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Adaptive Forecasting Models for Interrelated Time Series

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INTRODUCTION

The forecasting of interest rates has been a complex problem for many years, and satisfactory methods for accurately forecasting interest rates seem to be few and of limited generality. There are many reasons for this scarcity; theories of interest rates often involve expectations of the future and the statistical testing and validation of the theories has proved to be extraordinarily difficult. Moreover, there are several additional problems inherent in forecasting: (1) the problem of explaining movements of interest rates for sample time periods for which relevant data are available, (2) the problem of making and evaluating a forecast for future time periods, and (3) the problem of how a manager can use a forecast in his decision making.

This paper is concerned with forecasting short-term interest rates (3-month treasury bills and 4- to 6-month commercial paper) by means of econometric models. Some of these models are standard econometric models in that they consist of fixed coefficient regression equations estimated by ordinary least squares (OLS). Others are adaptive econometric models having regression equations in which coefficients are allowed to vary. The latter are models of the Kalman type and are discussed in Section 2 below. These models are relatively new in econometric research and have not, to our knowledge, been applied previously to the forecasting of interest rates.

Our main purpose here is to contrast the results of standard and adaptive econometric models on the basis of their performance with regard to forecast errors. Specifically, each type of model will be used to explain the movements of interest rates in the sample time period (the second quarter of 1962 through the fourth quarter of 1972) and each will be used to forecast short-term interest rates for the first and second quarters of 1973. It is not our intention to present forecasting models which, in their present stage of development, can be used directly in financial decision making; our view is that much research must be done before adequate models for decision making can be developed.

DISCUSSION OF THE MODELS

There are several well-known standard econometric models of the U.S. economy which include treasury bill and commercial paper rates as current endogenous variables in their financial subsectors. We have chosen to use the financial subsector of the Office of Business Economics (OBE) quarterly model as it appeared in [4]. This subsector contains nine equations and among these are two equations in which 3-month treasury bill rates are related to several aggregate monetary and fiscal variables and 4- to 6-month commercial paper rates are related in turn to treasury bill rates.[1] These two equations, considered in isolation from the rest of the financial subsector as well as from

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[1]This version of the OBE model contains 56 equations and is an outgrowth of an earlier 36-equation model described in [8]. The Office of Business Economics has been renamed the Bureau of Economic Analysis, and the most recent version of the model, reported on in [5], is called the BEA model. We refer to the version discussed in this paper as the OBE model.

the remaining subsectors of the full OBE model, will be taken as the standard econometric model for the purposes of the experimental studies in this paper.

The behavioral equations of this model are as follows, where t denotes the t-th quarter:

$$y_1(t) = \alpha_0 + \alpha_1 x_1(t) + \alpha_2 x_2(t) + \alpha_3 x_3(t) + e_1(t) \tag{1}$$

$$y_2(t) = \delta_0 + \delta_1 y_1(t) + \delta_2 y_1(t-1) + e_2(t) \tag{2}$$

and where

$y_1(t)$ denotes 3-month treasury bill market yields,

$y_2(t)$ denotes 4- to 6-month commercial paper rates,

$x_1(t)$ denotes the Federal Reserve discount rate,

$x_2(t)$ denotes the amount of free reserves divided by the sum of demand deposits and currency, both for the previous quarter, and

$x_3(t)$ denotes the fiscal balance, federal net surplus (NIA basis) divided by GNP in 1958 dollars for the previous quarter.

In [8], equation (2) also contains a dummy variable which assumes the value of 1 or 0, respectively, according to whether or not the quarter was one for which certificates of deposit were issued. The period for which our experiments were conducted (including the experiments for forecasting these interest rates into the future) was a period in which certificates of deposits were issued during each quarter, so the effect of the dummy variable is absorbed into the intercept term δ_0 in (2) and does not appear explicitly in this equation.

The behavioral equations (1) and (2) can be regarded as a system of stochastic equations and can be expressed in the standard matrix form for such (constant coefficient) econometric systems as

$$\begin{bmatrix} 1 & 0 \\ a_{21} & 1 \end{bmatrix} \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} + \begin{bmatrix} b_{10} & b_{11} & b_{12} & b_{13} & 0 \\ b_{20} & 0 & 0 & 0 & b_{24} \end{bmatrix} \begin{bmatrix} 1 \\ x_1(t) \\ x_2(t) \\ x_3(t) \\ y_1(t-1) \end{bmatrix} = \begin{bmatrix} e_1(t) \\ e_2(t) \end{bmatrix} \qquad (3)$$

or as

$$Ay(t) + Bz(t) = e(t), \quad (t = 1, \ldots, n). \qquad (4)$$

The conventional reduced form of (3) or (4) can be written

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} \beta_{10} & \beta_{11} & \beta_{12} & \beta_{13} & 0 \\ \beta_{20} & \beta_{21} & \beta_{22} & \beta_{23} & \beta_{24} \end{bmatrix} \begin{bmatrix} 1 \\ x_1(t) \\ x_2(t) \\ x_3(t) \\ y_1(t-1) \end{bmatrix} + \begin{bmatrix} \varepsilon_1(t) \\ \varepsilon_2(t) \end{bmatrix} \qquad (5)$$

or

$$y(t) = \Pi z(t) + \varepsilon(t), \quad (t = 1, \ldots, n). \qquad (6)$$

The adaptive econometric model used in our experiments is a version of the Kalman-Bucy model [7]. We write this model as follows,

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} 1 & x_1(t) & x_2(t) & x_3(t) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & x_1(t) & x_2(t) & x_3(t) & y_1(t-1) \end{bmatrix} \begin{bmatrix} \beta_{10}(t) \\ \beta_{11}(t) \\ \beta_{12}(t) \\ \beta_{13}(t) \\ \beta_{20}(t) \\ \cdot \\ \cdot \\ \cdot \\ \beta_{24}(t) \end{bmatrix} \tag{7}$$

$$+ \begin{bmatrix} \varepsilon_1(t) \\ \varepsilon_2(t) \end{bmatrix}$$

where

$$\begin{bmatrix} \beta_{10}(t) \\ \beta_{11}(t) \\ \cdot \\ \cdot \\ \cdot \\ \beta_{24}(t) \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} & \cdots & t_{19} \\ t_{21} & t_{22} & \cdots & t_{29} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ t_{91} & t_{92} & \cdots & t_{99} \end{bmatrix} \begin{bmatrix} \beta_{10}(t-1) \\ \beta_{11}(t-1) \\ \cdot \\ \cdot \\ \cdot \\ \beta_{24}(t-1) \end{bmatrix} + \begin{bmatrix} u_1(t) \\ u_2(t) \\ \cdot \\ \cdot \\ \cdot \\ u_9(t) \end{bmatrix} \tag{8}$$

Alternatively, (7) and (8) can be written in matrix form as

$$y(t) = X(t)\beta(t) + \varepsilon(t) \tag{9}$$

$$\beta(t) = T(t)\beta(t-1) + u(t). \tag{10}$$

The model represented by (9) and (10) requires that the matrix T(t) be specified, as well as an initial value $\beta(0)$ and the variances-covariances of the error vectors $\varepsilon(t)$ and $u(t)$. It should be noted that the vector $\beta(t)$ in (9) varies with t and is not a constant parameter. Thus, the model (9) and (10) is not a constant coefficient model as is the standard econometric model (3), (4) or (5), (6).

NATURE OF FORECASTING EXPERIMENTS

In forecasting with a standard econometric model for τ periods into the future, one uses an estimate $\hat{\pi}$ of the reduced form coefficient matrix π in (6) as well as an estimate $\hat{z}(t + \tau)$ of the values of the independent variables τ periods into the future and then develops a forecast $\hat{y}(t + \tau)$ of the dependent variables by

$$\hat{y}(t + \tau) = \hat{\pi}\ \hat{z}(t + \tau). \tag{11}$$

In contrast to this procedure, the Kalman-Bucy varying parameter regression model recognizes the changing coefficient vector $\beta(t)$ in (9) and adaptively updates this vector through (10) to obtain $\hat{\beta}(t + \tau)$. Then a forecast $\hat{X}(t + \tau)$ of the matrix of independent variables is developed and the forecast of the dependent variables is then given by

$$\hat{y}(t + \tau) = \hat{X}(t + \tau)\ \hat{\beta}(t + \tau). \tag{12}$$

As noted earlier, two kinds of forecast experiments are made in this paper: forecasts for the sample period (second quarter of 1962 through fourth quarter 1972) and forecasts for the quarters 1973.1 and 1973.2. In the former case, $X(t + \tau)$ is known and is, of course, taken to be the matrix of the actual values of the independent variables for the given quarter. In the latter case, the elements of this matrix must be forecast if they are not available. The elements of $X(t + \tau)$ could be forecast, for example, using exponential smoothing or some other time series forecasting method, or one could obtain them by solving the full OBE model. The latter would require reestimation of the OBE model through the fourth quarter of 1972. This reestimated model then would be used to forecast $\hat{z}(t + \tau)$ and this in turn would require the specification of the values of several other current endogenous variables which in the OBE model are assumed to be predetermined in a manner external to the model itself. Therefore these variables would have to be estimated in some way, perhaps by time series methods or by means of a so-called hidden model. Since in this paper our goal is to contrast the forecasting performance of fixed and varying parameter regression models, we used the actual values of the independent variables for our forecasting experiments for 1973.1 and 1973.2 rather than forecasting them by either of the above methods. We plan to examine the problem of forecasting independent variables and its effect upon the forecasting performance of varying parameter regression models more completely in a subsequent paper.

ESTIMATION IN THE KALMAN-BUCY MODEL

In order to use the Kalman-Bucy model (9), (10), we indicated earlier that one must specify the matrix $T(t)$, the initial value of the coefficient vector $\beta(0)$, and the variances and covariances of the error vectors $\varepsilon(t)$ and $u(t)$. Our computer experiments were conducted in two modes, a single-equation and a simultaneous-equation mode. In the former, we took the first equation of (7) for 3-month treasury bill rates (y_1) and placed it in the context of the Kalman-Bucy model (9), (10). A similar procedure was followed for the second equation of (7) in terms of 4- to 6-month commercial paper rates (y_2). This produced a single-equation Kalman-Bucy model for each of these interest rates. Because of the various specifications that are necessary, we characterize the former model by the parameter set ($\beta_1(0)$, $T_1(t)$, $Var(\varepsilon_1)$, $Cov(u_1)$) and the latter by ($\beta_2(0)$, $T_2(t)$, $Var(\varepsilon_2)$, $Cov(u_2)$). In the simultaneous-equation model (7), (8) or (9), (10) the two interest rates are considered jointly and this model is characterized by the parameter set ($\beta(0)$, $T(t)$, $Cov(\varepsilon)$, $Cov(u)$).

Estimation in either case began with an examination of the residuals around the OLS fit for each of the interest rate time series using the two individual equations implicit in (5). The sample variances of these residuals, s_1^2 and s_2^2 respectively, were calculated and $Var(\varepsilon_1)$ was set equal to $\frac{1}{2}s_1^2$ and $Var(\varepsilon_2)$ equal to $\frac{1}{2}s_2^2$. Then $\frac{1}{2}s_1^2$ was allocated equally to each of the four diagonal elements of the matrix $Cov(u_1)$--that is, each of these elements was set equal to $\frac{1}{8}s_1^2$--and $\frac{1}{2}s_2^2$ was allocated in similar fashion to the five diagonal elements of the matrix $Cov(u_2)$. All off-diagonal elements of the matrices $Cov(u_1)$ and $Cov(u_2)$ were set equal to zero (i.e., the coefficient disturbances were assumed to be mutually uncorrelated) giving

$$\mathrm{Cov}(u_1) = \begin{bmatrix} \frac{1}{8}s_1^2 & 0 & 0 & 0 \\ 0 & \frac{1}{8}s_1^2 & 0 & 0 \\ 0 & 0 & \frac{1}{8}s_1^2 & 0 \\ 0 & 0 & 0 & \frac{1}{8}s_1^2 \end{bmatrix}$$

and

$$\mathrm{Cov}(u_2) = \begin{bmatrix} \frac{1}{10}s_2^2 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{10}s_2^2 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{10}s_2^2 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{10}s_2^2 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{10}s_2^2 \end{bmatrix}$$

In the simultaneous-equation models Cov(ε) and Cov(u) were specified as above except that we used sample covariance s_{12} ($=s_{21}$) of the two OLS residuals as the off-diagonal elements of the matrix Cov(ε) instead of setting these values equal to zero--thus there is contemporaneous correlation between the equation errors $\varepsilon_1(t)$ and $\varepsilon_2(t)$. In other words, we used

$$\mathrm{Cov}(\varepsilon) = \begin{bmatrix} \frac{1}{2}s_1^2 & s_{12} \\ s_{21} & \frac{1}{2}s_2^2 \end{bmatrix} .$$

The matrix Cov(u) was taken to be the 9-by-9 matrix whose first four diagonal elements are equal to $\frac{1}{8}s_1^2$, whose remaining five diagonal elements are equal to $\frac{1}{10}s_2^2$, and whose off-diagonal elements are equal to zero. The vectors $\varepsilon(t)$ and $u(t)$ were assumed to be uncorrelated in the Kalman-Bucy models, and the problem of possible autocorrelation in $\varepsilon(t)$ and $u(t)$ has been ignored.

Since little is known about the sensitivity of the Kalman-Bucy model in economic applications to the specification of the vector $\beta(0)$, we estimated $\beta(0)$ in two different ways to investigate this matter.

1) OLS regressions using the individual equations implicit in (5) were calculated using the first 15 quarters of data (1962.2 through 1965.4) for each of the two interest rate series. The estimated values of $\beta_1(0)$ and $\beta_2(0)$ were then taken to be the components of the vector $\beta(0)$ in (9).

2) OLS regressions using the individual equations implicit in (5) for 43 quarters (1962.2 through 1972.4) were calculated and the resulting estimates $\beta_1(0)$ and $\beta_2(0)$ were taken to be the components of the vector $\beta(0)$.

The remaining parameter in the Kalman-Bucy model, the matrix T(t), offers some interesting challenges in the application of this model to economic time series. In an engineering context in which the Kalman-Bucy model first found applications, this matrix is typically given by known physical properties of some relevant real world system, but in econometrics one must estimate this matrix and a tractable and generally applicable method of estimation is not yet available. We assumed T(t) to be a constant matrix chosen to be either the identity matrix or a matrix estimated by OLS. In the latter case, two steps were necessary. First, the time-varying coefficients were estimated using the Kalman-Bucy model with T(t) specified as the identity matrix. Then these coefficient estimates were used as the actual values for $\beta(t)$ in (10), and the rows of T were estimated using stepwise regression (significance levels of .10 were used for the addition and deletion of variables).

RESULTS OF THE FORECASTING EXPERIMENTS

A summary of the results of our forecasting experiments are presented in Tables 1, 2, and 3 below. CalComp graphs of the two short-term interest rate time series and the four series of estimated regression coefficients for the varying parameter model for 3-month treasury bill rates are also presented, the latter to illustrate the behavior of a set of time-varying coefficients and to compare them with corresponding OLS estimates (dashed line in each graph). In Figures 1 and 2, the asterisks represent the actual values of the respective interest rates for the first two quarters of 1973. In Figures 3, 4, 5, and 6, the asterisks represent the values of the coefficients that were used in generating the forecasts for the model KB(1, OLS(2),I).

Single Equation Models

The results presented in Table 1 for single-equation models reveal that for both time series the varying parameter single-equation models of the Kalman-Bucy type fit the sample period data much better than the corresponding fixed coefficient regression models estimated by OLS. Moreover, these models appear to provide similar fits for the sample period data regardless of the choice made for the initial value $\beta(0)$ or the coefficient updating matrix T featured in the Kalman-Bucy models (however, the choices of $\beta(0)$ and T do have considerable effect on the time paths of the coefficients themselves).

In terms of the forecasting experiments related to predicting the actual levels of the two interest rates for the first and second quarters of 1973, however, only the models which used an initial value for $\beta(0)$ based on OLS(1) rather than OLS(2) provided reasonable forecast errors substantially smaller than those realized by using ordinary least squares. It is interesting to note that although the single-equation Kalman-Bucy models fit the commercial paper series better in the sample period, the 1973.1 and 1973.2 forecasts of treasury bill rates are better (in terms of average squared error for the two quarters) than the corresponding forecasts of commercial paper rates.

Simultaneous Equation Models

Table 2 contrasts the forecasting performance of simultaneous-equation varying parameter models of the Kalman-Bucy type for the two interest rate time series. The results are generally similar to those observed for the various single-equation models. Examination of Table 2 reveals that one simultaneous varying parameter model, denoted by KB(2, OLS(1),T), is clearly superior to any of the others in fitting the two interest rate time series over the sample period. Furthermore, this model performs well with respect to forecasting both interest rates for the first and second quarters of 1973, with the average squared forecast error for this period being smaller for the treasury bill rates than for the commercial paper rates.

Summary

Table 3 presents a comparison of how well 3-month treasury bill rates and 4- to 6-month commercial paper rates are forecast jointly (rather than individually) by the single-equation and simultaneous-equation models. For the purpose of providing a joint analysis of the movements of these two time series, the simultaneous-equation model KB(2,OLS(1),T) again fits the sample period data better than any other model. Furthermore, this model performs quite well when treasury bill rates and commercial paper rates are forecast jointly for the first and second quarters of 1973, although the simultaneous-equation model KB(2,OLS(1),I) provides slightly better joint forecasts of the two interest rates during this forecast period.

Table 1

SINGLE EQUATION MODELS

THREE-MONTH TREASURY BILL RATES

Time Period	1962.2-1972.4	1973.1	1973.2	1973.1-1973.2
Forecasting Method*	SSE(Sum Sq. Errors)	Forecast	Forecast	ASE(Average Sq. Error)
KB(1,OLS(1),I)	0.0933	5.878	6.454	0.0269
KB(1,OLS(2),I)	0.0911	6.654	7.308	0.7036
KB(1,OLS(1),T)	0.0792	5.833	6.350	0.0408
KB(1,OLS(2),T)	0.0816	6.568	7.142	0.5220
OLS(Ordinary Least Squares)	6.2893	6.470	7.104	0.4219
Actual Rates	--	5.700	6.603	--

Table 1 (Continued)

4- TO 6-MONTH COMMERCIAL PAPER RATES

Time Period	1962.2-1972.4	1973.1	1973.2	1973.1-1973.2
Forecasting Method*	SSE(Sum Sq. Errors)	Forecast	Forecast	ASE(Average Sq. Error)
KB(1,OLS(1),I)	0.0444	6.310	6.949	0.1345
KB(1,OLS(2),I)	0.0483	7.225	7.828	0.5088
KB(1,OLS(1),T)	0.0432	6.311	6.947	0.1356
KB(1,OLS(2),T)	0.0485	7.227	7.833	0.5125
OLS(Ordinary Least Squares)	6.1308	7.236	8.007	0.5999
Actual Rates	--	6.283	7.467	--

*In Tables 1, 2, and 3, OLS(1) refers to estimating β(0) by procedure 1 described on page 10; OLS(2) refers to estimating β(0) by procedure 2 described on page 10. KB(1,OLS(1),I) signifies single-equation Kalman-Bucy estimation with T(t) = I; KB(2,OLS(1),T) signifies simultaneous-Equation Kalman-Bucy estimation with T estimated by OLS (see page 11).

Table 2

SIMULTANEOUS EQUATION MODELS

THREE-MONTH TREASURY BILL RATES

Time Period	1962.2-1972.4	1973.1	1973.2	1973.1-1973.2
Forecasting Method*	SSE(Sum Sq. Errors)	Forecast	Forecast	ASE(Average Sq. Error)
KB(2,OLS(1),I)	0.0804	5.936	6.493	0.0339
KB(2,OLS(2),I)	0.0965	6.724	7.366	0.8154
KB(2,OLS(1),T)	0.0036	5.949	6.513	0.0351
KB(2,OLS(2),T)	0.0374	6.290	6.705	0.1793
OLS(Ordinary Least Squares)	6.2893	6.470	7.104	0.4219
Actual Rates	--	5.700	6.603	--

Table 2 (Continued)

4- TO 6-MONTH COMMERCIAL PAPER RATES

Time Period	1962.2-1972.4	1973.1	1973.2	1973.1-1973.2
Forecasting Method*	SSE(Sum Sq. Errors)	Forecast	Forecast	ASE(Average Sq. Error)
KB(2,OLS(1),I)	0.0629	6.310	7.027	0.0972
KB(2,OLS(2),I)	0.0830	7.185	7.903	0.5018
KB(2,OLS(1),T)	0.0031	6.299	6.977	0.1202
KB(2,OLS(2),T)	0.0592	7.127	7.767	0.4012
OLS(Ordinary Least Squares)	6.1308	7.236	8.007	0.5999
Actual Rates	--	6.283	7.467	--

*In Tables 1, 2, and 3, OLS(1) refers to estimating $\beta(0)$ by procedure 1 described on page 10; OLS(2) refers to estimating $\beta(0)$ by procedure 2 described on page 10. KB(1,OLS(1),I) signifies single-equation Kalman-Bucy estimation with $T(t) = I$; KB(2,OLS(1),T) signifies simultaneous-equation Kalman-Bucy estimation with T estimated by OLS (see page 11).

Table 3

SHORT TERM INTEREST RATES JOINTLY FORECAST

A. SINGLE EQUATION MODELS

Time Period	1962.2-1972.4	1973.1-1973.2
Forecasting Method*	Combined SSE	Combined ASE
KB(1,OLS(1),I)	0.1377	0.1614
KB(1,OLS(2),I)	0.1394	1.2124
KB(1,OLS(1),T)	0.1224	0.1764
KB(1,OLS(2),T)	0.1301	1.0345
OLS(Ordinary Least Squares)	12.4201	1.0218

B. SIMULTANEOUS EQUATION MODELS

Time Period	1962.2-1972.4	1973.1-1973.2
Forecasting Method*	Combined SSE	Combined ASE
KB(2,OLS(1),I)	0.1433	0.1311**
KB(2,OLS(2),I)	0.1795	1.3172
KB(2,OLS(1),T)	0.0067**	0.1553
KB(2,OLS(2),T)	0.0966	0.5805

*In Tables 1, 2, and 3, OLS(1) refers to estimating $\beta(0)$ by procedure 1 described on page 10; OLS(2) refers to estimating $\beta(0)$ by procedure 2 described on page 10. KB(1,OLS(1),I) signifies single-equation Kalman-Bucy estimation with $T(t) = I$; KB(2,OLS(1),T) signifies simultaneous-equation Kalman-Bucy estimation with T estimated by OLS (see page 11).

**Minimum values of SSE and ASE for forecast errors for the sample period and the forecast period when three-month treasury bill rates and 4- to 6-month commercial paper rates are considered jointly.

Table 3--Continued

THREE-MONTH TREASURY BILLS--MARKET YIELD
QUARTERLY DATA--1962.1 THROUGH 1973.2

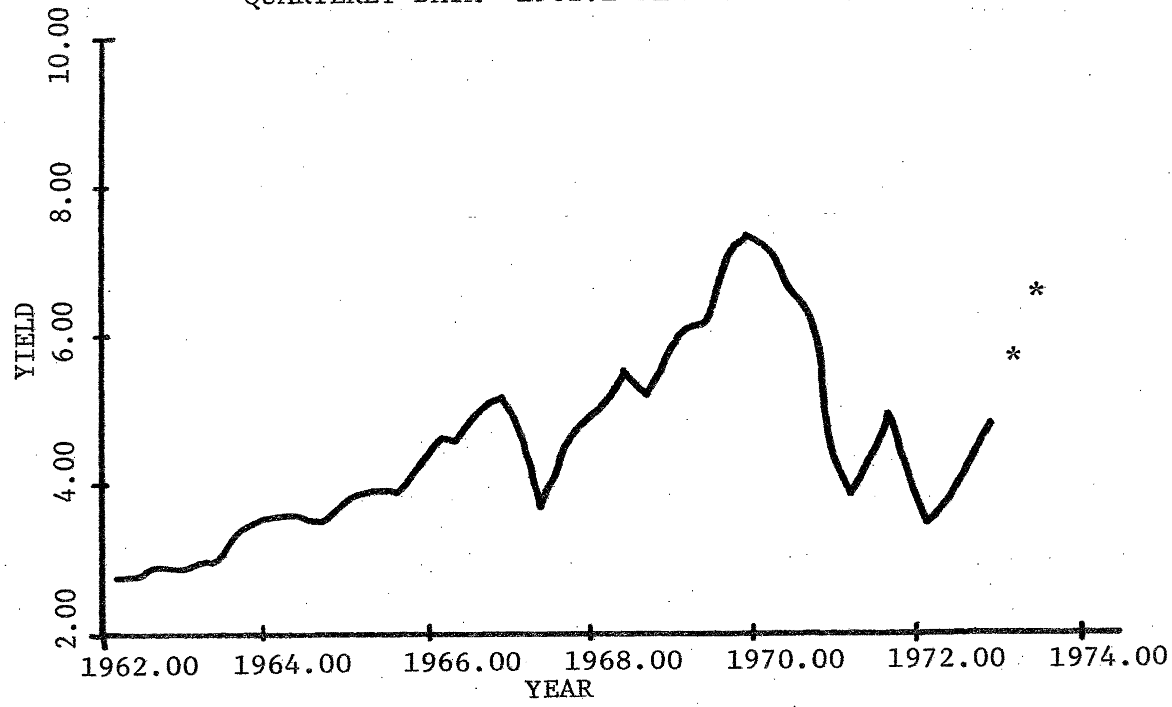

Figure 1

4- to 6-MONTH COMMERCIAL PAPER RATES
QUARTERLY DATA--1962.1 THROUGH 1973.2

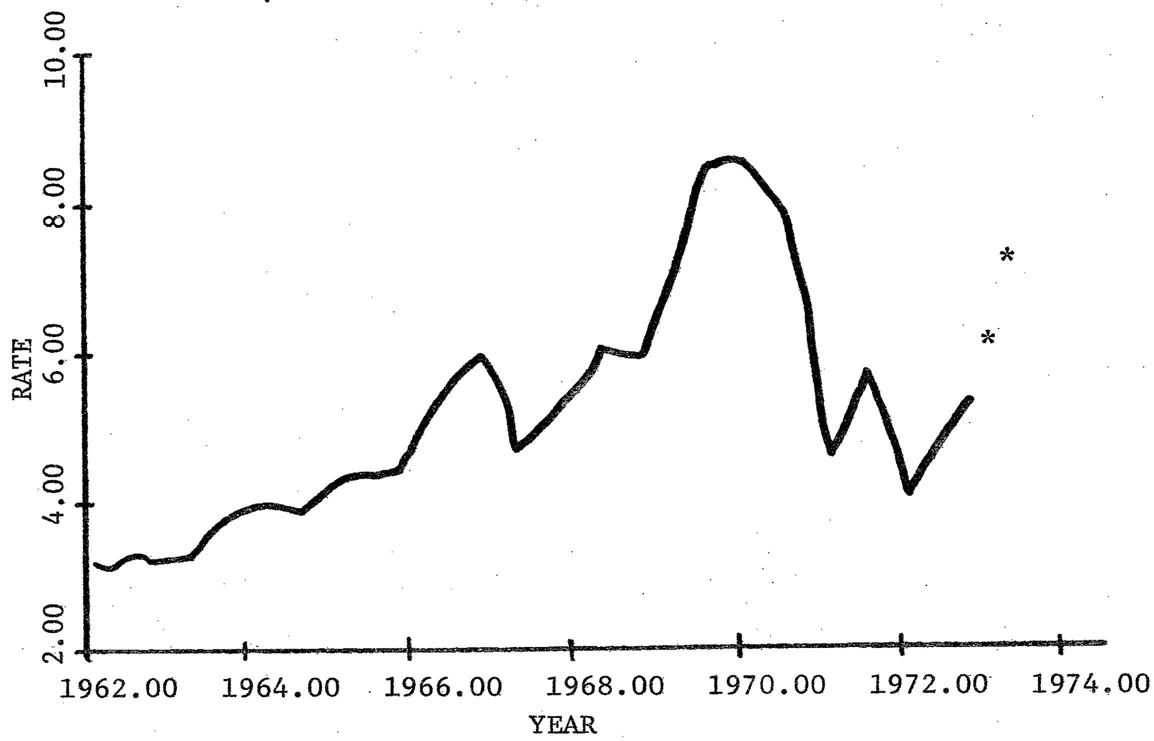

Figure 2

KALMAN-BUCY ESTIMATES OF B(0) IN TREASURY BILL EQUATION
(SINGLE EQUATION APPROACH WITH T=I)

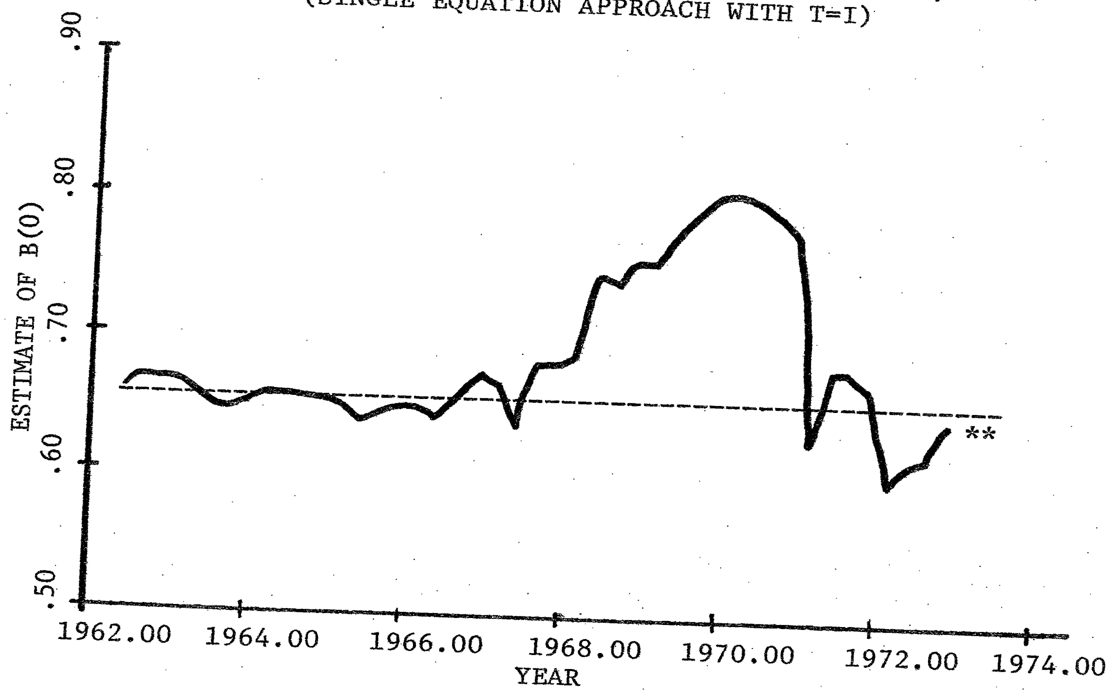

Figure 3

KALMAN-BUCY ESTIMATES OF B(1) IN TREASURY BILL EQUATION
(SINGLE EQUATION APPROACH WITH T=I)

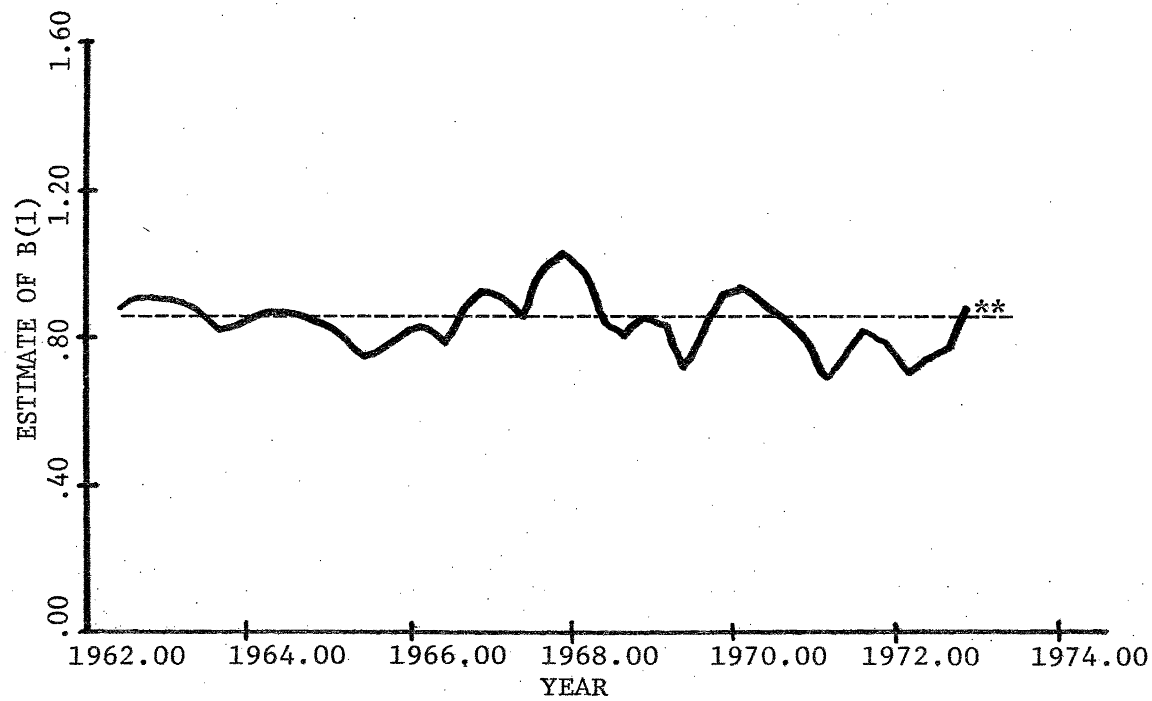

Figure 4

KALMAN-BUCY ESTIMATES OF B(2) IN TREASURY BILL EQUATION
(SINGLE EQUATION APPROACH WITH T=I)

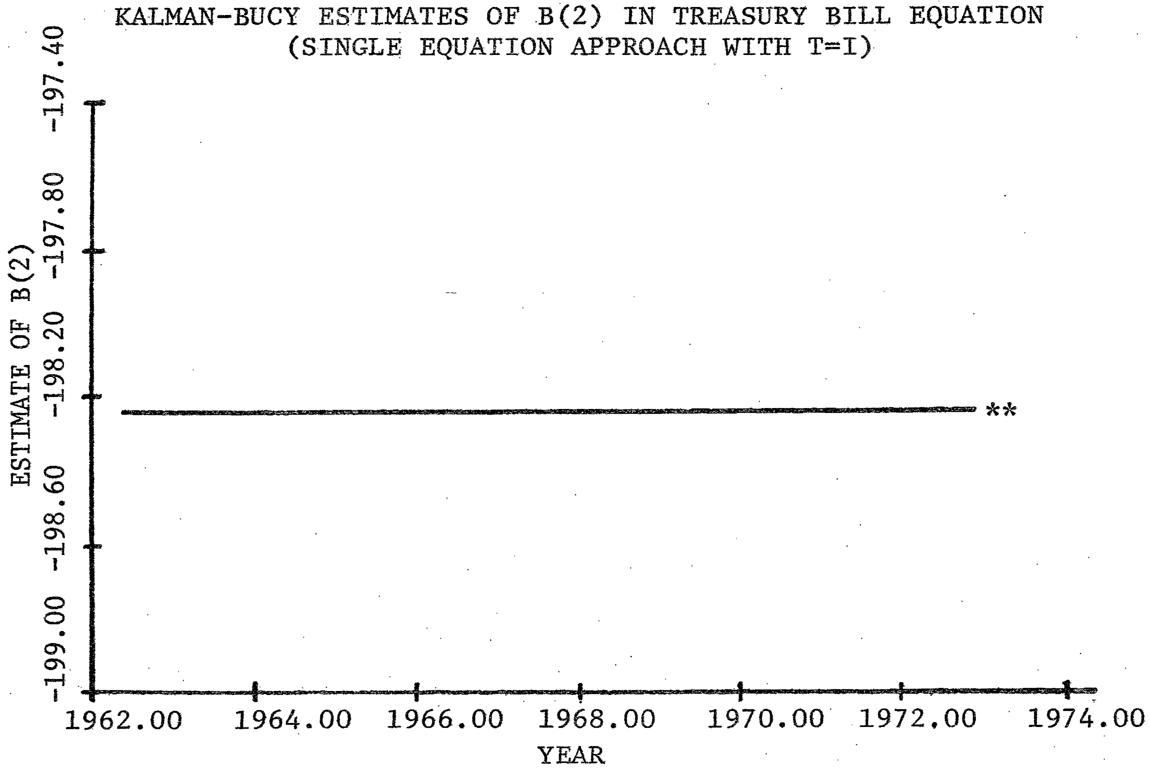

Figure 5

KALMAN-BUCY ESTIMATES OF B(3) IN TREASURY BILL EQUATION
(SINGLE EQUATION APPROACH WITH T=I)

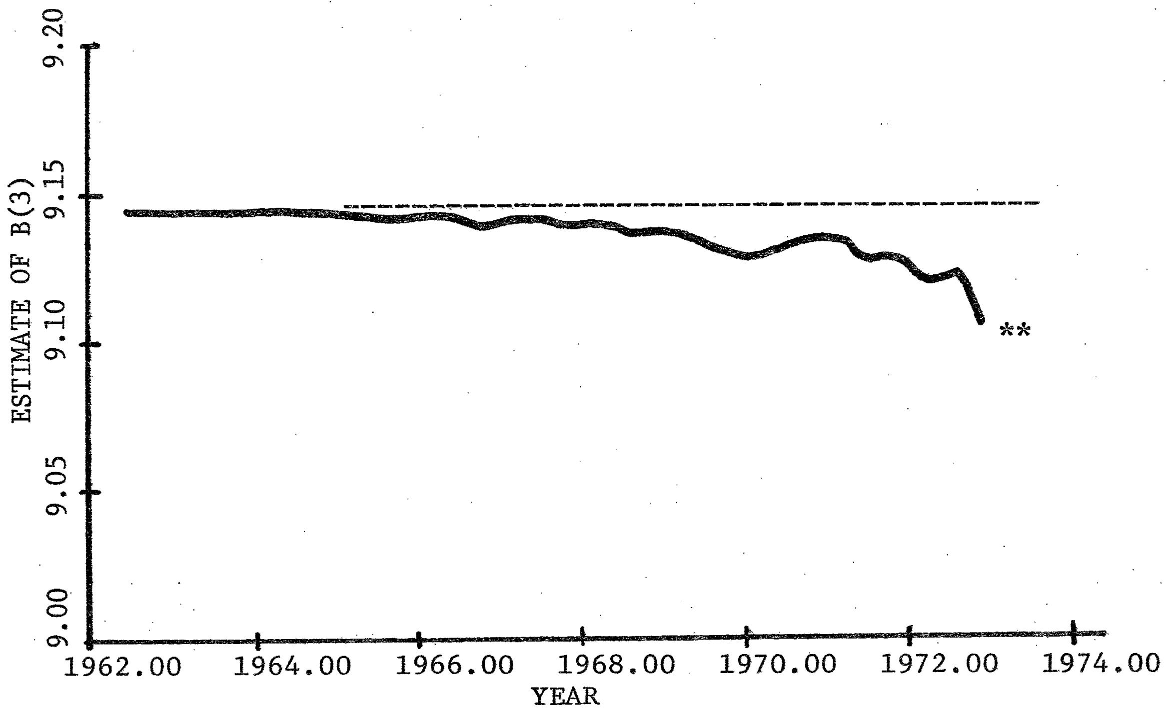

Figure 6

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