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TRANSFER BATCH SIZING IN FLEXIBLE MANUFACTURING SYSTEMS

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Abstract

This paper presents methods to determine the optimal transfer batch sizes of various part types between two machines so as to minimize the sum of the total relevant costs. Information required includes the expected production requirements, the operations' processing times, the travel times, the inventory carrying costs, and other information. The total relevant cost components include material handling, pallet, holding, and machine costs. The structure developed is relevant to an activity-based costing system. Scheduling is optimized so as to attain a good machine utilization. Mathematical programs are developed to minimize total costs subject to the appropriate constraints. Finally, comparisons to other possible and typical methods of transfer batch sizing are done in order to show the relevance of the new proposed methods.

Keywords : Transfer Batch Sizing, Material Handling, Flexible Manufacturing System, Cost Analysis, Optimization

1. Introduction

Given the production requirements and the processing times of operations, this paper presents a method to determine the transfer batch sizes of various part types between two machines so as to minimize the sum of the total relevant costs. Comparisons to other methods of transfer batch sizing are done. A major contribution of this paper is that *all* of the relevant costs that should be taken into account are considered. The method presented herein considers explicitly the indirect costs, which are traditionally taken care of by means of a global percentage added to the direct costs. Our approach is more akin to activity-based costing schemes. At the same time, we also optimize the scheduling of the parts in order to obtain good machine utilization.

The problem that we study consists of a two-stage production sub-system that processes one or two part types. The results found here could also be applied as a heuristic to two-machine subsets of larger systems. There are orders for r_i parts of type i. The sub-system is a flow shop, i.e., each part has to be processed on both machines in the same order. All orders are available at time zero. Parts are put on pallets in a preparation station and carried to the first machine for processing, then to the second machine, and finally back to the preparation station. We initially consider that the material handler is an automated guided vehicle system (AGVS). However, the subsequent analysis holds for any intermittant materials handler, such as lift trucks, conveyors, and trains of trays.

The problem studied here applies to flexible manufacturing systems (FMSs), where a make-to-order setup is implemented. A system such as the one considered can be found at the Laboratoire Universitaire de Recherche en Production Automatisée (L.U.R.P.A.) in Ecole Normale Supérieure de Cachan, France, a university laboratory of research in automated production. They constructed in the later eighties a two-machine FMS consisting of a milling center and a turning machine. The two machines are supplied by two AGVs carrying tools and parts from a manual preparation station where they are set up for processing. In this FMS, machining time is offered to external clients. Another eight-machine FMS like the type considered here is the Sundstrand/Caterpillar line in Peoria, Illinois. There are many FMSs with multiple parts on several fixture types. The methods proposed here can help to determine the appropriate number of parts that should be on a fixture.

The system can be schematized as in *Figure 1*. There are two machines, M1 and M2, with the input station (I) and the output station (O) of the manual preparation station.

Some related material handling literature includes Egbelu[1], which uses a dynamic programming-based heuristic algorithm to estimate the completion time of the requirements of a single part type. A sensitivity analysis is performed to observe the effect of changes in completion time due to changes in: (1) the transfer batch size, (2) the material-handling time, (3) the transfer batch preparatory time, and (4) the processing times of the operations. The model applies to the planning of production of parts of a single type through a multi-stage production system. However, this approach does not determine the optimal transfer batch sizes for part types.

Potts and Baker[2], Trietsch and Baker[3], and Baker and Jia[4] solve a similar problem: to split an order into different transfer batch sizes in order to minimize the maximum completion time. These authors consider optimizing the transfer batch sizes with or without intermittent idling of machines. Queues are allowed at the machines. Baker[5] considers setup times for a two machine flow shop.

Glass et al.[6] provide an analysis of lot streaming for a single order in three-stage production processes (for flow shops, job shops, and open shops). At each stage, the order is split into *s* batches. The authors propose algorithms to minimize makespan.

Mahadevan and Narendran[7] present an integer programming formulation for the problem of finding the optimal transfer batch size and the number of AGVs required to avoid both bottleneck situations as well as gross under-utilization of AGVs at the *FMS design phase.*

Askin and Madhavanur[8] examine a flow shop where workers are responsible for both machine operation and material handling. An efficient algorithm computes the cycle time for a single part type and this algorithm is used to help determine the optimal number of equal-sized transfer batches. Finally, these results are extended and tested for a flow shop producing multiple part types.

Kim et al.[9] solve a transfer batch scheduling problem for a two-stage flow shop with identical parallel machines at each stage. They suggest a rule similar to Johnson's to minimize makespan.

Smunt et al.[10] investigate transfer batch sizing for both stochastic job and flow shops using performance measures of mean flow time and standard deviation of mean flow time. They found that the number of transfer batches is more important than the exact form of splitting.

The previous papers may have considered a single cost component or none at all. We try to consider all relevant costs for the transfer batch size decision. In this paper, we believe that the decision on how to split the order should be based not only on the completion time and the limited transporter capacity as in Trietsch and Baker[3], but also on all other relevant costs. The makespan affects costs and the objective here is then to minimize the sum of total relevant costs.

The plan of the paper is as follows. The cost components are discussed in detail in section 2. We examine the problems of determining the optimal transfer batch size for a single type of part in section 3 and for two types of parts in section 4. Comparisons to other transfer batch sizing methods are presented. Section 5 concludes and outlines some future research needs.

2. Cost Components

Based on some cost discussions in Sule[11], Mahadevan and Narendran[7], and Lee[12], we consider that four different types of costs should be taken into account in the transfer batch size decision: material handling, pallet, holding, and machine costs. The underlying idea is that all used services (i.e., moving) have to be paid for and hence charged to the specific order. The percentage of indirects costs may vary from one order to another according to the actual required services. The detailed analysis of costs presented hereafter allows an evaluation of the amounts to be charged to each order. This information can then be used in activity-based cost accounting schemes. Activity-based costing ideas are useful for the cost categories that we identify here. Some useful related discussion can be found in Cooper[13] and Koltai et al.[14]. For our evaluation purposes, it is not necessary to consider the fixed costs as step functions. It is sufficient to allocate the fixed costs (overhead costs) to the appropriate material handling or machine costs. (See Cooper and Kaplan[15] and Koltai et al.[14].)

The system parameters are summarized in *Table 1*. There are two machines. Let N be the number of part types and r_i is the required number of parts of type i, for i=1 to N. For each part type i, the number of parts required r_i and the processing times p_{ij} for a part on machine j are given. Let u_i be the maximum number of parts of type i that can fit on a pallet. The AGVs travel from the preparation station to machine 1 in time $t_{I,M1}$, then from machine 1 to machine 2 in time $t_{M1,M2}$, and finally back to the preparation station in time $t_{M2,O}$.

Table 1
System Parameters.

R	:	number of round trips
N	:	number of part types
r_i	:	number of parts of type i required, i = 1,...,N
p_{ij}	:	processing time of a part of type i on machine j
$t_{I,M1}$	:	travel time from the input station to machine 1
$t_{M1,M2}$	:	travel time from machine 1 to machine 2
$t_{M2,O}$	:	travel time from machine 2 to the output station
MH$	:	charged cost of the handling equipment to the transfer batch per round trip
Hi$	:	inventory holding cost per time unit for a part of type i
P$	:	set-up cost per pallet + utilization cost per pallet
Pi$	:	value of a part of type i
M$	:	hourly machine rate
I	:	% of inventory value per time unit
u_i	:	upper bound on the number of parts of type i that can fit on a pallet

The decision variables are k_i, the transfer batch size for part type i. The parameters related to costs are: MH\$, the charged cost of the handling equipment per trip; P\$, the pallet cost; Hi\$, the inventory holding cost, and M\$, the hourly machine rate.

We can express the total cost as:

Total Cost = Material Handling Cost + Pallet Cost + Holding Cost + Machine Cost.

We now discuss each of the cost components in turn.The **material handling cost** accounts for the fixed and variable operating costs to use the material handler. It can be evaluated using the number of round trips (R) times the service cost of the handling equipment for one trip (MH\$). This service cost may be estimated considering the maintenance, energy, depreciation over its life period, and system operators' salary (Sule[11]).

Material Handling Cost = R x MH\$.

The **pallet cost** evaluates the expenses associated with the preparation and utilization of the handling containers. In the machining industries, the handling containers are pallets on which parts are positioned for (perhaps automated) pick-up and drop-off. The pallet cost is composed of fixed and variable components. The fixed component is the number of round trips, R, (corresponding to the number of pallet loads processed) times the pallet set-up and utilization cost (P\$). Mahadevan and Narendran[7] propose these types of costs in their model reviewed in section 1. The variable cost is the number of parts times the handling cost per part. The sum of the variable costs is a constant and thus may be eliminated from the model.

The preparation cost can then be written:

Pallet Cost = R x P$.

The **holding cost** corresponds to the opportunity cost (lost interest) of the capital required for the work-in-progress. The holding cost for a part of type i (Hi$) is calculated by multiplying the percentage of the inventory value per time unit with the value of the part. The *duration* (i.e., makespan, the total time in the system for all parts of all part types) is a function of the particular part type parameters, such as the processing times.

Holding Cost = *duration* x Σ_i (Hi$ x r_i).

The **machine cost** corresponds to the time during which the machines cannot be used to process anything else. It is given by the duration times the hourly machine rate (M$). Rather than duration, this should be the actual machining time. For the purposes of this paper, these are both the same. In our first case, we schedule a single part type. In the second case, we consider two part types. As long as we don't schedule additional production, the duration is equal to the actual machining time.

This $ per hour machine cost can be calculated based either on the cost of the available or on the cost of the used resources (see Cooper and Kaplan[15]). There are several ways to calculate this cost (see, for example, Hakala[16] and Koltai[17]). Also, Lee[12] shows how to evaluate this rate over the machine's anticipated life.

Machine Cost = *duration* x M$.

There is a relationship between the transfer batch size and the machine costs. The lead time, duration, and WIP costs change. The duration then impacts the machine costs. The material handling costs and the machine costs can be similar in scale. When they are not, one will dominate. Holding costs may be either signifiant or neglible.

Some parts in process can cost thousands of dollars. When neglible, other cost components will then dominate.

Next, we specify the cost components for the first situation considered, one part type, in Section 3.

3. Optimum Transfer Batch Size for a Single Type of Part

A characteristic of the system considered here is that there are no buffers at the machines: a pallet has to be carried away before another pallet can be brought in to a machine. We have a sufficient number of AGVs and pallets.

3.1 Cost Analysis

We now determine k_1, the number of parts to be moved on each pallet. As r_1 is the total number of parts to process, the number of pallet round trips, R, is given by $\lceil r_1/k_1 \rceil$. In what follows, we assume that r_1/k_1 is integer. The cost components are as follows.

Material handling cost = (r_1/k_1) x MH$

Pallet cost = (r_1/k_1) x P$

Holding cost = *duration* x r_1 x H1$

Machine cost = *duration* x M$.

To calculate *the duration*, one must consider two cases:

<u>**CASE a**</u> : If $p_1 \geq p_2$, then:

$$duration = t_{I,M1} + r_1 p_1 + t_{M1,M2} + k_1 p_2 + t_{M2,O.}$$

This corresponds to the time to carry the first pallet to the first machine ($t_{I,M1}$), plus the production time for all of the parts on the first machine ($r_1 p_1$), plus the time to move the last pallet from machine 1 to machine 2 ($t_{M1,M2}$), plus the time to process the last pallet on the second machine (k_1p_2), and finally the time to bring the last pallet to the output stage ($t_{M2,O}$). We assume that the time to transfer the pallet between the machine and the AGVs is negligible. See *Figure 2* for a schematic of this.

<u>**CASE b**</u> : If $p_1 < p_2$, then:

$$\textit{duration} = t_{I,M1} + k_1p_1 + t_{M1,M2} + r_1p_2 + t_{M2,O}.$$

This is analogous to the first case except that the processing time on machine 1 is smaller. See *Figure 3.*

3.2 Cost Optimization

The total cost function can now be written as follows.

<u>**CASE a**</u> : If $p_1 \geq p_2$, then

$$\textbf{TOTAL COST}(k_1) = (r_1/k_1)\ MH\$ + (r_1/k_1)\ P\$ + (t_{I,M1} + r_1p_1 + t_{M1,M2} + k_1p_2 + t_{M2,O}) \cdot r_1H1\$ + (t_{I,M1} + r_1p_1 + t_{M1,M2} + k_1p_2 + t_{M2,O})\ M\$.$$

<u>**CASE b**</u> : If $p_1 < p_2$, then

$$\textbf{TOTAL COST}(k_1) = (r_1/k_1)\ MH\$ + (r_1/k_1)\ P\$ + (t_{I,M1} + k_1p_1 + t_{M1,M2} + r_1p_2 + t_{M2,O})\ r_1H1\$ + (t_{I,M1} + k_1p_1 + t_{M1,M2} + r_1p_2 + t_{M2,O})\ M\$.$$

Both can be simplified by aggregating the constant terms as follows:

$$\text{TOTAL COST}(k_1) = A/k_1 + Bk_1 + C,$$

where $A=r_1(MH\$+P\$)$, $B=p_1(r_1H1\$+M\$)$, and $C=(t_{I,M1}+t_{M1,M2}+r_1p_2+t_{M2,O})(r_1H1\$+M\$)$. These B and C are the constants for **Case b**, in which $p_1<p_2$. For **Case a**, where $p_1 \geq p_2$, B and C can be derived similarly.

To find the optimal solution, one has only to find the value k_1 for which the derivative of the Total Cost function is null:

$$d(\text{TOTAL COST})/d(k_1) = -A/k_1^2 + B = 0.$$

$$\Rightarrow k_1 = (A/B)^{1/2}.$$

The TOTAL COST function is a convex function when $k_1 > 0$. Hence k_1^* is a minimum. If k_1^* isn't integer, one has to look at $\lfloor k_1^* \rfloor$ and $\lceil k_1^* \rceil$ to find the best integer solution. Let k_1^{OPT} be the best integer solution. If k_1^* is integer, then $k_1^{OPT} = k_1^*$. The optimal number of parts on a pallet is given by

$$\min \{u_1, k_1^{OPT}\},$$

where u_1 is the maximum number of parts that can fit on a pallet.

3.3 Experimental Design and Results

Tests have been performed to demonstrate the utility of this new method to determine transfer batch sizes. We let the cost parameters assume different values corresponding both to real cases as well as the literature. Some examples of these tests are now provided.

We have done about 500 test comparisons (Crepel et al.[18]) both to study the influence of the cost parameters on the optimal solution as well as to compare these results with the following two other possible methods of transfer batch sizing:

a) determining the transfer batch size to equal either the number of parts to be produced or the maximum number that can fit on a pallet, i.e., $k = \min \{u_1, r_1\}$, (a traditional and typical approach to material handling... this is thought to minimize AGV purchase costs);

b) determining the transfer batch size to equal one (similar to a just-in-time approach).

We present next a sampling of the tests. For the following examples the processing times are: p_1 = 3 minutes and p_2 = 5 minutes. Parameters are:

(1) For the **material handling cost** (MH$):

Buying cost of an AGVS:	$100,000
Life time:	5 years
Maintenance:	$1.50 per hour
Energy:	$10 for 8 hours
Distance:	9,750 feet per day
Wages:	$3 per hour (the system operator is paid $30 per hour and takes care of 10 vehicles)
Exchange time:	15 seconds
Total round trip:	780 feet
Speed:	104 feet per minute

2) For the **pallet cost** (P$)

Preparation time:	10 minutes
Wages:	$10 per hour
Multi-purpose pallet cost:	$1

(3) For the **holding cost** (H1$)

Price of part:	$100
Interest rate:	$0.10/$ per part per year

(4) For the **machine cost** (M$)

Machinery rate:	$50 per hour per machine.

For these costs :

MH$ = $8.14, P$ = $2.67, H1$ = $0.003472, M$ = $100.

With this particular system data, and for production requirements increasing from 10 to 1000 parts, the total cost function is developed as described previously. For each case, the optimal transfer batch size is calculated and provided in *Table 2*. Also contained in *Table 2* are the total costs of always transferring one part at the time (JIT) and the cost of transferring all the required parts at once. For each value of r, the optimal batch size and the corresponding optimal total cost are presented in bold.

Table 2
Results for One Part Type.

r	10	15	50	80	100	200	500	1000
k	4.65	5.69	10.39	13.13	14.68	20.72	32.60	45.71
$\lfloor k \rfloor$	4	5	**10**	**13**	14	20	32	45
Cost of $\lfloor k \rfloor$	142.06	192.51	**519.86**	**788.52**	965.29	1835.01	4416.39	8766.33
$\lceil k \rceil$	**5**	**6**	11	14	**15**	**21**	**33**	**46**
Cost of $\lceil k \rceil$	**141.66**	**192.11**	519.95	788.79	**965.16**	**1834.90**	**4416.36**	**8766.28**
(1)	11.5%	10.6%	7.8%	6.4%	5.6%	4.2%	2.8%	2.0%
(2)	3.8%	3.5%	2.6%	2.1%	1.9%	1.4%	0.9%	0.7%
(3)	0.0%	0.0%	0.2%	0.2%	0.3%	0.7%	1.6%	3.3%
(4)	84.7%	85.9%	89.4%	91.3%	92.2%	93.7%	94.7%	94.0%
Cost of k=1	208.13	302.22	961.23	1526.64	1903.85	3793.17	9494.95	19108.29
Cost of k=r	155.84	220.92	676.96	1068.75	1330.38	2643.89	6639.07	13477.53

(1) : Percentage of the *material handling cost* on the total cost
(2) : Percentage of the *pallet cost* on the total cost
(3) : Percentage of the *holding cost* on the total cost
(4) : Percentage of the *machine cost* on the total cost

The proportion of the four costs included in the production cost was calculated. As can be seen in *Table 2*, the total production cost computed for the optimal transfer batch size is, for any values of the parameters, *always smaller* than the costs obtained with the other two proposed methods of transfer batch sizing. However, the difference is significant mostly when there is a large number of parts to be produced (more than 200 parts) and there is a small pallet cost. Nonnegligible savings can be accrued by the methods proposed here when large production requirements are needed.

The largest cost component, regardless of the number of parts required to be produced, is the machine cost (see *Table 2*). All of the other costs are smaller, and their relative proportions vary with the numbers of parts to produce. In general, the inventory cost can account for up to 7%, the material cost for up to 15%, and the pallet cost for up to 9%.

Considering only the pallet, material handling, and inventory costs, the material handling cost represents more than 50% for small and medium production requirements (10 to 200 parts) but less than 35% for large batch sizes (500 to 1000 parts). This is true in case of a low pallet cost and long machining time. The higher the pallet cost is, the higher this percentage becomes.

The results demonstrated here are surprisingly robust. Similar results have been observed for over 500 cost structures (see Crepel et al.[18]).

4. Optimum Transfer Batch Size for Two Types of Parts

When the master production plan indicates that two types of parts have to be processed within a certain time horizon, one cannot just apply the method described in section 3 to each type. The scheduling is more complicated because of the two part types. The scheduling method affects total duration. Scheduling should also now be optimized to maximize the machine utilizations. Again, each part must be processed first on machine 1 (flow shop).

In some cases, because of morphological issues, it is impossible to combine two types of parts on the same pallet. It may also be possible that the two processes may require too many tools to be stored in the tool magazines at the same time. Under these conditions, there may be no advantage in interrupting the processing of the parts of one type to produce some of the other. This is a nonpreemptive case and is the case considered here.

In this nonpreemptive case, the optimal order (w.r.t. system utilization) in which the parts are processed can be determined by Johnson's algorithm. In the following, the type of parts to be processed first according to Johnson's algorithm is defined as type 1.

Some additional notation required is as follows.

r_1 : number of parts of type 1 required (given);

r_2 : number of parts of type 2 required (given);

k_1 : number of parts of type 1 in a transfer batch (to be determined);

k_2 : number of parts of type 2 in a transfer batch (to be determined).

Then we have r_1/k_1 (r_2/k_2) = number of round trips for parts of type 1 (type 2).

4.1 Cost Analysis

The four types of costs discussed in Section 2 have to be considered. They become :

Material handling cost = $(r_1/k_1 + r_2/k_2) \times MH\$$

Pallet cost = $(r_1/k_1 + r_2/k_2) \times P\$$

Holding cost = *duration* x $(r_1 H1\$ + r_2 H2\$)$

Machine cost = *duration* x M$.

With the parts of type 1 scheduled first according to Johnson's algorithm, then either p_{11} or p_{22} is the minimum of all p_{ij}. To derive the duration, we proceed as in the first problem, i.e., we use charts like *Figure 1* to develop the formulae for all possible cases of the ordering of the p_{ij}. The formulae are presented in *Table 3* and are more complex than in the first problem because of the possibility that the last pallet is not full. If we assume that r_1/k_1 is integer, the formulae for the duration reduce to the following four (depending on the value of the parameter δ in *Table 3*).

Case A : If the longest processing time (LPT) is on the second machine, there are two subcases :

A1 : If $r_1p_{11} + k_2p_{21} \leq k_1p_{11} + r_1p_{12}$, then

$$duration = k_1p_{11} + r_1p_{12} + r_2p_{22} + t, \quad (1)$$

where $t = t_{1,M1} + t_{M1,M2} + t_{M2,O}$ (time for a round trip).

A2 : If $r_1p_{11} + k_2p_{21} \geq k_1p_{11} + r_1p_{12}$, then

$$duration = r_1p_{11} + k_2p_{21} + r_2p_{22} + t. \quad (2)$$

Case B : If the LPT is on the first machine, there are again two subcases:

B1 : If $r_2p_{21} + k_2p_{22} \geq k_1p_{12} + r_2p_{22}$, then

$$duration = r_1p_{11} + r_2p_{21} + k_2p_{22} + t. \quad (3)$$

B2 : If $r_2p_{21} + k_2p_{22} \leq k_1p_{12} + r_2p_{22}$, then

$$duration = r_1p_{11} + k_1p_{12} + r_2p_{22} + t. \quad (4)$$

Note that when $r_1p_{11} + k_2p_{22} = k_1p_{11} + r_1p_{12}$ (Subcases **A1** and **A2**), equation (1) = equation (2). Similarly, when $r_2p_{21} + k_2p_{22} = k_1p_{11} + r_2p_{22}$ (Subcases **A3** and **A4**), equation (3) = equation (4).

Table 3

Duration Formulae for Two Types of Parts.

CASES with p_{11} = SPT	DURATION FORMULAE
$p_{11} \le p_{12} \le p_{21} \le p_{22}$	$k_1p_{11} + r_1p_{12} + r_2p_{22} + \delta$ $\delta = \max\{o,\ r_1p_{11} + k_2p_{21} - k_1p_{11} - r_1p_{12}\}$
$p_{11} \le p_{12} \le p_{22} \le p_{21}$	$r_1p_{11} + \lfloor r_2/k_2 \rfloor k_2p_{21} + k_2p_{22} + (r_2 \text{ modulo } k_2)\ p_{22}$
$p_{11} \le p_{21} \le p_{12} \le p_{22}$	$k_1p_{11} + r_1p_{12} + r_2p_{22} + \delta$ $\delta = \max\{o,\ r_1p_{11} + k_2p_{21} - k_1p_{11} - r_1p_{12}\}$
$p_{11} \le p_{21} \le p_{22} \le p_{12}$	$k_1p_{11} + r_1p_{12} + r_2p_{22} + \delta$ $\delta = \max\{o,\ r_1p_{11} + k_2p_{21} - k_1p_{11} - r_1p_{12}\}$
$p_{11} \le p_{22} \le p_{12} \le p_{21}$	$k_1p_{11} + r_1p_{12} + r_2p_{22} + \delta$ $\delta = \max\{o,\ r_1p_{11} + r_2p_{21} - \lfloor r_2/k_2 \rfloor k_2p_{22} - r_1p_{12} - k_1p_{11}\}$
$p_{11} \le p_{22} \le p_{21} \le p_{12}$	$k_1p_{11} + r_1p_{12} + r_2p_{22} + \delta$ $\delta = \max\{o,\ r_1p_{11} + k_2p_{21} - k_1p_{11} - r_1p_{12}\}$

CASES with p_{22} = SPT	DURATION FORMULAE
$p_{22} \le p_{21} \le p_{12} \le p_{11}$	$\lfloor r_1/k_1 \rfloor k_1p_{11} + k_1p_{12} + (r_1 \text{ modulo } k_1)\ p_{12} + r_2p_{22} + \delta$ $\delta = \max\{o,\ r_1p_{11} + r_2p_{21} - \lfloor r_1/k_1 \rfloor k_1p_{11} - k_1p_{12} - (r_1 \text{ modulo } k_1)\ p_{12}\}$
$p_{22} \le p_{21} \le p_{11} \le p_{12}$	$k_1p_{11} + r_1p_{12} + r_2p_{22} + \delta$ $\delta = \max\{o,\ r_1p_{11} + k_2p_{21} - k_1p_{11} - r_1p_{12}\}$
$p_{22} \le p_{12} \le p_{21} \le p_{11}$	$r_1p_{11} + r_2p_{21} + (r_2 \text{ modulo } k_2)\ p_{22}$
$p_{22} \le p_{12} \le p_{11} \le p_{21}$	$r_{p_{11}} + r_2p_{21} + (r_2 \text{ modulo } k_2)\ p_{22}$
$p_{22} \le p_{11} \le p_{21} \le p_{12}$	$k_1p_{11} + r_1p_{12} + r_2p_{22} + \delta$ $\delta = \max\{o,\ r_1p_{11} + k_2p_{21} - k_1p_{11} - r_1p_{12}\}$
$p_{22} \le p_{11} \le p_{12} \le p_{21}$	$r_1p_{11} + r_2p_{21} + (r_2 \text{ modulo } k_2)\ p_{22}$

4.2 Cost Optimization

The next step consists of formulating a mathematical program to evaluate the optimal k_1 and k_2. *Table 4* summarizes the objective functions ($C(k_1,k_2)$) and the constraints for the four possible cases.

Table 4
Mathematical Models to Minimize Total Costs.

Case	$C(k_1, k_2)$	Constraints
A1	$(MH\$ + P\$)(r_1/k_1 + r_2/k_2) +$ $(k_1p_{11} + r_1p_{12} + r_2p_{22} + t)$ $* (r_1H\$_1 + r_2H\$_2 + M\$)$	$r_1p_{11} + k_2p_{21} \le k_1p_{11} + r_1p_{12}$ $1 \le k_1 \le u_1$ $1 \le k_2 \le u_2$
A2	$(MH\$ + P\$)(r_1/k_1 + r_2/k_2) +$ $(r_1p_{11} + k_2p_{21} + r_2p_{22} + t)$ $* (r_1H\$_1 + r_2H\$_2 + M\$)$	$r_1p_{11} + k_2p_{21} \ge k_1p_{11} + r_1p_{12}$ $1 \le k_1 \le u_1$ $1 \le k_2 \le u_2$
B1	$(MH\$ + P\$)(r_1/k_1 + r_2/k_2) +$ $(r_1p_{11} + k_1p_{12} + r_2p_{22} + t)$ $* (r_1H\$_1 + r_2H\$_2 + M\$)$	$r_2p_{21} + k_2p_{22} \le k_1p_{12} + r_2p_{22}$ $1 \le k_1 \le u_1$ $1 \le k_2 \le u_2$
B2	$(MH\$ + P\$)(r_1/k_1 + r_2/k_2) +$ $(r_1p_{11} + r_2p_{21} + k_2p_{22} + t)$ $* (r_1H\$_1 + r_2H\$_2 + M\$)$	$r_2p_{21} + k_2p_{22} \ge k_1p_{12} + r_2p_{22}$ $1 \le k_1 \le u_1$ $1 \le k_2 \le u_2$

We now explore in detail, **Case A1**, i.e., when the *duration* = $k_1p_{11} + r_1p_{12} + r_2p_{22} + t$. We want to find the values of k_1 and k_2 that minimize the total cost. We have the following mathematical program:

Problem (P1)

$$\min C(k_1, k_2) = (MH\$ + P\$)\left[\frac{r_1}{k_1} + \frac{r_2}{k_2}\right] + (k_1p_{11} + r_1p_{12} + r_2p_{22} + t)(H1\$r_1 + H2\$r_2 + M\$)$$

subject to

$$r_1p_{11} + k_2p_{21} \le k_1p_{11} + r_1p_{12} \qquad (5)$$

$$1 \le k_1 \le u_1 \qquad (6)$$

$$1 \le k_2 \le u_2, \qquad (7)$$

where $C(k_1, k_2)$ is the total cost when the transfer batch sizes are k_1 and k_2. Recall that u_1 and u_2 are the maximum numbers of parts of each type that can fit on a pallet. The feasible solution set (S) to Problem (P1) is a convex and bounded polygon, and the objective function is convex over this polygon. Because

$$\frac{\partial C}{\partial k_2} = -(MH\$ + P\$)\frac{r_2}{k_2^2} < 0 \text{ for all } k_2,$$

the optimum is necessarily on the *upper* border of the solution set S. The upper border consists of all points (k_1, k_2) such that

$$k_2 = \max\{k: (k_1, k) \in S\}.$$

We have that $$\frac{\partial C}{\partial k_1} = -\frac{(MH\$ + P\$)r_1}{k_1^2} + p_{11}(H1\$r_1 + H2\$r_2 + M\$) = 0,$$

$$=> k_1 = \sqrt{\frac{(MH\$ + P\$)\, r_1}{p_{11}\ (H1\$r_1 + H2\$r_2 + M\$)}}$$

So if

$$\left[\sqrt{\frac{(MH\$ + P\$)r_1}{p_{11}\ (H1\$r_1 + H2\$r_2 + M\$)}}\ ,\ u_2\right] \tag{8}$$

satisfies equations (5 - 7), then it is the optimum solution, (k_1^*, k_2^*). Otherwise, the optimum is located at one of the vertices of the solution polygon S or possibly on the line segment defined by equation (5). If the optimum is not given by equation (8), then the possible solutions of Problem (P1) (shown in *Figure 4*) are:

$$(k_1, k_2) = \left[u_1, \frac{u_1 p_{11} + r_2 p_{12} - r_1 p_{11}}{p_{21}}\right] \tag{9}$$

or

$$\left[\frac{r_1 p_{11} + u_2 p_{21} - r_1 p_{12}}{p_{11}}, u_2\right] \tag{10}$$

or

$$(u_1\ ,\ u_2) \tag{11}$$

or

$$(1,\ u_2) \tag{12}$$

or

$$\left[1, \frac{p_{11} + r_1 p_{12} - r_1 p_{11}}{p_{21}}\right] \tag{13}$$

or

$$\left[\frac{r_1 p_{11} + p_{21} - r_1 p_{12}}{p_{11}}, 1\right]. \quad (14)$$

One has to check which of these points satisfies equations (5 - 7) and gives the best solution. One has also to check for a possible optimum on the edge of the solution polygon S corresponding to equation (5). (This could happen if, at one point of the segment, the gradient of $C(k_1,k_2)$ is orthogonal to the segment). To find the optimum, the objective function has to be expressed in terms of k_1 only (using equation (5)). Then its derivative is evaluated at the two ends of the line segment. If they are of different signs, there is an optimal solution between the two, which can be found by a dichotomic search along the segment. *Figure 4* illustrates all possible cases of Problem (P1) for **Case A1**.

A similar analysis can be performed for **Cases A2, B1**, and **B2**. Table 5 presents a summary of the results.

Then when the LPT of the operations is on the second machine (**Case A**) and (k_1^{A1}, k_2^{A1}) and (k_1^{A2}, k_2^{A2}) are the local optimum solutions for **Cases A1** and **A2**, respectively, then the global optimal solution is simply the lower cost of the two. In other words, the optimum transfer batch sizes are:

$$(k_1^*, k_2^*) = \mathit{argmin}\ \{ C(k_1, k_2) : [(k_1, k_2) = (k_1^{A1}, k_2^{A1}) \text{ or } (k_1^{A2}, k_2^{A2})]\}.$$

A similar analysis holds for Case B.

Table 5
Possible Optimal Points for the Four Cases.

	Case A1	Case A2	Case B1	Case B2
Possible optimal solution	$\left(\sqrt{\frac{(M\$+P\$)\,r_1}{p_{11}(r_1 H1\$ + r_2 H2\$ + M\$)}},\ u_2\right)$	$\left(u_1,\ \sqrt{\frac{(M\$+P\$)\,r_2}{p_{21}(r_1 H1\$ + r_2 H2\$ + M\$)}}\right)$	$\left(\sqrt{\frac{(M\$+P\$)\,r_1}{p_{12}(r_1 H1\$ + r_2 + H2\$ + M\$)}},\ u_2\right)$	$\left(u_1,\ \sqrt{\frac{(M\$+P\$)\,r_2}{p_{22}(r_1 H1\$ + r_2 H2\$ + M\$)}}\right)$
otherwise, if the preceding point is not feasible, one of these vertices of the solution polygon S is optimal	$(1,\ u_2)$	$(u_1,\ 1)$	$(1,\ u_2)$	$(u_1,\ 1)$
	$(u_1,\ u_2)$	$(u_1,\ u_2)$	$(u_1,\ u_2)$	$(u_1,\ u_2)$
	$\left(\frac{r_1 p_{11} + u_2 p_{21} - r_1 p_{12}}{p_{11}},\ u_2\right)$	$\left(\frac{r_1 p_{11} + u_2 p_{21} - r_1 p_{12}}{p_{11}},\ u_2\right)$	$\left(\frac{u_2 p_{21} + u_2 p_{22} - r_2 p_{22}}{p_{12}},\ u_2\right)$	$\left(\frac{u_2 p_{21} + u_2 p_{22} - r_2 p_{22}}{p_{12}},\ u_2\right)$
	$\left(u_1,\ \frac{u_1 p_{11} + r_2 p_{12} - r_1 p_{11}}{p_{21}}\right)$	$\left(u_1,\ \frac{u_1 p_{11} + r_2 p_{12} - r_1 p_{11}}{p_{21}}\right)$	$\left(u_1,\ \frac{u_1 p_{12} + r_2 p_{22} - r_2 p_{21}}{p_{22}}\right)$	$\left(u_1,\ \frac{u_1 p_{12} + r_2 p_{22} - r_2 p_{21}}{p_{22}}\right)$
	$\left(1,\ \frac{p_{11} + r_1 p_{12} - r_1 p_{11}}{p_{21}}\right)$	$\left(1,\ \frac{p_{11} + r_1 p_{12} - r_1 p_{11}}{p_{21}}\right)$	$\left(1,\ \frac{p_{12} + r_2 p_{22} - r_2 p_{21}}{p_{22}}\right)$	$\left(1,\ \frac{p_{12} + r_2 p_{22} - r_2 p_{21}}{p_{22}}\right)$
	$\left(\frac{p_{21} + r_1 p_{11} - r_1 p_{12}}{p_{11}},\ 1\right)$	$\left(\frac{p_{21} + r_1 p_{11} - r_1 p_{12}}{p_{11}},\ 1\right)$	$\left(\frac{p_{22} + r_2 p_{21} - r_2 p_{22}}{p_{12}},\ 1\right)$	$\left(\frac{p_{22} + r_2 p_{21} - r_2 p_{22}}{p_{12}},\ 1\right)$
	or on the edge defined by the first constraint of Case A1	or on the edge defined by the first constraint of Case A2	or on the edge defined by the first constraint of Case B1	or on the edge defined by the first constraint of Case B2

4.3 Experimental Results

Extensive testing of the method presented in the section 4.2 has been conducted using various values for the parameters and are reported in Marcoux et al.[19]. We present a sample of the results in *Table 6*. In this example, the values of the parameters are the same as those used in section 3.3, except for the following: P1$ = $50, P2$ = $50, p_{11} = 8 minutes, p_{12} = 20 minutes, p_{21} = 10 minutes, and p_{22} = 25 minutes. The upper bounds on the number of parts of each type that can fit on a pallet is set equal to 25.

Table 6
Results for Two Types of Parts.

		Optimal solution			Integer Solution			Cost($) of k_1,k_2=	
r_1	r_2	Case	k_1	k_2	Cost($)	k_1	k_2	1,1	r_1,r_2
10	10	A1	2.847	10.000	852.12	3	10	994.8	920.26
10	15	A1,A2	2.920	14.336	1061.39	3	14	1257.35	1128.77
15	15	A1	3.486	15.000	1245.3	3	15	1478.23	1362.31
15	25	A1,A2	3.550	20.840	1664.53	4	21	2003.49	1779.50
25	15	A1	4.501	15.000	1605.92	4	15	1920.10	1829.53
25	25	A1	4.500	25.000	2023.22	4	25	2445.49	2246.87
50	50	A1	6.361	25.000	3963.38	6	25	4865.89	4460.96
100	100	A1	8.989	25.000	7824.86	9	25	9716.47	8900.64
100	200	A1	8.981	25.000	12069.75	9	25	14998.94	13104.50
200	100	A1	12.701	25.000	11288.58	13	25	14161.20	13606.91
200	200	A1	12.690	25.000	15546.58	13	25	19456.69	17826.00
500	500	A1	19.962	25.000	38924.00	20	25	48989.82	44970.11
1000	1000	A1	25.000	25.000	78829.22	25	25	99253.32	91437.06

Conclusions similar to those presented in section 3.3 can be drawn for this two part types problem. Material handling costs are nonnegligible. They account for up to 7% of the total costs and if we consider only the pallet, inventory, and material handling costs, the material handling cost accounts for up to 85%. Much savings can be obtained by choosing the appropriate transfer batch size instead of strategies such as transferring one part at a time (JIT) or all of the required parts at once.

5. Conclusions

Methods to determine optimal transfer batch sizes between two machines are developed for several simple cases. All relevant cost components are considered. For two part types, optimal scheduling is done to maximize machine utilizations. Mathematical programs are developed to minimize the sum of the relevant costs. Comparisons to other methods of transfer batch sizing show the usefulness of considering costs. These costs could be part of an activity-based cost accounting system. Also, the methodology should be applicable to different cost elements and assumptions, which leads to a robust methodology.

Other possible uses of the methods are for operational purposes, in allocating costs to make such transfer decisions. The methods should also be useful for design purposes, in determining the appropriate number of AGVs to buy.

Methods are presented here for two part types and two machines. These methods can also serve as heuristics for larger problems, in particular, when there are more than two machines. The methodology may be applied to a larger class of more complex problems in the future.

Another extension to consider is when more than one part type at a time can be transported. This possibility is a function of the fixturing capabilities, tool magazine capacities, tooling requirements, and relative sizes and weights of the part types. Methods also need to be developed when there are many part types to be processed.

In all cases, costs and utilizations should be considered. When relevant, due dates also need to be included.

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FIGURES

BIOGRAPHIES

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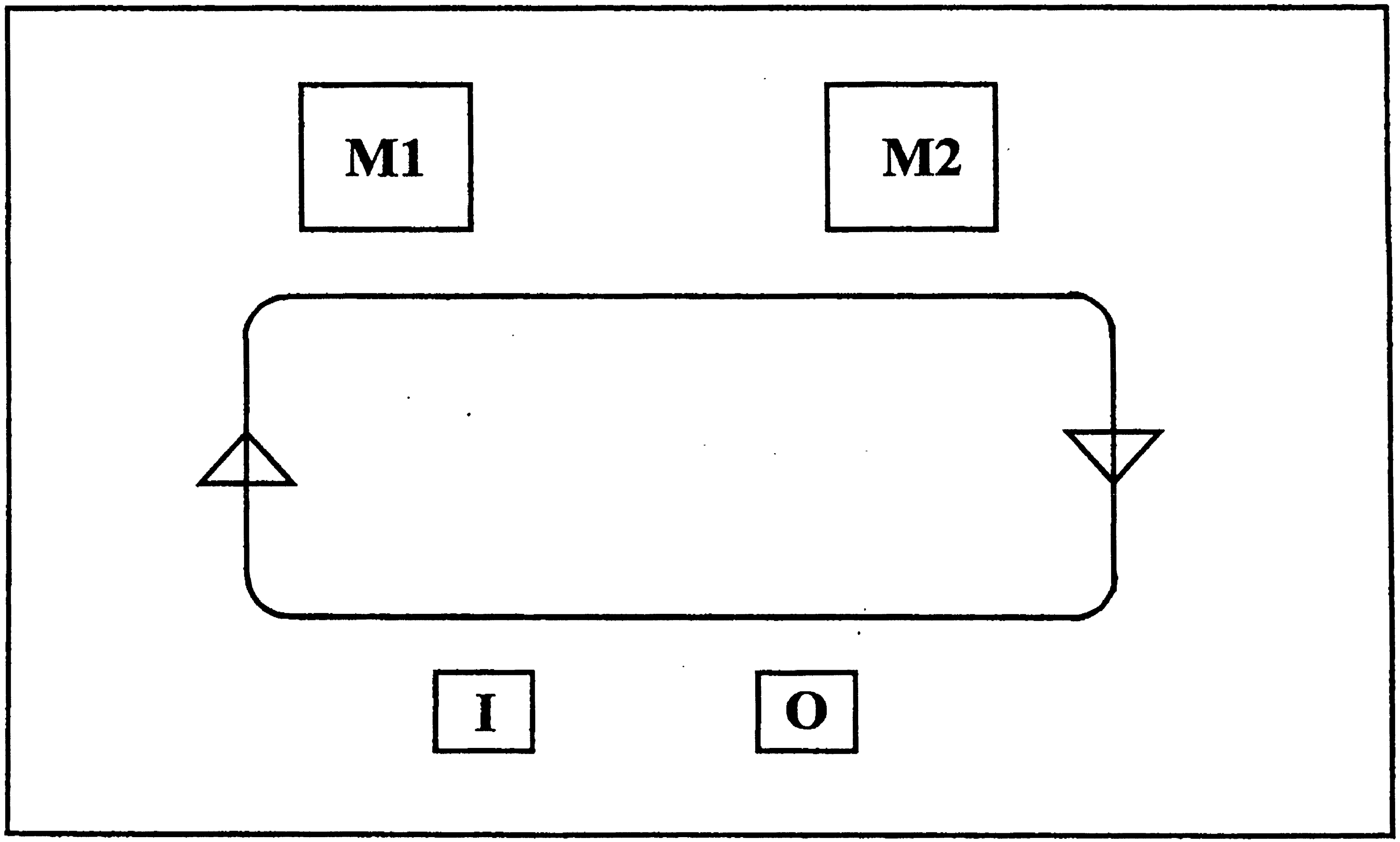
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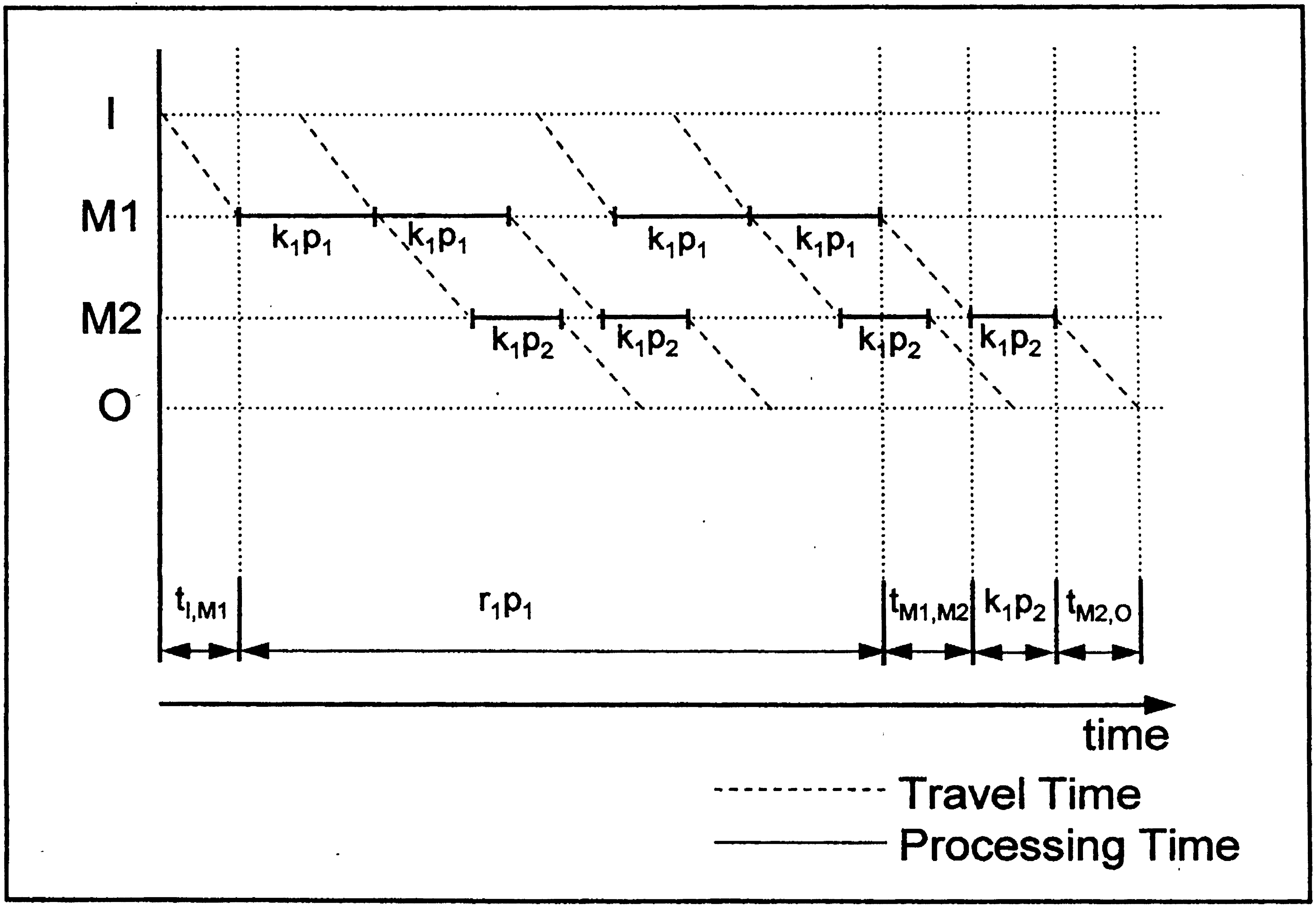
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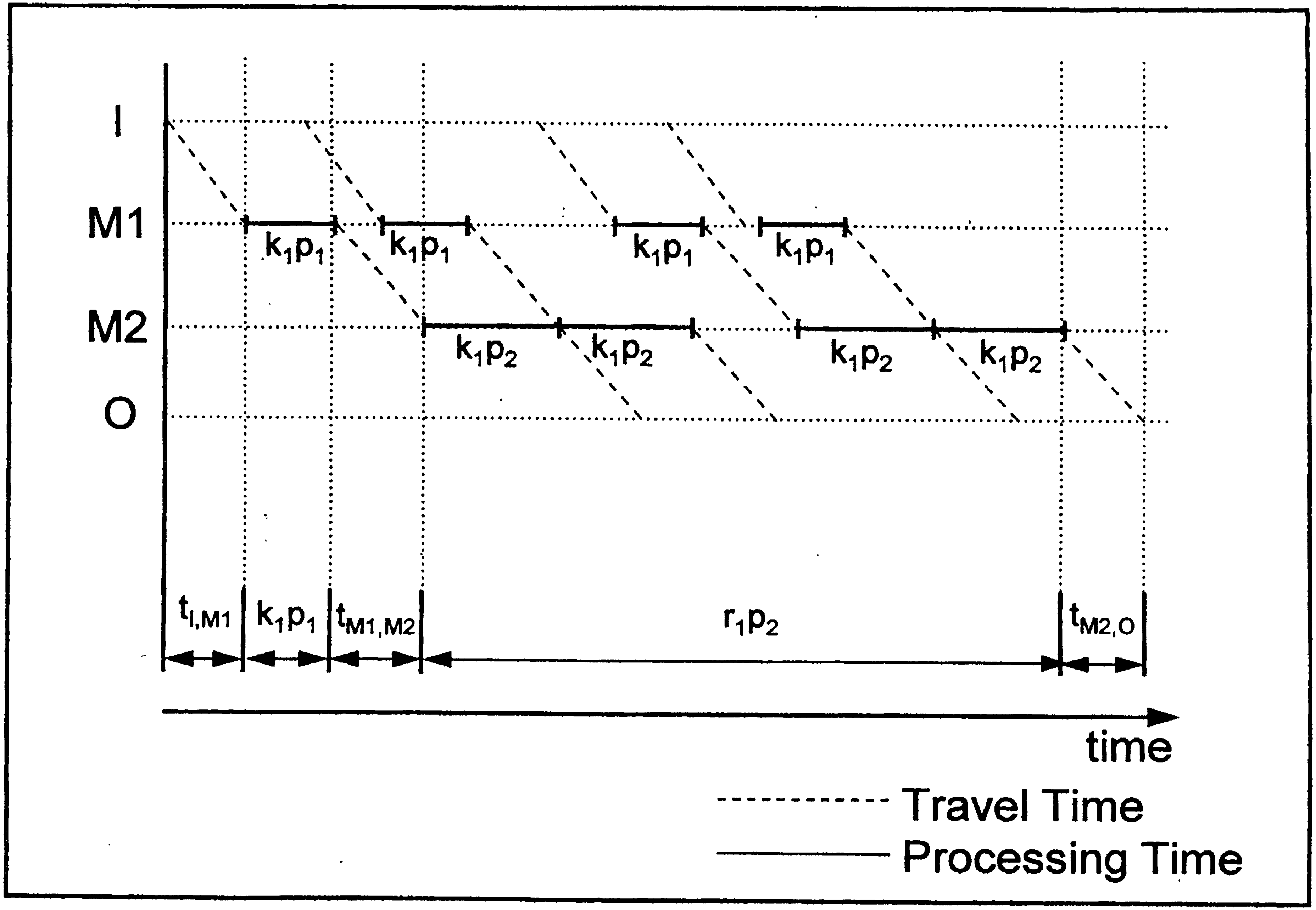

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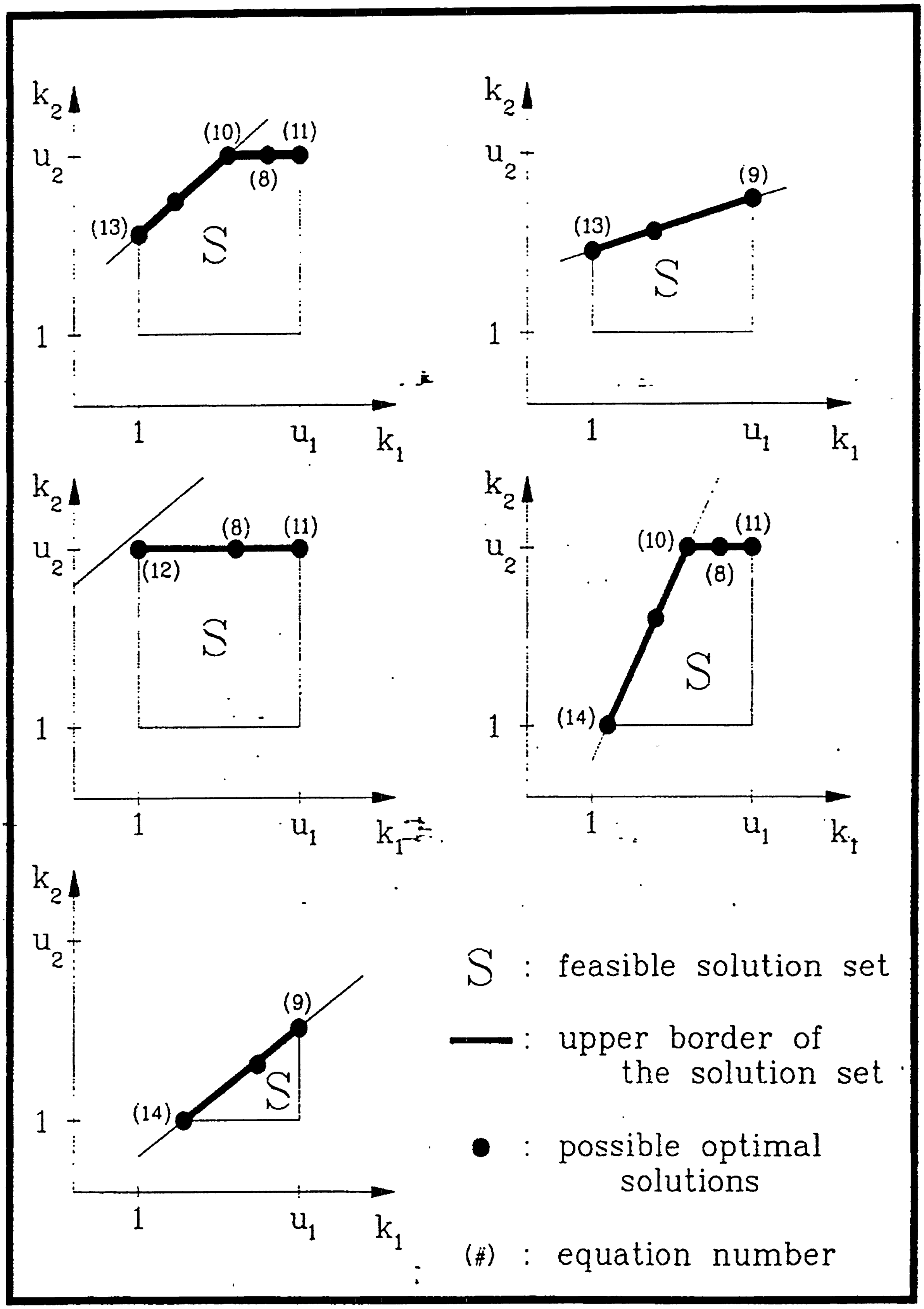

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S : feasible solution set
: upper border of the solution set
: possible optimal solutions
(#) : equation number