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August 1984

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Working Paper No. 387

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It has been claimed very often in the Mass Media that a corporate income tax cut will cause higher investment. But on the other hand, economic and financial literature declaring the tax neutrality on investment can also be easily found (e.g., Boadway and Bruce [1]). The former argues that a lower corporate income tax increases the cash inflows in the capital budgeting calculation and hence makes more investment projects profitable. The latter's argument in the final analysis boils down to the familiar economic proposition that profit tax does not affect input and output quantities, or, the effect of a corporate income tax rate t on profit is equivalent to multiplying all input and output prices by (1-t) and thus, by the neoclassical economic proposition that input demand and output supply are homogeneous of degree zero in prices, does not affect input and output decisions.

As the above two arguments are both so very convincing and popular and yet they lead to contradicting conclusions, this paper seeks to add an observation.

Our observation starts from the economics and finance logic that if a percentage tax rate t were directly applied to the net present value (NPV) of an investment project, then such taxation would not affect the acceptability of the project. Because a project with positive pre-tax NPV (= $-C + \sum_{j=1}^{n} \frac{EBDIT_j}{(1+k)^j}$, where C is the initial cash outflow, $EBDIT_j$ is the project's earnings before deducting the expenses of depreciation D, interest I, and tax T in time period j , and $k = W_i k_i + (1-W_i) ke$ is[1] the marginal weighted average cost of capital, with the interest rate k_i weighted by the debt ratio of financing W_i and the required rate of return on equity k_e weighted by $(1-W_i)$) would still have a positive after-tax NPV $\left(= (1-t)\left(-C + \sum_{j=1}^{n} \frac{EBDIT_j}{(1+k)^j}\right)\right)$, although smaller.

Now, the actual tax treatment is such that after-tax NPV =

$$- C + \sum_{j=1}^{n} \frac{EBDIT_j - t(EBDIT_j - D_j - I_j)}{(1+k)^j}$$

Then, does the actual treatment:

$$-C + \sum_{j=1}^{n} \frac{EBDIT_j - t(EBDIT_j - D_j - I_j)}{(1+k)^j} \text{, which is}$$

$$-C + \sum_{j=1}^{n} \frac{EBDIT_j(1-t)}{(1+k)^j} + t\sum_{j=1}^{n} \frac{D_j+I_j}{(1+k)^j} \text{, equal}$$

the hypothetical treatment:

$$(1-t)(-C + \sum_{j=1}^{n} \frac{EBDIT_j}{(1+k)^j}) \text{, which is}$$

$$-C + ct + \sum_{j=1}^{n} \frac{EBDIT_j(1-t)}{(1+k)^j} \text{ ?}$$

On a comparison, we find that they equal each other if $\sum_{j=1}^{n} \frac{D_j+I_j}{(1+k)^j} = C$. But since C in general is financed by both debt and equity, on theoretical ground, C should equal $\sum_{j=1}^{n}[D_j + I_j +$ required dollar return on equity capital (in economic terms, the normal profit)] x $(1+k)^{-j}$. In other words, if the required net income (normal profit) were tax deductible like the interest expense, then after-tax NPV = $-C + \sum_{j=1}^{n}$ [$EBDIT_j$ - $t(EBDIT_j - D_j - I_j$ - required net income or normal $profit_j)]$ x $(1+k)^{-j}$

$$= - C + Ct + (1-t) \sum_{j=1}^{n} \frac{EBDIT_j}{(1+k)^j}$$

$$= (1-t)(-C + \sum_{j=1}^{n} \frac{EBDIT_j}{(1+k)^j}) \text{, and then}$$

t would not affect the acceptability of the project.

The above seems to suggest that due to the required equity return being wrongfully taxed, part of the initial cost of the project C is unduly taxed. I.e., the current tax treatment is such that

After-tax NPV = $(1-t)\ (-C + \sum_{j=1}^{n} \frac{EBDIT_j}{(1+k)^j}) - t\sum_{j=1}^{n}$ (required net income)$_j$x$(1+k)^{-j}$;

and, therefore, a lower tax rate t may make a previously unprofitable project turn to a profitable one and thus lead to higher investment activity. But on a closer examination, we see that[2] even with straight-line depreciation so that $D_j = \frac{C}{n} = D$, $\sum_{j=1}^{n} \frac{D + I_j + \text{Required net income}_j}{(1+k)^j} > C$.

This means that there exists the possibility of $\sum_{j=1}^{n} \frac{D+I_j}{(1+k)^j} \gtreqless C$. This happens when

$$\sum_{j=1}^{n} \frac{I_j}{(1+k)^j} \geqq C - \frac{C}{n} \sum_{j=1}^{n} \frac{1}{(1+k)^j}$$

. Assuming that I_j is constant across all the n periods and W_i is the debt ratio of C , we can factor out $I_j = I = CW_i k_i$, (where k_i is the interest rate) from the left-hand summation .

I.e., $$CW_i k_i \sum_{j=1}^{n} \frac{1}{(1+k)^j} \geqq C - \frac{C}{n} \sum_{j=1}^{n} \frac{1}{(1+k)^j} .$$

After simplification , we find that[3]

$$k_i \geqq \frac{1}{W_i} \left(\frac{k}{1-\frac{1}{(1+k)^n}} - \frac{1}{n} \right)$$ is the condition .

The above shows that either

$$k_i \geqq \frac{1}{W_i} \left(\frac{k}{1-\frac{1}{(1+k)^n}} - \frac{1}{n} \right)$$ with straight-line depreciation or an accelerated depreciation may make $\sum_{j=1}^{n} \frac{D_j + I_j}{(1+k)^j} \geqq C$ and thus C may be completely tax deductible after all ; and therefore a change in the corporate income tax rate may not affect investment decisions.

Footnotes

1. The treatment of using $k = W_i k_i (1-t) + (1-W_i) k_e$ as the discount rate, with debt being artifically treated on the same footing as equity, together with using [EBIT (1-t) + Depreciation expense] as the after-tax artificial cash inflow is equivalent to the treatment here -- using $k = W_i k_i + (1-W_i) k_e$ as the discount rate together with using (EBDIT - the actual dollar taxation) as the after-tax cash inflow. The following demonstrates the equivalence between the two treatments.

First, we note $$\sum_{j=1}^{n} \frac{1}{(1+k)^j} = \frac{1 - \frac{1}{(1+k)^{n+1}}}{1 - \frac{1}{(1+k)}} - 1$$

$$= \frac{1+k - \frac{1}{(1+k)^n}}{k} - 1 = \frac{1 - \frac{1}{(1+k)^n}}{k}$$

Next, we make the following notation:

$k = W_i k_i + (1-W_i)k_e$; and $\hat{k} = W_i k_i(1-t) + (1-W_i)k_e$.

Third, we note that (the interest expense I + the required after-tax net income π_o) = the initial cash outflow C x k ; and (I + the required before-tax equity income $\frac{\pi_o}{1-t}$) = $CW_i k_i + \frac{C(1-W_i)k_e}{1-t} = \frac{c\,\hat{k}}{1-t}$.

Fourth, we compare the two approaches:

(i) the k approach --

$$\left(\sum_{j=1}^{n} \frac{I+\pi_o}{(1+k)^j}\right) + \frac{C}{(1+k)^n} = Ck\left(\frac{1-\frac{1}{(1+k)^n}}{k}\right) + \frac{C}{(1+k)^n} = C ;$$

(ii) the $\hat{k}$ approach --

$$\left(\sum_{j=1}^{n} \frac{(I+\frac{\pi_o}{1-t})(1-t)}{(1+\hat{k})^j}\right) + \frac{c}{(1+\hat{k})^n} = C\hat{k}\left(\frac{1-\frac{1}{(1+\hat{k})^n}}{\hat{k}}\right) + \frac{C}{(1+\hat{k})^n} = C$$

Therefore, $C = \sum_{j=1}^{n} \frac{I+\Pi_o}{(1+k)^j} + \frac{C}{(1+k)^n} = \sum_{j=1}^{n} \frac{(I+\frac{\Pi_o}{1-t})(1-t)}{(1+\hat{k})^j} + \frac{C}{(1+\hat{k})^n}$.

Thus, $\sum_{j=1}^{n} \frac{I+\Pi_o}{(1+k)^j} + \frac{C}{(1+k)^n} + X = \sum_{j=1}^{n} \frac{(I+\frac{\Pi_o}{1-t})(1-t)}{(1+\hat{k})^j} + \frac{C}{(1+\hat{k})^n} + X$

= Present value PV of a project ; the project is acceptable if and only if $X \geqq 0$; so that if the NPV based on k is nonnegative, it must be that $X \geqq 0$, which guarantees the NPV based on $\hat{k}$ is also nonnegative.

2. We want to show $\sum_{j=1}^{n} \frac{D+I+\Pi_o}{(1+k)^j} > C$, where

$D = \frac{c}{n}$, $I+\Pi_o = ck$.

That is, $\sum_{j=1}^{n} \frac{\frac{c}{n} + ck}{(1+k)^j} > C$. This is so since

$$\sum_{j=1}^{n} \frac{\frac{c}{n} + ck}{(1+k)^j} = C(\frac{1}{n} + k) \times \frac{1-\frac{1}{(1+k)^n}}{k} \text{ (refer to footnote 1)}$$

$$= C\left[\frac{1-\frac{1}{(1+k)^n}}{nk} + 1 - \frac{1}{(1+k)^n}\right] > C ,$$

which in turn is because $\frac{1-\frac{1}{(1+k)^n}}{nk} - \frac{1}{(1+k)^n}$

$= \frac{(1+k)^n - 1 - nk}{nk(1+k)^n} > 0$ (by binomial theorem).

3. $$k_i \geqq \left(\frac{1}{W_i}\right)\left[\left(\sum_{j=1}^{n} \frac{1}{(1+k)^j}\right)^{-1} - \frac{1}{n}\right] = \frac{1}{W_i}\left(\frac{k}{1 - \frac{1}{(1+k)^n}} - \frac{1}{n}\right)$$

(refer to footnote 1) .

Reference

[1] Boadway, R.W. and Bruce, N., "Depreciation and Interest Deductions and the effect of the Corporation Income Tax on Investment," Journal of Public Economics, February 1979.