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ON THE CALCULATION OF THE CRITICAL ROLLING SPEED OF A PNEUMATIC TIRE

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ON THE CALCULATION OF THE CRITICAL ROLLING SPEED OF A PNEUMATIC TIRE

V. L. Biderman

As the angular velocity of a pneumatic tire increases, the state of rolling of the tire changes abruptly at a certain limit. Waves appear on the surface of the tire (Fig.1), stationary in the area, but moving with the speed of rolling relative to the tire. The speed at which waves appear on the surface of the tire may be called the critical speed of rolling of the tire. As the rolling speed of the tire approaches the critical point, rolling loss in the tire abruptly increases, and correspondingly the heating of the tire increases. The life of the tire at a speed near the critical point, therefore, is quite short.

(It is not possible to secure a satisfactory reproduction of this photograph.)

Fig. 1. Wave-like deformations of a 7.50-20 tire during the time of rolling on a stand at a speed of 180 km/hr.

In this way, phenomena appearing at the critical speed restrict the use of tires at high speeds. Consequently, it is necessary to plan special tires with possibly a higher critical speed for speeding automobiles.

General trends of the design of tires used at high speeds are important. An increase of the critical rolling speed of a tire is attained with an increase of internal pressure in the tire and with a reduction of its mass.

However, the quantitative influence of these factors on critical speed has not been sufficiently investigated. There is also a lack of information on the dependence of critical speed on the stiffness of rubber and cord, and on the angle of cord fibers and other constructional features of the tire.

Consequently it has been of practical interest to develop a method for the calculation of the critical rolling speed of a pneumatic tire.

In a published work* it was demonstrated that the appearance of the critical speed of a tire is completely analagous to the appearance of lateral oscil-

*S. D. Ponomarev, V. L. Biderman, K. K. Lixarez, V. M. Makyshin, N. N. Malinin, V. I. Feodosev, Fundamental contemporary methods of calculation on reliability in mechanical engineering, Mashgiz, 1952.

lations of turbine-driven discs. Oscillations of a tire appear when its rolling speed becomes equal to the speed of travel of a running wave of deformation along the circumference of the tire. Moreover, forces of resilience of the tire are equalized with forces of inertia, and therefore the exterior load may be thought of as due to the presence of internal friction in the material.

With this is associated the increasing loss of power as the rolling speed of the tire approaches the critical point.

Thus the critical rolling speed of the tire is equal to the minimum speed of travel of the wave along its circumference.

The tire represents a rubber-cord shell with extremely strong anisotropic elastic properties. Deformation in the circumferential and meridional directions of this rubber-cord shell are relatively small due to a change in rhombus angles (formed by fiber cords of adjacent layers), and the deformation is restricted in the direction of the fiber cords due to the greater rigidity of the fiber cords.

Using this as a basis, we can now take as a principle of calculation a hypothesis in which fiber cords do not elongate during the time of membranous deformation of the shell.

Investigation of tire deformation during the time at which waves appear (see Fig. 1) shows that, in a given cross section of a tire, normal movements have an identical sign, that is, the increase of diameter of a tire in a given cross section matches the simultaneous increase of profile width. The same sort of wave deformations may extend not only along the toroidal-shaped tire casing, but also along a straight rubber-cord sleeve. It is evident that the speed of propagation of waves along the tire may in the first approximation be assumed to be equal to the speed of the waves in a sleeve of the same cross section and in the presence of the same internal pressure. However, this does not take into account the curvature near the rim of the tire. The influence of this factor may be evaluated by approximate corrections.

Let us examine the progress of the deformation waves along the straight rubber-cord cylindrical shell (Fig. 2).

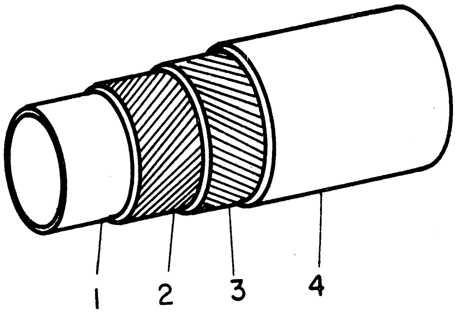

Fig. 2. Two-layered rubber-cord hose:
1. Airtight inward layer;
2,3. Layer of cord;
4. Protective device.

The critical speed is calculated with the aid of Rayleigh's method. This method, as is well known, is based upon equating values of potential and kinetic energy of the system.

Component parts of the potential energy system are:

(1) Energy of compressed air in the inner cavity of the shell at the time of its oscillation;

(2) Energy of extension of the walls of the shell in connection with its deformation;

(3) Bending energy of the walls of the shell.

To calculate the value of potential and kinetic energy, we must get expressions for the displaced points of the shell. Interest is directed to the case where there is no tangential deformation of the shell, inasmuch as, as was shown above, longitudinal nodal lines are absent at the time of the extension of waves in the tire.

In this case of deformation in which the tangential component of deformation is zero, any point of the shell may be defined by two dimensions: radial displacement ω and axial displacement u (Fig. 3, a).

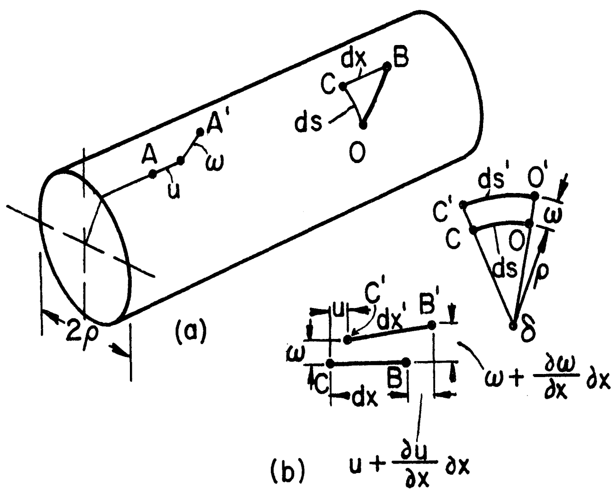

Fig. 3. On the derivation of conditions of nonextensibility in tire cords.

Inasmuch as the hypothesis of unextensible tire cord has been accepted, dimensions u and ω cannot be independent.

Let us examine the element $d\ell$ of the length of tire cord, the projection of which on circumferential and axial directions equals ds and dx, respectively, until the shell begins to deform. After deformation, ds changes to ds' (Fig. 3b):

$$ds' = ds\left(1 + \frac{\omega}{\rho}\right)$$

but dx becomes (Fig. 3b)

$$dx' = dx\sqrt{\left(1 + \frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial \omega}{\partial x}\right)^2}$$

The new length of element $d\ell$ is determined from the equation:

$$(d\ell')^2 = (ds')^2 + (dx')^2 = ds^2\left(1 + \frac{\omega}{\rho}\right)^2 + dx^2\left[\left(1 + \frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial \omega}{\partial x}\right)^2\right]$$

Since the cord length is invariant,

$$(d\ell')^2 = d\ell^2 ,$$

or

$$d\ell^2 = ds^2\left(1 + \frac{\omega}{\rho}\right)^2 + dx^2\left[\left(1 + \frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial \omega}{\partial x}\right)^2\right]$$

Dividing both parts of the equation by $d\ell^2$ and taking into account that $ds/d\ell = \cos\beta$ and $dx/d\ell = \sin\beta$ (where β—angle constructed by fibers of cords with alignment of profile), we find:

$$1 = \left(1 + \frac{\omega}{\rho}\right)^2 \cos^2\beta + \left[\left(1 + \frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial \omega}{\partial x}\right)^2\right] \sin^2\beta ,$$

or after simplification,

$$0 = \left[2\frac{\omega}{\rho} + \left(\frac{\omega}{\rho}\right)^2\right] \cos^2\beta + \left[2\frac{\partial u}{\partial x} + \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial \omega}{\partial x}\right)^2\right] \sin^2\beta . \qquad (1)$$

Equation (1) espresses the relationships between u and ω, when the length of the tire cord is constant. In the presence of small movements, quadratic terms in Eq. (1) appear small in comparison with linear terms. Therefore in the first approximation, Eq. (1) gives the function:

$$\frac{\omega}{\rho} = -\frac{\partial u}{\partial x} \tan^2\beta \qquad (1a)$$

However, this expression does not appear exact enough for the problem with which we are faced. Substituting the approximate expression for ω, (1a), we find the expression for ω corrected for terms of second order as:

$$\frac{\omega}{\rho} = -\frac{\partial u}{\partial x} \tan^2\beta - \frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2 \tan^2\beta\,(1 + \tan^2\beta) - \frac{1}{2}\rho^2\left(\frac{\partial^2 u}{\partial x^2}\right) \tan^6\beta . \qquad (2)$$

We let u express the sinusoidal running waves:

$$u = u_o \sin \frac{2\pi}{L} (x - ct) \tag{3}$$

where L = the length of the wave;

c = the speed of propagation;

u_o = amplitude dimension of displacement.

Then we obtain a suitable expression for:

$$\frac{\omega}{\rho} = -u_o \frac{2\pi}{L} \tan^2\beta \cos \frac{2\pi}{L} (x - ct) - \frac{1}{2} u_o^2 \frac{4\pi^2}{L^2} \tan^2\beta\cdot(1 + \tan^2\beta)\cos^2 \frac{2\pi}{L} (x - ct)$$

$$-\frac{1}{2} \rho^2 u_o^2 \frac{16\pi^4}{L^4} \tan^6\beta \sin^2 \frac{2\pi}{L}(x - ct) \tag{4}$$

In this expression the first term is dominant, while the remaining terms are small. They must be taken into account only for calculation of compressed air effects, inasmuch as in this case the first term drops out.

Let us now proceed to the calculation of kinetic energy of the oscillating shell.

The element of a shell with the dimension dx·ds has a mass dm = qdx·ds, where q is mass per unit surface area.

Components of the velocity are, omitting a few terms of higher order:

$$\frac{\partial u}{\partial t} = -c \frac{2\pi}{L} u_o \cos \frac{2\pi}{L} (x - ct)$$

$$\frac{\partial \omega}{\partial t} = \rho c \frac{4\pi^2}{L^2} u_o \tan^2\beta \sin \frac{2\pi}{L} (x - ct)$$

The incremental kinetic energy is given by:

$$dT = \frac{1}{2} dm \left[\left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial \omega}{\partial t}\right)^2\right]$$

$$= \frac{1}{2} qc^2 u_o^2 \frac{4\pi^2}{L^2} \left[\cos^2 \frac{2\pi}{L} (x - ct) + \frac{4\pi^2}{L^2} \rho^2 \tan^4\beta \sin^2 \frac{2\pi}{L} (x - ct)\right] dxds$$

Integrating dT over s from zero to $2\pi\rho$ and over x from zero to L, we find the kinetic energy in the limits of one wave of oscillation:

$$T = \frac{2\pi^3}{L} c^2 \rho q u_o^2 \left(1 + \frac{4\pi^2}{L^2} \rho^2 \tan^4\beta\right) \tag{5}$$

The potential energy of the air pressure changes during the time of deformation $U_B = -p\Delta V$ where ΔV is the change of the volume of the shell.

The volume of the cylindrical length dx (Fig. 4) before deformation is:

$$dV_o = \pi\rho^2 dx \quad ,$$

and after deformation,

$$dV' = \pi(\rho + \omega)^2 dx \left(1 + \frac{\partial u}{\partial x}\right) .$$

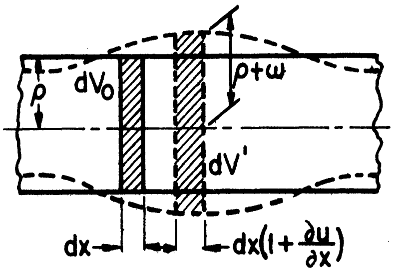

Fig. 4. On the determination of the change of air volume during deformation of the hose.

The change of volume is:

$$\Delta dV = \pi\rho^2 \left[2\frac{\omega}{\rho} + \frac{\partial u}{\partial x} + \left(\frac{\omega}{\rho}\right)^2 + 2\frac{\omega}{\rho}\cdot\frac{\partial u}{\partial x} + \left(\frac{\omega}{\rho}\right)^2 \frac{\partial u}{\partial x}\right] dx \quad .$$

Substituting the value of ω and u and integrating x between the limits of zero to L, we find a change of volume within the confines of one wave of oscillation:

$$\Delta V = -u_o^2 \pi\rho^2 \frac{2\pi^2}{L}\left(3 + 4\frac{\pi^2\rho^2}{L^2}\tan^4\beta\right)\tan^2\beta \quad .$$

According to this value of ΔV, the potential energy of pressure equals:

$$U_B = p u_o^2 \pi\rho^2 \frac{2\pi^2}{L}\left(3 + 4\frac{\pi^2\rho^2}{L^2}\tan^4\beta\right)\tan^2\beta \qquad (6)$$

Let us now examine the deformation energy of the shell; this energy is made up of two parts:

(1) Energy U_M of deformation of the membrane type, and

(2) Energy U_e of bending deformation.

This leads to the supposition that, in the presence of membrane deformation, tire cords remain unextensible, and all energy in this case will be consumed by the deformation of the rubber.

Average deformation of the shell is determined by using the terms

circumferential deformation $\epsilon_t = \omega/\rho$

and lengthwise deformation $\epsilon_x = \partial u/\partial x$

These expressions represent rubber deformation in the tread and interlayer areas. In layers where the rubber occupies only space between the fibers, actual deformation of the rubber is greater and approximately

$$\epsilon_{tp} = k\epsilon_t \; ; \; \epsilon_{xp} = k\epsilon_x \; ,$$

where

$$k = 1/1 - \frac{\pi}{4}\, id \; ,$$

where:**

i = end count per centimeter
d = diameter of tire cord

The volume of rubber in the layers on one surface of the shell is:

$$\frac{1}{k} \cdot d \cdot n$$

where n = the number of cord layers and the volume of the rubber in the tread and interlayers is:

$$(h - d \cdot n)$$

where h = the overall thickness of the shell walls.

Deformation energy of a single volume of rubber in the presence of plane stress conditions and deformation, ϵ_t and E_x, amounts to:

$$a = \frac{E_p}{2(1 - \mu^2)} \left(\epsilon_x^2 + \epsilon_t^2 + 2\mu\epsilon_x \cdot \epsilon_t \right) \; ,^{***}$$

where E_p = modulus elasticity of the rubber;
μ = Poisson's Ratio.

The energy of deformation of the tread and interlayers over an element of the shell's surface, dx·ds, is:

$$dU_1 = (h - d \cdot n) \frac{E_p}{2(1 - \mu^2)} (\epsilon_x^2 + \epsilon_t^2 + 2\mu\epsilon_x\epsilon_t) \, dxds \; ,$$

and the energy of the rubber in the plies is:

**Derivation of a reduced formula based on a substitution of an actual cross section of fiber with a like size taken at right angles, d·(πd/4).

***Translator's Note: The reader is referred to an article by Bleich, H. H. and DiMaggio, F., "A Strain-Energy Expression for Thin Cylindrical Shells," Jour. Appl. Mech., 20, n. 3 (Sept., 1953), 448, for a discussion of the implications of this expression.

$$dU_2 = kd \cdot n \frac{E_p}{2(1 - \mu^2)} (\epsilon_x^2 + \epsilon_t^2 + 2\mu\epsilon_x\epsilon_t) \, dxds \quad .$$

Summing up these values, we find:

$$dU_M = \frac{E_p h^*}{2(1 - \mu^2)} (\epsilon_x^2 + \epsilon_t^2 + 2\mu\epsilon_x\epsilon_t) \, dxds \quad . \tag{7}$$

where h^* = the reduced thickness of the rubber;

$$h^* = h + (k - 1)d \cdot n = h + \frac{\pi d^2 in}{4 - \pi id} \tag{7a}$$

Substituting the values ϵ_x and ϵ_t in expression (7) and integrating s from zero to $2\pi\rho$, and x from zero to L, we find the energy of membrane deformation of a shell within the limits of one wave:

$$U_M = \frac{E_p h^*}{2(1 - \mu^2)} \cdot \frac{4\pi^3\rho}{L} u_o(1 + \tan^4\beta - 2\mu \tan^2\beta) \quad . \tag{8}$$

Now let us calculate the energy of bending of the shell walls.

As a result of deformation of the shell, its generatrix, not having curvature originally, acquires a curvature approximately equalling:

$$\chi_1 = - \frac{\partial^2 \omega}{\partial x^2} \quad ,$$

but the circumference, which originally had a curvature $1/\rho$, acquires a new curvature $1/\rho+\omega$; in this way the change of curvature of the circumference amounts to:

$$\chi_2 = \frac{1}{\rho + \omega} - \frac{1}{\rho} \approx - \frac{\omega}{\rho^2} \quad .^{****}$$

At a distance y from the neutral surface of deflection in relation to the change of curvature, deformations occur:

$$\epsilon_x = \chi_1 y = - \frac{\partial^2 \omega}{\partial x^2} y \quad ,$$

$$\epsilon_t = \chi_2 y = - \frac{\omega}{\rho^2} y \quad .$$

In view of the rigidity of the cords, the middle surface of the tire carcass will be used as the neutral axis and y measured from there.

During the time of deflection it is necessary to take into account not only the deformation of the rubber, but also the deformation of the cords.

****Translator's Note: Within the assumption of small displacements.

In the layer found at a distance y from the neutral surface, tire cord is elongated:

$$\epsilon_k = \epsilon_x \sin^2\beta + \epsilon_t \cos^2\beta = -\left(\frac{\partial^2\omega}{\partial x^2}\sin^2\beta + \frac{\omega}{\rho^2}\cos^2\beta\right) y .$$

The deformation energy of tire cord of this layer on the section dxds of the surface amounts to:

$$dU_k' = i\,\frac{E_k\epsilon_k^2}{2}\,dxds = i\,\frac{E_k}{2}\,y^2\left(\frac{\partial^2\omega}{\partial x^2}\tan^2\beta + \frac{\omega}{\rho^2}\right)^2 \cos^4\beta\; dxds ,$$

where E_k = the ratio of stress in the cord to its deformation. Summarizing energy for all layers, we find:

$$dU_k = \frac{1}{2}\, i\, E_k \sum y^2 \left(\frac{\partial^2\omega}{\partial x^2}\tan^2\beta + \frac{\omega}{\rho^2}\right)^2 \cos^4\beta\; dxds . \qquad (9)$$

Whereupon $\sum y^2$—this is the sum of the squares of the distance of all cord layers from the neutral layer.

Substituting in expression (9) the values ω and $\partial^2\omega/\partial x^2$ and integrating s from zero to $2\pi\rho$ and x from zero to L, we find the energy of deformation of tire cord during the time of deflection:

$$U_k = A_k\,\frac{2\pi^3}{\rho L}\left(1 - \frac{4\pi^2\rho^2}{L^2}\tan\beta\right)^2 u_o^2 , \qquad (10)$$

where

$$A_k = i\, E_k \sum y^2 \sin^4\beta .$$

Deformation energy of rubber during the time of deflection of the shell may be determined in the following way:

The energy of deformation of the element ds dx dy amounts to:

$$\frac{E_p}{2(1-\mu^2)}\left(\epsilon_x + \epsilon_t + 2\mu\epsilon_x\epsilon_t\right) ds\,dx\,dy = \frac{E_p}{2(1-\mu^2)}\left[\left(\frac{\partial^2\omega}{\partial x^2}\right)^2 + \frac{\omega^2}{\rho^4} + 2\mu\,\frac{\partial^2\omega}{\partial x}\cdot\frac{\omega}{\rho^2}\right] y^2\, ds\,dx\,dy .$$

Disregarding the influence of cords on the deformation of rubber in the layers of the carcass and performing the integration over y, we find:

$$dU_{ep} = A_p\,\frac{1}{2}\left[\left(\frac{\partial^2\omega}{\partial x^2}\right) + \frac{\omega^2}{\rho^4} + 2\mu\,\frac{\omega}{\rho^2}\,\frac{\partial\,\omega}{\partial x^2}\right] dsdx , \qquad (11)$$

where

$$A_p = \frac{E_p(h_1^3 + h_2^3)}{3(1 - \mu^2)} \tag{11a}$$

h_1 and h_2 are the distances from the neutral layer to the exterior and interior surface of the shell.

Substituting in expression (11) the values ω and $\partial^2\omega/\partial x^2$ and performing the integration over s and x, we find the deformation energy of rubber inside the limits of one wave due to deflection as:

$$U_{ep} = A_p \frac{2\pi^3}{\rho L} \tan^4\beta \left(1 + 16\pi^4 \frac{\rho^4}{L^4} - 8\pi^2\mu \frac{\rho^2}{L^2}\right) u_o^2 \quad . \tag{12}$$

We find the velocity of wave travel by equating the kinetic energy change to total potential energy:

$$T = U_B + U_M + U_K + U_{ep} \quad ,$$

where T is determined according to formula (5), but the remaining values according to formulas (6), (8), (10), and (12), respectively.

Thus we find:

$$c^2 = \frac{p\rho}{q} \cdot \frac{a + b\lambda^2 + c\lambda^4}{1 + d \cdot \lambda^2} \quad , \tag{13}$$

where

$$\lambda = 2\pi\rho/L \quad ,$$

$$a = 3 \tan^2\beta + \frac{E_p h^*}{p\rho(1 - \mu^2)} (1 + \tan^4\beta - 2\mu \tan^2\beta) + \frac{A_k}{p\rho^3} + \frac{A_p}{p\rho^3} \tan^4\beta \;,$$

$$b = \tan^6\beta - \frac{A_k}{p\rho^3} \cdot 2 \tan^2\beta - \frac{A_p}{p\rho^3} 2\mu \tan^4\beta \quad ,$$

$$c = \frac{A_k + A_p}{p\rho^3} \tan^4\beta \quad , \text{ and}$$

$$d = \tan^4\beta \quad .$$

From expression (13) it is clear that the speed of wave propagation depends on its wavelength, since λ is the ratio of the perimeter of a cross section of the shell to the length of the wave (Fig. 5).

From expression (13) we find that the minimum value of c occurs when

$$\lambda^2 = \lambda_*^2 = \sqrt{(c - bd + ad^2)/cd^2} - 1/d \quad . \tag{14}$$

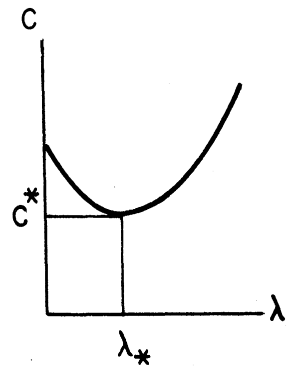

Fig. 5. The relation of wave speed to the ratio of the perimeter of a cross section of the shell to the length of the wave.

From expression (14) the length of the wave L^* in the presence of oscillation is determined as:

$$L^* = \frac{2\pi\rho}{\lambda_*}$$

The speed of the wave is determined by substituting λ^2 from expression (14) into formula (13).

For the calculation of the critical speed of a rolling tire, according to the previous expressions, an effective value of specific tire mass q and of the angle of cord fibers β must be derived.

By means of an example we are going to show how to calculate the critical speed of a rolling tire of size 7.50 x 16 in the presence of an internal pressure $p = 2.5\ kg/cm^2$.

In this example, the radius of the tire profile is $\rho = 8$ cm; average angle of tire cord can be taken as $\beta = 45°$. The tire has 6 layers of cord 9T; the average size of cord layer in the tire tread amounts to 1.4 mm, end-count of the fibers is i = 8 fiber/cm, and the size of cord fibers d = 0.8 mm.

The average mass per unit surface area of the tire is found, assuming that the thickness of a wall on the average equals 2 cm, and the specific weight of material $\gamma = 1.5 \cdot 10^{-3}\ kg/cm^3$. Then

$$q = \frac{2(1.15 \times 10^{-3})}{981} = 2.35 \times 10^{-6} \frac{kg\text{-}sec^2}{cm^3}$$

Inasmuch as the fiber in the casing under the effect of inner pressure has initial deformation of the order of 2%, the modulus of the fiber must be determined at about this value of elongation, whereupon the following formula may be used:

$$E_k \approx (N_3 - N_2)100 \quad ,$$

where N_3 = stress in the fiber in the presence of 3% elongation;
N_2 = stress in the fiber in the presence of 2% elongation.

For cord 9T, $(N_3 - N_2)$ = i kg and consequently E_k = 100 kg/fiber. Modulus of rubber resilience $E_p = 10\ kg/cm^2$; Poisson's Ratio $\mu = 0.5$.

Let us calculate dimensions necessary for the computation of critical speed.

According to formula (7a) we find the equivalent thickness of the rubber:

$$h^* = h + \frac{\pi d^2 i \cdot n}{4 - \pi i d} = 2 + \frac{\pi(0.8)^2 \cdot 8 \cdot 6}{4 - \pi \cdot 8 \cdot 0.8} = 2.48 \text{ cm} .$$

According to formula (10a), rigidity at deflection of cord layers is determined as:

$$A_k = iE_k \sum y^2 \sin^4\beta ,$$

where y is the distance of each of the layers from the neutral surface. Figuring that the neutral surface runs through the center between the third and fourth layers, we get

$$y_1 = 0.35 \text{ cm}; \quad y_2 = 0.21 \text{ cm}; \quad y_3 = 0.07 \text{ cm};$$
$$y_4 = 0.07 \text{ cm}; \quad y_5 = -0.21 \text{ cm}; \quad y_6 = 0.36 \text{ cm}.$$

Then

$$A_k = 8(100)(2)(0.35^2 + 0.21^2 + 0.07^2) = 68 \text{ kg·cm}$$

The rigidity of the rubber is determined according to Eq. (11a), realizing that $h_1 = 0.5$ cm, $h_2 = 1.5$ cm;

$$A_p = \frac{E_p(h_1^3 + h_2^3)}{3(1 - \mu^2)} = \frac{10[(0.5)^3 + (1.5)^3]}{3(0.75)} = 15.6 \text{ kg} \cdot \text{cm} .$$

The coefficients a, b, c, and d that appear in Eq. (13) are determined:

$$a = 3 \tan^2\beta + \frac{E_p h^*}{p\rho(1 - \mu^2)} (1 + \tan^4\beta - 2\mu \tan^2\beta) + \frac{A_k}{p\rho^3} + \frac{A_p}{p\rho^3} \tan^4\beta$$

$$= 3(1)^2 + \frac{10(2)(48)}{(2.5)(8)(0.75)} [1 + (1)^4 - 2(0.5)(1)^2] + \frac{68}{(2.5)(8)^3} + \frac{15.6}{(2.5)(8)^3}(1)^4$$

$$= 4.7;$$

$$b = \tan^6\beta - \frac{A_k}{p\rho^3} 2 \tan^2\beta - \frac{A_p}{p\rho^3} 2\mu \tan^4\beta$$

$$= (1)^6 - \frac{68(2)(1)^2}{(2.5)(8)^3} - \frac{15.6}{(2.5)(8)^3} (2)(0.5)(1)^4 = 0.88;$$

$$c = \frac{A_k + A_p}{p\rho^3} \tan^4\beta = \frac{68 + 15.6}{(2.5)(8)^3} (1) = 0.065;$$

$$d = \tan^4\beta = 1.$$

λ_*^2 is found according to (14):

$$\lambda_*^2 = \sqrt{\frac{c - bd + ad^2}{cd^2}} - \frac{1}{d} = \sqrt{\frac{0.065 - 0.88(1) + 4.7(1)}{0.065(1)}} - 1 = 6.74 .$$

$$\lambda_* = 2.6 \; .$$

Length of wave deformation:

$$L = \frac{2\pi\rho}{\lambda} = \frac{2\pi(8)}{2.6} = 19.3 \text{ cm} \; .$$

The critical speed is found according to (13):

$$c^2 = \frac{p\rho}{q} \times \frac{a + b\lambda^2 + c\lambda^4}{1 + d\lambda^2} = \frac{2.5(8)}{(2.35 \times 10^{6})} \times \frac{4.7 + 0.88(6.74) + 0.065(6.74)^2}{1 + 1(6.74)^2}$$

$$= 13.1 \times 10^6 \text{ cm}^2/\text{sec}^2 \; ;$$

$$c = 3.62 \times 10^3 \text{ cm/sec} = 36.2 \text{ m/sec} = 130 \text{ km/hr} \; .$$

The actual critical rolling speed of this tire, found experimentally by V. I. Novopolsk, amounts to 160 km/hr, and the wave length was L = 17 cm.

It is possible to assume some increase of critical speed and decrease of wave length compared to calculated values primarily due to the influence of fixity at the rim.

The expressions derived allow us to analyze the dependence of the critical rolling speed of a tire on inner pressure, angle of tire cord, and rigidity of rubber.

Appropriately calculated graphs are presented in Figs. 6, 7, and 8. From the graphs it is obvious that to increase the critical speed one can increase

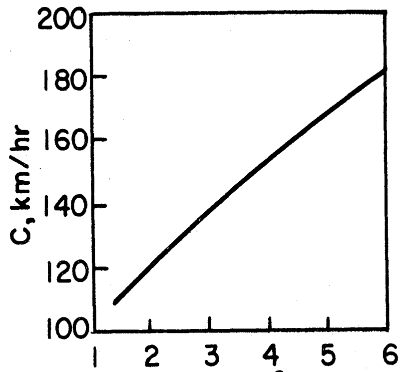

Fig. 6. Dependence of critical speed on inner pressure.

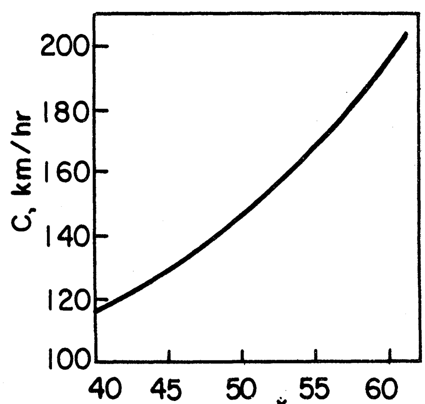

Fig. 7. Dependence of critical speed on tire cord angle.

the inner pressure and lower the tires' mass, as well as to increase the angle of cord fibers and to raise the rigidity of the rubber.

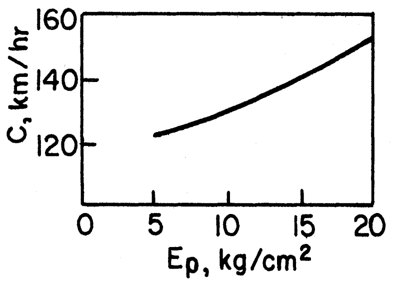

Fig. 8. Dependence of critical speed on modulus of rubber resilience.

The method just used for the determination of critical speed may be developed without using a simplified hypothesis about the equivalence of the rubber-cord shell and tire. However, inasmuch as deformation will of necessity include tangential movement, the mathematical expressions are considerably more complicated. In conclusion it should be pointed out that the influence of dynamic processes on the performance of a tire begin to tell in the presence of speed essentially less than critical. In view of this, the permissible operation speed of a rolling tire must be designated as

$$V_{max} = \eta \, c_{kp} ,$$

where η is the coefficient of allowance, a number smaller than one.

The value of the coefficient η must be chosen with due regard to tire load and the required mileage in the presence of speed V max.