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TIME VARIATION IN EXPECTED RETURNS

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Abstract

In this paper, we attempt to characterize the stochastic behavior of expected returns on common stock. We assume market efficiency, and postulate an autoregressive process for conditional expected returns. We use weekly returns on ten-sized based portfolios over the 1962-1985 period, and employ signal extraction methodology to extract the expected returns. Our major findings are: (1) the behavior of expected returns are better characterized by a stationary (as opposed to non-stationary) process, (2) the relative time-variation in expected returns has a monotonic (inverse) relation with size: the smallest portfolio exhibits the maximum variation, and (3) the degree of variation in expected returns also changes systematically over time across most portfolios.

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1. Introduction

Tests of market efficiency have invariably been associated with a particular model of capital market equilibrium, however naive. [For a detailed discussion, see Fama (1976), ch. 5.] Nevertheless, in the implementation of most tests of market efficiency and/or a particular equilibrium model, expected returns are assumed to remain constant over some period of time.[1]

On the other hand, theoretical models of asset pricing have put few, if any, restrictions on the behavior of expected returns over time. In the development of the CAPM, expected returns on risky securities (given risk aversion) are simply presumed to contain positive risk premiums. More recent models of market equilibrium [e.g., Breeden (1979), Cox, Ingersoll and Ross (1985), and Merton (1973)] imply that market efficiency imposes no obvious restrictions on expected return movements. In fact, these models do not rule out even negative expected returns.

Given the disparity between the equilibrium models and their empirical implementation, we are left with the puzzle of whether to accept/reject the pricing model, the joint assumption of market efficiency, or the other assumptions used to test the model, in particular, stationarity. There are two approaches in resolving this problem. The first is to devise tests of pricing models which do not require assumptions of stationarity. Gibbons and Ferson (1985) take this approach. They design a test which specifically relaxes the assumption of constant risk premiums (although they do assume

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[1] For example, see Black, Jensen and Scholes (1972), Fama and Macbeth (1973), and Gibbons (1982). In addition, most event studies also assume stationarity over some time interval. Constant expected returns are not necessarily inconsistent with capital market equilibrium--however, the motivation for this assumption has largely arisen due to empirical (rather than theoretical) considerations.

returns on ten size-based portfolios. Based on the findings of recent empirical papers, we hypothesize that expected returns follow an autoregressive process. We consider two special cases of this process. We first constrain the first order autoregressive parameter to be equal to 1.0 and all higher order parameters to be equal to zero, which implies a (non-stationary) random walk process for expected returns. This, in turn, implies that realized returns follow a random walk plus noise process--a model used to characterize many economic time series. We then relax the constraint and consider a (stationary) first order autoregressive process for expected returns in which the parameter is allowed to vary across portfolios.

We use weekly security returns on the ten size-based portfolios over the 1962-1985 period, and attempt to eliminate market micro-structure biases through careful sample selection. We find that the autocorrelation structure of the realized portfolio returns appears to be consistent with a slowly moving expected return component. We use a Kalman filter technique proposed by Ansley (1980) to extract expected returns and find that constancy is strongly rejected for all but the largest portfolios. Moreover, there is a monotonic relation between the size-ranking of the portfolios and the relative time-variation in expected returns: the smallest portfolio exhibits the maximum variation. The significant time-variation, and its relation to size, is found during each of the five-year sub-periods, even when we separately allow for a January dummy. Hence, movements in expected returns are not simply a January phenomenon. In the sub-period analysis, there is also strong evidence that the relative variation in expected returns changes systematically over time, across portfolios. Specifically, the magnitude of the relative variation is much larger during the seventies, while the eighties have parameters similar to the sixties.

time. Fama and French (1987) find that the dividend/price ratio has significant predictive ability for long-horizon returns. Elsewhere, Fama (1981) and Fama and Gibbons (1982) show that stock returns vary systematically with ex ante estimates of expected inflation and expected real returns on T-bills. Campbell (1987) finds that various measures of the term structure can be used to predict stock returns. Keim and Stambaugh (1985) find evidence of predictability of stock and bond returns based on past price variables, and various other predictive measures have been used in the literature [see, e.g., Ferson (1986), Ferson, Kandel and Stambaugh (1987), Gibbons and Ferson (1985), and Huizinga and Mishkin (1984, 1985)]. Perhaps the most striking evidence of time-varying expected returns may be the well-known phenomenon of seasonals in returns, particularly the "January effect".

Note that if one assumes rationality, then predictability is not necessarily an anomaly but simply evidence that expected returns are conditional on all information (including whether it is January or not). As Gibbons and Ferson (1985) note, "...by assuming efficiency, statistical association of returns with a predetermined variable is evidence that expected returns are changing...". (p. 225).

2.2 Autocorrelation

More evidence consistent with time-varying expectations may be found in the literature analyzing autocorrelations in returns. Fama (1965) finds autocorrelations in daily returns on 30 large stocks which are predominantly of the same sign. He indicates that these patterns could be caused by autocorrelation in market returns and by the contemporaneous correlation between the returns on each stock and the market portfolio.

expected returns process. This approach also enables us to decompose realized returns into an expected return component and a serially uncorrelated 'noise' term at each point in time. We model the expected returns as a first order autoregressive process.[2] Specifically,

$$R_t = E_{t-1}(R_t) + \varepsilon_t \tag{1}$$

$$E_{t-1}(R_t) = \phi E_{t-2}(R_{t-1}) + u_{t-1} \tag{2}$$

where R_t = realized return on a particular security over period t-1 to t,

$E_{t-j}(R_{t-j+1})$ = expected return for a security over period t-j to t-j+1 as of period t-j,

$\varepsilon_t \sim$ iid $N(0, \sigma_\varepsilon^2)$,

$u_t \sim$ iid $N(0, \sigma_u^2)$,

and $\phi \leq 1$.

Recent empirical evidence suggests that the autoregressive parameter, ϕ, may be close to one. A majority of the predetermined variables which have been found to have significant correlations with realized stock returns can themselves be characterized by slowly wandering (highly autocorrelated) behavior. For example, Fama (1981) and Fama and Gibbons (1982) find a strong negative relation between stock returns and expected inflation [see also Kaul (1987)], and a strong positive relation between stock returns and expected real returns on treasury bills. In these papers, both expected inflation and expected real returns are modeled as random walks - a model which appears to

[2] We could specify a higher order autoregressive process for expected returns, but a parsimonious model is more desirable if it adequately characterizes the "true" process. Our results, discussed later, indicate that an AR(1) process appears to be well-specified.
Ohlson and Rosenberg (1982) employ such an approach in estimating the stochastic behavior of the systematic risk of the equal-weighted common stock index.

Therefore, realized returns can be written as:[4]

$$R_t = R_{t-1} + a_t - \theta a_{t-1} \tag{6}$$

which implies that in the inverted form:

$$E_{t-1}(R_t) = (1-\theta)R_{t-1} + \theta(1-\theta)R_{t-2} + \theta^2(1-\theta)R_{t-3} + \cdots \tag{7}$$

and in the random shock form:

$$E_{t-1}(R_t) = (1-\theta)a_{t-1} + (1-\theta)a_{t-2} + (1-\theta)a_{t-3}\cdots \tag{8}$$

Hence, expected returns can be represented as an exponentially weighted sum of past returns [equation (7)], where the weights add up to 1.0 [see Nelson (1973), ch. 4]. Conversely, expected returns can be written as an equal-weighted sum of all past shocks [equation (8)], where the weights are equal to $(1-\theta)$.

We estimate the constrained model for three specific reasons. First, as mentioned above, the predetermined variables which have been found to have significant correlations with realized returns can themselves be characterized by highly autocorrelated behavior. Second, the constrained model has a very appealing interpretation. Following Muth (1960), we can think of the shock to security returns at time t-1, a_{t-1}, as being composed of two parts, a permanent and a temporary one. We can interpret $(1-\theta)a_{t-1}$ as the permanent contribution of a shock to realized returns in the sense that it affects <u>all</u> future expected returns by this amount. Correspondingly, θa_{t-1} can be viewed as the temporary contribution. Hence, estimates of θ alone can give us a good idea of the degree of relative variation in expected returns across portfolios. Finally, the constrained model is robust to alternative specifications [see Monte Carlo experiments of Cooley and Prescott (1973)].

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[4] Cooley and Prescott (1973, 1976) were the first to study the estimation of such a time-varying model. Note that realized returns follow a non-stationary process with undefined moments. However, <u>changes</u> in returns can be written as a stationary MA(1) process.

In this case expected returns are an exponentially weighted sum of past returns, where the weights add up to less than 1.0 [equation (14)]. Conversely, expected returns can be expressed as a weighted sum of all past shocks, but (unlike the random walk model) the weights given to past shocks decline exponentially [equation (15)]. In other words, if expected returns are stationary a shock at t-1, a_{t-1}, has a progressively smaller effect on future expected returns (unlike the random walk in which all future expected returns are affected by the same amount).[5,6]

The autocorrelations in realized returns implied by an autoregressive expected return component may also be consistent with other explanations. Specifically, market micro-structure biases caused by nonsynchronous trading and bid-ask spreads (for example) could also lead to such autocorrelations. Fisher (1966) shows that nonsynchronous trading can induce negative autocorrelations in individual security returns and positive autocorrelations in portfolio returns. [See also Scholes and Williams (1977) and Cohen et al. (1983).] However, our sample selection procedure, described later, minimizes

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[5] Rosenberg (1973) first developed such a convergent parameter model, as opposed to the random walk model in which expected returns have no tendency to converge. We can rewrite our model in the following form:

$$R_t = E_{t-1}(R_t) + \varepsilon_t$$

$$E_{t-1}(R_t) = (1-\delta)\bar{R} + \delta E_{t-2}(R_{t-1}) + u_{t-1}$$

where δ is the covergence parameter, and $\bar{R}$ is the population "norm" towards which the expected return process converges.

We can then rewrite realized returns as:

$$R_t = (1-\delta)\ \bar{R} + \delta R_{t-1} + \varepsilon_t - \delta\varepsilon_{t-1} + u_{t-1}$$

which is an ARMA (1,1) process.

[6] The two models for expected returns have different implications for the realized and expected return processes. However, the autocorrelation structure for realized returns implied by both cases could be similar, depending upon the values of the autoregressive and moving average parameters [see Box and Jenkins (1970), ch. 4].

holding period returns for each security are then constructed by compounding daily returns. The security returns in each portfolio are equally-weighted to form ten series of portfolio returns, and we also construct an equal-weighted "market portfolio" return using all our sample securities.[8] Hence, we have a total of 1226 weekly returns across ten size-based portfolios and one market portfolio from July 1962 to December 1985.

4.2. Autocorrelations

Table 1 shows the summary statistics for the weekly returns for the 1962-1985 period. The first order autocorrelations are significant and decline slowly at higher order lags.[9] The returns on the equal-weighted market portfolio, EWMR, exhibit similar persistence in autocorrelations. [We replicate all our results using excess returns, i.e., returns in excess of the weekly risk-free return. The results are virtually identical.]

An interesting aspect of the autocorrelation structures across portfolios is the consistent pattern observed as we go from the smallest portfolio (R1) to the largest (R10): the persistence and magnitude of higher order autocorrelations decline monotonically. However, higher order autocorrelations remain significant for all but the largest portfolio (which exhibits only first order autocorrelation).[10]

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[8] Our method of computing weekly holding period returns minimizes the bias in earlier studies which use arithmetic averages of returns within the review period [see Roll (1983)]. We construct our own market portfolio because compounding the daily market returns in the CRSP files would also lead to bias.

[9] Wichern's (1973) simulation evidence shows that sample autocorrelations for non-stationary series could emanate from a (first order) value considerably less than one.

[10] Nelson and Schwert (1977) show that the first order autocorrelation in realized returns, for example, depends on the first order autocorrelation coefficient of expected returns and the ratio of the variance of unexpected returns to the variance of expected returns. If the latter ratio is large it could lead to low autocorrelation in realized returns even though expected returns are highly correlated.

concentrate mainly on the first order autocorrelation. Thus there exists an inconsistency between the characteristics of the short- and long-run holding period returns which needs to be resolved. Such a resolution, however, is beyond the scope of this paper.

4.3 The model estimates

4.3.1 Case I

The restricted version ($\phi = 1 \ \forall$ portfolios) is estimated for each of the ten portfolios. Following Ansley (1980), we extract the expected return, $E_{t-1}(R_t)$, using a two-step procedure. First, we utilize the fact that differences in returns can be written as an MA(1) process. For convenience we rewrite equations (1) - (3):

$$\Delta R_t = \Delta E_{t-1}(R_t) + \varepsilon_t - \varepsilon_{t-1} \tag{3}$$

$$= u_{t-1} + \varepsilon_t - \varepsilon_{t-1} \tag{4}$$

$$= a_t - \theta a_{t-1} \tag{5}$$

where u_{t-1} are the steps of the random walk process followed by expected returns. We use the Marquardt maximum likelihood procedure to estimate the parameters of the model.

The second step is to develop a time series of expected returns. Using the identifying restriction that u_{t-1} and ε_t are independent, the Kalman filter technique is used to extract a time series of estimates of u_{t-1} from estimates of $a_t - \theta a_{t-1}$ in (5). We then simply cumulate the estimates of u_{t-1} to obtain a time series of the smoothed expected return, $E_{t-1}(R_t)$.

The filtering technique generates an estimate of the ratio (λ) of the variances of the random walk step of expected returns (u_t) and the disturbance term (ε_t). If $\hat{\lambda}$ is significantly different from zero, we can reject the hypothesis that the expected returns are constant. We also present corresponding estimates of θ, the MA(1) parameter in (5).

This result is consistent with the evidence reported in other recent papers. For example, Fama and French (1986a) find that autocorrelations in 3- to 5- year returns explain almost twice the proportion of return variance for small firms as for large firms. Keim and Stambaugh (1985) find that various predetermined variables also explain a (relatively) significantly larger proportion of the return variance of small firms. In fact, in almost all empirical work relating to movements in ex ante expected returns, the proportion of return variance explained by ex ante information decreases monotonically as the market value (size) of the portfolio increases. Hence, not only is the variance of realized returns for small firms systematically higher than for large firms (see Table 1), but the relative variance of expected returns as well. This, in turn, enables the signal extraction technique to detect significant variation in the expected returns of (especially) the smaller firms.

Next, we re-estimate the model with a dummy variable for the first week of January[15]; the results are shown in Table 2. The dummy variable is significant for the smallest five market value portfolios. This, in turn, implies that: (i) the seasonality is apparently not just a small firm phenomenon, and (ii) to the extent that there is a discrete jump in expected returns in the first week of January, our Kalman filter (smoothing) technique will not pick it up.

Table 2 also shows the first order residual autocorrelations. There is significant residual autocorrelation, especially for the large portfolios. [Higher order residual autocorrelations are generally close to zero.] One possible reason for this is the (potential) time-variation in the parameters

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[15] Keim (1983) reports that the January premium is attributable to large abnormal returns during the first week of trading in the year.

4.3.2 Case II

We use the maximum likelihood technique to estimate the model in which ϕ, the first order autoregressive parameter, is allowed to vary across portfolios.[17] The results are reported in Table 3. The evidence again indicates a monotonic relation between the estimates of ϕ and size: the magnitude of $\hat{\phi}$ declines as we go from the smallest to the largest portfolio. This pattern remains even when we introduce a dummy for the first week of January, which is significant for all but the largest portfolio.

The stationary autoregressive process for expected returns appears to be well-specified. The estimates of ϕ are significantly different from 1.0 for all portfolios, and the residuals behave like white noise. This evidence is consistent with the results in a recent paper by Fama and French (1987), who find positive autocorrelations in expected returns which are documented in their regressions of long-horizon returns on dividend/price ratios. The pattern in the coefficients of these regressions for different holding periods are suggestive of a mean-reverting expected return process. However, the autocorrelations in long-horizon realized returns are negative. As Fama and French point out, time-varying expected returns do not necessarily imply negatively autocorrelated returns. In their dividend valuation model, if shocks to expected returns are positively correlated with shocks to expected dividends (u_t and ε_t, respectively, in our model), then positively autocorrelated expected returns would imply positive autocorrelation in returns.

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[17] We obtain similar estimates of ϕ when we use the nonlinear least squares technique [see Box and Jenkins (1970), ch. 7] to fit an ARMA (1,1) model to realized returns.

These results give further support to the hypothesis that even though expected returns could be moving slowly over time, their stochastic behavior is better characterized by a stationary autoregressive process.

4.5 The information content of the extracted expected returns

We now consider the information content of the extracted expected returns from the stationary model, $ERAR_{t-1}$, with respect to other relevant ex ante information. It is by no means necessary that the expected returns conditioned solely on each portfolio's past returns should also incorporate other information. However, we can get an idea of the economic content of the forecasts by testing whether the information in other relevant ex ante variables is already impounded in them.

The choice of the predetermined variables is largely dictated by the findings in other papers, and data availability considerations. Among other variables, we consider the nominal risk-free rate[19] [Fama and Schwert (1977) and Ferson (1986)] and the lagged return on the equal-weighted market portfolio [Gibbons and Ferson (1985)]. We also include a dummy variable for the first week of January in all regressions since our filtering process cannot pick up this (apparently) discrete jump in expected returns.

Table 5 shows the regression estimates for the overall period. The risk-free rate, RF_{t-1}, is significantly negatively related to the returns of all portfolios [see regressions (i)]. There is residual autocorrelation in all the regressions, which implies that the standard errors are biased. However,

[19] Since new treasury bills are introduced every Thursday in the Wall Street Journal quotations, the risk-free rate series is consistently for an 8-day instrument. In calculating the returns we adjust for the skip-day settlement, and continuously compound assuming 365 days in a year. The price used to calculate the return is an average of the bid and ask prices derived from the quoted bid/ask discount rates.

Finally, the results in Table 5 confirm our earlier finding that a monotonic (inverse) relation exists between the relative variation in expected returns and size (see Table 2). The coefficient of determination [in regression (iii)] for the smallest portfolio is about 23%, and its magnitude drops consistently to about 0.8% for the largest portfolio.[21]

5. Summary and conclusions

In this paper, we attempt to characterize the stochastic behavior of expected returns on common stock. We assume market efficiency, and postulate an autoregressive process for conditional expected returns. Recent empirical evidence on the predictability of stock returns suggests that expected returns change slowly over time. Accordingly, we consider two specific cases of the autoregressive process for expected returns: (1) we first constrain the first order autoregressive parameter to be equal to 1.0 and all higher order parameters to be equal to zero, which implies a random walk process for expected returns, (2) we then relax the constraint and consider a (stationary) first order autoregressive process in which the parameter is allowed to vary across portfolios.

We use weekly returns on ten size-based portfolios over the 1962-1985 period, and extract expected returns for all portfolios. In implementing our signal extraction methodology, we attempt to eliminate market micro-structure biases through careful sample selection. Our major findings are: (1) there is significant variation in expected returns across portfolios and it has a monotonic (inverse) relation with size: the smallest portfolio exhibits the

[21] Of course, absolute values of R-squares should be interpreted with caution because of potential misspecifications caused by heteroskedasticity, non-normality, and/or bias from correlation of residuals with lagged regressors [Stambaugh (1986)]. Nevertheless, their relative magnitudes are suggestive of a distinct pattern.

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Table 1

Summary Statistics of Weekly Returns of Ten Equal-Weighted Portfolios of New York and American Stock Exchange Common Stocks, Formed by Decile Rankings of Market Value of Equity Outstanding at the End of the Previous Year, 1962-1985 (1226 weeks).[a]

Variable(x)	$\hat{\rho}_1$	$\hat{\rho}_2$	$\hat{\rho}_3$	$\hat{\rho}_4$	$\hat{\rho}_5$	$\hat{\rho}_6$	$\hat{\rho}_7$	$\hat{\rho}_8$	$\bar{x}$[b]	s(x)[b]
R1(Smallest)	0.41	0.24	0.17	0.11	0.03	0.03	-0.02	-0.03	0.1797	2.650
ΔR1[c]	-0.36	-0.07	-0.01	0.02	-0.08	0.05	-0.04	-0.03	0.0014	2.887
R2	0.36	0.20	0.14	0.10	0.02	0.03	-0.04	-0.04	0.1495	2.481
ΔR2	-0.38	-0.06	-0.03	0.04	-0.07	0.06	-0.05	-0.05	-0.0006	2.815
R3	0.31	0.17	0.12	0.08	0.01	0.03	-0.01	-0.04	0.1595	2.431
ΔR3	-0.40	-0.07	-0.01	0.02	-0.07	0.05	-0.01	-0.06	-0.0018	2.854
R4	0.30	0.15	0.09	0.06	0.01	0.01	-0.01	-0.04	0.1632	2.407
ΔR4	-0.39	-0.07	-0.01	0.00	-0.03	0.02	-0.00	-0.06	-0.0014	2.854
R5	0.29	0.14	0.08	0.05	0.00	0.01	0.00	-0.03	0.1474	2.401
ΔR5	-0.39	-0.07	-0.02	0.01	-0.04	0.01	0.02	-0.04	-0.0021	2.851
R6	0.27	0.13	0.08	0.05	-0.00	-0.00	0.01	-0.04	0.1630	2.327
ΔR6	-0.40	-0.06	-0.01	0.01	-0.03	-0.01	0.04	-0.06	-0.0015	2.812
R7	0.24	0.11	0.07	0.04	-0.00	-0.00	0.00	-0.03	0.1651	2.214
ΔR7	-0.41	-0.06	-0.01	0.01	-0.03	-0.00	0.03	-0.04	-0.0014	2.722
R8	0.22	0.10	0.07	0.04	0.00	-0.01	0.01	-0.03	0.1547	2.133
ΔR8	-0.42	-0.07	0.00	0.00	-0.01	-0.02	0.04	-0.05	-0.0004	2.657
R9	0.18	0.07	0.06	0.02	-0.02	-0.03	0.01	-0.04	0.1585	2.050
ΔR9	-0.43	-0.06	0.02	0.00	-0.02	-0.03	0.06	-0.07	-0.0007	2.620
R10(Largest)	0.09	-0.00	0.05	0.01	-0.02	-0.03	0.05	-0.04	0.1363	1.965
ΔR10	-0.45	-0.08	0.05	-0.01	-0.00	-0.05	0.10	-0.08	-0.0003	2.652
EWMR[d]	0.29	0.14	0.10	0.06	0.00	0.01	-0.00	-0.04	0.1583	2.194
ΔEWMR	-0.40	-0.07	-0.01	0.01	-0.04	0.01	0.02	-0.06	-0.0009	2.618

Notes:

(a) R1-R10 are the continuously compounded weekly returns on ten size-based portfolios in ascending order from smallest-largest. $\bar{x}$ and s(x) are the sample mean and standard of the deviation variable, and $\hat{\rho}_t$ is the sample autocorrelation at lag t. Under the hypothesis that the true autocorrelations are zero, standard errors of the estimated autocorrelations are about 0.03.

(b) The returns are rates of return per week, in decimal fraction units x 10^{-2}.

(c) The operator Δ denotes first differences.

(d) EWMR is the equal-weighted market portfolio return.

Table 3

Weekly Estimates of the Parameters of the Model in which Expected Returns follow a Stationary AR(1) process.[a]

$$R_t = E_{t-1}(R_t) + \varepsilon_t \qquad (1)$$
$$E_{t-1}(R_t) = \phi E_{t-2}(R_{t-1}) + u_{t-1} \qquad (2)$$
$$\text{where } \phi < 1.$$

Portfolio	Overall Period: July 1962 - December 1985, n = 1226						
	Without January Dummy			With January Dummy (D)			
	$\hat{\phi}$ (b)	$s(\varepsilon)$(c)	$\hat{\rho}_1$(d)	$\hat{\phi}$	$\hat{\delta}$(e)	$s(\varepsilon)$	$\hat{\rho}_1$
1 (Smallest)	0.611 (0.052)(f)	0.02410	-0.003	0.690 (0.037)	0.0476 (0.004)	0.02307	0.006
2	0.585 (0.060)	0.02310	-0.003	0.656 (0.045)	0.0361 (0.004)	0.02249	0.007
3	0.565 (0.069)	0.02304	-0.003	0.629 (0.054)	0.0307 (0.004)	0.02260	0.007
4	0.499 (0.078)	0.02295	-0.006	0.593 (0.062)	0.0254 (0.004)	0.02267	0.007
5	0.452 (0.082)	0.02292	-0.007	0.544 (0.069)	0.0218 (0.005)	0.02273	0.003
6	0.461 (0.088)	0.02238	-0.007	0.543 (0.076)	0.0171 (0.004)	0.02227	0.003
7	0.462 (0.096)	0.02145	-0.007	0.544 (0.081)	0.0157 (0.004)	0.02134	0.002
8	0.429 (0.107)	0.02077	-0.006	0.506 (0.095)	0.0113 (0.004)	0.02072	0.001
9	0.338 (0.137)	0.02015	-0.010	0.462 (0.117)	0.0097 (0.004)	0.02011	-0.003
10 (Largest)	0.094 (0.028)	0.01958	-0.003	0.093 (0.028)	0.0048 (0.004)	0.01957	-0.004

Notes:

(a) R_t = continuously compounded realized return for week t;

$E_{t-j}(R_{t-j+1})$ = expected return over week t-j to t-j+1 as of week t- j;

$\varepsilon_t \sim N(0,\sigma_\varepsilon^2)$; $u_t \sim N(0,\sigma_u^2)$.

(b) $\hat{\phi}$ = the estimated autoregressive parameter in equation (2).

(c) $s(\varepsilon)$ = residual standard error.

(d) $\hat{\rho}_1$ = residual autocorrelation at lag 1. Under the hypothesis that the true autocorrelations are zero, the standard errors of the residual autocorrelations are about 0.03.

(e) $\hat{\delta}$ = estimated coefficient on the dummy varible (D) where D = 1 for first week in January
= 0 ∀ other weeks.

(f) The numbers in parentheses below the estimated coefficients are standard errors.

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Portfolio	Constant	(b) $ERRW_{t-1}$	(c) $ERAR_{t-1}$	(d) $\bar{R}^2$	(e) $s(\eta)$	(f) $\hat{\rho}_1$
7 (i)	0.00110 (0.00098)	0.305 (0.301)		0.001	0.02212	0.24
(ii)	0.00113 (0.00064)		0.979 (0.148)	0.06	0.02144	-0.002
(iii)	0.00142 (0.00094)	-0.176 (0.150)	1.004 (0.301)	0.06	0.02144	-0.004
8 (i)	0.00138 (0.00111)	0.094 (0.437)		0.000	0.02134	0.22
(ii)	0.00111 (0.00062)		0.979 (0.155)	0.05	0.02077	-0.002
(iii)	0.00169 (0.00108)	-0.366 (0.156)	1.002 (0.431)	0.05	0.02077	-0.003
9 (i)	0.00221 (0.00115)	-0.366 (0.432)		0.000	0.02050	0.19
(ii)	0.00123 (0.00059)		0.971 (0.184)	0.03	0.02015	-0.005
(iii)	0.00247 (0.00114)	-0.705 (0.184)	0.999 (0.429)	0.04	0.02014	-0.006
10 (i) (Largest)	0.00198 (0.00101)	-0.389 (0.391)		0.000	0.01966	0.09
(ii)	0.00123 (0.00057)		0.954 (0.392)	0.007	0.01959	0.001
(iii)	0.00203 (0.00101)	-0.495 (0.391)	0.978 (0.388)	0.007	0.01958	0.001

Notes:

(a) R_t = continuously compounded realized returns for week t; $E_{t-1}(R_t)$ = expected return for week t-1 to t as of week t-1; η_t random disturbance term.

(b) $ERRW_{t-1}$ = estimated expected return from the model in which expected returns follow a (non-stationary) random walk process.

(c) $ERAR_{t-1}$ = estimated expected return from the model in which expected returns follow a (stationary) first order autoregressive process.

(d) $\bar{R}^2$ = (adjusted) coefficient of determination.

(e) $s(\eta)$ = residual standard error.

(f) $\hat{\rho}_1$ = residual autocorrelation at lag 1. Under the hypothesis that the true autocorrelations are zero, the standard errors of the residual autocorrelations are about 0.03.

(g) The numbers in parentheses below the estimated regression coefficients are standard errors based on White's (1980) consistent heteroskedasticity correction.

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Portfolio	$\hat{\alpha}$	$\hat{\delta}$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\bar{R}^2$ (b)	$s(\eta)$ (c)	$\hat{\rho}_1$ (d)
7 (i)	0.0045 (0.0013)	0.013 (0.004)	-2.916 (1.183)			0.010	0.02203	0.23
(ii)	0.0010 (0.0006)	0.013 (0.004)		0.239 (0.038)		0.062	0.02144	-0.01
(iii)	0.0028 (0.0013)	0.012 (0.004)	-1.791 (1.153)	-0.035 (0.135)	1.084 (0.523)	0.067	0.02138	-0.00
8 (i)	0.0043 (0.0012)	0.009 (0.003)	-2.776 (1.145)			0.006	0.02126	0.22
(ii)	0.0010 (0.0006)	0.009 (0.003)		0.201 (0.037)		0.045	0.02085	0.01
(iii)	0.0028 (0.0013)	0.008 (0.003)	-1.725 (1.125)	-0.163 (0.123)	1.646 (0.525)	0.056	0.02073	0.01
9 (i)	0.0039 (0.0012)	0.007 (0.003)	-2.345 (1.120)			0.004	0.02045	0.18
(ii)	0.0012 (0.0006)	0.007 (0.003)		0.147 (0.034)		0.026	0.02023	0.03
(iii)	0.0028 (0.0012)	0.007 (0.003)	-1.665 (1.100)	-0.164 (0.097)	1.810 (0.517)	0.039	0.02010	-0.00
10 (i) (Largest)	0.0037 (0.0012)	0.004 (0.003)	-2.219 (1.125)			0.002	0.01963	0.08
(ii)	0.0012 (0.0006)	0.004 (0.003)		0.072 (0.032)		0.006	0.01960	0.02
(iii)	0.0033 (0.0012)	0.003 (0.003)	-1.944 (1.118)	0.004 (0.065)	0.857 (0.797)	0.008	0.01958	0.00

Notes:

(a) R_t = continuously compounded realized returns for week t; D_t = dummy variable (= 1 for first week of January, = 0 ∀ other weeks); RF_{t-1} = the risk-free rate calculated as the continuously compounded return on a one-week treasury bill; $EWMR_{t-1}$ = continuously compounded return on the equal-weighted market portfolio for week t-1; $ERAR_{t-1}$ = estimated expected return extracted from the model in which expected returns follow an AR(1) process; η_t = random disturbance term.

(b) $\bar{R}^2$ = (adjusted) coefficient of determination.

(c) $s(\eta)$ = residual standard error.

(d) $\hat{\rho}_1$ = residual autocorrelation at lag 1. Under the hypothesis that the true autocorrelations are zero, the standard errors of the residual autocorrelations are about 0.03.

(e) The numbers in parentheses below the estimated regression coefficients are standard errors based on White's (1980) consistent heterskedasticity correction.