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A NOTE ON INTERACTION CLASSIFICATION

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Abstract

Further classification is made of Lindquist's dichotomy of interaction effects. The extension hopefully reduces errors of interpretation and provides a simple, accurate means of summarizing interactions obtained.

A NOTE ON INTERACTION CLASSIFICATION

A thorough review of the textbooks and reference manuals on experimental design reveals an absence of any discussion of interaction effects which extends beyond the customary treatment of computational requirements, necessary assumptions, and general interpretation. The purpose of this paper is to supplement these design-oriented treatments with the discussion of an analysis-oriented issue: interaction classification.

Theoretical Basis

Lubin (1961, p. 808) makes reference to Lindquist (1953) for distinguishing between "two important subclasses of significant interaction effects: the 'ordinal' case, where the rank order of the treatments is constant; and the 'disordinal' case, where the rank order of the treatments changes." If the cell means relevant to a particular interaction are plotted on a graph with the levels of one factor on the abscissa and the dependent variable levels on the ordinate axis, disordinal interaction is characterized by crossed lines connecting treatment combinations for each profile displayed (Glass and Stanley 1970, p. 411). The rank order is non-monotonic between two or more adjacent abscissa factor levels. Glass and Stanley specify further that ordinal interaction is apparent in those cases where the lines do not cross. This condition occurs only when the rank order of levels of the factor displayed remains constant over all levels of the abscissa factor.

These authors point out, however, that some interactions may be ordinal with one factor as the abscissa and disordinal with a different contributing factor. For this reason they caution that the terminology relates solely to properties of graphs and recommend plotting all possible graphs of an interaction to determine its basic nature. We agree strongly with their premise and suggestion, but we also believe students and casual researchers may continue to identify interactions incorrectly unless further classification of interaction types is made. We offer the following three categories: pure ordinal, hybrid and pure disordinal interactions. An example of each type is displayed in Figure 1.

[Figure 1 about here]

Pure ordinal interaction characterizes the situation in which, regardless of the factor used as the abscissa, a consistent rank order relationship exists for levels of a factor or factor combination between levels of the abscissa factor. The "slopes" of <u>all</u> lines representing profiles vary in the same direction between adjacent abscissa levels on every possible graph depicting the interaction. This definition does not require that the slopes are always positive or negative across levels of every possible abscissa factor; rather, pure ordinal interaction is a restricted form of ordinal interaction that includes only those cases in which <u>every</u> graph that depicts the interaction is ordinal.

Hybrid interaction differs from pure ordinal interaction in that the rank order of treatment profiles is invariant between levels of one or more factors, and it varies between levels of one or more remaining factors. The slopes of two or more lines will vary inversely between two or more levels of any of the possible factors, but they do not necessarily have to cross. Since negatively related slopes between two abscissa levels imply that, for a two-factor interaction, the largest and smallest mean values occurred at the same abscissa factor level, the lines must cross on the remaining graph. This relationship will exist for higher-order interactions in which at least one graph will display crossed lines for the interaction contrast and at least one graph will display uncrossed lines; the exact number of each depends on effects present.

Pure disordinal interaction occurs only when a non-monotonic relationship characterizes at least one possible contrast for each factor combination. Regardless of the factor used as the abscissa, lines connecting treatment combinations for two levels of the other component(s) cross at least once. For a particular pure disordinal contrast, the contribution to variation explained made by the interaction parameter is much larger than main effect contributions, and it can be absolute (Lubin 1961, p. 815).

From any of the possible graphs describing a particular situation, it is relatively easy to detect that pure ordinal interaction does or does not characterize the relationship under consideration. However, if the rank order varies between levels of the abscissa factor, the researcher should either construct the remaining possible graphs, investigate the planar relationships in hypergeometric space (for higher order interactions), or compute main effect and interaction contributions to variation explained. In this case, the two-dimensional graph is preferred as an exploratory tool because it is easily constructed and read. It should not, however, serve as a substitute for other means of analyzing relationships.

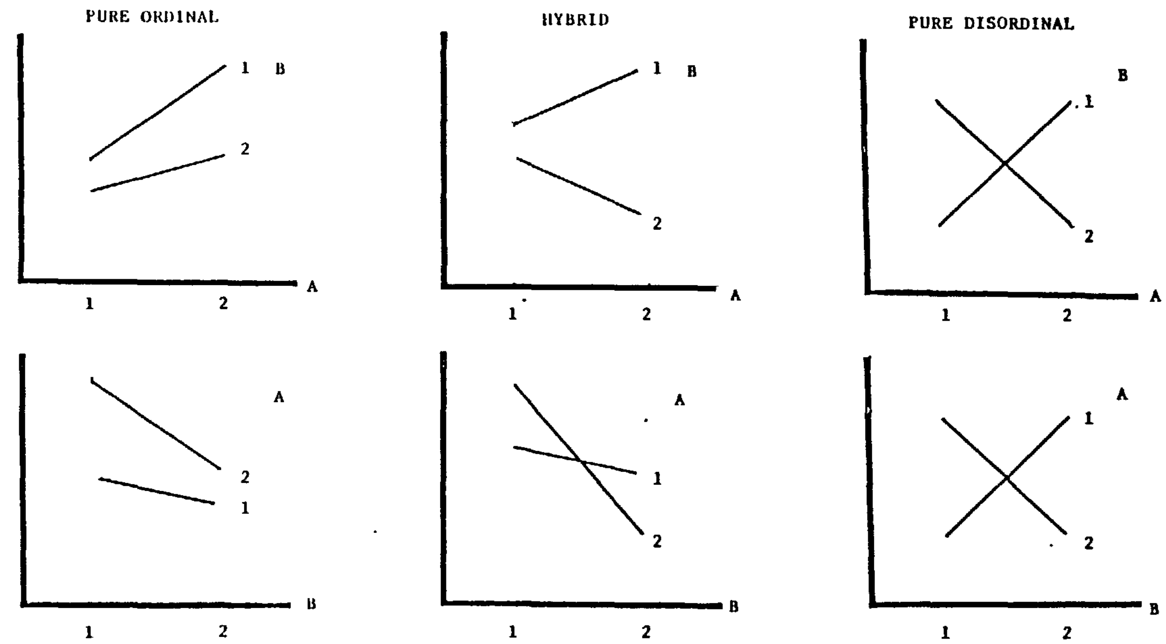
Discussion

The typology developed here is designed to be applicable to all levels of interaction analysis. Entire interaction effects may be classified accordingly to assess the appropriateness of interaction-reducing transformations. Most books on experimental design and linear models indicate the possibility of making the interaction effect non-significant and an additive model applicable by means of a nonlinear transformation on the criterion measure. Without examining issues relating to this practice, we should mention that, although the rank orders for non-intersecting factor combinations may be consistent between abscissa levels, negatively related slopes between two levels indicate one factor may be mediating the effects of others. In this case an effective transformation would sacrifice potentially important information. However, we do not believe transformations can be made profitably on hybrid and pure disordinal interactions unless the disordinal portions are trivial. Pure ordinal interaction is the only type for which transformations are both mathematically and theoretically appealing.

This typology is particularly useful in the investigation of individual interaction contrasts. The researcher normally attempts to understand how various treatment combinations interact as well as the overall effect. Classifying contrasts by type and by the respective magnitudes and confidence intervals seems to summarize adequately the important relationships and should complement displayed graphs. This suggested summary is applicable to interactions of intervally scaled predictor variables in addition to the ANOVA application addressed in this paper.

FIGURE 1

EXAMPLES OF THREE TYPES OF INTERACTION



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