Division of Research
Graduate School of Business Administration
The University of Michigan

August 1984

DEPRECIATION: THE AGING OF CAPITAL STOCK

Working Paper No. 388

Lu Wang

DEPRECIATION: THE AGING OF CAPITAL STOCK

Lu Wang
Assistant Professor of Finance
School of Management
The University of Michigan-Dearborn

This paper seeks to present an explanation of the mysterious stagnation of the productivity of the economy. Our observation is that capital stock is subject to an aging process and the physically relatively old capital stock dilutes the productivity of the new. For example, a 20-year-old truck breaks down on the highway, blocking the traffic and diminishing the productivity of a brand new truck. This situation is similar to our familiar experience of the following: When a cup of coffee gets cold, we dispose of it and replace it with a hot one. If we don't dispose of the cold coffee, in order to warm up the coffee, we have to add more hot coffee than otherwise necessary. As time goes by, the coffee will get cold again; and we will then have to add even more hot coffee in order to warm it up, as the quantity of coffee is now even larger. As such, in the economy, on the surface we may have an ever expanding capital stock, but underneath the average age of the capital stock may be increasing. That the average age of the capital stock is increasing need not be a matter of concern so long as the physical durability of the capital stock has been likewise extended by the original engineering construction. But if not, an increase in the average age of the capital stock clearly indicates a dilution of the average productivity of the capital stock.

The above dilution aspect of depreciation is not well recognized. And we are not exaggerating. Consider the following incidents One was that "St. Louis - (UPI) - The west wall of an 80-year-old brick building collapsed onto a busy highway during a thunderstorm Friday, smashing a passing car, killing a teenage girl and injuring two adults. (Detroit Free Press, August 6, 1983.) The other was

"Fire Blacks Out Large Part Of New York Garment Area ... The chain of events that led to the fire and black out began about 1:30 a.m., when a 68-year-old, 12-inch water main, apparently weakened by age, stress and vibration, burst underneath 38th Street and Seventh Avenue, city officials said." (The New York Times, August 11, 1983.) And the essence of such phenomena is that in the general production process of the economy, all the various production inputs are mutually interdependent. Just as an unskilled worker may damage a machine, so may a used-up machine harm a worker. And thus, in the entire capital ensemble, a physically relatively old part may prove to serve as a nagative production input, generating a net negative service to the economy.

A point worthy of note concerning the above two cited incidents has to be made here, lest we should be misunderstood. We are not suggesting that 68 years, 80 years, or even 1,000 years of age is "old" and thus the capital equipment should get disposed of. On the contrary, we do urge that capital equipments be made as durable as possible to improve the productivity of the economy. But once the equipment has passed its economically useful age, it should either be maintained further or be disposed of. So that it won't become a negative production input, severely diluting the general productivity. Yes, there are also things of time value. The older they are, the more valued they become, regardless of their physical deterioration. In this case, the user or owner of the equipment need bear the cost of maintenance or the cost of insurance against the risk of its potential disruption to the economy.

So we see that capital stock is subject to an aging process like human beings. But human productivity may increase with age due to experience, while capital stock, not. And in the following we will further assume that the productivity of capital stock declines in a "straight-line" fashion. So that a combination of a zero-year-old brand new machine and a 10-year-old one is economically equivalent to two 5-year-old ones. Following this assumption, the average age of the capital stock naturally becomes a matter of concern. And we wonder: given a constant growth rate of investment, a constant disposal rate of the capital stock -- with the disposed parts having a certain weighted average age (as weighted according to their original costs), what will the equilibrium average age of the capital stock be? And how will the size of the capital stock be changing over time?

The above is similar to the following question: given a constant growth rate g of babies (investment), a constant death rate d of the population (the disposal rate of the capital stock), a constant average death age D (the average disposal age of the capital stock), what is the equilibrium average age A of the population? And what is the equilibrium growth rate of the population P (the equilibrium growth rate of the size of the capital stock)? These can be answered as follows:

Suppose that on January 1, 1981, in a certain region, there were P persons, with an average age A. Then, clearly, the total age then was PA. Assume first that there was neither birth nor death in the year 1981. Then, clearly, one year later, on January 1, 1982, everyone of the population P would become 1 year older and the new average age would become (A+1), resulting in a new total age of P(A+1). Now, to simplify our modelling, assume hypothetically that all births and deaths took place on the same day, January 1, 1982, and that B number of babies were born and d proportion of P died at an average age of D. So that starting January 1, 1982, the new population would then become P-Pd+B; and the new total age of the population would become P(A+1) - PdD. As a result, the new average age would become $\frac{P(A+1)-PdD}{P-Pd+B}$. Now, recall that we are interested in determining the equilibrium age of the population. Hence, the new average age should equal the old average age A. I.e., the new average age = $\frac{P(A+1)-PdD}{P-Pd+B}$ = A (the old average age)[1]. Solving the equation for the unknown A, we have P(A+1) - PdD = (P-Pd+B)A, or, PA+P-PdD = PA - PdA + BA, i.e., P - PdD = -PdA + BA = (-Pd+B)A, implying that $A = \frac{P-PdD}{-Pd+B} = \frac{1-dD}{-d+\frac{B}{P}}$. But this is not the final solution, since the birth rate $(\frac{B}{P})$ in the denominator may not stay constant over time. And we must thus further ascertain the equilibrium birth rate $(\frac{B}{P})$ in order to have an equilibrium average age A. To this end, we note that for the ratio $(\frac{B}{P})$ to stay constant, if the numerator B is growing at a constant annual rate of g as assumed and hence having the scenario of B x (1+g) x (1+g) x ..., then the denominator P must also behave likewise, having the same geometric progression of P x (1+g) x (1+g) x ..., i.e., share the same annual growth rate g (as that of B). Otherwise, either the ratio $(\frac{B}{P})$ is to rise or to fall, as time goes by. Knowing that[2] the equilibrium population growth rate = the given baby growth rate g, we next utilize the familiar relationship of population growth rate = birth rate - death rate. So that $g = \frac{B}{P} - d$, and the equilibrium birth rate $\frac{B}{P} = g+d$.

Now, we can further simplify the solution for the equilibrium population average age A. In the above, we first derived $A = \frac{1-dD}{-d+\frac{B}{P}}$;

we next derived $\frac{B}{P} = g+d$. Clearly then, $A = \frac{1-dD}{-d+\frac{B}{P}} = \frac{1-dD}{-d+g+d} = \frac{1-dD}{g}$.

But in the above solution for A, the disposal rate d apparently is related to the average disposal age D. For intuitively, if the average death age D (which is the same thing as the average life span) increases, the death rate d should decrease. Let's now examine this relationship by considering a special[3] case, in which the whole population shares the same disposal age D. And for expository neatness, we will just stick to this simplified assumption in our following analysis. Under such condition, the disposal rate d =

$$\frac{\text{that part of the population having now the last possible age } (D-1)}{\text{the whole population}},$$

where the whole population is the sum of all the age groups, from age (D-1) back to 0, with each younger age successively having its group population growing at an annual rate of g, due to the assumed constant baby growth rate of g.

So that, d =

$$\frac{1 \text{ (population of age } D-1)}{1 \text{ (population of age } D-1) + (1+g) + (1+g)^2 + \ldots + (1+g)^{D-1} \text{(of age 0)}}$$

$$= \frac{1}{\frac{1-(1+g)^D}{1-(1+g)}} = \frac{1}{\frac{1-(1+g)^D}{-g}}$$

$= \frac{g}{(1+g)^D-1}$. And indeed, <u>as D increases, d decreases</u>.

And then, our earlier derived equation $A = \frac{1-dD}{g}$ can now be finalized to be $A = \left(1-\frac{gD}{(1+g)^D-1}\right)/g = \frac{1}{g} - \frac{D}{(1+g)^D-1}$

As an illustration, suppose g = 4%, D = 4 years old. Then

$A = \frac{1}{4\%} - \frac{4}{1.04^4-1} = 25 - \frac{4}{0.17} = 25 - 23.5 = 1.5$ years old.

Now, if D = 8 years old, then $A = \frac{1}{4\%} - \frac{8}{1.04^8-1} = 25 - \frac{8}{0.36} =$ 25-22 = 3 years old. <u>So we see that as D increases, A also increases.</u>[4]

By now, we clearly see the dilemma: on the one hand, a prolonging of the life span or the disposal age D can lower the disposal rate d and thus increase the population size P (since $P = \frac{B}{g+d}$, from p. 4 , where we derived the equilibrium birth rate $\frac{B}{P} = g + d$); but on the other hand, an increase in D also causes an increase in the equilibrium average age A of the population, reflecting a dilution of the average productivity of the population. Simply put, given the same growth rate g of investment and the same engineering durableness of the investment goods, we can either have a larger but more diluted capital stock or a smaller but more efficient capital stock.

With this, we conclude that: (1) With a given amount of investment, only when we increase the original engineering durability of the capital stock, can we have larger and/or more efficient capital stock; (2) otherwise, we could try to maximize the size of the capital stock by extending the disposal age of the individual capital stock as much as possible, but not beyond the point where the individual capital stock starts to provide a net negative service to the economy.[5]

Footnotes

1. This equilibrium condition can be put more formally as follows:

   Let t stand for time point. Then, in a continuous model, $\frac{dA}{dt} = \frac{PA+P-Pd\cdot D}{P-Pd+B} - A$; and whenever $B-Pd>0$, the equilibrium A is a stable one; since under such condition $(\frac{PA}{P-Pd+B})$ of the first term changes (in the same direction as but) less than the second term A.

2. More formally, the change in population $\frac{dP(t)}{dt} = -d\cdot P(t)+B(t)$. Also, we have assumed that $B(t)=B(0)e^{gt}$. So that the solution of this differential equation is:

   $P(t)=(\frac{B(0)}{g+d})e^{gt} + (P(0)- \frac{B(0)}{g+d})e^{-dt}$. And as $t \to \infty$, $P(t) \to (\frac{B(0)}{g+d})e^{gt} = \frac{B(t)}{g+d}$; so that the equilibrium population growth rate = the baby growth rate g. And here we note the distinction between the population size P and the population growth rate g; and in this model, a lower d makes a higher P but does not affect the equilibrium population growth rate g.

3. The original general case is that the disposal rate is a weighted average of the individual disposal rates, i.e., $d=w_1d_1+w_2d_2+ \dots +w_nd_n$, where w_i , $i=1,2, \dots, n$, is the proportion of the population P that has a separate individual disposal rate d_i, with its corresponding disposal age D_i (as determined from

the relationship $d_i = \frac{g}{[(1+g)^{D_i}]-1}$, derived in p.5);

and the overall average disposal age D

$$= \frac{w_1d_1D_1+w_2d_2D_2+ \; \dots \; +w_nd_nD_n}{w_1d_1+w_2d_2+ \; \dots \; +w_nd_n}$$

$$= \frac{w_1d_1D_1+w_2d_2D_2+ \; \dots \; +w_nd_nD_n}{d}$$

$$= (\frac{w_1d_1}{d})D_1+(\frac{w_2d_2}{d})D_2+ \; \dots \; +(\frac{w_nd_n}{d})D_n.$$

Under such circumstances, the overall disposal rate d is not a function of the overall average disposal age D any more(just consider the case where $d_1=d_2= \dots =d_{n-1} =0$; then, $d=w_1d_1+w_2d_2+ \dots +w_{n-1}d_{n-1}+w_nd_n=w_nd_n$, and $D=\frac{w_nd_nD_n}{w_nd_n} = D_n$; so that with the same $D=D_n$, $d=w_nd_n = \frac{w_ng}{(1+g)^{D_n}-1}$ can still vary with the weight w_n); and we will rely on the original equation $A= \frac{1-dD}{g}$, which shows that, to minimize d (so as to maximize the size of the capital stock P) subject to an acceptable A (so as to maintain a certain level of the overall efficiency of the capital stock), the disposal d should be made to have a high average disposal age D, consistent with the conclusion given in the main body of the paper.

4. $A= \frac{1}{g} - \frac{D}{(1+g)^D-1}$ implies that $\frac{dA}{dD}>0$. This is so since $dA/dD = \{-((1+g)^D-1)+D(1+g)^D\ln(1+g)\}/((1+g)^D-1)^2$; and the numerator $\{1+D(1+g)^D\ln(1+g)-(1+g)^D\}$ $=1+(1+g)^D(D\ln(1+g)-1)>0$, if $g>0$, and $D>0$.

5. A pioneer work of applying the human population analogy to the capital stock similar to the treatment given here can be dated in 1937 (Kurtz [1]). There, the author for the first time gave a very thoughtful engineering analysis concerning how the productivity of capital assets declines due to aging. And in several places, the author exactly used the phrase "the renewal of renewals" of capital assets -- a major notion on which this current paper is based. But Kurtz's emphasizing point is not the external diseconomy associated with the overage capital asset, which is the central point of this paper. After Kurtz, the bulk of the literature research about depreciation has been mainly devoted to the measurement of capital stock (without specifying the age dimension), the accounting treatment, tax policy, or inflation effect. Recently, for the housing industry, a "filtering hypothesis" has been brought up (Margolis [2]) to explain how houses filter down from high to low quality. But the aging process, as Kurtz initially noted and this paper has stressed, is a general phenomenon. And it is hoped that this paper can draw the public attention to this important issue, realizing that a tiny worn-out part in our environment may cause a serious incident. For one thing, we can never allow a communication breakdown in this nuclear age.

References

1. Kurtz, Edwin B. The Science of Valuation and Depreciation, 1937.

2. Margolis, S.E., "Depreciation of Housing: An Empirical Consideration of the Filtering Hypothesis," Review of Economics and Statistics, February 1982.