Division of Research
Graduate School of Business Administration
The University of Michigan

August 1984

A NOTE ON CORPORATE FINANCIAL MANAGEMENT

Working Paper No. 384

Lu Wang

A NOTE ON CORPORATE FINANCIAL MANAGEMENT

Lu Wang
Assistant Professor of Finance
School of Management
The University of Michigan-Dearborn

(I wish to thank my colleague, Professor Cedric V. Fricke, for his helpful comments.)

Abstract

The premise of corporate financial management as taught in some textbooks is the maximization of shareholders' wealth, or more precisely, the market price per share of the stock. Following this premise, the paper derives the optimal investment and financing condition, which shows not to be equating marginal internal rate of return to marginal cost of capital and securing the minimum cost of capital over the debt/equity ratios, as concluded in the textbooks.

INTRODUCTION

The theory of corporate financial management as taught in some textbooks[1] shows an inconsistency in the logical link between maximizing price per share of the company's common stock and the investment criterion of equating marginal internal rate of return to marginal cost of capital. Textbooks often begin with the emphasis that the goal of corporate financial management is to maximize shareholders' wealth, or more precisely, the market price per share of the stock[2], not corporate profit as in economics. But later when discussing investment decision, the rule of acceptance essentially follows the conclusion of standard microeconomics, viz., to exhaust investment opportunities until the marginal internal rate of return equals the marginal cost of capital - which is derived from maximizing profit for a firm. It is doubtful that replacing the maximizing shareholders' wealth assumption with the maximizing profit assumption can lead to the same conclusion. The following will first give a simple numerical example to show this discrepancy. Then, the investment rule based on maximizing the market price per share of the stock will be introduced. Next, a revised financing rule will also be shown to be necessary. Finally, a more comprehensive numerical illustration will be given.

I. A Simple Illustrative Example

Assume for simplicity that a newly-created corporation is considering whether to have one plant or two plants. Plant A can yield a marginal internal rate of return 20%; plant B can yield a marginal internal rate of return 15%. The company has a constant cost of capital 10%. Then, according to the investment criterion of comparing marginal internal rate of return with marginal cost of capital, both plant A and plant B should be accepted. Now, suppose both plants cost the same \$1,000,000. Then after one year's operation, plant A can contribute an excess profit of \$1,000,000 x (20% - 10%) = \$100,000; plant B can contribute an excess profit of \$1,000,000 x (15% - 10%) = \$50,000. And the total excess profit belonging to the shareholders is then \$100,000 + \$50,000 = \$150,000. Assume that the corporation has a debt-equity ratio one to one. So that the shareholders finance 1/2 x (\$1,000,000 + \$1,000,000) = \$1,000,000. And so they earn an excess profit rate of $\frac{\$\ 150,000}{\$1,000,000} = 15\%$. But wouldn't it make the shareholders better off if only plant A is bought? Yes, this is indeed the case. If the corporation only buys plant A, then, true, the total excess profit decreases; but this smaller excess profit of \$100,000 as generated by plant A, when divided by a now also shrunk equity financing of \$500,000 (= the same equity ratio 50% x the cost of plant A \$1,000,000), yields an excess profit rate of $\frac{\$100,000}{\$500,000} = 20\%$, which is higher than that of the two-plant case. Critics may say that the 1-plant case has fewer shareholders. But so far as the market price per share is concerned, there is no doubt that the 1-plant case yields a higher market price per share.

Thus, a case can be made that if a corporation is immune from competition in its product market, then in order to maximize its existing shareholders' wealth, the corporation may achieve this objective by deliberately holding back from further investment (despite that the marginal internal rate of return is still higher than the marginal cost of capital). So, let us now pursue the condition of maximizing the market price per share of the stock.

II. The Condition of Maximizing Market Price Per Share

A. For Investment

The condition of maximizing stock price per share is as follows[3]:

$$\frac{p}{ac} = /\varepsilon/ \cdot \left(1 - \frac{1}{s}\right) \tag{1}$$

where $p \equiv$ the average percentage internal rate of return on total capital assets \$K, i.e., $\$K \cdot p$ = net income + interest on debt = the total dollar return on capital;

$ac \equiv$ weighted average percentage cost of capital; thus, $\$K \cdot ac$ = the required dollar return on equity + interest on debt = the total dollar cost of capital \$TC;

$\varepsilon \equiv \frac{\%\Delta K}{\%\Delta p} < 0$ is the elasticity measuring the percentage change in capital assets \$K needed in order to increase the average internal rate of return p by one percent; i.e., capital assets are subject to the law of diminishing marginal return; as \$K increases, p decreases; so that $\varepsilon<0$;

$\frac{1}{s} \equiv \frac{\%\Delta TC}{\%\Delta K} > 0$ is another elasticity measuring the percentage change in the total dollar cost of capital \$TC as a result of a one-percent increase in the total capital assets \$K.

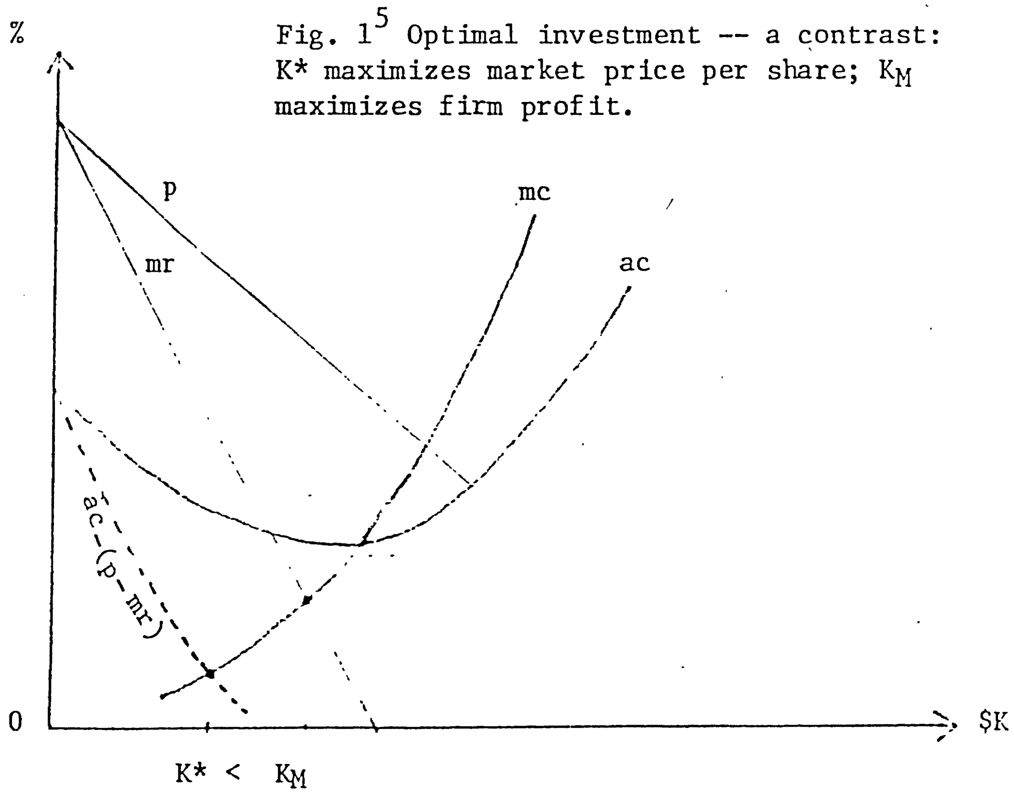
The above equation $\frac{p}{ac} = /\varepsilon/ \cdot (1 - \frac{1}{s})$ can be rewritten as follows. First let mr denote the marginal (percentage) internal rate of return on capital assets \$K, i.e., $mr = \frac{d}{dK}(K \cdot p)$; also, let mc denote the marginal percentage cost of capital, i.e., $mc = \frac{d}{dK}(TC) = \frac{d}{dK}(K \cdot ac)$. Then, following the familiar economic relations[4]: $p - mr = \frac{p}{/\varepsilon/}$ and $s = \frac{ac}{mc}$, and rearranging equation (1) $\frac{p}{ac} = /\varepsilon/ \cdot (1-\frac{1}{s})$ as $\frac{p}{/\varepsilon/} = ac\ (1-\frac{1}{s})$, we have:

$p - mr = ac - ac(\frac{mc}{ac}) = ac - mc$; or

$$p - ac = mr - mc. \qquad (2)$$

The preceding equation shows that except in the case of p = ac (where only a normal profit is earned), the marginal internal rate of return mr must be greater than the marginal cost of capital mc, i.e., $p - ac = mr - mc > 0$, in order for the company to maximize its market stock price per share. This in turn means that a corporation with the goal of maximizing its stock price per share should invest less than one with the goal of maximizing firm profit, since the investment is to be stopped before mr = mc. This is illustrated in the following figure:

Fig. 1[5] Optimal investment -- a contrast: K* maximizes market price per share; K_M maximizes firm profit.



B. For Financing

Moreover, the assumption of maximizing the market price per share of the stock also has implication on the corporate financing decision, i.e., the optimal debt or equity ratio. Now, let w denote equity ratio, and $\sigma \equiv \frac{\%\Delta TC}{\%\Delta w}$ denote another elasticity measuring the percentage change in the total dollar cost of capital due to a one-percent increase in the equity ratio w, holding capital assets \$K constant. Then, the financing condition of maximizing the market price per share of the stock is [6]:

$$\frac{p}{ac} = 1 - \sigma, \tag{3}$$

or, $$\sigma = 1 - \frac{p}{ac} \tag{4}$$

But once again so long as the corporation earns an excess profit, we have $\frac{p}{ac} > 1$. This means that $\sigma = 1 - \frac{p}{ac} < 0$. But $\sigma \equiv \frac{\%\Delta TC}{\%\Delta w} < 0$ means that if the corporation increases its equity ratio w, it can still cut down its total dollar cost of capital TC further. I.e., the optimal equity ratio w* maximizing the market price per share is short of reaching the point where the minimum total cost of capital is attained. This is illustrated in the following figure:

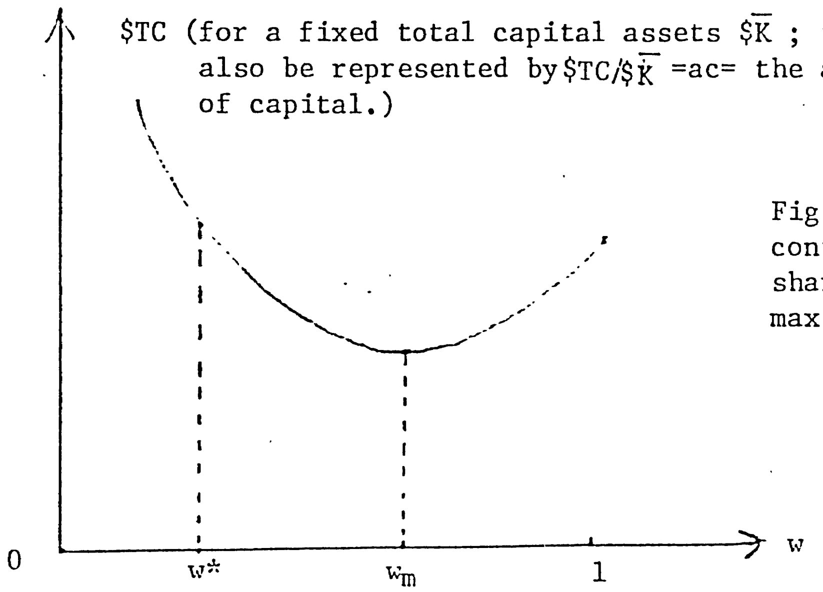


Fig.2 Optimal financing -- a contrast: Maximizing price per share implies a point like w^*; maximizing profit implies w_m.

So once again we see that the assumption of maximizing the market price per share of the stock leads to a different financing conclusion, viz., the corporation should employ less equity than that as derived from maximizing firm profit.

Summarizing the above optimal investment and financing decisions for maximizing the market price per share of the corporation's stock, we have the following combined equation (from equation (1)and(3)on p.6 & 8)

$$\frac{p}{ac} = /\varepsilon/ \cdot (1 - \frac{1}{s}) = 1 - \sigma . \tag{5}$$

We will now design a concrete numerical example to illustrate the above relations. But first, we notice that the above conditions are expressed in terms of elasticities. So to ease the computation burden, we will design the relevant given functions having some constant elasticities. Also, in order to have some neat final numerical solutions, we shall make up the functions in a somewhat unnatural way.

III. A Comprehensive Numerical Illustration

(i) Assume that the average internal rate of return on total capital assets \$K is:

$p = 2e^{-2}K^{-0.25} \approx 0.3K^{-0.25}$ (where $e \approx 2.71$ is the natural number; and the constant power (-0.25) of K yields a constant elasticity of $/\varepsilon/ \equiv / \frac{d\log K}{d\log p}/ = \frac{1}{0.25} = 4.$)

This equation essentially has the following picture:

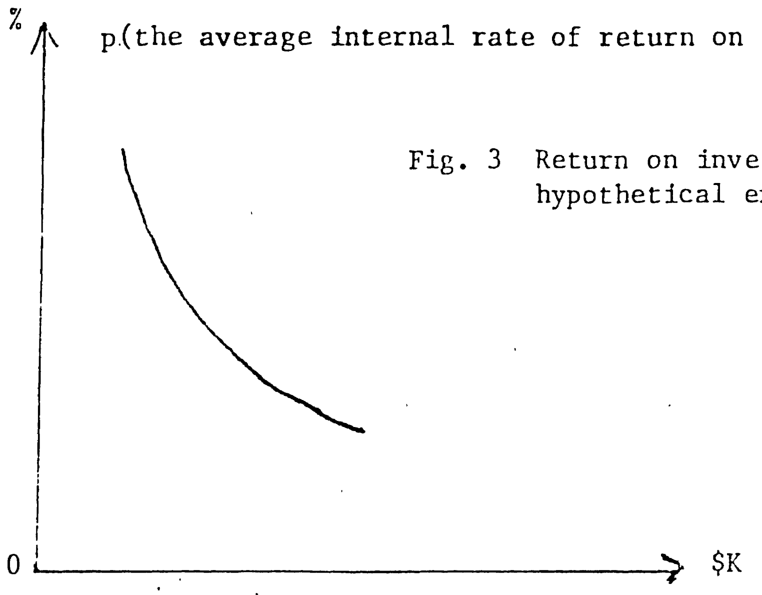


Fig. 3 Return on investment of the hypothetical example.

So that the equation reflects the law of diminishing marginal (and thus average) product of capital from economics.

(ii) Assume that the average percentage cost of capital

$$ac = K^{-0.5}\ (10ew)^{-2 + \log \log (10ew)}$$

$$\approx K^{-0.5}\ (27w)^{-2 + \log \log (10ew)}$$

$$TC = K \cdot ac = K^{0.5}\ (10ew)^{-2 + \log \log (10ew)}$$

$$\approx K^{0.5}\ (27w)^{-2 + \log \log (27w)}$$

(where the log is of base e ; $0 < w \leq 1$; $\frac{1}{s} \equiv \frac{\partial \log TC}{\partial \log K}$

$= 0.5$, the power attached to K; and [7] $\sigma \equiv \frac{\partial \log TC}{\partial \log w} = -1 + \log \log (10ew)$.)

So that the average percentage cost of capital ac is a function of both the capital investment $K and the equity ratio w. For the relationship between ac and $K, the above equation yields the following picture:

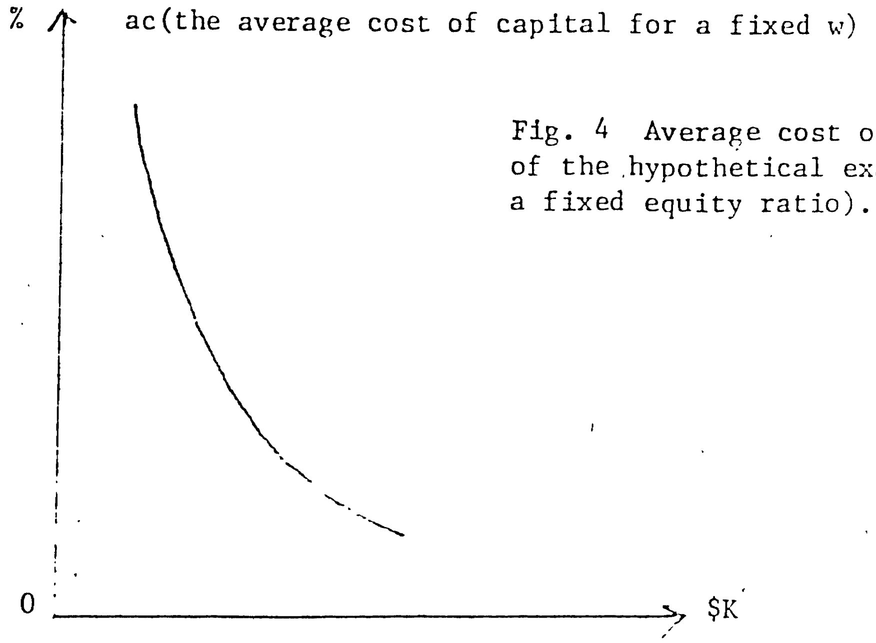


Fig. 4 Average cost of capital of the hypothetical example (for a fixed equity ratio).

Here, we note that the above all-the-way downward sloping ac curve is attributed to the constant power of (-0.5) attached to K and is only supposed to depict part of the U-shaped whole ac curve (of fig. 1.) Here we only need the downward sloping part because this is where the optimal K* lies (since recall from page 7, ac - mc = p - mr>0, implying that ac > mc, so that the ac at K* is still decreasing, from standard economics).

For the relationship between ac and w, the above ac equation does result in a U-shaped ac curve along the w-axis, as shown in the following:

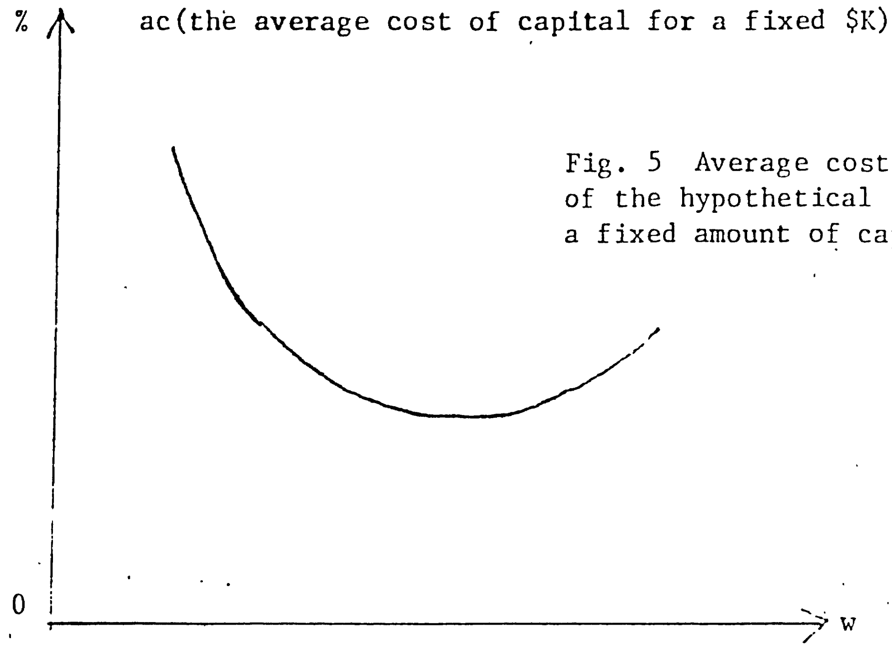


Fig. 5 Average cost of capital of the hypothetical example (for a fixed amount of capital assets).

Given the above equations of p and ac, we now apply to them our earlier derived conditions:

$$\frac{p}{ac} = /\varepsilon/ \cdot (1 - \frac{1}{s}) = 1 - \sigma \ . \qquad (5)$$

Since from the parenthetical remarks on p. 9 & 10, we have already obtained $/\varepsilon/ = 4$, $\frac{1}{s} = 0.5$, and $\sigma = -1 + \log \log(10ew)$, we therefore have: $/\varepsilon/ \cdot (1 - \frac{1}{s}) = 4 \times 0.5 = 2 = 1 - \sigma$, implying $-1 = \sigma = -1 + \log \log(10ew)$, implying in turn the optimal equity ratio $w^* = 0.1$, or 10% . Substituting $w^* = 0.1$ into the ac equation, we then have:

$$ac = K^{-0.5}(10e \times 0.1)^{-2} + \log \log(10e \times 0.1)$$

$$= K^{-0.5}e^{-2}; \text{ accordingly,}$$

$$\frac{p}{ac} = (2e^{-2}K^{-0.25}) \ / \ (e^{-2}K^{-0.5}) = 2K^{0.25} \ .$$

But we have just obtained

$\frac{p}{ac} = /\varepsilon/ \cdot (1 - \frac{1}{s}) = 4 \times (1 - 0.5) = 2$; thus,

$2 = \frac{p}{ac} = 2K^{0.25}$, implying the optimal investment $K^* = \$1$.

Under such circumstances, the average percentage cost of capital $ac = e^{-2} \approx 14\%$; the average percentage internal rate of return $p = 2e^{-2} \approx 28\%$; the excess profit rate over equity capital

$$= \frac{\$K \cdot p - \$K \cdot ac}{\$K \cdot w}$$

$$= \frac{\$\ (2e^{-2} - e^{-2})}{\$\ 0.1} \approx \frac{\$0.14}{\$0.1}$$

$$= 140\% = 1.4 ;$$

so that the market price per share of the stock = the book value per share of the stock x (1 + 140%) = the book value per share x 2.4.

Now, if we apply the profit-maximizing financing and investment rules to the above numerical example, we then have $w_m = \frac{e^e}{10e} \approx 55\%$ as the equity ratio incurring the minimum cost of capital (where the elasticity $\sigma = 0$, thus implying $0 = \sigma = -1 + \log\log(10ew)$, so that $w_m = \frac{e^e}{10e} \approx 55\%$) , and $K_M = \$\infty$ as the profit-maximizing investment (due to in this example mr>mc despite $K \to \$\infty$, meaning practically that the firm should invest as much as possible until mr = mc). Under such circumstances, on substituting $w_m = \frac{e^e}{10e}$ into ac, we have the excess profit rate over equity capital $= \lim_{K\to\infty} \frac{(K \cdot p - K \cdot ac)}{K \cdot w} = \lim_{K\to\infty} \frac{p - ac}{w}$

$$= \lim_{K\to\infty} \frac{2e^{-2}K^{-0.25} - e^{-e}K^{-0.5}}{e^e/10e} = \lim_{K\to\infty} \left(\frac{20e^{-1-e}}{K^{0.25}} - \frac{10e^{1-2e}}{K^{0.5}}\right)$$

= 0 ; so that the market price per share of the stock = the book value per share of the stock, which apparently is lower than that of the earlier case.

IV. Summary

From the above example and the earlier discussions, we clearly see that the investment rule of achieving the condition of marginal internal rate of return equal to marginal cost of capital as well as the financing rule of securing the minimum cost of capital over the debt/equity ratios does not result in the maximization of the market price per share of the corporation's common stock.

References

1. Charles W. Haley & Lawrence D. Schall. The Theory of Financial Decisions. McGraw-Hill, 1979.

2. James C. Van Horne. Financial Management and Policy, 6th ed., Prentice-Hall, 1983.

3. J. Fred Weston & Eugene F. Brigham. Essentials of Managerial Finance, 6th ed., The Dryden Press, 1982.

Notes

1. E.g., Van Horne, p.6, ". . . we assume that the objective of the firm is to maximize its value to its shareholders. Value is represented by the market price of the company's common stock, . . . "
2. E.g., in Weston & Brigham, p.6, ". . . the goal of financial management is generally expressed in terms of maximizing the value of the ownership shares of the firm - in short, maximizing share price." But in Haley & Schall, the authors refer to "share value" as the value of total shares outstanding, not value per share. This was made explicit in p. 282 of the text, where definite mathematical notation was assigned and the following was remarked: "It will now be shown that if a decision by management results in a higher total value for the firm, it will be to the shareholders' benefit to make such a decision even if share prices fall."(Emphasis original.)
3. To maximize the market price per share of the corporation's stock, we maximize the corporation's profit per equity share, which is $\frac{K \cdot p - K \cdot ac}{K \cdot w} = \frac{p - ac}{w}$, where w is the equity ratio. Thus, we have the following formulation:

$$\max_{K,w} \frac{p(K) - ac(K, w)}{w} .$$

First, the first-order conditions (for a mathematical interior solution) are:

$$\begin{cases} \frac{\partial}{\partial K}\left[\frac{p(K) - ac(K,w)}{w}\right] = 0 & \ldots . (1), \text{ and} \\ \frac{\partial}{\partial w}\left[\frac{p(K) - ac(K,w)}{w}\right] = 0 & \ldots . (2) \end{cases}$$

Equation (1) implies

$$\frac{d}{dK}\left(\frac{p(K)}{w}\right) = \frac{\partial}{\partial K}\left(\frac{ac(K,w)}{w}\right), \text{ i.e.,}$$

$$\frac{d}{dK}[p(K)] = \frac{\partial}{\partial K}[ac(K,w)] \quad . \; . \; . \; (3)$$

But the left-hand side of (3) is

$$\frac{d}{dK}[p(K)] = \left(\frac{K}{p}\;\frac{dp}{K}\right)\frac{p}{K} = \frac{-1}{/\varepsilon/} \cdot \frac{p}{K} \quad . \; . \; . \; (4)$$

And the right-hand side of (3) is

$$\frac{\partial}{\partial K}[\,ac(K,w)\,] = \frac{K\left(\frac{\partial ac}{\partial K}\right) + ac - ac}{K}$$

$$= \frac{\frac{\partial}{\partial K}(K \cdot ac) - ac}{K}$$

$$= \frac{ac \cdot \dfrac{\frac{\partial (K \cdot ac)}{\partial K} \cdot K}{ac \cdot K} - ac}{K}$$

$$= \frac{ac \cdot \frac{K}{TC}\;\frac{\partial TC}{\partial K} - ac}{K}$$

$$= \frac{ac \cdot \left(\frac{1}{s}\right) - ac}{K}$$

$$= \frac{ac\left(\frac{1}{s} - 1\right)}{K} \quad . \; . \; . \; (5)$$

Then, equating equation (4) with (5), we have:

$$\frac{-1}{/\varepsilon/}\;\frac{p}{K} = \frac{ac\left(\frac{1}{s}-1\right)}{K}, \quad \text{i.e.} ,$$

$$\frac{p}{ac} = /\varepsilon/ \cdot \left(1-\frac{1}{s}\right) , \quad . \; . \; . \; . \; . \; (6)$$

which is the necessary condition for the optimal investment as mentioned in p. 6.

Continuing the derivation of the optimal financing condition, we proceed from equation (2), which implies

$$0 = \frac{\partial}{\partial w} \left(\frac{p-ac}{w}\right)$$

$$= \frac{w \cdot -(\frac{\partial ac}{\partial w}) - (p-ac)}{w^2} ; \quad \text{hence} \quad ,$$

the numerator of the above

$$w(-\frac{\partial ac}{\partial w}) - (p - ac) = 0 ; \quad \text{i.e.} \; ,$$

$$w \frac{\partial ac}{\partial w} = ac - p ; \quad \text{so that,}$$

when dividing both sides of the above by ac, we have

$$\frac{w}{ac} \frac{\partial ac}{\partial w} = 1 - \frac{p}{ac} ; \quad \text{but}$$

$$\frac{w}{ac} \frac{\partial ac}{\partial w} = \frac{w}{K \cdot ac} \quad \frac{\partial (K \cdot ac)}{\partial w}$$

$$= \frac{w}{TC} \cdot \frac{\partial TC}{\partial w} \equiv \sigma ; \quad \text{thus,}$$

$$\sigma = 1 - \frac{p}{ac} , \quad \text{or}$$

$$\frac{p}{ac} = 1 - \sigma , \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (7)$$

which is the necessary condition for the optimal financing as mentioned in p. 8.

Finally, the second-order condition is:

$$\frac{\partial^2 ac}{\partial w^2} > 0 \quad \text{and} \quad \frac{\partial^2 ac}{\partial w^2} \left(\frac{\partial^2 ac}{\partial K^2} - \frac{d^2 p}{dK^2}\right) > \left(\frac{\partial^2 ac}{\partial K \partial w}\right)^2 \quad ,$$

as from calculus.

4. (a) $mr = \frac{d}{dK} (K \cdot p)$

$$= p + K \cdot \frac{dp}{dK} = p\ (1 + \frac{K}{p}\ \frac{dp}{dK})$$

$$= p\ (1 + \frac{1}{\varepsilon}) = p\ (1 - \frac{1}{/\varepsilon/})$$

$$= p - \frac{p}{/\varepsilon/} \quad ; \quad \text{hence} \quad ,$$

$$p - mr = \frac{p}{/\varepsilon/} \quad .$$

(b) $\frac{1}{s} \equiv \frac{\%\Delta TC}{\%\Delta K} = \frac{\partial TC}{\partial K}\ \frac{K}{TC}$

$$= mc \cdot (\frac{TC}{K})^{-1} = \frac{mc}{ac} \quad ; \quad \text{hence} \quad ,$$

$$s = \frac{ac}{mc}$$

5. The rationale of the U-shaped ac curve following the \$K-axis as drawn here in fig. 1 is as follows:
The downward sloping part is due to some fixed cost associated with either bond or stock financing. And the upward sloping part is due to the perceived higher business risk.

6. See the above 3, equation (7).

7. Applying log function to both sides of the TC equation, we have:
$\log TC = 0.5 \log K + (-2 + \log\log(10ew)) \log(10ew)$.
Then, taking total differential of the above with K being treated as a constant, we have:

$$d \log TC = \frac{dTC}{TC} = \frac{10e \cdot dw}{[\log(10ew)] \cdot 10ew} \cdot \log(10ew)$$

$$+ (-2 + \log \log(10ew)) \cdot \frac{10edw}{10ew}$$

$$= \frac{dw}{w} + (-2 + \log \log(10ew)) \cdot \frac{dw}{w}$$

$$= \frac{dw}{w} (1 - 2 + \log \log(10ew))$$

$$= \frac{dw}{w} \cdot (\log \log(10ew) - 1)$$

$$= (d \log w) \cdot (\log \log(10ew) - 1) \; ; \quad \text{hence} ,$$

$$\sigma \equiv \frac{\partial \log TC}{\partial \log w} = - 1 + \log \log (10ew) \quad .$$

Figures

1. Optimal investment -- a contrast: K^* maximizes market price per share: K_M maximizes firm profit.

2. Optimal financing -- a contrast: Maximizing market price per share implies a point like w^*; maximizing firm profit implies w_m .

3. Return on investment of the hypothetical example.

4. Average cost of capital of the hypothetical example (for a fixed equity ratio).

5. Average cost of capital of the hypothetical example (for a fixed amount of capital assets).