimated over the first three years. Clearly this influences the number of funds included in the analysis in contrast to data under Set A.[4]

The second step to develop the data under Set B involves the following cross-sectional regression.

$$\alpha_{i,t} = a_t + b_t x_{i,t} + \epsilon_{i,t} \qquad i = 1, ..., n \qquad t = 1, ..., T, \tag{5}$$

where $\alpha_{i,t}$ is an estimate obtained from the first step, $x_{i,t}$ is a set of fund's characteristics, i indexes the number of funds in the sample, and t is the index of time. The explanatory variables in equation (5) are: *ln(Total Net Assets)*, turnover (*Mturn*), maximum load fees, and expense ratio.[5]

Similar to Carhart (1997) *TNA* is lagged one year to avoid spurious correlation. *Mturn* denotes modified turnover reported as turnover plus one-half the rate of change in *TNA*. Load fees are lagged one year to avoid the possibility of funds changing their fees in response to performance. *Mturn* and expense ratio are measured contemporaneously with the *alphas*. Consequently, the characteristics we use span the period from 1988 to 1998. The choice of independent variables is similar to the methodology used by Pástor and Stambaugh (2000a). Unlike Carhart, we do not include two additional variables constructed from turnover measure as they seem to be useful mainly in the ex-post analysis.[6] Separate regressions of the equation (5) are run for each period t, and the residuals from the least-squares regressions are kept and used as our data. No generality is lost in working with residuals; it merely simplifies the mathematics somewhat. Details are provided in the Appendix.

In summary, two sets of data, Set A and Set B were created. These data are residuals from the appropriate regressions described above. Set A corresponds to investors who prefer returns as *evidence*, while Set B should appeal to investors who prefer a fund's performance – as measured by *alpha* – as *evidence*. The details of the samples under both Set A and Set B are reported in Table I.

Insert Table I about here

III. Hierarchical Bayesian Models

This section along with the Appendix, provides the details of the methodology used in the paper. Section III.A considers the case of the correlated random effects model. In Section III.B we discuss the error components model. Section III.C discusses predictive inference for the models presented in the first two subsections. Before delving into the details, we note that the Bayesian framework used in this study makes it possible to avoid the survivorship bias problem; see BMW (2000) for explanations.

A. Correlated Random Effects Model

It is obvious that mutual funds, over time, differ from each other with respect to their performance and in particular with respect to the stock picking ability of their managers. This fund-to-fund difference in statistical terminology is called a random effect. Clearly, a prudent investor would like to account for the uncertainty underlying such effects in their investment strategy. Additionally, as described previously, there has been considerable interest in the idea of persistence of performance of a fund; persistence could be positive or negative. In statistical terms, this is simply the autoregressive parameter in a time-series regression. Denoting this parameter as ρ, persistence (positive or negative) is measured by the probability description of ρ, whose range, from standard econometrics, is constrained to be less than one in absolute value. Employing notation, a parametric version of such a *random effects model*, treated initially as a simple AR(1) (*persistence*) process, is given by:

$$y_{i,t} = \gamma_i + \rho y_{i,t-1} + \epsilon_{i,t}, \qquad i = 1, ..., n \qquad t = 2, ..., T. \tag{6}$$

where γ_i is a fund specific component.

For the moment we assume that both the errors and the random effects are normal and mutually independent:

$$\epsilon_{i,t} \sim N(0, \tau^{-1}), \tag{7}$$

$$\gamma_i \sim N(\psi, \Omega) \tag{8}$$

τ, ψ and Ω are hyperparameters.

Under Set A – the intercepts obtained from raw returns – γ_i, can be identified as a performance measure, whose value may be dependent on the fund specific characteristics such as loads, expenses, value of the fund, etc. Under Set B – the intercepts obtained via the two step regression – γ_i represents the manager specific value, which can be understood as the stock picking skill of a manager. This value has *already* been adjusted for the fund specific characteristics.

This suggests that if the distribution of γ is similar for the data under Sets A and B then the stock picking ability is the main factor explaining the performance of the fund. If these distributions are substantially different then the performance of the fund is a composition of various factors, including ability of the managers. In this respect, our approach sheds new light on the problem of the decomposition of the performance measure.[7]

Managers of mutual funds are constantly adjusting their decisions based on past *evidence*. But, as it stands, Equation (6) suffers from a drawback. The dependent variable starting with the second time period is conditional on y_{i1}. This means, the random effect γ_i does *not* depend on y_{i1}. This is clearly a restrictive assumption. For, given γ_i, if y_{i1} is drawn from its stationary distribution, there would be *strong* correlation between γ_i and y_{i1}. To bypass this difficulty, the Bayesian approach allows one to build in the *correlation* by conditioning γ_i on y_{i1}. That is, the random effects parameter is given an initial probability distribution that respects this correlation:

$$\gamma_i \mid y_{i1} \sim N(\psi, \Omega). \tag{9}$$

Equation (6) in conjunction with the above prior for γ_i is used in the analysis. We refer to this modeling set-up as the Correlated Random Effects Model (CREM); see description of (c) in the Introduction.

In order to be relatively non-informative; i.e., allowing little or no influence of prior values on the analysis, we defined the following prior distributions.

Improper flat priors for ρ and ψ were assumed and independent scaled chi-square priors for τ and Ω^{-1}:

$$\tau \sim \frac{\chi^2(1)}{.01} \tag{10}$$

$$\Omega^{-1} \sim \frac{\chi^2(1)}{.01} \tag{11}$$

See Box and Tiao (1973) for details.

Since the posterior distribution of the parameters for the hierarchical model would be difficult to derive analytically, we use a Markov chain Monte Carlo (MCMC) approach, initially proposed by Metropolis et al.(1953) and further extended by Hastings

(1970). Deatils are provided in Appendix A. Also, the details of the MCMC algorithm for obtaining the posterior distributions of the parameters in equation (6) are given in Appendix A.

B. Error Components Model

Just as there are competing performance measures and pricing models, there are competing statistical approaches to modeling uncertainty. And just as no one performance approach or pricing model is "right", so also no statistical model is "right". They all have to be taken with a large pinch of salt. The size of the pinch will depend on the investor, the risk, and the time and effort investors are willing to commit. At best, one can put forth a *process* based on *evidence* that will present the investor with a set of credible alternatives, from which he or she subjectively chooses one that carries the most appeal. One component in this decision process is the statistical model. In this section, we propose a second parametric model that is flexible and useful from an investor's perspective. Why? The CREM is adequate in many cases, but from the specification in (6) it is clear that predictive distributions obtained from this model can be quite sensitive to the behavior of the returns (or *alpha*) in the last period; i.e., there is little or no "smoothing" of the predictions for time periods following the last time period. To temper such predictions, one could consider the model described below.

$$y_{i,t} = \gamma_i + v_{i,t} + \epsilon_{i,t}, \qquad i = 1, ..., n, \quad t = 1, ..., T; \tag{12}$$

$$v_{i,t} = \rho v_{i,t-1} + w_{i,t}, \qquad i = 1, ..., n, \quad t = 1, ..., T; \tag{13}$$

where:

$$v_{i,1} \sim N(0,\ \sigma_v^2), \tag{14}$$

$$\epsilon_{i,t} \sim N(0,\ \sigma^2), \tag{15}$$

$$w_{i,t} \sim N(0,\ \sigma_w^2), \tag{16}$$

$$\gamma_i \sim N(0,\ \sigma_\gamma^2). \tag{17}$$

This Error Components Model (ECM) is equivalent to an $ARMA(1,1)$ up to second moments. A problem with this formulation is that the fund components γ_i become unidentified when $\rho = 1$. The posterior distribution is still well-defined if a proper prior distribution is used for all the parameters, but for diffuse priors the posterior can be ill-behaved. Dropping then the individual effect we have:

$$y_{i,t} = v_{i,t} + \epsilon_{i,t}, \tag{18}$$

$$v_{i,t} = \rho v_{i,t-1} + w_{i,t}, \tag{19}$$

$$v_{i,1} \sim N(0,\ \sigma_v^2) \tag{20}$$

$$w_{i,t} \sim N(0,\ \sigma_w^2), \tag{21}$$

$$\epsilon_{i,t} \sim N(0,\ \sigma^2). \tag{22}$$

Heterogeneity; i.e., fund-to-fund differences, is still retained in the model, because $v_{i,1}$ will vary across funds. Note, however, if ρ is much smaller than 1 in absolute value, this heterogeneity will shrink over time. That is, if persistence of performance is negligible, then, over time, one can expect to see little differences across the performance of funds. This inference, of course, would vary depending on investment objective.

As under CREM, for the ECM, to be non-informative, we use independent $\frac{\chi^2(1)}{.01}$ priors for σ^{-2}, σ_v^{-2}, and σ_w^{-2} and a uniform prior for ρ. The MCMC details are given in Appendix B.

The *process* of reaching an informed decision under CREM and ECM is the same. The *evidence* however could be different, leading to potentially different insights into the data. The illustrative analyses will exemplify this feature somewhat dramatically.

C. Predictive Distributions

Prediction is a critical factor for an investor to devise any investment strategy. Market volatilities require one to capture the uncertainty in future predictions. The Bayesian models developed earlier can be easily used to obtain predictive distributions for hypothetically observable periods $t = T+1, ..., T+H$. In practice, we want to obtain predictive distributions for the returns or performance measure (*alpha*), depending on the investor's choice.

Using θ to denote the vector of parameters, $z \equiv \{y_{j,t} : \quad j = 1, ..., n \quad t = 1, ..., T\}$, the past data, the predictive distribution, in canonical notation, is given by

$$p(y_{i,T+1}, ..., y_{i,T+H} \mid z) = \int p(y_{i,T+1}, ..., y_{i,T+H} \mid z, \theta) dP(\theta \mid z) \qquad (23)$$

In (23), note that the *parameter* is being integrated out. The consequences of this is important and will be described later under *REMARK 9*.

In the illustrative analyses, we will show that these predictive distributions can vary quite dramatically depending on investment objective, and the particular EBDP used in the analysis. Knowledge of future performance is thus critical if an investor wants to be more consistent in picking "winners". As is to be expected, the farther one attempts to forecast the future, the greater the uncertainty and hence higher the risk. While this paper does not address the issue of optimal portfolios, it is clear that questions of optimality will hinge on obtaining reasonably "good" predictions.

From a computational perspective, obtaining predictive distributions is straightforward since *no* additional simulations are needed. One merely uses the samples from the posterior distribution of θ, obtained via MCMC, to obtain a close approximation to the above integral.[8] Details pertaining to the specific form of the integral under CREM and ECM are given in Appendix C and D, respectively.

IV. Empirical Results

In order to illustrate the impact of different factors on the DMP we analyze the sample of the equity mutual funds constructed under Section II. For seven broad investment groups we successively measure the impact of the performance measure, the modeling tool, and the pricing formula adjusting for the risk selected by the investor. We compare all surviving funds for the period 1989 – 1998.[9] Additionally, we conduct predictive analysis to show how investor strategies are influenced from a "looking-ahead" perspective.

To facilitate understanding, we have classified the results in terms of CREM and ECM. Within each we have provided discussion on Set A and Set B response measures (defined earlier), for each of the three pricing models. Predictive inference is described later.

A. CREM Analysis

Unless otherwise noted, the description that follows is across all investment objectives.

Set A (Raw Returns) Results

(A1) The persistence parameter, ρ, is somewhat larger under the CAPM model compared to the 3 - and 4-factor models. A reason for this may be because more of the variation in returns is accounted for by the inclusion of additional independent variables in the model, thereby diminishing the lag effect in the response variable. The estimates of ρ across the different groups of funds do not differ by more than 0.03, and the errors of those estimates, measured by their standard deviations, are relatively small. It means for the investor following a strategy based on persistence, qualitatively, the inference is invariant to the pricing model.

(A2) The *distributions* of the standard deviations, $\tau^{-1/2}$, of the error component, ϵ, are, in general, not different across pricing models. This implies that all three pricing models describe approximately the same amount of variation in raw returns. Likewise, the distributions of the standard deviation of the random effect's term, across all investment objectives and pricing models, are similar.

(A3) The random effect's mean ψ is negative under each of the pricing models and for each of the funds. This provides evidence against abnormal performance of equity funds. Perhaps the most telling feature here is the standard deviation of the random effect for the Other Aggressive Growth investment objective. Regardless of the pricing model used, this value is substantially different in comparison to the same statistic under all the other investment objectives. The reason for this is the much higher volatility one can expect in this class of equity funds. The EBDP nicely captures this feature.

Set B *(alpha)* Results

(B1) The distribution of ρ under CAPM is similar to that obtained under Set A. But for the 3-factor and 4-factor models, the distribution of ρ is centered around positive values. In particular, there is a substantial difference in the mean of the persistence parameter under the three pricing models for the Income objective (0.1697), when compared to the other objectives. This is quite interesting because it sheds new light on the persistence phenomenon and its consequences for investor strategies. If investors believed in *alpha* as a more appropriate measure of fund performance, then they would choose to invest in the very funds that they might have discarded if raw returns (as in Set A) were under consideration. In other words, it may be inadequate to define persistence in terms of a mathematical relationship alone; the choice of the response variable may be at least as important. This result is more stark when we look at the analysis under ECM.

(B2) As in Set A, the *distributions* of the standard deviations, $\tau^{-1/2}$, of the error component ϵ are in general not different across pricing models. This implies that all three pricing models describe about the same amount of variation in *alpha*. Likewise, the distributions of the standard deviation of the random effect's term, across all investment objectives, are similar.

(B3) The random effect's mean ψ is positive under each of the pricing models and for each of the funds in sharp contrast to Set A. This reiterates the strategy investors might take based on which performance measure is under consideration; see (B1) above. Also, this might resolve the puzzle as to why investors are willing to invest in the equity mutual fund sector even though their performance may not be abnormal; see (A3). Again, as under (A3), regardless of the pricing model used, the standard deviation of the mean of the random effect, ψ, is substantially different for the Other Aggressive Growth class. Regardless of the performance measure used, the investor should factor this higher volatility in their portfolio selection.

The detailed results for all groups of funds have been presented in Table II.

Insert Table II about here

B. ECM Analysis

Unless otherwise noted, the description that follows is across all investement objectives.

<u>Set A (Raw Returns) Results</u>

(A1) The striking differences are to be noted in the distributions of the persistence parameter, ρ, which is dramatically higher than what resulted under CREM. Also, here we see the impact of the pricing model on the investment objective more clearly. Consider Small Company Growth. CREM values for ρ appear in paranthesis. For CAPM: 0.4137 (0.1124); for 3-factor model: -.0432 (-.0056); for 4-factor model -.0454 (-.0061). Now consider Growth. For CAPM .3776 (.0728); for 3-factor model .2085 (.0401); for 4-factor model .2024 (.0387). Consequences to investors will be examined after we discuss the impact of ECM using Set B.

(A2) The *distributions* of the standard deviations, σ, of the error component, ϵ, are in general not different across pricing models. This implies that all three pricing models describe about the same amount of variation in raw returns.

Set B (*alpha*) Results

(B1) Consider ρ for the investment objective, Small Company Growth. Numbers in parantheses are from CREM. For CAPM: .3431 (.1174); for 3-factor model: .0878 (.0246); for 4-factor model: -.1113 (-.0340). Now consider Growth. For CAPM: .4265 (.0761); for 3-factor model: .4414 (.0931); for 4-factor model: .4258 (.0801). Contrast these results with (A1) above. It is clear that any definition of persistence must consider the response variable. Thus, under Growth, regardless of the pricing model (except, perhaps CAPM), conclusions about the persistence of performance of funds in this sector are dependent on the response variable used. Coupling this feature with the type of model one uses in the EBDP, leads to conclusions about persistence that are diametrically opposite. This finding is of utmost importance to investor stratgeies. It shows that one must be extremely careful in understanding the various components of the EBDP while constructing portfolios. It also shows that the task of quantifying managerial skill and its subsequent impact on fund performance is a very difficult and imprecise task as it stands.

(B2) The *distributions* of the standard deviations, σ, of the error component, ϵ, are in general not different across pricing models. This implies that all three pricing models describe about the same amount of variation in *alpha*.

The details of the ECM analysis have been presented in Table III.

Insert Table III about here

Based on the results described above, we discuss some consequences of, and note some features to, the EBDP for an investor.

REMARK 1 For most investment objectives, we can observe the dramatic increase in the absolute value of persistence for Set and Set B data using ECM. The magnitude of change is very similar for all pricing models with the parameter of ρ increasing about 3 to 4 times. These results suggest a very significant dependence of the decision strategies on the econometric techniques and/or the data investors use in their analysis. Thus, based on CREM and Set A data, an investor might conclude little or no presence of persistence, when in fact such a dependence might exist as implied by ECM and Set B data. The selection of mathematical tools in this paper is meant to be illustrative. In practice, the range of econometric models used by investors is vast. The purpose of this comparison is to illustrate the nature of EBDP on investor strategies. Note that all the analyses were carried out using *non-informative* priors. In practice, it is clear from the mathematical development in the Appendix, that the strength of an EBDP is its capacity to incoporate prior beliefs.

REMARK 2 In the comparison of pricing models under CREM we can observe one dominant feature in the behavior of the persistence measure, ρ. For the performance measure based on Set A, the estimates of persistence differ only between the CAPM model and two other multiple factor models. This result is important for at least two reasons. First, it suggests that in this case the 4-factor model of Carhart(1997) does not provide information which would be marginally significant for the EBDP as compared to the 3-factor model. Since the resulting decision will be almost the same it is a matter of preference as to which of these two models investors should consider in their EBDP. However, as we mentioned before, if we relate the "factor models" to the standard CAPM, in most cases, the results will be significantly different. In particular, the 3-factor model gives substantially lower estimates of ρ for other aggressive growth, growth, maximum capital gains, and the small company growth funds. Especially, for the last group, the differences are extremely significant and suggest completely different decisions depending on the pricing model an investor assumes.

One of the reasons for a lower trend in persistence both for CREM and ECM may be a better pricing ability of the multiple factor models. It means their construction helps to explain more of the systematic risk as compared to the simple CAPM model. However, this study does not aim at justifying the reasons for using any of the pricing models used in this analysis.[10]

REMARK 3 In this study, we separate our samples according to seven broad investment objectives as reported in Table I. From a statistical perspective, unequal samples across the groups is not an issues since the sample sizes are substantially large; besides pooling helps in "borrowing strength" from data across funds within the Bayesian framework.

REMARK 4 Based on the measure of persistence, we can observe meaningful differences among funds under ECM. Under CREM, for all pricing models using Set A and Set B data, the persistence parameter, ρ, varies in absolute value between 0.0056 and 0.1873. Correspondingly, under ECM this range is 0.0432 to 0.5167. In particular, the higher values of persistence for the growth, income, maximum capital gains, and the sector funds suggest that investment in those four groups are dependent on past performance.

REMARK 5 The inference based on the measure of *alpha* shows more heterogeneity among the funds. Recall that ψ is the mean of the random effects γ_i. For the CAPM model, ψ attains the highest mean value of 0.0518 for the maximum capital gains funds, which suggests a significantly positive average performance of that group. At the same time, the lowest value -0.0378 for the income funds suggests a considerable underperformance of that particular group; consequently, the decision rule is self-evident for an investor. Note, however, if one were to consider the same decision question using Set A data, the investor would be faced with the inference that there are differences in the perfromance of the same two funds!

C. Predictive Analysis

Under ECM and CREM, for each of the pricing models and performance measures considered in this paper, one can obtain the posterior predictive distributions of the residuals. For illustration,we discuss the analysis under CAPM risk adjustment for Set A and Set B data under both ECM and CREM. A similar analysis can be obtained for the 3-factor and 4-factor pricing models. Previous analysis in this paper focused on fund histories. That analysis revealed many differences. Now using a predictive tool, the EBDP shows some stark differences in the future performance of each of the funds.

(P1) Consider Figures 1 and 3. At a glance, it is clear that there are substantial differences in how the funds will behave one period ahead, depending on the performance measure under consideration; i.e., Set A vs. Set B. This is particularly the case for Other Aggressive Growth Funds and Small Company Growth Funds. Note also the sharp differences in the *variances* of the distributions of the funds future performance. The volatility in the future performance of Growth and Incime and Growth funds are much smaller than the rest. It is an appealing feature of the EBDP that these differences can be nicely quantified, leaving the investor a better way of making decisions. To aid the investor, snapshot summaries of the distributions via their means and standard deviations are provided immediately below the graphs.

(P2) Consider Figures 1 and 2. This comparison illustrates the differences in the one-period ahead predictions depending on the modeling strategy employed in the EBDP; i.e, CREM vs. ECM. Again, note the remarkable differences in the behavior of all except Growth and Income, Sector and Growth funds. What is surprising is that when the same comparison was executed using Set B data, namely, the *alpha* measure, the Growth and Income and Growth are strikingly different as well. Once again, first and second moment summaries are provided in tables below the graphs.

(P3) Comparing, somewhat randomly, the tables under Figures 1 through 4, it is clear that there are reversals in sign for the means of some of the groups of funds.

(P4) In our study, we provide predictive inference for up to 12 periods ahead, which is equivalent to one year. We report predictive .1, .25, .5, .75, and .9 quantiles to illustrate the predictive inference in Figures 5 through 8. For CREM, we can observe that the predictions of residuals originating from Set A differ mainly only up to two periods ahead. Investors may may increase their valuation as in the case of Growth & Income and Income funds. On the other hand, they may behave in the opposite manner for the Growth, and Maximum Capital Gains funds. In the long run, however, their behavior will not be significantly different. This, obviously, is not surprising. But note that the strength of the Bayesian method is if, as is typically the case, investors have access to prior knowledge abput the future, such information can be included straightforwardly into the EBDP. For the ECM, the quantile analysis is similar to the CREM.

What does the above mean for an investor?

REMARK 6 Selection of perfomance measure is critical.

REMARK 7 Having selected a performance measure, one must carefully assess the consequences of the mathematical model employed in the analysis. Here, for illustration, we have considered ECM and CREM. The rich modeling possibilities open to the investor via a careful input of prior knowledege can go a long way in feeling confident to some degree about eventual portfolio selection.

REMARK 8 At the present time, quantifying mangerial skill in selecting funds is a very difficult challenge. Diametrically opposite conclusions can be drawn in this context as exemplified by the EBDP analysis.

REMARK 9 While fund history data analysis is the first step in assessing fund performance, a predictive analysis is needed. Why? If the goal is to model investor decisions, it is unclear whether they have the *same* information set as the financial analyst, or access to the same data and/or subjective prior knowledge. In reality, the EBDP analysis points out that no investor can know the "true" stochastic process underlying performance, be it raw returns or *alpha*. Which information set should the investor rely on? Clearly the EBDP points to many credible alternatives. One way of tackling this question is via the predictive distribution. Recall from Section III C, that the predictive distribution is *not* conditioned on any unknown parameters since these are integrated out, but they are conditioned on past fund history. Said differently, the uncertainty in the parameters are removed when the investor assesses a strategy using the predictive distribution. Having said that, the EBDP analysis in this paper echoes a sentiment due to Jimmy Savage: "no model is true, some models are useful." And that's the best an investor can hope for.

Insert Figures 1-4 about here

Insert Figures 5-8 about here

D. Classical vs. Bayesian Inference

While we favor the Bayesian approach for many reasons as argued in Smith (1984), here, for completeness, we contrast the results with those obtained via standard procedures (a.k.a classical statistics).

<u>CREM Comparisons</u>

We executed a classical analysis, where appropriate, for illustration. First, we estimate the parameters ρ and $\tau^{-1/2}$ for the CREM.[11] We obtain these estimates using an AR(1) model for the residuals obtained in the classical analysis. We also use the residuals obtained in the two-stage regression that includes the cross sectional regression. Since the series for Set A and Set B is covariance stationary and ergodic, we can apply standard time-series procedures. Some highlights are given below. Table IV contains the details of the classical analysis.

(A1) The estimates of ρ and τ are very similar to the corrsponding Bayesian estimates. This is because we used non-informative priors for our analysis. But the standard errors based on the standard analysis are typically larger. This is also to be expected because this analysis gives us asymptotic estimates, a somewhat unpleasant feature of the classical approach.

(A2) Estimates of persistence are also quite comparable, although some differences could be meaningful to an investor. As an example, for Sector funds, with the Fama-French pricing model, and using Set B data (*alpha*), the Bayesian estimate of persistence equals 0.1682, while the classical approach yields 0.1158.

ECM Comparisons

For the ECM, classical estimation is more complicated. Basically, the equivalent of the Bayesian representation described by equations (18) and (19) is the random effects model with an additional autocorrelated term. In general, this model can be presented as follows:

$$r_{i,t} = \alpha + \beta' \, x_{i,t} + v_{i,t} + \epsilon_{i,t} \tag{24}$$

where

$$v_{i,t} = \rho v_{i,t-1} + w_{i,t} \tag{25}$$

$r_{i,t}$ is the raw return/performance measure of the mutual fund i in time t, and x is the set of regressors from the regression on the respective factors coming either from CAPM, 3-factor or 4-factor models.

We obtain the estimate of persistence by calculating the residuals $v_{i,t}$ from the *Least Squares Dummy Variable* (LSDV) model and then performing an AR(1) regression (without an intercept) on those residuals. This estimate will be consistent in nT as well as in T. For details of this approach, see Baltagi (1995) and Green (2000). The autoregressive process will also provide us with the estimate of innovation variance σ^2, equivalent to the value of τ from the Bayesian CREM. Again, the additional two hyperparamters σ_w and σ_v do not enter the frequentist analysis. Some highlights:

(B1) For the performance measured both by raw returns and *alpha* the estimates of persistence are substantially lower when compared to the Bayesian estmates. One reason may be because the Bayesian ECM does not have an exact counterpart in the classical methodology. The random effects model we estimate here is only a close approximation to the Bayesian approach.

(B2) The variance of the innovation term σ is very similar to the results obtained from the Bayesian analysis. It means that both methods provide similar explanation of the factors driving the performance of the mutual fund. What is interesting, regardless of the Bayesian or Classical approach, is that the standard errors for the persistence parameters under ECM are smaller than those under CREM. *Ceteris paribus*, this is generally a desirable feature.

Insert Table IV about here

V. Concluding Remarks

The selection ability of investors is a complex process. In this paper, we have developed a process to enable investors to choose from among various mutual funds. The *evidence* in EBDP is comprised of three interdependent components.

- The response variable used, namely, raw returns or a fund's performance (*alpha*).
- The pricing model used: CAPM, Fama-French 3-factor, and Carhart's 4-factor models.

- The statistical model used: CREM and ECM, both designed to model performance *persistence*.

Previous research in this area has focused on a subset of the above. In this paper, we consider all the components simultaneously. This consideration is the *process* in EBDP. The illustrative analyis using data from the equity mutual funds sector are quite surprising and informative. The upshot is that selection ability of investors will depend heavily on the interactions between various combinations of the components that define the EBDP.

While the Bayesian approach developed in this paper models parameter uncertainty, a key feature that could potentially influence the EBDP is volatility of returns; i.e., uncertainty in the variance of the returns. Modeling volatilities simultaneously with expected returns could be useful in arbitrage pricing. This could be another feature to add on to the EBDP. Finally, since the choice of pricing model could have a significant impact in the EBDP, it would be interesting to see if one can build a model selection feature into the EBDP framework. These will be reported elsewhere.

AppendixA. Parametric Correlated Random Effects

Gibbs Sampling (Smith and Roberts, 1993) is a special type of MCMC; it is by far the most widely used algorithm in Bayesian analysis at the present time. Gibbs sampling is a Markovian updating scheme that proceeds as follows. Given an arbitrary starting set of values $V_i^{(0)}, ..., V_k^{(0)}$ we draw $V_1^{(1)}$ from $[V_1 \mid V_2^{(0)}, ..., V_k^{(0)}]$, then $V_2^{(1)}$ from $[V_2 \mid V_1^{(1)}, ..., V_k^{(0)}]$, and so on up to $V_k^{(1)}$ from $[V_k \mid V_1^{(1)}, ..., V_{k-1}^{(1)}]$ to complete one iteration of the scheme. After t such iterations we arrive at $(V_1^{(t)}, ..., V_k^{(t)})$. Geman and Geman (1984) showed under mild conditions that $V_s^{(t)} \to V_s \sim [V_s]$ as $t \to \infty$. Thus, for t large enough we can regard $V_s^{(t)}$ as a simulated observation from $[V_s]$.

Thus in the context of the models in this paper, the above implies we need the full conditional distributions, up to proportionality, of all the unknown random variables in the model. Successively drawing random variates from these conditional distributions, and iterating the process a large number of times, will result in draws from the posterior marginal distributions of the parameters of interest; see Smith and Roberts (1993) for examples.

For the models described in the text, deriving these conditional distributions is analytically involved. However, these are fairly standard results based on the theory of the normal distributions. Hence these derivations are omitted, and the resulting conditional distributions are given below.

The Gibbs sampler successively samples for blocks of parameters according to the following set of distributions:

Blocks:(ρ, τ), (γ), (ψ, Ω^{-1})

(ρ, τ): Define

$$\hat{\rho} = \frac{\sum_{i=1}^{n} \sum_{t=2}^{T} y_{i,t-1} \, (y_{i,t} - \gamma_i)}{\sum_{i=1}^{n} \sum_{t=2}^{T} y_{i,t-1}^2} \tag{A1}$$

Then draw

$$\tau \sim \frac{\chi^2(n\ (T-1))}{[.01 + \sum_{i=1}^{n}\ \sum_{t=2}^{T}\ (y_{i,t} - \gamma_i - \hat{\rho}\ y_{i,t-1})^2]}, \tag{A2}$$

$$\rho \sim N(\hat{\rho}, (\tau \sum_{i=1}^{n} \sum_{t=2}^{T} y_{i,t-1}^2)^{-1}). \tag{A3}$$

(γ_i): Define

$$H = (\Omega^{-1} + (T-1)\ \tau) \tag{A4}$$

and

$$\gamma_i^* = H^{-1}\ [\Omega^{-1}\psi\ y_{i,1} + \tau \sum_{t=2}^{T} (y_{i,t} - \rho\ y_{i,t-1})]. \tag{A5}$$

Then

$$\gamma_i \sim N(\gamma_i^*, H^{-1}). \tag{A6}$$

(ψ, Ω^{-1}): Define

$$\hat{\psi} = \frac{\sum_{i=1}^{n} \gamma_i\ y_{i,1}}{\sum_{i=1}^{n}\ y_{i,1}^2}. \tag{A7}$$

Then

$$\Omega^{-1} \sim \frac{\chi^2(n)}{[.01 + \sum_{i=1}^{n}\ (\gamma_i - \hat{\psi}\ y_{i,1})^2]}, \tag{A8}$$

$$\psi \sim N(\hat{\psi}, \Omega\ (\sum_{i=1}^{n}\ y_{i,1}^2)^{-1}). \tag{A9}$$

AppendixB. Parametric Error Components Model

Define $v_i \equiv\ (v_{i,1}, ..., v_{i,T})$, and $v \equiv\ (v_1, ..., v_n)$.

Blocks: (σ_v^{-2}), (ρ, σ_w^{-2}), (σ^{-2}), $(v_1, ..., v_n)$.

(σ_v):

$$\sigma_v^{-2} \sim \frac{\chi^2(1+n)}{[.01 + \sum_{i=1}^{n}\ v_{i,1}^2]} \tag{B1}$$

(ρ, σ_w): Define

$$\hat{\rho} = \frac{\sum_{i=1}^{n}\ \sum_{t=2}^{T}\ v_{i,t}\ v_{i,t-1}}{\sum_{i=1}^{n}\ \sum_{t=2}^{T}\ v_{i,t-1}^2} \tag{B2}$$

Then

$$\sigma_w^{-2} \sim \frac{\chi^2\ (n\ (T-1))}{[.01 + \sum_{i=1}^{n}\ \sum_{t=2}^{T}\ (v_{i,t} - \hat{\rho}\ v_{i,t-1})^2]} \tag{B3}$$

$$\rho \sim N(\hat{\rho}, \sigma_w^2 (\sum_{i=1}^{n} \sum_{t=2}^{T} v_{i,t-1}^2)^{-1}) \tag{B4}$$

(σ):

$$\sigma^{-2} \sim \frac{\chi^2\ (nT+1)}{\sum_{i=1}^{n}\ \sum t = 1^T\ (y_{i,t} - v_{i,t})^2} \tag{B5}$$

(v_i): Define $y_i \equiv (y_{i,1}, ..., y_{i,T})'$. By examining the kernel of the posterior distribution for terms involving v_i and simplifying the resulting expression to get the kernel of a multivariate normal density:

$$v_i \sim N(v_i^*, H^{-1}), \tag{B6}$$

where

$$H = \sigma^{-2}\ I_T + \Lambda^{-1} \tag{B7}$$

$$\Lambda = A\ diag\{\sigma_v^2, \sigma_w^2, ..., \sigma_w^2\}\ A' \tag{B8}$$

$$A = \begin{bmatrix} 1 & & & & 0 \\ \rho & 1 & & & \\ \rho^2 & \rho & 1 & & \\ \vdots & & & \ddots & \\ & & & & 1 \end{bmatrix} \tag{B9}$$

and

$$v_i^* = H^{-1}\ \sigma^{-2}\ y_i. \tag{B10}$$

AppendixC. Predictive Distributions in the Autoregressive Model

We simulate the predictive distribution of ($y_{i,T+1}, ..., y_{i,T+H}$). By recursive substitution,

$$y_{i,T+h} = (1 + \rho + ... + \rho^{h-1})\ \gamma_i + \rho^h\ y_{i,T} + \epsilon_{i,T+h} + \rho\ \epsilon_{i,T+h-1} + ... + \rho^{h-1}\ \epsilon_{i,T+1}. \qquad \text{(C1)}$$

So in the models with $\epsilon_{i,t} \sim N(0, \tau^{-1})$ we can write

$$\begin{pmatrix} y_{i,T+1} \\ \vdots \\ y_{i,T+H} \end{pmatrix} \sim N(\mu_i, \Sigma_i), \qquad \text{(C2)}$$

where

$$\mu_i = \begin{pmatrix} 1 \\ \vdots \\ \sum_{h=0}^{H-1}\ \rho^h \end{pmatrix} \gamma_i + \begin{pmatrix} \rho \\ \vdots \\ \rho^H \end{pmatrix} y_{i,T}, \qquad \text{(C3)}$$

$$A = \begin{bmatrix} 1 & & & & 0 \\ \rho & 1 & & & \\ \rho^2 & \rho & 1 & & \\ \vdots & & & \ddots & \\ & & & & 1 \end{bmatrix} \qquad \text{(C4)}$$

and

$$\Sigma_i = \frac{AA'}{\tau} \qquad \text{(C5)}$$

This suggests the following method for evaluating the predcitve distributions using Monte Carlo simulation. Let $(\rho^{(j)}, \tau^{(j)}, \gamma_i^{(j)})$ denote draws for the parameters from the j^{th} iteration of the Gibbs sampling algorithm, after discarding some initial set of iterations to allow for burn-in. We form $\mu_i^{(j)}$ and $\Sigma_i^{(j)}$ according to the previous expressions, and construct the predictive distribution as:

$$F(y_{i,T+1},...,y_{i,T+H} \mid z) \approx \frac{1}{J} \sum_{j=1}^{J} \Phi(y_{i,T+1},...,y_{i,T+H} \mid \mu_i^{(j)}, \Sigma_i^{(j)}). \tag{C6}$$

where $\Phi(\cdot \mid \mu, \Sigma)$ denotes the distribution function of the multivariate normal random variable with mean variance μ and variance matrix Σ.

AppendixD. Predictive Distributions in the Error Components Model

Similar to the autoregressive model we use recursive substitution to write

$$y_{i,T+h} = \rho^h \, v_{i,T} + w_{i,T+h} + \rho \, w_{i,T+h} + ... + \rho^{h-1} \, w_{i,T+1} + \epsilon_{i,T+h}. \tag{D1}$$

So, we can write.

$$\begin{pmatrix} y_{i,T+1} \\ \vdots \\ y_{i,T+H} \end{pmatrix} \sim N(\mu_i, \Sigma_i), \tag{D2}$$

where

$$\mu_i = \begin{pmatrix} \rho \\ \vdots \\ \rho^H \end{pmatrix} v_{i,T}, \tag{D3}$$

and

$$\Sigma_i = \sigma_w^2 \, AA' + \sigma^2 \, I, \tag{D4}$$

and A was defined for the autoregressive random effects model. To construct the Monte Carlo approximation to the predictive distribution, let $(\rho^{(j)}, v_{i,T}^{(j)}, \sigma_w^{2\ (j)}, \sigma^{2\ (j)})$ denote the j^{th} draw from the Gibbs sampling algorithm, after discarding some initial set of iterations to allow for burn-in. For each draw, we form $\mu_i^{(j)}$, $\Sigma_i^{(j)}$, $A^{(j)}$, and construct the predictive distribution as:

$$F(y_{i,T+1}, ..., y_{i,T+H} \mid z) \approx \frac{1}{J} \sum_{j=1}^{J} \Phi(y_{i,T+1}, ..., y_{i,T+H} \mid \mu_i^{(j)}, \Sigma_i^{(j)}). \tag{D5}$$

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Notes

[1]CRSP, Center for Research in Security Prices, Graduate School of Business, The University of Chicago 1999, crsp.com. Used with permission. All rights reserved.

[2]We are grateful to Klaas Baks, Andrew Metrick, and Ľuboš Pástor for helpful comments regarding the data.

[3]We use the factors provided by Kenneth French and publicly accessible from his personal webpage.

[4]Additionally, the sample size is also diminished, because for some funds the CRSP database does not report values for the characteristics used in the second step.

[5]Because of computational reasons we exclude from the sample all funds which had a reported TNA value of zero.

[6]Compare Pástor and Stambaugh (2000a)

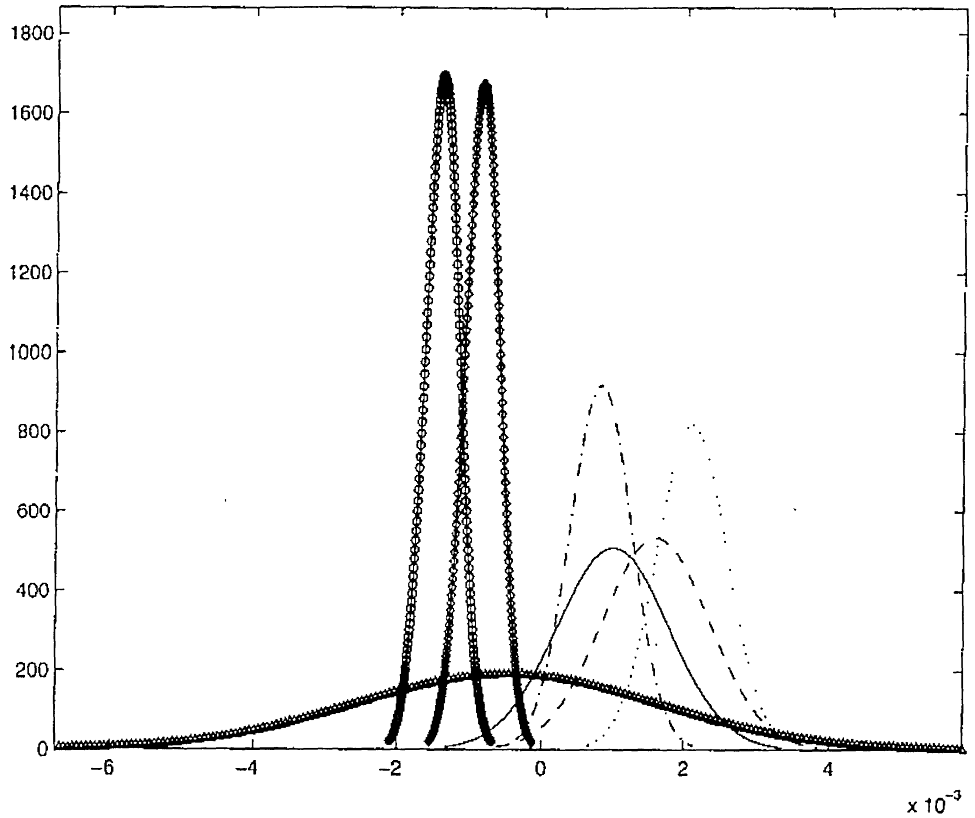
[7]For the comprehensive reference of the performance decomposition, compare for example Wermers (2000).

[8]See: Geman & Geman (1984); Smith and Roberts (1993)

[9]We also conducted analyses for the horizon of 15 and 20 years for the samples following Table I (Panel A). For the group of funds, which existed at this time we obtained similar results; in the interest of space we have not reported them here.

[10]For a comprehensive study of pricing models see Pástor and Stambaugh (2000b)

[11]We cannot estimate the two remaining parameters ψ and Ω because they do not enter the classical model.



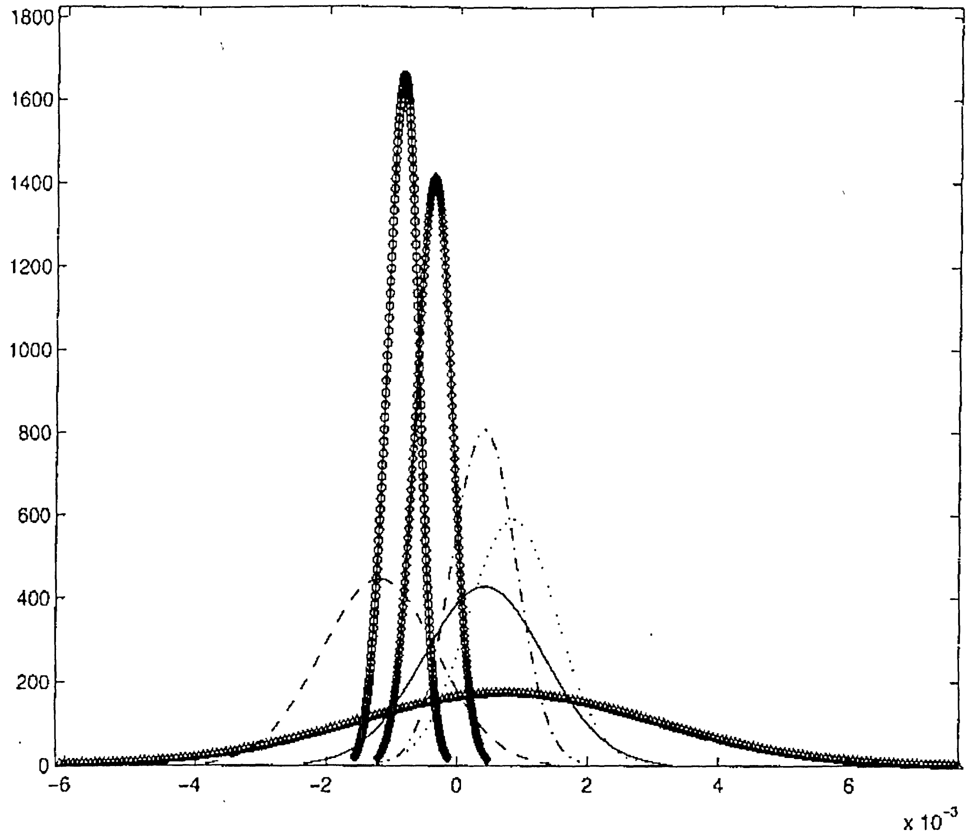
Objective of the fund	Mean value of the distribution	Standard deviation of the distribution
Other Aggressive Growth	0.0016	0.00075
Growth	0.00082	0.00043
Growth & Income	-0.00085	0.00024
Income	-0.0014	0.00023
Maximum Capital Gains	0.0021	0.00049
Small Company Growth	0.001	0.00079
Sector	-0.00046	0.0021

Figure 1. One-month predictive distributions of residuals for the correlated random effects (SetA /CAPM)

This figure presents the one-month ahead predictive distribution functions of residuals for all groups of the mutual funds. The predictions have been performed for the correlated random effects model given CAPM and raw return as a basis of performance measure.

--- represents Other Aggressive Growth funds, -.- Growth funds, ◊◊◊ Growth & Income funds, °°° Income funds, ... Maximum Capital Gains funds, - Small Company Growth funds, and ΔΔΔ Sector funds.

The table below the graph shows the mean and the standard deviation of the distribution for each of the objective group plotted.



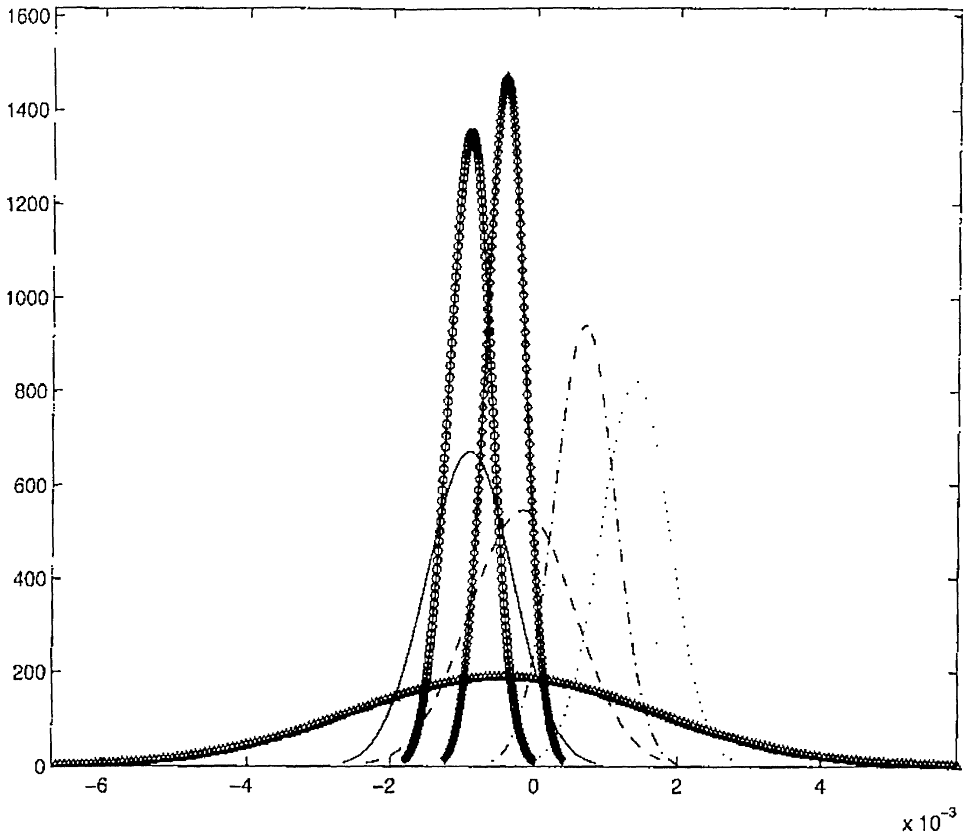
Objective of the fund	Mean value of the distribution	Standard deviation of the distribution
Other Aggressive Growth	-0.0012	0.00089
Growth	0.00041	0.00049
Growth & Income	-0.00038	0.00028
Income	-0.00086	0.00024
Maximum Capital Gains	0.00085	0.00068
Small Company Growth	0.00042	0.00093
Sector	0.00075	0.0023

Figure 2. One-month predictive distributions of residuals for the error components (SetA /CAPM)

This figure presents the one-month ahead predictive distribution functions of residuals for all groups of the mutual funds. The predictions have been performed for the error components model given CAPM and raw return as a basis of performance measure.

--- represents Other Aggressive Growth funds, -.- Growth funds, ◊◊◊ Growth & Income funds, ººº Income funds, ... Maximum Capital Gains funds, - Small Company Growth funds, and ΔΔΔ Sector funds.

The table below the graph shows the mean and the standard deviation of the distribution for each of the objective group plotted.



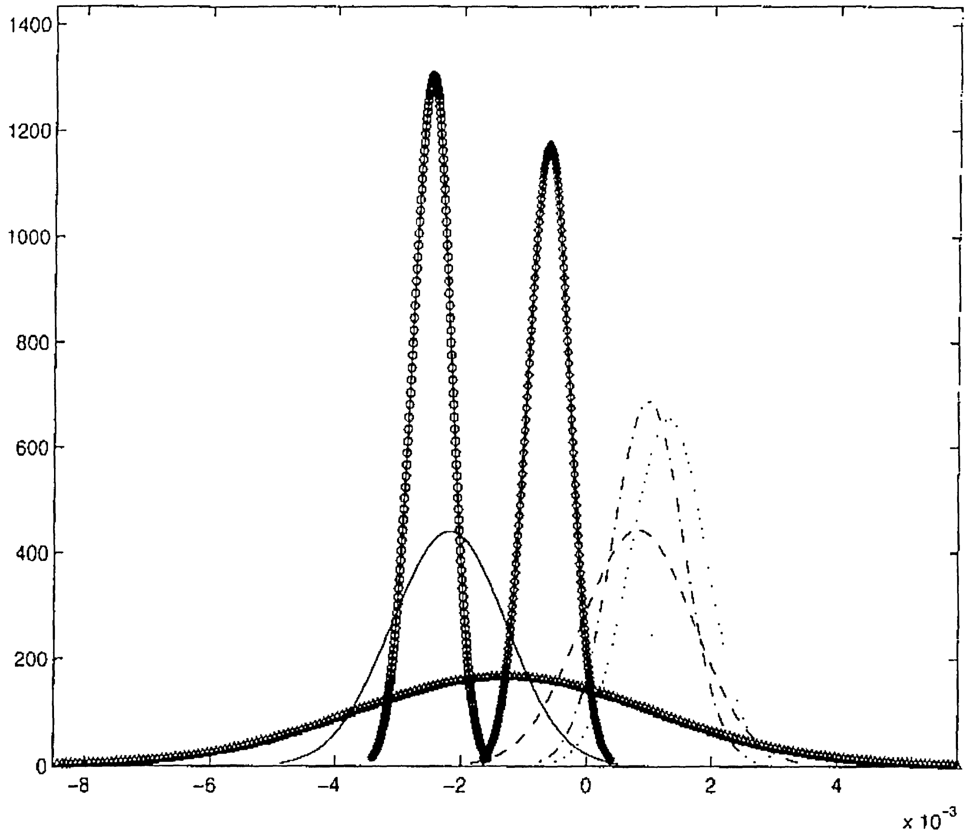
Objective of the fund	Mean value of the distribution	Standard deviation of the distribution
Other Aggressive Growth	-0.00017	0.00073
Growth	0.00069	0.00042
Growth & Income	-0.00042	0.00027
Income	-0.00091	0.00029
Maximum Capital Gains	0.0014	0.00048
Small Company Growth	-0.0009	0.00059
Sector	-0.00042	0.0021

Figure 3. One-month predictive distributions of residuals for the correlated random effects (SetB /CAPM)

This figure presents the one-month ahead predictive distribution functions of residuals for all groups of the mutual funds. The predictions have been performed for the correlated random effects model given CAPM and "alpha" as a basis of performance measure.

--- represents Other Aggressive Growth funds, -.- Growth funds, ◊◊◊ Growth & Income funds, °°° Income funds, ... Maximum Capital Gains funds, - Small Company Growth funds, and ΔΔΔ Sector funds.

The table below the graph shows the mean and the standard deviation of the distribution for each of the objective group plotted.



Objective of the fund	Mean value of the distribution	Standard deviation of the distribution
Other Aggressive Growth	0.00082	0.0009
Growth	0.00097	0.00058
Growth & Income	-0.00063	0.00034
Income	-0.0025	0.00031
Maximum Capital Gains	0.0013	0.00061
Small Company Growth	-0.0022	0.00091
Sector	-0.0013	0.0024

Figure 4. One-month predictive distributions of residuals for the error components (SetB /CAPM)

This figure presents the one-month ahead predictive distribution functions of residuals for all groups of the mutual funds. The predictions have been performed for the error components model given CAPM and "alpha" as a basis of performance measure.

--- represents Other Aggressive Growth funds, -.- Growth funds, ◊◊◊ Growth & Income funds, ººº Income funds, ... Maximum Capital Gains funds, - Small Company Growth funds, and \\\ Sector funds.

The table below the graph shows the mean and the standard deviation of the distribution for each of the objective group plotted.

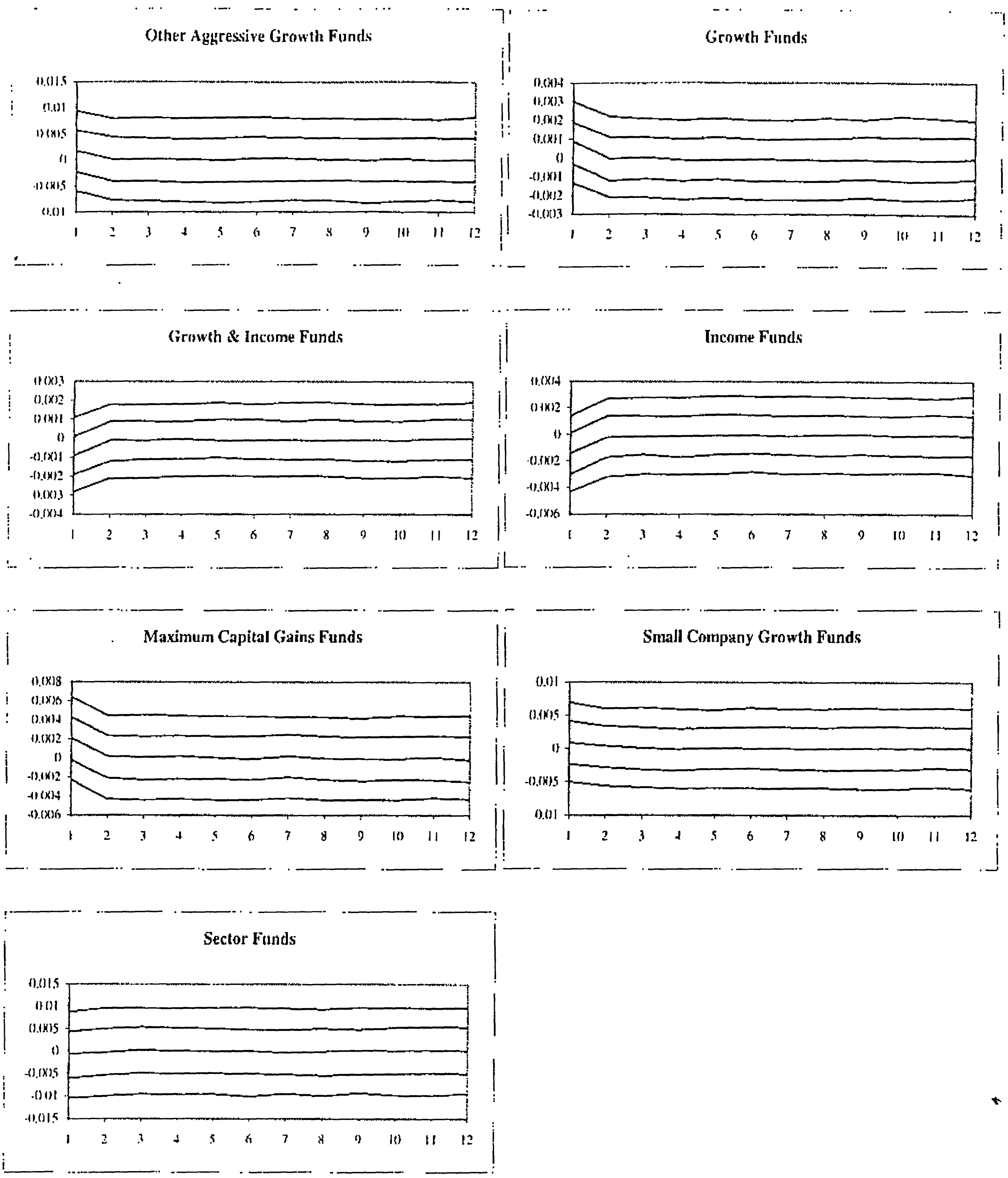


Figure 5. Predictive Quantiles - Correlated Random Effects Model

This figure presents the predictive .1, .25, .5, .75, and .9 quantiles for the correlated random effects model. The quantiles have been obtained for the CAPM model given that the performance is measured using the raw returns. The plots include all groups of the mutual funds and the predictions are calculated up to 12 months ahead.

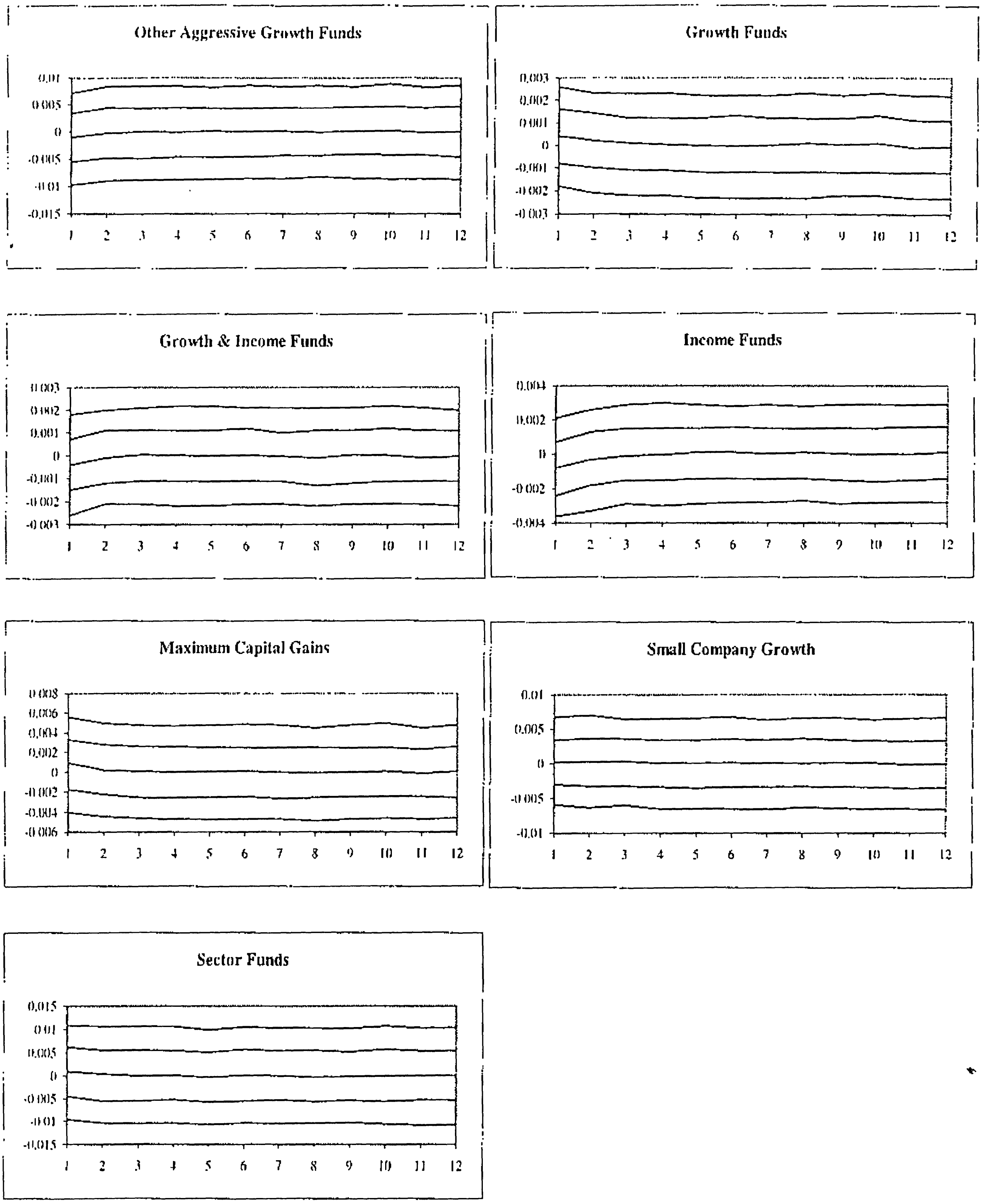


Figure 6. Predictive quantiles for the error components model.

This figure presents the predictive .1, .25, .5, .75, and .9 quantiles for the error components model. The quantiles have been obtained for the CAPM model given that the performance is measured using the raw returns. The plots include all groups of the mutual funds and the predictions are calculated up to 12 months ahead.

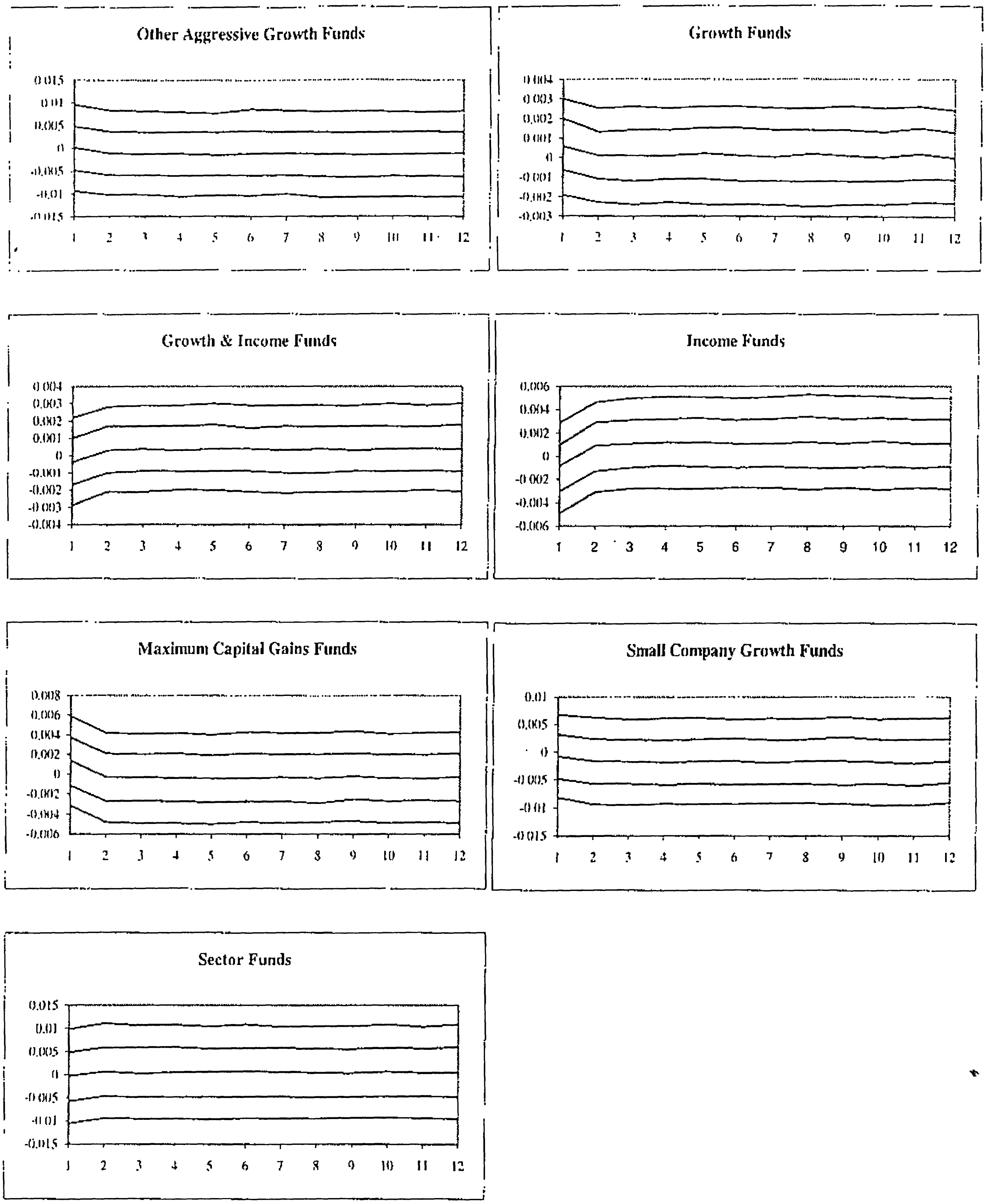


Figure 7. Predictive quantiles for the correlated random effects model.

This figure presents the predictive .1, .25, .5, .75, and .9 quantiles for the correlated random effects model. The quantiles have been obtained for the CAPM model given that the performance is measured by "alpha". The plots include all groups of the mutual funds and the predictions are calculated up to 12 months ahead.

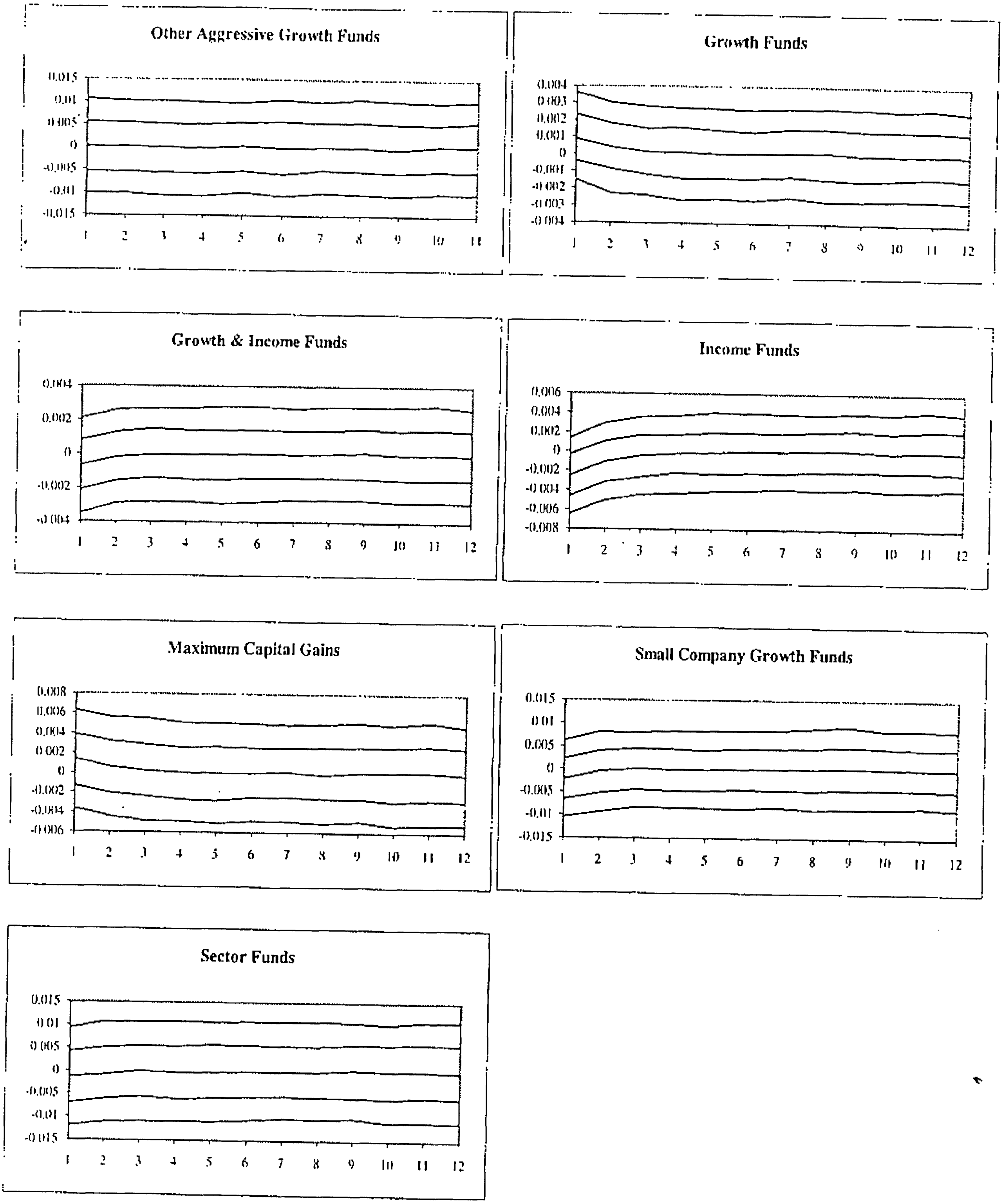


Figure 8. Predictive quantiles for the error components model.

This figure presents the predictive .1, .25, .5, .75, and .9 quantiles for the error components model. The quantiles have been obtained for the CAPM model given that the performance is measured by "alpha". The plots include all groups of the mutual funds and the predictions are calculated up to 12 months ahead.

Table I

The distribution of the samples by investment objective and horizon of the analysis

This table presents the classification of the sample of equity funds according to the investment objectives by Wiesenberger, ICDI, and Strategic Insight as reported by CRSP. We classify funds based on the horizon of the analysis: 10, 15 and 20 years back from 1998 respectively for the data derived from the pure returns /Panel A/, and 10 years for the data derived from the performance measure alpha /Panel B/. We include only the funds which survived until the end of 1998. Multiple share classes for the same fund are excluded. The sample constructed for the raw returns is further denoted as Set A whereas the sample constructed for the performance measure *alpha* as Set B.

A. The distribution of funds based on the raw returns (Set A)

Investment objective	Horizon of the analysis		
	10 years	15 years	20 years
Small-company growth	37	3	0
Other aggressive growth	20	3	0
Growth	162	86	64
Growth and income	102	44	36
Income	47	19	17
Maximum capital gains	43	37	32
Sector funds	36	6	3
ALL FUNDS	447	198	152

B. The distribution of funds based on the performance measure alpha (Set B)

Investment objective	Horizon of the analysis
	10 years
Small-company growth	17
Other aggressive growth	14
Growth	116
Growth and income	71
Income	32
Maximum capital gains	39
Sector funds	35
ALL FUNDS	320

Table II

Posterior Inference for Performance Measures Based on the Raw Returns and Alphas (Correlated Random Effects Model)

This table reports the posterior means (standard deviations) of the parameters obtained by MCMC for the correlated random effects model given that the investor makes her decision based on the raw returns and performance measure alpha. The estimates are given for each potentially considered pricing model: CAPM, Fama-French 3-factor model, and Carhart's 4-factor model. ρ is the measure of persistence, $\tau^{-1/2}$ measures the standard deviation of the error component ϵ; ψ and $\Omega^{1/2}$ are the mean and standard deviation of the individual component γ respectively. The posteriors are given for the horizon of 10 years (1989-1998) for each of seven broad groups of funds classified by their investment objective.

	Investment objective	CAPM				Fama-French 3-factor model				Carhart's 4-factor model			
		ρ	$\tau^{-1/2}$	ψ	$\Omega^{1/2}$	ρ	$\tau^{-1/2}$	ψ	$\Omega^{1/2}$	ρ	$\tau^{-1/2}$	ψ	$\Omega^{1/2}$
Raw Returns	Small Company Growth	0.1124	0.028	-0.0084	0.0167	-0.0056	0.0211	-0.0084	0.0164	-0.0061	0.0210	-0.0084	0.0167
		(0.0017)	(0.0004)	(0.013)	(0.0021)	(0.0005)	(0.0005)	(0.0099)	(0.0021)	(0.0008)	(0.0005)	(0.0099)	(0.0021)
	Other Aggressive Growth	0.0924	0.0277	-0.0120	0.0235	0.0216	0.0233	-0.0116	0.0235	0.0218	0.0232	-0.0116	0.0235
		(0.0021)	(0.0197)	(0.1412)	(0.0202)	(0.002)	(0.0198)	(0.1254)	(0.0202)	(0.0023)	(0.0198)	(0.1254)	(0.0202)
	Growth	0.0728	0.0210	-0.0086	0.008	0.0401	0.0185	-0.0085	0.008	0.0387	0.0184	-0.0084	0.008
		(0.0013)	(0.0079)	(0.0019)	(0.0023)	(0.0005)	(0.0079)	(0.0019)	(0.0081)	(0.0005)	(0.0079)	(0.0019)	(0.0081)
	Growth and Income	0.0673	0.0155	-0.0088	0.0100	0.0861	0.0136	-0.0086	0.0100	0.0872	0.0134	-0.0086	0.0100
		(0.0008)	(0.0017)	(0.0055)	(0.0019)	(0.0011)	(0.0017)	(0.0088)	(0.0019)	(0.001)	(0.0017)	(0.0088)	(0.0019)
	Income	0.1171	0.0154	-0.009	0.0148	0.1135	0.0141	-0.0082	0.0148	0.1093	0.0140	-0.0082	0.0148
		(0.0015)	(0.0043)	(0.0613)	(0.0046)	(0.0013)	(0.0043)	(0.0827)	(0.0046)	(0.0014)	(0.0043)	(0.0827)	(0.0046)
	Maximum Capital Gains	0.1157	0.0221	-0.0085	0.0155	0.0495	0.0180	-0.0083	0.0155	0.0514	0.0179	-0.0083	0.0155
		(0.0005)	(0.0042)	(0.0151)	(0.0046)	(0.0023)	(0.0011)	(0.0124)	(0.0046)	(0.0025)	(0.0043)	(0.0124)	(0.0046)
	Sector	0.0872	0.0455	-0.0083	0.0170	0.0819	0.0436	-0.0082	0.0170	0.0795	0.0434	-0.0082	0.0170
		(0.0002)	(0.0004)	(0.0131)	(0.0021)	(0.0006)	(0.0005)	(0.0131)	(0.0021)	(0.0006)	(0.0005)	(0.0131)	(0.0021)
Alphas	Small Company Growth	0.1174	0.0244	0.0263	0.0254	0.0246	0.0206	0.0298	0.0254	-0.0340	0.0209	0.0162	0.0254
		(0.0018)	(0.0011)	(0.0396)	(0.0048)	(0.0009)	(0.0011)	(0.0429)	(0.0048)	(0.0016)	(0.0011)	(0.0422)	(0.0048)
	Other Aggressive Growth	0.0668	0.0270	0.0259	0.0284	0.0126	0.0241	0.0796	0.0285	-0.0056	0.0243	0.0603	0.0285
		(0.0012)	(0.0008)	(0.2016)	(0.0059)	(0.0008)	(0.0009)	(0.2866)	(0.0059)	(0.0013)	(0.0009)	(0.2796)	(0.0059)
	Growth	0.0761	0.0206	0.0309	0.0096	0.0931	0.0194	0.0195	0.0096	0.0801	0.0196	0.0139	0.0096
		(0.0014)	(0.0020)	(0.0047)	(0.0022)	(0.0001)	(0.0020)	(0.0048)	(0.0022)	(0.0006)	(0.0020)	(0.0048)	(0.0022)
	Growth and Income	0.0587	0.0165	0.0455	0.0122	0.0885	0.0148	0.0291	0.0121	0.0902	0.0148	0.0226	0.0121
		(0.0022)	(0.0016)	(0.0134)	(0.0020)	(0.0007)	(0.0017)	(0.0099)	(0.0020)	(0.0008)	(0.0017)	(0.0098)	(0.0020)
	Income	0.1484	0.0172	-0.0378	0.0182	0.1780	0.0162	0.0166	0.0182	0.1697	0.0162	0.0059	0.0182
		(0.0006)	(0.0005)	(0.0816)	(0.0024)	(0.0004)	(0.0005)	(0.001)	(0.0024)	(0.0004)	(0.0005)	(0.0465)	(0.0024)
	Maximum Capital Gains	0.1081	0.0221	0.0518	0.0165	0.0776	0.0193	0.0526	0.0165	0.0568	0.0197	0.0441	0.0165
		(0.0013)	(0.0042)	(0.0176)	(0.0047)	(0.0032)	(0.0043)	(0.0146)	(0.0047)	(0.0036)	(0.0042)	(0.0144)	(0.0047)
	Sector	0.1098	0.0459	0.0306	0.0177	0.1682	0.0457	0.0205	0.0177	0.1476	0.0460	0.0035	0.0177
		(0.0006)	(0.0006)	(0.0107)	(0.0023)	(0.0004)	(0.0006)	(0.0127)	(0.0023)	(0.0005)	(0.0006)	(0.0124)	(0.0023)

Table III

Posterior Inference for Performance Measures Based on the Raw Returns and Alphas (Error Components Model)

This table reports the posterior means (standard deviations) of the parameters obtained by MCMC for the error components model given that the investor makes her decision based on the raw returns and performance measure alpha. The estimates are given for each potentially considered pricing model: CAPM, Fama-French 3-factor model, and Carhart's 4-factor model. ρ is the measure of persistence, σ measures the standard deviation of the error component ϵ; σ_w is the standard deviation of the parameter w; σ_v is the standard deviation of the parameter v. The posteriors are given for the horizon of 10 years (1989-1998) for each of seven broad groups of funds classified by their investment objective.

	Investment objective	CAPM				Fama-French 3-factor model				Carhart's 4-factor model			
		ρ	σ_w	σ_v	σ	ρ	σ_w	σ_v	σ	ρ	σ_w	σ_v	σ
Raw Returns	Small Company Growth	0.4137	0.0111	0.0168	0.0282	-0.0432	0.0093	0.0169	0.0211	-0.0454	0.0092	0.0169	0.0210
		(0.1193)	(0.0016)	(0.0020)	(0.0013)	(0.1299)	(0.0016)	(0.0021)	(0.0013)	(0.1297)	(0.0016)	(0.0021)	(0.0013)
	Other Aggressive Growth	0.2750	0.0124	0.0231	0.0274	0.0672	0.0111	0.0233	0.0230	0.0665	0.0111	0.0233	0.0229
		(0.1125)	(0.0079)	(0.0065)	(0.0048)	(0.1134)	(0.0079)	(0.0065)	(0.0048)	(0.1132)	(0.0079)	(0.0065)	(0.0048)
	Growth	0.3776	0.0070	0.0079	0.0209	0.2085	0.0065	0.0079	0.0184	0.2024	0.0065	0.0079	0.0183
		(0.0925)	(0.0011)	(0.0014)	(0.0008)	(0.0906)	(0.0011)	(0.0014)	(0.0008)	(0.0905)	(0.0011)	(0.0014)	(0.0008)
	Growth and Income	0.1984	0.0064	0.01	0.0155	0.2576	0.0059	0.01	0.0136	0.2627	0.0058	0.0100	0.0135
		(0.1003)	(0.0036)	(0.004)	(0.0029)	(0.1003)	(0.0036)	(0.004)	(0.0029)	(0.1004)	(0.0036)	(0.0040)	(0.0029)
	Income	0.2839	0.0073	0.0148	0.0154	0.2745	0.0069	0.0148	0.0140	0.2660	0.0069	0.0148	0.0139
		(0.0819)	(0.0016)	(0.0055)	(0.0005)	(0.0816)	(0.0016)	(0.0055)	(0.0005)	(0.0816)	(0.0016)	(0.0055)	(0.0005)
	Maximum Capital Gains	0.3664	0.0092	0.0155	0.0222	0.1840	0.0081	0.0155	0.0180	0.1905	0.0081	0.0155	0.0179
		(0.0519)	(0.0011)	(0.0043)	(0.0033)	(0.0578)	(0.0011)	(0.0043)	(0.0034)	(0.0574)	(0.0011)	(0.0043)	(0.0034)
	Sector	0.3681	0.0154	0.0171	0.0498	0.3372	0.0150	0.0170	0.0479	0.3343	0.0149	0.0170	0.0477
		(0.0619)	(0.0015)	(0.0021)	(0.3039)	(0.0591)	(0.0015)	(0.0021)	(0.3039)	(0.0603)	(0.0015)	(0.0021)	(0.3039)
Alphas	Small Company Growth	0.3431	0.0117	0.0251	0.0247	0.0878	0.0106	0.0258	0.0208	-0.1113	0.0106	0.0258	0.0211
		(0.0591)	(0.0061)	(0.0044)	(0.0075)	(0.0773)	(0.0061)	(0.0046)	(0.0075)	(0.0828)	(0.0061)	(0.0046)	(0.0075)
	Other Aggressive Growth	0.2272	0.0129	0.0278	0.0270	0.1161	0.0121	0.0277	0.0242	0.0411	0.0122	0.0278	0.0243
		(0.1500)	(0.0033)	(0.0058)	(0.0012)	(0.1563)	(0.0033)	(0.0058)	(0.0013)	(0.1597)	(0.0033)	(0.0058)	(0.0013)
	Growth	0.4265	0.0073	0.0094	0.0208	0.4414	0.0071	0.0094	0.0196	0.4258	0.0071	0.0094	0.0198
		(0.0769)	(0.001)	(0.0030)	(0.0020)	(0.0749)	(0.001)	(0.0030)	(0.0020)	(0.0758)	(0.001)	(0.0030)	(0.0020)
	Growth and Income	0.2418	0.0071	0.0120	0.0166	0.3124	0.0066	0.0120	0.0149	0.3241	0.0066	0.0120	0.0150
		(0.1321)	(0.0016)	(0.0024)	(0.0022)	(01304)	(0.0016)	(0.0024)	(0.0022)	(0.1302)	(0.0016)	(0.0024)	(0.0022)
	Income	0.4387	0.0084	0.0180	0.0174	0.5354	0.0080	0.0180	0.0166	0.5388	0.0079	0.0180	0.0166
		(0.0437)	(0.0072)	(0.0024)	(0.001)	(0.0421)	(0.0072)	(0.0024)	(0.001)	(0.0427)	(0.0072)	(0.0024)	(0.001)
	Maximum Capital Gains	0.3919	0.0094	0.0164	0.0224	0.3930	0.0085	0.0164	0.0196	0.3474	0.0086	0.0164	0.0199
		(0.0642)	(0.0015)	(0.0060)	(0.0032)	(0.0657)	(0.0015)	(0.0060)	(0.0032)	(0.0699)	(0.0015)	(0.0060)	(0.0032)
	Sector	0.4596	0.0155	0.0173	0.0463	0.5472	0.0159	0.0173	0.0465	0.5540	0.0156	0.0173	0.0466
		(0.0788)	(0.0031)	(0.0031)	(0.0007)	(0.0707)	(0.0031)	(0.0031)	(0.0007)	(0.0716)	(0.0031)	(0.0031)	(0.0007)

Table IV

Classical Inference for Performance Measures Based on the Raw Returns and Alphas

This table reports the means (standard errors) of the parameters obtained in a classical inference for the correlated random effects using least squares regression and error components model using random effects panel data model given that the investor makes her decision upon the raw returns or performance measure alpha. The estimates are given for each potentially considered pricing model: CAPM, Fama-French 3-factor model, and Carhart's 4-factor model. Fist two columns for each pricing model consider autocorrelated random effects whereas two other error components model ρ is the measure of persistence, $\tau^{1/2}$ and σ measure the standard deviation of the error component for the correlated random effects and error components, respectively. The estimates are given for the horizon of 10 years (1989-1998) for each of seven broad groups of funds classified by their investment objective.

	Investment objective	CAPM				Fama-French 3-factor model				Carhart's 4-factor model			
		ρ	$\tau^{-1/2}$	ρ	σ	ρ	$\tau^{1/2}$	ρ	σ	ρ	$\tau^{-1/2}$	ρ	σ
Raw Returns	Small Company Growth	0.0769	0.0267	0.0940	0.0298	-0.0103	0.0202	0.0537	0.0256	-0.0105	0.0201	0.0533	0.0256
		(0.0917)	(0.0087)	(0.0149)	(0.0089)	(0.0918)	(0.0059)	(0.0150)	(0.0082)	(0.0917)	(0.0059)	(0.0150)	(0.0082)
	Other Aggressive Growth	0.0609	0.0264	0.0846	0.0284	-0.0073	0.0221	0.0588	0.0263	-0.008	0.0219	0.0604	0.0263
		(0.0928)	(0.0073)	(0.0204)	(0.0076)	(0.0922)	(0.0064)	(0.0204)	(0.0071)	(0.0921)	(0.0064)	(0.0204)	(0.0071)
	Growth	0.077	0.0194	0.1062	0.0235	0.0446	0.0168	0.1139	0.0249	0.043	0.0167	0.1164	0.0249
		(0.0928)	(0.0079)	(0.0071)	(0.0065)	(0.0923)	(0.0077)	(0.0071)	(0.0078)	(0.0922)	(0.0077)	(0.0071)	(0.0078)
	Growth and Income	0.0715	0.0148	0.1639	0.021	0.1028	0.0126	0.1712	0.0252	0.1043	0.0124	0.1746	0.0252
		(0.093)	(0.0045)	(0.0089)	(0.0056)	(0.0925)	(0.0051)	(0.0089)	(0.0077)	(0.0922)	(0.0051)	(0.0089)	(0.0077)
	Income	0.1142	0.0141	0.0557	0.0175	0.1131	0.0125	0.0538	0.0170	0.1112	0.0125	0.0536	0.0170
		(0.0922)	(0.0061)	(0.0133)	(0.0076)	(0.0917)	(0.0062)	(0.0133)	(0.0075)	(0.0917)	(0.0062)	(0.0133)	(0.0075)
	Maximum Capital Gains	0.1003	0.0206	0.1140	0.0248	0.0462	0.0166	0.0943	0.0236	0.0505	0.0166	0.0944	0.0236
		(0.0933)	(0.0083)	(0.0138)	(0.0087)	(0.0926)	(0.0068)	(0.0139)	(0.0083)	(0.0925)	(0.0068)	(0.0139)	(0.0083)
	Sector	0.1017	0.0392	0.1099	0.0466	0.0649	0.0371	0.1110	0.0464	0.0678	0.0368	0.1101	0.0464
		(0.0923)	(0.0237)	(0.0151)	(0.0243)	(0.0922)	(0.0236)	(0.0151)	(0.0237)	(0.0921)	(0.0237)	(0.0151)	(0.0237)
Alphas	Small Company Growth	0.0764	0.0234	0.0827	0.0250	0.0218	0.02	0.0324	0.0224	-0.0261	0.0202	-0.0404	0.0228
		(0.0913)	(0.0062)	(0.0221)	(0.0068)	(0.0919)	(0.0048)	(0.0221)	(0.0065)	(0.0917)	(0.0051)	(0.0221)	(0.0066)
	Other Aggressive Growth	0.0387	0.0261	0.0583	0.0279	-0.0815	0.0233	0.0069	0.0247	-0.027	0.0235	-0.0158	0.0249
		(0.0923)	(0.0081)	(0.0244)	(0.0083)	(0.092)	(0.0055)	(0.0244)	(0.0072)	(0.0919)	(0.0055)	(0.0244)	(0.0073)
	Growth	0.0842	0.019	0.1547	0.0220	0.0879	0.0176	0.1034	0.0205	0.0772	0.0177	0.0921	0.0206
		(0.0927)	(0.0081)	(0.0084)	(0.0087)	(0.092)	(0.0083)	(0.0084)	(0.0082)	(0.092)	(0.0085)	(0.0084)	(0.0082)
	Growth and Income	0.0608	0.0158	0.0764	0.0162	0.0797	0.0136	0.1372	0.0160	0.078	0.0136	0.1403	0.0160
		(0.0933)	(0.0047)	(0.0108)	(0.0049)	(0.0921)	(0.0057)	(0.0107)	(0.0048)	(0.0918)	(0.0057)	(0.0107)	(0.0048)
	Income	0.1187	0.0157	0.1376	0.0161	0.1288	0.0142	0.1874	0.0161	0.1217	0.0142	0.1740	0.0162
		(0.0927)	(0.0069)	(0.0160)	(0.0071)	(0.0914)	(0.0078)	(0.0159)	(0.0071)	(0.0916)	(0.0078)	(0.0159)	(0.0071)
	Maximum Capital Gains	0.1028	0.0208	0.0917	0.0230	0.0825	0.0179	0.0406	0.0203	0.0826	0.0181	0.0030	0.0206
		(0.0932)	(0.0076)	(0.0146)	(0.0079)	(0.0922)	(0.0072)	(0.0146)	(0.0074)	(0.0919)	(0.0074)	(0.0146)	(0.0075)
	Sector	0.1161	0.0397	0.0957	0.0459	0.1158	0.0389	0.1603	0.0460	0.1158	0.039	0.1463	0.0463
		(0.0922)	(0.0237)	(0.0154)	(0.0267)	(0.0912)	(0.0243)	(0.0152)	(0.0267)	(0.0913)	(0.0249)	(0.0153)	(0.0269)