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INTRODUCTION TO A MORE FRUITFUL USE OF COMPARATIVE STATIC ANALYSIS

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This paper seeks to serve as a minor supplement to Professor Samuelson's [1] well elaborated general research methodology of comparative static analysis, in the hope that this powerful tool of analysis might be used by the economic and business professions with additional insight.

Suppose we have an arbitrary differentiable function $Q = f(X,Y,Z)$. Then we can have the following mathematical derivation:

$$dQ = \frac{\partial Q}{\partial X} dX + \frac{\partial Q}{\partial Y} dY + \frac{\partial Q}{\partial Z} dZ$$

$$= X \frac{\partial Q}{\partial X} \frac{dX}{X} + Y \frac{\partial Q}{\partial Y} \frac{dY}{Y} + Z \frac{\partial Q}{\partial Z} \frac{dZ}{Z} \quad , \quad \text{thus}$$

$$\frac{dQ}{Q} = \left(\frac{X}{Q} \frac{\partial Q}{\partial X}\right)\frac{dX}{X} + \left(\frac{Y}{Q} \frac{\partial Q}{\partial Y}\right) \frac{dY}{Y} + \left(\frac{Z}{Q} \frac{\partial Q}{\partial Z}\right) \frac{dZ}{Z} \quad . \quad \text{or} \quad ,$$

in simpler notation, $Q^* = Q_X X^* + Q_y Y^* + Q_Z Z^*$,
where a '*' over a variable means a percentage change in that variable and a coefficient such as Q_X means the partial elasticity of Q with respect to X . In this way, the familiar concept of elasticity, which has the nice feature of being unit free, can be well utilized. The following will utilize Professor Samuelson's example[1] to demonstrate such a use of elasticity.

Given the demand equation $D(x,\alpha) - P = 0$, where D, the demand price, is a function of the quantity X and a parameter α , and the supply equation $S(x) - P = 0$, where S, the supply price, is likewise a function of the quantity x, we wonder what will happen to the equilibrium values of the quantity (x°) and the price (p°) if there is a change in the parameter α .
And we have the answer:

$$\left(\frac{\partial X}{\partial \alpha}\right)^\circ = - \frac{\frac{\partial D}{\partial \alpha}}{\frac{\partial D}{\partial X} - S'} \quad ,$$

$$\left(\frac{\partial P}{\partial \alpha}\right)^{\circ} = - \frac{S' \frac{\partial D}{\partial \alpha}}{\frac{\partial D}{\partial X} - S'}$$

So that we can see the directions of change in X° and P° by examining the signs of the right-hand sides of the above two equations.

But as a supplement, we might also be interested in the quantitative changes in X° and P°. This can be done as follows by utilizing our earlier derivation and notations. I.e.,

$P^* = D^* = D_X X^* + D_\alpha \alpha^*$, for the demand equation;

$P^* = S^* = S_X X^*$, for the supply equation .

Therefore,

$$X^{\circ *} = \left(- \frac{D_\alpha}{D_X - S_X} \right) \alpha^* \quad \text{, and}$$

$$P^{\circ *} = \left(- \frac{S_X \cdot D_\alpha}{D_X - S_X} \right) \alpha^*$$

Thus, if $D_\alpha = 3$, $D_X = -4$, $S_X = 6$, and $\alpha^* = 10\%$, then $X^{\circ *} = \left(- \frac{3}{-4-6} \right) \times 10\% = 3\%$, and $P^{\circ *} = \left(- \frac{6 \times 3}{-4-6}\right) \times 10\% = 18\%$ (of course, for a meaningful calculation such as above, the stability condition must first be satisfied).

The above supplemental quantitative comparative static analysis can often be so very convenient in use[2] , as it linearizes system of equations in the concrete form of percentage changes and elasticities , that we feel it our necessary duty to present it to the public.

Footnotes

1. This example is completely copied from page 17 to page 19 of Professor Samuelson's book [1].

2. In particular, we record the following special relationships:

   Assuming that a,b, and c are constant, then of course

   $(cX^aY^b)^* = aX^* + bY^*$; also ,

$$(aX + bY)^* = \left(\frac{aX}{aX+bY}\right)X^* + \left(\frac{bY}{aX+bY}\right)Y^*$$

Reference

[1] Samuelson, P.A., "Foundations of Economic Analysis," 1976.