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**PERCEPTION OF ANALOGY**

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When the solution to a difficult problem is available as the solution to an analogous problem in a different semantic domain, subjects often fail to notice or use it. Previous work has emphasized the induction of an abstract 'schema' as the prerequisite for analogical transfer between semantic domains. By using isolated semantic links to align problem and analog in different ways, we explore the possibility that direct analogy provides an alternative way of arriving at such analogies. Our results show that this is not the case, and support the belief that appropriate prior abstraction is necessary.

## INTRODUCTION

Analogy plays an important role in problem solving and creative discovery. Even in routine investigation and everyday life, small problems must sometimes be reformulated through a parallel to a previously unrelated context so that new kinds of information can be brought to bear, or known solution steps can be applied.

Valid and useful analogy may be drawn between cases which lie in entirely distinct semantic domains. While the parallels involved may be easy to demonstrate after the fact, the ability to notice such an analogy in the first place is not so easily explained.

An experimental approach to this question designed by Gick and Holyoak (1980, 1983), combined a difficult problem with a story in which the analogous problem is solved. While story and analog are drawn from different semantic domains, they share a common abstract pattern which allows a common solution. Simple enough for some degree of experimental control, the combination does approximate a realistic case of problem-solving by analogy. Consistent with previous findings (e.g. Duncker 1926), many subjects did not spontaneously notice and use the analogy (30% given the analogy solved the problem compared to 10% able to solve it unaided). Modifications which increased the proportion noticing the analogy might then reveal the process at work in the other cases too.

## 1. Schema Interpretation

Gick and Holyoak (1980, 1983) conjecture that the analogy should not be apparent until the common 'schema' has been abstracted from details of the story, and given a form applicable to the problem as well. It is the shared schema with its abstract pattern of causal relations which justifies transfer of the solution from one domain to the other. They point out that a semantically driven retrieval system would have no incentive to review the story in searching for solutions to the problem until a) the story has been formulated as an instance of problem and solution, b) this formulation has been made abstract enough to describe the problem as posed in a different domain. Gick and Holyoak thus propose schema abstraction as the prerequisite for analogical transfer across semantic domains.

Devices to promote abstraction of the schema were expected to increase the proportion of subjects who noticed and used the analogy. Subjects were clearly led to 'reason from a schema' when given multiple analogs, especially when these were combined with an abstract presentation of the principle involved. However, devices to promote abstraction had no effect when a single story analog was used. The authors concluded that two different analogs must be mapped together to effect schema abstraction. Subjects who notice the analogy and profited from it when only one story analog is given are presumed to

employe a second analog known to them previously, cross-map this with the story provided, and thus generated a schema which could apply to their problem.

## 2. Direct Analogy

'Reasoning from a schema' is a justified and dependable procedure, with an important place in educational strategy. But since it does not come into play until subjects have been trained with multiple examples, it is a rigid and ponderous procedure. We might propose that an alternative procedure, more flexible and opportunistic, also plays an important role in analogy. Rather than supplying and cross-mapping a second analog, subjects who notice the analogy without special prompting might be bypassing the 'schema abstraction' process and finding their way more directly to the analogy.

In direct analogy, a parallel between the two cases may be noticed before a neutral, abstract expression of common principles has been formulated. Recognition that the story analog contains a solution for the problem might itself prompt abstraction of the justifying schema. In this process it may be necessary to allow a large place for chance and contingent factors to explain why the problem and its analog are brought together at all. But we can consider how they must be brought together in order for the analogy to be noticed.

To bring problem and analog together, arbitrary semantic links can be used to join them at different points. Differential effects on performance would help to reveal a 'direct analogy' process involved in subjects' alertness to an available analogy.

## ANALOGY

### 1. Analogy.

Our point of departure was the original story/problem pair used by Gick and Holyoak (1980). The story reads:

A small country was ruled from a strong fortress by a dictator. The fortress was situated in the middle of the country, surrounded by farms and villages. Many roads radiated outwards from the fortress like spokes on a wheel. A rebel general vowed to capture the fortress. He gathered his army at the head of one of the roads. However, he learned that the dictator had planted mines on each of the roads. The mines were set so that small bodies of men could pass over them safely, since the dictator needed to move his troops and workers to and from the fortress. However, any large force would detonate the mines. Not only would this blow up the road, but it would also destroy many neighboring villages. It therefore seemed impossible to capture the fortress.

However, the general devised a simple plan. He divided his army into small groups

and dispatched each group to the head of a different road. Each group continued down its road to the fortress so that the entire army finally arrived at the fortress at the same time. The fortress fell and the dictator was overthrown.

The problem (from Duncker, 1945) reads:

A doctor is faced with the following problem. A patient is brought in with a malignant tumor in his stomach. It is impossible to operate on the dictator, but unless the tumor is destroyed he will die. There is a kind of ray which can be used to destroy the tumor. If the rays reach the tumor all at once at a sufficient intensity, the tumor will be destroyed. Unfortunately, at this intensity the healthy tissue that the rays pass through on the way to the tumor will also be destroyed. At lower intensities the rays are harmless to healthy tissue, but they will not affect the tumor either.

What procedure might be used to destroy the tumor with the rays, and at the same time avoid destroying the healthy tissue?

The analogy suggests a 'convergence solution', in which many weak rays might converge on the tumor just as weak columns of troops converged on the fort. A 'schematic' representation of the story as a problem and solution, in which troops are an instance of force and the fort is an instance of target, applies equally well to the medical problem.



## 2. Modifications

Modified versions of this story/problem pair were produced by the addition of an arbitrary semantic link.

The first version ('rebel doctor') altered the story's wording to read "a doctor who had taken command of the rebel forces", instead of "a rebel general".

The second version ('sick dictator') altered the problem to read: "A doctor is faced with the following problem. A ruthless dictator is brought in with a malignant tumor in his stomach..." The story introduced the 'dictator' as "a ruthless dictator".

The third version ('ruthless doctor') altered the story's wording to read "ruled...by a doctor who had seized dictatorial powers".

The fourth version ('sick general') altered the problem to read: "A doctor is faced with the following problem. A rebel general is brought in with a malignant tumor in his stomach..."

In addition, one version combined the first and second links given, another combined the third and fourth.

All of these links can be considered 'arbitrary' from the point of view of schema abstraction. They do not imply the required formulation of the story as a problem and solution, they do not provide any of the crucial equivalences which are necessary in 'reasoning from the schema' to the analogy.

For purposes of direct analogy, each connection corresponds to a cross-mapping between story and problem. Indurkya (1985) shows how such a mapping can be represented as substitution of an element from one analog into arguments of the other. Because the elements of each text are interrelated among themselves, a single mapping of this kind can be extended to suggest further correspondences.

A consistent extension from the initial mapping provided by the semantic links may or may not produce the useful analogy. Links provided by the first two versions are consistent with the analogy and lead to it if they are extended; links provided by the second two versions do not lead to anything useful with further extension.

There are many paths by which the consistent versions can be extended and still arrive at the same result. Thus, if the doctor is matched with the general, his means (rays) might be equated with the general's (troops), or his actions (unknown in the problem) might be equated with the general's (divide and reassemble something). In the course of these extensions, the 'convergence' solution to the medical problem will appear. Equating dictator and patient is not perfectly consistent with the analogy, but its extension would place the patient at the center of converging rays at which point the solution is difficult to miss.

There seems less scope for extension when doctor and dictator are equated, and possibilities do not give any useful result. Defensive actions of the dictator suggest only bizarre behavior if they are applied to the doctor. When general and patient are equated, effective transfer of

the general's solution is blocked by the assumption that a patient will not seize control of the ray machine or design novel forms of treatment for himself.

There are then three possibilities.

If only the consistent links are effective, this suggests that once the two examples are linked at any point, subjects will tend to extend the mapping, perhaps unconsciously, in a rather automatic and exploratory way. Doing so, they would find that the story contains a solution to the problem provided that the starting point is one of the 'consistent' links. If the starting point is one of the 'inconsistent' links, nothing will come of the process.

If all links are equally effective, this suggests a less automatic and more subtle alertness to direct analogy: generally poor performance by subjects would reflect a strong tendency to insulate the two analogs, overcome if they are brought together in any way at all.

If none of these links are effective, this suggests that subjects are not likely to notice parallels fortuitously through a direct analogy process, and will not be alert to the analogy unless an appropriate schema has been abstracted.

## METHODS

Subjects were 122 males and females drawn from a volunteer subjects' pool at the University of Michigan, Ann Arbor. The pool includes students and others reflecting the makeup of the university community. Gick and Holyoak (1980) had determined that about 10% of the subjects drawn from a similar pool were able to produce the 'convergence' solution with no analogy given; 25% failed to produce it given the story analog with a direct suggestion to use it as a guide to problem solution.

Subjects were first asked to read the story and write a short statement 'telling in your own words what the story is about' to ensure that they read it attentively.

Subjects were next asked to read the problem and write down 'the best solution you can think of'.

Subjects were then given an irrelevant task requiring about twenty minutes that involved sorting words and supplying opposites. This task which required moderate attention but not great concentration was meant to provide a break from the problem and allow a second fresh approach.

Subjects were then asked to recall the original story and write another statement 'from memory' telling 'what the story was about'. This was meant to return their attention to the story.

Subjects were then asked to reread the problem and provide a second solution, 'different from the first one you suggested', then to note

'which solution you think is the better one'. This was intended to initiate a new search for a solution, unconstrained by the first solution even if it had seemed adequate.

Finally, subjects who did not offer the 'convergence solution' on either attempt were asked: "did you see any relation between the story and the problem", "could you have solved the problem the way the general solved his problem in the story", "what would happen if weak rays were to converge on the tumor from many directions", and "how does that compare to the solutions you suggested?"

The first question was to ensure that subjects were in fact noticing the intended semantic links. The others were to check our assumption that the 'convergence' solution would be used if noticed (beliefs about tumors or rays might have led subjects to reject it, or they might have preferred the different solutions they suggested).

## RESULTS

Figure 1 shows the percentage of subjects in each condition who produced the solution by analogy on either the first or second attempt:

.....Fig.1.....

Version	%solution by analogy	N
no link	38%	21
consistent link	45%	42
inconsistent link	31%	39
double consistent link	50%	10
double inconsistent link	50%	10

.....

None of the conditions shows performance which differs significantly from the original story/problem pair containing no semantic link.

A chi-square test comparing the 'consistent link' and 'inconsistent link' conditions fails to reveal any differential effect (chi square = 1.79, not significant).

Nonsignificant tendencies in the data which might otherwise be suggestive contradict one another: consistent and inconsistent link provided singly appear to work in opposite directions, but doubled appear to work in the same direction.

It seems clear that none of the variations on the original story/problem pair has any appreciable effect on performance at all.

## DISCUSSION

The results support the original premise of Gick and Holyoak (1983) that schema abstraction is prerequisite for analogical transfer between unlike semantic domains.

We initially questioned this assumption and tested the effect of semantic links which did not promote abstraction, but should have been effective if 'direct analogy' was an important alternative way for subjects to notice the available parallel.

'Consistent' links joined points of the problem and story analog so that a simple extension of the mapping would reveal the solution needed for the problem. Subjects showed no automatic tendency to extend the mapping in an exploratory fashion this way: it made no significant difference whether the mapping was consistent with the analogy or not.

The presence of semantic links in and of itself ensured that subjects would think of the story while searching for a problem solution and would note at least some interconnection of the contents. But performance when semantic links were present did not differ significantly from performance when story and problem were given with no semantic links.

Subjects who did not find the solution by analogy had noticed the intended semantic links, evidenced by responses to the question "did

you notice any connection between the story and the problem" (e.g., "only that doctor that was in both of them"). On the whole these subjects appeared to accept the 'convergence' solution and prefer it to both their suggestions when it was suggested; most were not happy with their solutions and had tried to think of alternatives without noticing the possible 'convergence' solution.

Thus, the failure of these semantic links, which seem at first sight heavy-handed, to produce any significant effect bears out the assumption by Gick and Holyoak (1983). Subjects are unlikely to notice useful analogies between domains until the appropriate "schema" abstraction has taken place.

#### FURTHER SUGGESTIONS

Gick and Holyoak showed that schema abstraction is surprisingly difficult to induce, while we have found that the alternative is ineffective. There is evidently no simple manipulation which will prompt subjects to perceive analogy easily across semantic domain boundaries.

If, however, semantically similar or identical domains are



used, we may observe a significant effect. To justify this expectation, we need to describe the basic problem and the experimental task rigorously. The effect under study is the conditional probability of solving a problem given prior exposure to an 'analogous problem and its solution'. To make this somewhat more precise than the verbal problem used in this experiment, suppose that a subject is exposed to  $\langle P, S(P) \rangle$ :

Problem P: It would take John 2 hours to paint a wall by himself, and Mary 3 hours to paint the same wall by herself. How long would it take them if they worked cooperatively and harmoniously together without overlap?

Solution,  $S(P)$ : In one hour, John working alone would only do half of the job, and Mary  $1/3$ . Together, they would complete  $1/2 + 1/3 = 3/6 + 2/6 = 5/6$  of the job. To do the whole job would take  $1/(5/6)$  hours or  $6/5$  hr. or  $6 \times 60/5 = 6 \times 12 = 72$  minutes (Lindsay & Kochen 1985).

Let  $t$  be the time elapsed between exposure to  $\langle P, S(P) \rangle$  and the presentation of a new problem:

$P'$ : One of two partners in a firm charges \$200 for preparing a will; the other charges \$300. How much should a client pay to have both work together?

Note that  $\langle P, S(P) \rangle$  plays exactly the same role as the story about the general and  $P'$  that of the doctor. What can be said about the probability  $(P' / \langle P, S(P) \rangle, t)$ ? In an experiment to test the hypothesis that failures to recall are due not so much to failures of retrieval but of attention, an experiment was conducted to expose subjects to items from a dictionary with no hint that they would be asked questions about these items at the end of an hour. At that time, many subjects tried to answer the question as if they had never been exposed to the answer. The prior

exposure simply did not always come to mind (Kochen, 1978). We should expect such an effect to occur to an even greater extent when  $P'$  is not identical to  $P$ . Thus  $P$ , the probability of solving  $P'$  is  $\text{Prob}(P'/\langle P, S(P) \rangle)$  is attended to and the relevance of  $P'$  to  $P$  is noticed and  $P'$  is relevant to  $P$  to a given degree and there was exposure to  $\langle P, S(P) \rangle$   $t$  time units ago). The probability that  $\langle P, S(P) \rangle$  is attended to given a degree of relevance between  $P$  and  $P'$  and prior exposure to it should increase with the strength of links in  $P'$  as clues to connect  $P$  and  $P'$  provided that  $P$  and  $P'$  were correspondingly close according to the degree of relevance.

Two concepts require explication: the degree of relevance or semantic proximity between  $P$  and  $P'$  or the domains they are from; the strength of a link serving as a clue. To illustrate the first, note that problem  $P'$ , "In one hour it would take John, working alone, to prepare a will and Mary 3 hours to prepare the same will by herself. How long would it take them if they worked cooperatively and harmoniously together without overlap?" is intuitively more relevant or closer to  $P'$  than  $P$ . What makes it so? The presence of the words "prepare a will"? In what sense is  $P'$  very close to  $P$ ? Minor substitutions can transform one into the other. They are both instances of a general, abstract problem - class that is invariant under such substitutions and other kinds of transformations. The number of transformations needed to convert one to the other, weighted by their salience (in a non-cognitive situation, affect would play a role, too: Pool and Kessler 1963), is a plausible measure of distance.

The strength of a clue-bearing link can also be measured by the extent to which it reduces that needed number of transformations.

A sufficiently explicit formulation of experimental tasks following the above line of thought will make the experimental outcome nearly obvious. As an explanation of the role of analogy in real

problem-solving, this approach is promising and settles the issue raised in this paper by positing that a process of generalization is used, but that it is independent of analogizing as already noted by Polya (1954), which involves primarily attending to appropriate prior exposures and clues to notice the number of transformations needed to convert two problem-statements.

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