Division of Research
Graduate School of Business Administration
The University of Michigan

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THE RELATIONSHIP BETWEEN TAX AND REAL INTEREST RATE

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Lu Wang

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Lu Wang
Assistant Professor of Finance
School of Management
The University of Michigan-Dearborn

This paper seeks to serve as a simple footnote of an earlier article of this journal (Shapiro [2]), writing down an algebraic condition for an income tax cut to result in a lower real market interest rate and thus higher investment expenditure to contribute more capital stock and thus higher aggregate supply capacity to the economy.

As pointed out by Shapiro, "whether a tax cut will raise or lower the growth rate of real income through its effects on the supply side is a question.... The answer to the question posed depends on a multitude of factors and their complex interrelationships." [2] This is indeed the case. Before making a simple footnote concerning the above-mentioned interrelationships, we will first cite a recent empirical study (Arak [1]) that "In conclusion, tax cuts appear to raise consumption even more than the standard Keynesian model would predict." This empirical finding suggests that the IS curve shifts out after a tax cut and gives an affirmative answer to the title question of this article by Arak "Are Tax Cut Stimulatory?" This answer contradicts the assertion of those who believe that an income tax cut, viewed from all potential capital suppliers, means a higher post-tax real interest yield, making an incentive to supply more capital to the market (or, equivalently, making it more expensive to hold back the capital supply by doing activities such as consumption). The following will take into account this interest-rate effect on savings and answer how large the elasticity of real savings with respect to the real interest rate would have to be in order to make the IS curve shift inward (or downward) after an income tax cut.

What we will do is to make a comparative static analysis of the goods - market equilibrium condition: investment + government expenditure + export = savings + taxes + import. Since the derivation is somewhat mechanical, we will save it in the following footnote[1]. And here we will just write down the conclusion.

The conclusion is as follows: For an income tax cut to shift the IS curve inward (or downward), we need $s_{r'} > \frac{MPC}{APS}$, where $s_{r'}$ is the elasticity of real savings with respect to the after-tax real interest rate (i.e., $s_{r'} \equiv$ percentage change in real savings/percentage change in the after-tax real interest rate), MPC $\equiv$ the marginal propensity to consume, and APS $\equiv$ the average propensity to save.

Footnote

1. First, we will note the following general mathematical relationships, as they will be repeatedly used in the derivation.

For any arbitrary differentiable function

$Q = f(X,Y,Z)$, we have

$$dQ = \frac{\partial Q}{\partial X} dX + \frac{\partial Q}{\partial Y} dY + \frac{\partial Q}{\partial Z} dZ$$

$$= X \frac{\partial Q}{\partial X} \frac{dX}{X} + Y \frac{\partial Q}{\partial Y} \frac{dY}{Y} + Z \frac{\partial Q}{\partial Z} \frac{dZ}{Z} \quad , \text{ thus}$$

$$\frac{dQ}{Q} = \left(\frac{X}{Q} \frac{\partial Q}{\partial X}\right) \frac{dX}{X} + \left(\frac{Y}{Q} \frac{\partial Q}{\partial Y}\right) \frac{dY}{Y} + \left(\frac{Z}{Q} \frac{\partial Q}{\partial Z}\right) \frac{dZ}{Z} \quad , \text{ or in}$$

simpler notations,

$Q^* + Q_X X^* + Q_Y Y^* + Q_Z Z^*$, where a '*' over a variable means the percentage change in that variable and a symbol such as Q_X means the partial elasticity of Q with respect to X.

Next, in particular, if $Q = \frac{aXY}{Z}$, then

$Q^* = X^* + Y^* - Z^*$, by logarithmic differentiation (where a is a constant).

Also, if $Q = aX + bY$, where a and b are constants, then

$$Q^* = [\frac{X}{aX+bY}\frac{\partial}{\partial X}(aX + bY)]X^* + [\frac{Y}{aX+bY}\frac{\partial}{\partial Y}(aX + bY)]Y^*$$
$$= \frac{aX}{aX+bY} X^* + \frac{bY}{aX+bY} Y^*$$

Now, we proceed to make the comparative static analysis:

Let $i \equiv$ real investment (which is a function of the real market interest rate r), $g \equiv$ real government expenditure, $X \equiv$ real export, $s \equiv$ real savings (which is a function of (i) the real after-tax interest rate $r' \equiv r(1-t)$, t being the tax rate, and (ii) the real after-tax income $y' \equiv y(1-t)$, y representing the pre-tax real income) , $t_i \equiv$ other real indirect taxation (so that $t_i + yt$ = total real taxation) , and $m \equiv$ real import.

So the goods-market equilibrium condition is:

$$i(r) + g + X = s(r',y') + yt + t_i + m$$

And this 'quantitative comparative static analysis' continues as follows.

$$(i(r) + g + X)^* = (s(r',y') + yt + t_i + m)^*;$$

assuming $o = g^* = X^* = t_i^* = m^*$ and

$W_i \equiv \frac{i}{i+g+X}$, $W_s \equiv \frac{s}{s+yt+t_i+m}$, and

$W_{yt} \equiv \frac{yt}{s+yt+t_i+m}$ representing the three weights ,

We have:

$$i^*W_i = s^*W_s + (yt)^* W_{yt} \quad ;$$

but $i^* = i_r r^*$ (shere i_r is the real-interest-rate elasticity of real investment demand) ,

and $s^* = s_{r'}r'^* + s_{y'}y'^* = s_{r'}[r(1-t)]^* + s_{y'}[y(1-t)]^*$

$$= s_{r'}[r^* + (\frac{-t}{1-t})t^*] + s_{y'}[y^* + (\frac{-t}{1-t})t^*];$$

thus, $i^* W_i = s^*W_s + (yt)^*W_{yt}$ implies

$$W_i i_r r^* = W_s \left\{ s_{r'}[r^* + (\frac{-t}{1-t})t^*] + s_{y'}[y^* + (\frac{-t}{1-t})t^*] \right\}$$
$$+ W_{yt}(y^* + t^*)$$

$= W_s s_{r'} r^* + W_s s_{r'} \left(\frac{-t}{1-t}\right) t^* + W_s s_{y'} y^*$

$+ W_s s_{y'} \left(\frac{-t}{1-t}\right) t^* + W_{yt} y^* + W_{yt} t^*$, and

$r^*(W_i i_r - W_s s_{r'}) = t^*[W_s s_{r'} \left(\frac{-t}{1-t}\right) + W_s s_{y'} \left(\frac{-t}{1-t}\right) + W_{yt}]$

$+ y^* [W_s s_{y'} + W_{yt}]$;

let $D \equiv W_i i_r - W_s s_{r'}$ then $D < 0$, since $i_r < 0$;

transposing D to the right - hand side, we have

$r^* = t^* D^{-1} [W_s s_{r'} \left(\frac{-t}{1-t}\right) + W_s s_{y'} \left(\frac{-t}{1-t}\right) + W_{yt}]$

$+ y^* D^{-1} [W_s s_{y'} + W_{yt}]$.

Now, for a tax cut, which means $t^* < 0$, to result in a lower equilibrium market real interest rate , which means $r^* < 0$, we need

$[W_s s_{r'} \left(\frac{-t}{1-t}\right) + W_s s_{y'} \left(\frac{-t}{1-t}\right) + W_{yt}] < 0$;

i.e., $W_s \left(\frac{-t}{1-t}\right) (s_{r'} + s_{y'}) + W_{yt} < 0$,

or, $W_s \left(\frac{t}{1-t}\right) (s_{r'} + s_{y'}) - W_{yt} > 0$,

i.e., $W_s \left(\frac{t}{1-t}\right) (s_{r'} + s_{y'}) > W_{yt}$,

or, $s_{r'} + s_{y'} > W_{yt} \left(\frac{1-t}{tW_s}\right) = \left(\frac{W_{yt}}{W_s}\right) \left(\frac{1-t}{t}\right) = \left(\frac{yt}{s}\right) \left(\frac{1-t}{t}\right)$,

hence, $s_{r'} > \left(\frac{yt}{s}\right) \left(\frac{1-t}{t}\right) - s_{y'} = \left(\frac{y}{s}\right) (1-t) - s_{y'}$

$= \left(\frac{y}{s}\right) (1-t) - \frac{\partial s}{\partial y'} \frac{y'}{s} = \frac{y'}{s} - \frac{y'}{s} \frac{\partial s}{\partial y'} = \left(\frac{y'}{s}\right) \left(1 - \frac{\partial s}{\partial y'}\right)$

$= \left(\frac{1}{APS}\right) (1-MPS) = \frac{MPC}{APS}$.

Therefore, other things being equal, for $t^* < 0$ to result in $r^* < 0$, the economy needs to have $s_{r'} > \frac{MPC}{APS}$.

References

[1] Arak, M., "Are Tax Cut Stimulatory?" Review of Economics and Statistics, February 1982.

[2] Shapiro, E., " "Supply-Side" Economics: A Diagrammatic Exposition," The Nebraska Journal of Economics and Business, Spring 1981.