

# Prestige, Intrafirm Tournaments and Failure Aversion in Corporate Decisions\*

Todd T. Milbourn<sup>†</sup>

Richard L. Shockley<sup>‡</sup>

Anjan V. Thakor<sup>§</sup>

February 1999

---

\*We would like to thank Eli Berkovitch (our discussant at the 1997 WFA meetings), Sudipto Bhattacharya, Francesca Cornelli, Michel Habib, Haizhou Huang, Lars Stole, seminar participants at London School of Economics, University of Amsterdam, Birkbeck College, Norwegian School of Management, Stockholm School of Economics, and Oxford University, and participants at the 1997 WFA Meetings for their helpful comments. Any remaining errors are solely our responsibility.

<sup>†</sup>Assistant Professor of Finance, London Business School, Sussex Place, Regent's Park, London, NW1 4SA, UK. Tel: +44 171 262 5050, Fax: +44 171 724 3317, E-mail: [tmilbourn@lbs.ac.uk](mailto:tmilbourn@lbs.ac.uk) <http://www.lbs.ac.uk/faculty/tmilbourn>

<sup>‡</sup>Assistant Professor of Finance, Kelley School of Business, Indiana University, 1309 E 10th Street, Bloomington, IN 47405, USA. Tel: 812 855 8568, Fax: 812 855 5875, E-mail: [rishockl@indiana.edu](mailto:rishockl@indiana.edu)

<sup>§</sup>Edward J Frey Professor of Banking and Finance, University of Michigan Business School, 701 Tappan Street, Ann Arbor, MI 48109-1234, USA. Tel: 734 647 6434, Fax: 734 647 6861, E-mail: [anjan\\_thakor@umich.edu](mailto:anjan_thakor@umich.edu)

# Prestige, Intrafirm Tournaments and Failure Aversion in Corporate Decisions

## Abstract

The pervasive “tournament” style of organization is beneficial in that it allows shareholders to place the most talented managers in senior positions. We demonstrate that this benefit is achieved only at a cost: competition for promotions to prestigious positions induces excessive failure aversion among competing managers. Failure aversion emanates from the interaction between the huge rewards promised to the winning manager (the CEO) – the “winner-take-all” phenomenon – and the limited opportunities for managers to influence perceptions about their abilities. Failure aversion in turn corrupts corporate decisions, resulting in distorted project and effort choices. Importantly, it may not be possible to remove the resulting distortions through wage contracts as the organization architecture itself creates implicit contracts that constrain the efficacy of explicit compensation contracts.

# 1 Introduction

The “tournament” style of organization is pervasive. In such an organisation, managers compete for promotions to scarce prestigious positions, and this competition leads to a sorting process whereby the most talented managers rise to the top. But what is the cost of getting managers to compete in a tournament? We show that the tournament induces excessive failure aversion on the part of individual managers. This failure aversion in turn leads to distortions in decision making (e.g., non-value-maximizing project choices) that may not be possible to correct through incentive-based wage contracts.

We distinguish between failure aversion and risk aversion. In our model, a manager optimally behaves in an inordinately “safe” manner even though he is risk neutral, cannot be fired, and is paid a periodic wage that does not induce risk aversion. Rather, the manager’s dislike of failure is a direct consequence of the way the typical organization sifts its talent pool to determine who will be promoted to prestigious managerial positions.

The idea that managers may somehow act more averse to risk than shareholders would like has been explored in various papers. Holmstrom and Ricart i Costa (1986), Lambert (1986), and Brander and Poitevin (1992) show that excessive managerial risk aversion can distort project choices and capital structure. Greenwald and Stiglitz (1990) explain that such managerial risk aversion may be due to asymmetric information between suppliers of capital and managers. For example, managers may know their own abilities while shareholders can only infer them from their firms’ payoffs. A manager may be fired after a poor payoff even though he may be able. Since managers have firm-specific human capital invested in their current employment, and since this capital is neither transportable nor diversifiable, they dislike being fired. They may thus try to reduce the personal risk to their human capital by making corporate decisions that are distorted away from first best. For example, Amihud and Lev (1981) show that managers may diversify through conglomerate mergers even if they diminish shareholder wealth.

In our model, there is no investment in firm-specific capital during the tournament, nor is there a possibility of losing one’s job; it is the perception-building process, combined with the limited number of opportunities and the huge ultimate rewards to winning, that generate the excessive

caution.

Our idea is as follows. Most hierarchical organizations exhibit a tournament structure under which managers compete for promotions. The ultimate stage of this process is the promotion to CEO. The winner of the overall tournament is thus ultimately rewarded not only with ample monetary compensation, but also, perhaps more importantly, with the non-pecuniary benefits of the position (including the prestige and other private benefits) derived from control over corporate assets. In the tournament, the winners at each stage are those whose “abilities” are judged to be the highest at that stage, and the principal determinant of how a manager’s ability is perceived is whether the manager’s project succeeded or failed at that stage. However, opportunities for managers to influence perceptions of their abilities are scarce in that there are more competing managers than opportunities at any stage. Failure in an assignment is, therefore, privately costly because it decreases the manager’s probability of receiving future opportunities (projects) – “staying in the game” – and thereby reduces his probability of ultimate success. Thus, all managers exhibit an aversion to failure, with each attempting to minimize the probability of interim exclusion from the tournament.

Our analysis is built around a parsimonious multi-stage model of an intrafirm tournament. It is a variant of the classic “two-armed bandit” problem, similar to MacLeod and Malcolmson (1986), but simpler. We also focus on a distortion that is related to the managerial failure aversion that arises in this setting. We show that it distorts effort/project choice, and that even “optimally-designed” incentive contracts may not provide an attenuation.<sup>1</sup> Shareholders are thus stuck between the “rock” of inefficient decisions and the “hard place” of suspending the tournament that tells them who is best equipped to run the firm.

While shareholders dislike this distortion in corporate decisions, they may have no choice but to tolerate it. An incentive-compensation scheme designed to realign managers’ and shareholders’ interests would need to encourage risk-taking, and this can only be done by rewarding failure, with the manager receiving pecuniary payments large enough to offset his perceived non-pecuniary benefits of winning the tournament. In many instances, shareholders will find it cheaper to simply

---

<sup>1</sup>This is simply an example drawn from a much broader class of distortions which could be demonstrated in a similar fashion.

accept the project-choice and other distortions rather than relying on even more costly contractual resolutions.

This brings up an important issue for the optimal contracting literature – the contracting problem must be considered within the constraints of the overall organizational form. For example, Dybvig and Zender (1991) argue that sub-optimal managerial behavior in most models is driven by assumed inefficiencies in contracting.<sup>2</sup> However, Dybvig and Zender’s focus is on *explicit* contracts between shareholders and managers, and not on the incentives arising from informal tournaments among managers. Our paper is an example of a situation in which the organization architecture creates *implicit* contracts that affect the effectiveness of explicit managerial compensation contracts. The organization architecture includes a tournament that is necessary for placing the highest-ability manager as the CEO. But the large benefit of becoming CEO distorts managerial incentives along the way. The non-pecuniary component of the “prize” exists outside the explicit boundaries of the shareholder-manager relationship, and cannot be contracted away by the shareholders.<sup>3</sup>

Our model of an intrafirm tournament should not be taken as a literal description of the *formal* structure of performance appraisal within a firm. Very few firms, if any, tell their managers that they are engaged in a race to the top. Yet, it is commonly understood that merit-based promotion in a hierarchy inevitably results in those perceived to be the most capable rising to the top. That is, the tournament we model is typically quite informal and embedded in the unwritten “rules of the game”. Recognition of this induces managers to attempt to influence perceptions of their abilities.

This raises two obvious questions. First, why have a tournament-style hierarchy? Second, why should the prize for becoming CEO be so large that the resulting distortions cannot be overcome at reasonable cost?

The first question is addressed by MacLeod and Malcolmson (1988), who build on the earlier work of Lazear and Rosen (1981), Green and Stokey (1983), Nalebuff and Stiglitz (1983),

---

<sup>2</sup>Jaggia and Thakor (1994) show that long-term contracts that insure managers against involuntary termination will provide the desired incentives for managers to invest in firm-specific human capital. Hirshleifer and Suh (1992) emphasize the role of the curvature of the managerial compensation contract in inducing managerial risk preference.

<sup>3</sup>Of course, in a more general sense, the prize for winning the tournament can be viewed as a part of the manager’s (stochastic) future compensation. In our model, however, this prize need not be reflected in the manager’s monetary wages and perks, and may be limited to the increase in innate job satisfaction and private benefits associated with holding the top job.

Mookherjee (1984) and Radner (1985).<sup>4</sup> MacLeod and Malcolmson demonstrate that an organization characterized by “job ladders” with a finite number of ranks and a system of promotions is an incentive-compatible and self-selecting mechanism when employee actions and abilities are not observable. The critical element in the model is reputation – success at lower ranks provides information to employers about the quality of employees, and poor performance leads to reputation loss. In the end, the higher-ability employees reach the top ranks.<sup>5</sup>

What we show is that the excellent sorting capabilities of a tournament-style organization are achieved at a cost; the large reward for winning the CEO position corrupts corporate decisions at every stage of the tournament. In the tournament literature discussed above, the magnitude of the reward for winning the tournament is endogenously determined by the incentive-compatibility considerations that shape the design of the tournament. Our departure from this literature is the recognition that shareholders may be unable to define the boundaries of the prize for becoming CEO, since a substantial portion of the prize may come from the non-pecuniary and non-contractible private benefits of the position.

The question of why the perceived private benefits of being in control of the firm are often very large is outside the scope of this paper. This issue, however, appears to be merely an example of a broader “winner-take-all” social phenomenon. In their recent book, *The Winner-Take-All Society*, Frank and Cook (1995) state,

Rabo Karelian and Kathleen Battle sell their services in what we call “winner-take-all markets”. ... The markets in which these people and others like them work are very different from the ones economists normally study. We call them winner-take-all markets because the value of what gets produced in them often depends on the efforts of only a small number of top performers, who are paid accordingly.

---

<sup>4</sup>Lazear and Rosen (1981), in the seminal work on the topic, show that prizes based on rank-order outcomes are optimal incentive structures under universal risk neutrality. They point out that even though corporate profitability is a cardinal measure, the dominant incentive strategy may be an ordinal ranking when it is difficult to measure the productivity of an individual employee. Green and Stokey (1983) show that a tournament can dominate individual incentive contracts even under employee risk aversion if the number of employees is large or if there is a “common error” component to individual outputs. When there is a common error, the cumulative output of all agents provides information about the shock and thus eliminates this source of noise. Nalebuff and Stiglitz (1983) argue that tournaments may be desirable for their flexibility in reward structures, their incentive provision, and their risk-sharing capabilities.

<sup>5</sup>There is a growing literature which empirically examines the efficiency of tournament-based organizations. Ehrenberg and Bognanno (1990) document that tournaments have incentive effects. In particular, they find “strong support for the proposition that the level and structure of prizes in the PGA [Professional Golf Association] tournaments influence players’ performance”. Nalbantian and Schotter (1997) conclude from an experimental study of various incentive programs that “tournament-based group incentive mechanisms that create competition between subgroups in the organization for a fixed set of prizes (i.e., which create internal tournaments) determine higher mean outputs than all target-based mechanisms examined ...”

These high stakes have created a new class of “unknown celebrities”: those pivotal players who spell the difference between corporate success and failure. Because their performance is crucial, and because modern information technology has helped build consensus about who they are, rival organizations must compete furiously to hire and retain them.

The widening gap between the winners and losers is apparently not new<sup>6</sup>... What is new is that the phenomenon has spread so widely and that so many of the top prizes have become so spectacular. The lure of these prizes, we will argue, has produced several important distortions in modern industrial economies.

The rest of the paper is organized as follows. In Section 2 we develop a general model of an intrafirm tournament that leads to managerial failure aversion. Section 3 shows that this failure aversion can distort the manager’s effort (project) choice, and even optimally designed compensation contracts may be ineffective in ameliorating this distortion. We also introduce managerial risk aversion in this section to illustrate the difference between failure aversion and risk aversion. Section 4 concludes. All proofs are in the Appendix.

## 2 Intrafirm Tournaments and Managerial Failure Aversion

### 2.1 The Model

There are many points in time between now ( $t = 0$ ) and the end of the relevant time horizon ( $t = T$ ). Thus,  $t = 0, 1, 2, \dots, T$ . At  $t = 0$ , there are  $N$  risk-neutral managers in the firm, all observationally identical to the Chief Executive Officer (CEO). The CEO’s objective is to maximize the value of the firm.

At each date  $t$ , there will be  $M$  projects available.<sup>7</sup> Each project requires one manager and pays off at the next date.<sup>8</sup> The project payoff takes one of the two values:  $S$  denoting “success” and  $F$  denoting “failure”, where  $\infty > S > F \geq 0$ . The probabilities of success,  $\Phi(S|i)$ , and failure,  $\Phi(F|i)$ , are functions of the manager’s “type”  $i \in \{G, B\}$ , where  $G$  denotes “good” and  $B$  denotes “bad”. Nobody (including the manager himself) knows the manager’s true type, but it is common knowledge that the prior probability that the manager is good at  $t = 0$  is  $\gamma \in (0, 1)$ . We assume

<sup>6</sup> Alfred Marshall wrote over a century ago, “... the relative fall in the incomes to be earned by those of moderate ability, however carefully trained, is accentuated by the rise in those that are obtained by many men of extraordinary ability. There never was a time at which moderately good oil paintings sold more cheaply than now, and there never was a time at which first-rate paintings sold so dearly.”

<sup>7</sup> Alternatively, one could view this assumption as there being only enough capital to fund  $M$  projects.

<sup>8</sup> That is, a project assigned to a manager at  $t = 0$  pays off at  $t = 1$ , at which time it is terminated. Another project then becomes available at  $t = 1$  and it yields a terminal payoff at  $t = 2$ , and so on.

that  $M < N$ , so that at every  $t$  there is a scarcity of projects relative to managers available to manage them.<sup>9</sup> At any date, the CEO assigns  $M$  managers to  $M$  projects based on her beliefs about the managers' types. Since managers are observationally indistinguishable at  $t = 0$ , the choice of managers to be assigned to projects is made randomly, with each manager facing a probability  $\frac{M}{N}$  of being given a project. Over time, the CEO's beliefs will change as she observes the payoff outcomes of the projects managed by different managers. Thus, at any  $t > 0$ , the CEO will assign projects to managers based on their perceived qualities, where a manager's quality is the CEO's posterior belief that the manager is good. Even though managerial wages may be increasing in perceived quality, we assume that the manager does not capture all of the surplus from higher quality, so that the NPV of the project to the firm is increasing in managerial quality. Thus, the  $M$  managers chosen will be those with the highest qualities from those available. We assume that no new managers are hired and no old managers are fired during  $\{0, \dots, T\}$ , so there are always  $N$  managers available. For ease of notation, we will assume henceforth that  $\Phi(S|G) = \Phi(F|B) > \frac{1}{2}$ .

At  $t = T$ , a "prize" is given to one of the best  $Z < N$  managers at that time. This prize may represent a significant promotion or control of significant assets, additional power, etc. What matters is that the prize has a value  $\Omega > 0$  that is very large for any manager. A good example of the prize is being appointed CEO. In that case,  $\Omega$  would represent the additional utility the manager would derive from being appointed CEO and would capture the value the manager attaches to the incremental power, prestige, job satisfaction, perks and direct monetary compensation associated with the job. Of course,  $\Omega$  could be large even without an increase in monetary compensation for being CEO.

We assume that the CEO's type affects the value of the entire firm. Let the value contribution of a good CEO be  $V_G$  and that of a bad CEO be  $V_B$ , where  $0 < V_B < V_G < \infty$ . Thus, if a CEO must be appointed at  $t = T$ , the firm's value is maximized by appointing the highest quality manager to the job. If multiple managers have the same perceived quality at  $T$ , then one is chosen at random from that set to be CEO. That is, if  $Z$  is the number of managers with the highest perceived quality at  $T$ , then for each manager in that set, the probability of receiving the prize is  $\frac{1}{Z}$ . We assume

---

<sup>9</sup>All that we want to depict is a situation in which managers do not have unlimited opportunities and organizational resources to demonstrate their abilities.



that a person who is CEO at  $t = 0$  will remain the CEO until  $t = T$ , at which time a new CEO must be appointed.

## 2.2 Analysis of Project Allocation Decisions

Consider first what happens at  $t = 1$ . When first-period projects expire, there are potentially three sets of managers: those who succeeded, those who failed, and those ( $N - M$  in number) who were not assigned projects at  $t = 0$ .

For the managers whose first-period projects succeeded, the posterior probability that a randomly drawn manager from this set is good is:

$$\begin{aligned}\mu(G|S) &= \frac{\Pr(G) \times \Pr(S|G)}{\sum_{i \in \{G,B\}} \Pr(i) \times \Pr(S|i)} \\ &= \frac{\gamma \Phi(S|G)}{\gamma \Phi(S|G) + [1 - \gamma][1 - \Phi(F|B)]}.\end{aligned}\tag{1}$$

Similarly, using Bayes rule,

$$\mu(G|F) = \frac{\gamma[1 - \Phi(S|G)]}{\gamma[1 - \Phi(S|G)] + [1 - \gamma]\Phi(F|B)}.\tag{2}$$

It is transparent that  $\mu(G|S) > \gamma > \mu(G|F)$ , and that  $\gamma$  is the posterior belief at  $t = 1$  that a manager drawn at random from the set of managers not assigned projects at  $t = 0$  is good.

At  $t = 1$ , the firm must allocate  $M$  new projects. Let  $\mathcal{S}_1$  be the set of managers whose projects succeeded at  $t = 1$ ,  $\mathcal{F}_1$  the set of managers whose projects failed at  $t = 1$ , and  $\mathcal{T}_1 = \{1, \dots, N\}$  the total set of managers. Since firm value is maximized by assigning projects to the highest quality managers, from (1) and (2) we know that the probability that manager  $i \in \mathcal{T}_1$  will be assigned a project at  $t = 1$  is:

$$\left\{ \begin{array}{ll} 1 & \text{if } i \in \mathcal{T}_1 \\ \frac{M - c(\mathcal{S}_1)}{N - M} & \text{if } i \in \mathcal{F} \setminus (\mathcal{S}_1 \cup \mathcal{F}_1) \text{ and } M - c(\mathcal{S}_1) \leq N - M \\ 1 & \text{if } i \in \mathcal{T} \setminus (\mathcal{S}_1 \cup \mathcal{F}_1) \text{ and } M - c(\mathcal{S}_1) > N - M \\ 0 & \text{if } i \in \mathcal{F}_1 \text{ and } M - c(\mathcal{S}_1) \leq N - M \\ \frac{2M - [c(\mathcal{S}_1) + N]}{M - c(\mathcal{S}_1)} & \text{if } i \in \mathcal{F}_1 \text{ and } M - c(\mathcal{S}_1) > N - M \end{array} \right.\tag{3}$$

where  $c(\cdot)$  is the counting measure<sup>10</sup> and  $A \setminus B = \{x | x \in A, x \notin B\}$ . Basically (3) says that at  $t = 1$ , all managers within a particular group are treated alike, but managers across different groups are assigned second-period projects with different probabilities. First, *all* managers who succeeded are assigned second-period projects with probability (w.p.) 1 because  $c(S_1) \leq M$  and  $\mu(G|S) > \gamma > \mu(G|F)$ . After such managers are all assigned projects, if there are still second-period projects left over, the CEO turns to those who were not assigned projects at  $t = 0$ . The probability with which a manager in this group receives a project depends on the number of projects left over to be allocated to the  $N - M$  managers in this group. If the number of projects left over to be allocated is less than  $N - M$ , then every manager in this group receives a project with probability less than 1, and all managers who failed in the first period are excluded from consideration for second-period projects. If the number of projects left over to be allocated to those inactive in the first period exceeds  $N - M$ , then every manager in this group receives a project w.p. 1, and the remaining projects can be allocated to some managers who failed in the first period. The probability with which each manager in the group of first-period failures is allocated a project then depends on the size of this group and how many projects remain for such managers. The main point of (3) is that the probability of being awarded a second-period project is highest for managers who succeed in the first period, next highest for inactive managers and the least for those who fail in the first period.

Let us now examine what happens at  $t = 2$  and beyond. Observe that the number of distinct groups of managers at  $t = 1$  is 3. The next lemma tells us how to compute this number for  $t \geq 2$ .

**Lemma 1**      *For any  $t \in \{0, 1, \dots, T\}$ , the number of distinct groups of managers, each with a different quality assessment (i.e., the CEO's posterior belief that a manager in that group is good), is  $2t + 1$ .*

We can use *Lemma 1* to establish the following useful result.

---

<sup>10</sup>Formally,  $c(A)$  is the cardinality of set  $A$ ; that is,  $c(A) = (\# \text{ of distinct elements in set } A)$ .

**Lemma 2**      *At any date  $t \leq T$ , the posterior belief that a manager is good is determined solely by  $n(t)$ , which is the difference between the number of successes and the number of failures that the manager has experienced through  $t$ . Moreover, the posterior belief that a manager is good is strictly increasing in  $n(t)$ .*

Armed with *Lemma 2*, we can show that the project assignment rule in (3) generalizes to any arbitrary  $t$ . Before doing that, however, we need some additional notation. At any date  $t$ , let  $n_i(t)$  be the  $n(t)$  for manager  $i$ . Then  $\mathcal{N}_m^*(t)$  is the set of managers at date  $t$  who have the maximum  $n$ 's among their peers,  $\mathcal{N}_{m-1}^*(t)$  is the set of managers with the next highest  $n$ 's, and so on. That is,

$$\mathcal{N}_{m-1}^*(t) = \{i \in \mathcal{T} | n_j(t) > n_i(t) > n_k(t); \forall j \in \mathcal{N}_m^*(t), k \in \mathcal{T} \setminus (\mathcal{N}_m^*(t) \cup \mathcal{N}_{m-1}^*(t))\}.$$

This leads to the following result.

**Lemma 3**      *The value-maximizing project assignment policy for the CEO at any date  $t$  is to first assign a project to every manager  $i \in \mathcal{N}_m^*(t)$ , then to assign the remaining projects to managers in  $\mathcal{N}_{m-1}^*(t)$ , and so on, until all projects are assigned. No project is assigned to a manager in  $\mathcal{N}_{m-k-1}^*(t)$  until all managers in  $\mathcal{N}_{m-k}^*(t)$  have been assigned projects, where  $k = 0, 1, 2, \dots, m-1$ . If there are projects available to be assigned to managers in  $\mathcal{N}_{m-k}^*(t)$  and  $c(\mathcal{N}_{m-k}^*(t))$  exceeds the number of projects available for that group, then each manager in the group is assigned a project with equal probability less than one.*

Clearly, when the managers are engaged in a tournament like this, it is in each manager's best interest to be able to participate in the tournament, and the only way to do this is to be assigned a project to manage. Thus, each manager is concerned about the probability of being assigned a project at any  $t$ , as well as the prospective assessment of the likelihood of being able to compete in the tournament in the future. The following collection of results will be useful to characterize the main result of this section.

#### Lemma 4

1. *At any time  $t > 0$ , the probability that manager  $i$  is assigned a project is strictly increasing in  $n_i(t)$ . Moreover, the probability that manager  $i$  will never be assigned another project in the future is (weakly) greater if the manager's last project failed than if it succeeded.*
2. *When viewed at date  $t < T - 1$ , the probability of manager  $i$  being assigned a project at any future date  $t^*$  (where  $t < t^* \leq T - 1$ ) is strictly increasing in  $n_i(t)$ , and the probability that the manager will never be assigned another project is monotonically decreasing in  $n_i(t)$ .*
3. *At any date  $t < T$ , manager  $i$ 's expected waiting time is monotonically decreasing in  $n_i(t)$ .*
4. *At any date  $t < T$ , manager  $i$ 's expected waiting time until a project is assigned to him next is longer if the manager's date  $t - 1$  project failed at  $t$  than if it succeeded.*

Thus, every project success shortens the number of periods the manager expects to wait before being assigned another project. Moreover, as successes mount and  $n_i(t)$  increases for manager  $i$ , the manager's risk of being shut out of the tournament until its end also diminishes. We can now prove the main result for this subsection.

**Proposition 1**      *At any date  $t < T$ , manager  $i$ 's expected number of future project assignments is monotonically increasing in  $n_i(t)$ , and the manager's probability of receiving the ultimate prize  $\Omega$  never decreases after a success and never increases after a failure. Moreover, the probability that manager  $i$  will receive the ultimate prize  $\Omega$  is monotonically increasing in  $n_i(t)$ .*

This proposition indicates that, at every stage in the game, the manager is better off with a widening gap between his successes and failures. The reason is simple. As this gap widens, the CEO's posterior assessment of the manager's ability goes up, and this increases the likelihood that the manager will himself become CEO one day. This means that the manager will be averse to failure at any date  $t$ , and this aversion will be greater than that which can be justified on the basis of lower expected wages between  $t$  and  $T$  due to failure at  $t$ .

### 3 Effort Distortions and Failure versus Risk Aversion

For simplicity, we did not give the manager a project or effort choice in the previous section, so the manager's failure aversion was of no consequence to the shareholders. Moreover, with managerial risk neutrality, it is difficult to distinguish between failure and risk aversion. We now extend the previous model to deal with these issues.

#### 3.1 The Extended Model

Suppose we have multiple managers engaged in an implicit tournament that will last for one period and at the end of which only one manager will become CEO. We will allow managers to be risk averse and, for simplicity, assume that the CEO is risk neutral. The rule for determining who will become CEO is the same as in the previous section. Each manager can choose an action (effort)  $\alpha \in \{0, 1, 2\}$ . If  $\alpha = 0$ , then the project payoff  $x$  is 0 w.p. 1. If  $\alpha = 1$ , the project payoff  $x$  is  $L > 0$  w.p. 1. If  $\alpha = 2$ , the project payoff  $x$  is 0 w.p.  $1 - \Phi(i)$ , and positive w.p.  $\Phi(i)$ , where  $i \in \{G, B\}$  denotes the manager's type. Conditional on the payoff being positive, it will be  $H$  w.p.  $p \in (0, 1)$  and  $L$  w.p.  $1 - p$ , where  $H > L > 0$ . As before, we assume that good managers enjoy higher success probabilities, i.e.,  $\Phi(G) > \Phi(B)$ . Since nobody knows the manager's type, define  $\bar{\Phi} \equiv \gamma\Phi(G) + [1 - \gamma]\Phi(B)$ . The CEO can observe the project payoff but not the manager's type or his action choice.

The manager's utility function is given by

$$u(W, \Omega, \alpha) = U(W) + \Omega - D(\alpha), \quad (4)$$

where  $W$  represents wealth,  $U' > 0, U'(0) = \infty, U'' \leq 0, D' > 0$  and  $\Omega > 0$ . Here  $\Omega$  represents the utility that the manager attaches to being CEO and  $D(\alpha)$  represents his disutility of effort. The manager's reservation utility is  $\bar{U} > 0$ . Let  $D(2) > D(1) > D(0) = 0$ . We also assume that managerial compensation must be nonnegative and:

$$L > U^{-1}(\bar{U} + D(1)) \quad (5)$$

$$\Phi(B) [pH + [1 - p]L] > L. \quad (6)$$

Note that (5) ensures that shareholders find the payoff  $L$  preferable to 0 and (6) says that  $\alpha = 2$  chosen by even the bad manager produces a higher expected payoff than  $\alpha = 1$  by any manager.

### 3.2 Interpreting the Model

Our interpretation of this model is that each manager can choose one of three major actions. He can choose to shirk by largely ignoring his job ( $\alpha = 0$ ), in which case the neglected project yields the worst possible payoff of zero. That is, if the manager chooses to do nothing, the division he manages experiences a poor payoff almost surely. The manager's second option is to choose the "status quo", a safe course of action that yields an average payoff ( $L$ ) with a high degree of certainty. This may be viewed as "keeping the ship afloat" by continuing with the division's existing products at roughly the same scale in the same markets. For example, a hamburger division operating 50 outlets in Chicago selling hamburgers, cheeseburgers and fries continues to sell the same products from those 50 outlets. The manager has to do some work to maintain the status quo, but he doesn't need to work very hard ( $\alpha = 1$ ). There is virtually no risk here, but the reward is mediocre. Finally, the manager could take some more risk and introduce a new product or venture into a new market. For example, our hamburger store manager could experiment with chicken sandwiches and pizza or spicy burgers with garlic and hot peppers. This calls for hard work ( $\alpha = 2$ ) by the manager, but if he succeeds, the payoff is very high ( $H$ ). However, despite this hard work, the strategy is risky in that the payoff may be mediocre ( $L$ ) or the venture may fail altogether (0).

### 3.3 Solving for the Optimal Contract Ignoring $\Omega$

Suppose we initially ignore the tournament between the agents, i.e.,  $\Omega$ . To solve for the optimal contract, note first that the CEO can motivate the managers only by conditioning his wage  $W$  on the realized project payoff. Thus, a wage contract is a triplet  $\langle W(H), W(L), W(0) \rangle$ .

Now, if the CEO wanted to elicit  $\alpha = 0$ , the optimal contract is transparently seen to be  $W(H) = W(L) = 0$ , and  $W(0) = U^{-1}(\bar{U})$ . With this the manager chooses  $\alpha = 0$ . If the CEO wanted to elicit  $\alpha = 1$ , the optimal contract is obtained as a solution to:

$$\begin{aligned} & \underset{(W(H), W(L), W(0))}{Max} L - W(L) \\ & \text{subject to} \end{aligned}$$

$$U(W(L)) - D(1) \geq \bar{U} \quad (7)$$

$$U(W(L)) - D(1) \geq U(W(0)) \quad (8)$$

$$U(W(L)) - D(1) \geq \left\{ \begin{array}{l} \bar{\Phi}[pU(W(H)) + [1-p]U(W(L))] \\ + [1 - \bar{\Phi}]U(W(0)) - D(2) \end{array} \right\}. \quad (9)$$

Here (7) is the individual rationality (I.R.) constraint, and (8) and (9) are the incentive compatibility (I.C.) constraints. The following lemma can now be proved.

**Lemma 5** *The optimal contract to elicit  $\alpha = 1$  involves  $W(H) = W(0) = 0$  and  $W(L) = U^{-1}(\bar{U} + D(1))$ . Moreover, the CEO strictly prefers the manager to choose  $\alpha = 1$  over  $\alpha = 0$ .*

Now suppose the CEO wants  $\alpha = 2$ . Then, she solves

$$\underset{(W(H), W(L), W(0))}{Max} \bar{\Phi}[p[H - W(H)] + [1-p][L - W(L)]] - [1 - \bar{\Phi}]W(0)$$

subject to

$$\left\{ \begin{array}{l} \bar{\Phi}[pU(W(H)) + [1-p]U(W(L))] \\ + [1 - \bar{\Phi}]U(W(0)) - D(2) \end{array} \right\} \geq \bar{U} \quad (10)$$

$$\left\{ \begin{array}{l} \bar{\Phi}[pU(W(H)) + [1-p]U(W(L))] \\ + [1 - \bar{\Phi}]U(W(0)) - D(2) \end{array} \right\} > \max\{U(W(L)) - D(1), U(W(0))\}. \quad (11)$$

Again (10) is the I.R. constraint and (11) represents two I.C. constraints. We can now prove the following.

**Lemma 6** *The optimal contract to elicit  $\alpha = 2$  is  $W(0) > 0$ ,  $W(L) > 0$  and  $W(H) > \max\{W(0), W(L)\}$ . Moreover, there exists an  $H$  high enough such that the CEO strictly prefers the manager to choose  $\alpha = 2$  over  $\alpha = 1$ .*

The intuition is simple. All three payoff values (0,  $L$  and  $H$ ) are possible with  $\alpha = 2$ . Since the manager is risk averse, he prefers to sacrifice some consumption in the  $H$  state in order to gain some consumption in the 0 and  $L$  states. Also, if  $H$  is high enough, the CEO would prefer each manager to choose  $\alpha = 2$  since  $H$  is possible only with  $\alpha = 2$ .

### 3.4 The Impact of a Tournament to Win $\Omega$

Suppose now that we have two managers, each vying to be CEO. The manager who will become CEO at the end of the period is the one about whom there is a higher posterior belief that he is type  $G$ . We will first compute these posterior beliefs for the three different payoff states, *conditional on the equilibrium effort choices of both managers being  $\alpha = 2$* . The conditioning of the posterior probabilities is on the effort choices the CEO *assumes* the manager will make. Thus, if we say  $Pr(\text{mgr is } G|x = H, \alpha = 2)$ , we mean the posterior probability that the manager is good, conditional on  $x = H$  and the CEO *assuming* that the manager chose  $\alpha = 2$ . However, if we want to condition on the manager's *actual* effort choice when this choice differs from what the CEO assumes, we will distinguish this case by writing the manager's actual effort choice in curly brackets. That is,  $Pr(\text{mgr is } G|x = H, \alpha = 2, \{\alpha = 1\})$  is the manager's *ex ante* expected probability of being considered good when  $x = H$  and the CEO assumes that the manager chose  $\alpha = 2$  when he actually chose  $\alpha = 1$ . Turning back to the CEO's type-inference process, if  $x = 0$ , then

$$\begin{aligned} Pr(\text{mgr is } G|x = 0, \alpha = 2) &= \frac{Pr(x = 0|G, \alpha = 2) Pr(G)}{Pr(x = 0|\alpha = 2)} \\ &= \frac{[1 - \Phi(G)]\gamma}{[1 - \Phi(G)]\gamma + [1 - \Phi(B)][1 - \gamma]}. \end{aligned} \quad (12)$$

If  $x = L$ , then

$$\begin{aligned} Pr(\text{mgr is } G|x = L, \alpha = 2) &= \frac{Pr(x = L|G, \alpha = 2) Pr(G)}{Pr(x = L|\alpha = 2)} \\ &= \frac{\Phi(G)[1 - p]\gamma}{\Phi(G)[1 - p]\gamma + \Phi(B)[1 - p][1 - \gamma]}. \end{aligned} \quad (13)$$

If  $x = H$ , then

$$\begin{aligned} Pr(\text{mgr is } G|x = H, \alpha = 2) &= \frac{Pr(X = H|G, \alpha = 2) Pr(G)}{Pr(x = H|\alpha = 2)} \\ &= \frac{\Phi(G)p\gamma}{\Phi(G)p\gamma + \Phi(B)p[1 - \gamma]}. \end{aligned} \quad (14)$$



We now have the following result.

**Lemma 7**      *If the CEO ignores the impact of  $\Omega$  on each manager's effort choice and designs the optimal contract to elicit  $\alpha = 2$ , there is no Nash equilibrium in which both managers choose  $\alpha = 2$ .*

The intuition is as follows. The posterior belief that the manager is good is equal for  $x = H$  and  $x = L$  and higher for both these payoffs than for  $x = 0$  (follows from (12), (13) and (14)). Thus, the manager maximizes this posterior belief by choosing  $\alpha = 1$  since then  $x = L$  with probability one. That is, the manager always has a higher probability of winning  $\Omega$  with  $\alpha = 1$  than with  $\alpha = 2$ . This result means that the CEO must respond to these distorted managerial incentives by adjusting the incentive contract.

We will assume that the wage contract by itself must satisfy the manager's participation (I.R.) constraint, regardless of  $\Omega$ . This immediately reveals two points. First, if  $\Omega$  is sufficiently large, then designing a wage contract that elicits  $\alpha = 2$  will necessarily lead to the individual rationality constraint being slack. The reason is that the manager always has a higher probability of winning  $\Omega$  by choosing  $\alpha = 1$  than by choosing  $\alpha = 2$  (see *Lemma 7*). Thus, in making his effort choice, he trades off the higher expected wage from choosing  $\alpha = 2$  against the higher probability of winning  $\Omega$  from choosing  $\alpha = 1$ . To ensure a choice of  $\alpha = 2$  calls for increasing the manager's expected wage from choosing  $\alpha = 2$ . The most extreme adjustment would be to set  $W(L) = 0$  (by *Lemma 6*, this is not optimal) and increase  $W(H)$  and  $W(0)$ . The higher  $\Omega$  is, the higher will  $W(H)$  and  $W(0)$  have to be, so that for an  $\Omega$  high enough,  $W(H)$  and  $W(0)$  will be so high that the manager's expected utility (independently of  $\Omega$ ) will exceed  $\bar{U}$ . Second, even though the shareholders would have preferred that each manager choose  $\alpha = 2$  when  $\Omega$  could be ignored, they may now prefer that wage contracts be designed to only elicit  $\alpha = 1$ . The reason is that eliciting  $\alpha = 2$  requires such a high expected wage for the manager that it exhausts all of the shareholders' surplus from having  $\alpha = 2$  chosen over  $\alpha = 1$ . This discussion is summarized in the following result.

**Proposition 2**      *There exists  $\Omega$  high enough such that the CEO prefers to design managerial incentive contracts that elicit  $\alpha = 1$  from each manager in a Nash equilibrium. Thus, the intrafirm tournament between the managers distorts effort choices.*

The effort choice in this model should be viewed as an allegory for a more general class of decisions the manager could make. For example, the manager could have a project choice, in which case his desire to become CEO could distort his selection in favor of possibly lower-valued projects with lower probabilities of conspicuous failure.

### 3.5 Risk Aversion Versus Failure Aversion

A natural question our analysis raises is: what is the difference between risk aversion and failure aversion? This question can be answered most clearly in the context of the model with risk-averse managers developed in this section. Suppose it is optimal for the CEO to elicit  $\alpha = 2$  from each manager, i.e.,  $\Omega$  is present but is not too large. Then, from *Lemma 6* we know that the manager's wage under the optimal incentive contract will be positive in every payoff state. This result has nothing to do with failure aversion; it arises solely from managerial risk aversion. On the other hand, if the manager was risk neutral, the optimal incentive contract would involve the manager paying the firm a fixed amount and absorbing all the residual risk associated with the project payoff.<sup>11</sup> Thus, the risk neutral manager is indifferent to risk/variability in the project payoff *per se*. However, to the extent that  $\Omega > 0$  matters, he will still be failure averse. This means that if  $\Omega$  is large enough, the manager will display failure aversion and avoid  $x = 0$  by choosing  $\alpha = 1$  regardless of whether he is risk averse or risk neutral.

This also illuminates another point. At first blush, one might think that an extreme reward for winning the tournament would generate a convex payoff structure, which would then lead to *risk-seeking* behavior on the part of the manager. However, this does not happen in our model because a higher probability of project failure in any period always lowers the manager's probability of ultimately winning the tournament, without affecting the size of the prize if he wins. Thus,

---

<sup>11</sup>Technically speaking, this arrangement requires relaxing the restriction that the managerial wage be nonnegative in every state.

taking on a higher probability of failure is not the same as investing in a riskier project where the probabilities of both extremely high and extremely low outcomes are higher.

This discussion can be summarized as follows. Risk aversion is sufficient to generate failure aversion, but it is not necessary. The manager's desire to avoid conspicuous failure is a property of his behavior that transcends his risk preferences. The role of managerial risk preferences is therefore to affect the details of the manager's incentive contract and (possibly) the strength of his failure aversion, without eliminating this aversion.

## 4 Conclusion

The basic point of our analysis is this: the pyramid structure of most organizations makes it necessary to select the person at the top based on an *implicit* winner-take-all intrafirm tournament among candidates. *Perceptions* of ability thus become important, leading to failure aversion among managers because failure lowers perception of one's ability. We have explicitly modeled one implication of this failure aversion: distorted effort/project choices. These distortions sacrifice shareholder value.

There are possibly many other distortions that are attributable to excessive managerial failure aversion. For example, managers may avoid empowering subordinates because an unsupervised subordinate is more likely to fail, and this may reflect poorly on the manager. It would be interesting to explore this and related issues in future research. Such an agenda should help us better understand the richness of moral hazard within corporations and the implications of this richness for performance assessment and organization design.

## 5 Appendix

### 5.1 Proof of Lemma 1

This result follows directly from the symmetry assumption  $\Phi(S|G) = \Phi(F|B)$ . First, consider time  $t = 2$  where there have been two rounds of project allocations and project outcomes. At this point project allocations and outcomes produce nine sets of managers, where each group can be identified by the pair  $(i, j)$ , where

$$\begin{aligned} i &\in \{S(\text{success in } t = 0 \text{ project}), F(\text{fail in } t = 0 \text{ project}), \text{ and } X(\text{No } t = 0 \text{ project allocated})\} \\ j &\in \{S(t = 1 \text{ project}), F(t = 1 \text{ project}), \text{ and } X(\text{No } t = 1 \text{ project})\}. \end{aligned}$$

The groups are  $(S, S)$ ,  $(S, X)$ ,  $(S, F)$ ,  $(X, S)$ ,  $(X, X)$ ,  $(X, F)$ ,  $(F, S)$ ,  $(F, X)$ , and  $(F, F)$ . To calculate the  $t = 2$  posterior probability that a given manager is  $G$ , one needs the  $t = 1$  posterior *after* having observed the  $t = 0$  project outcomes. This  $t = 1$  posterior belief is  $\mu(G|\cdot)$ . Thus, the  $t = 2$  posterior for a manager in the  $(S, F)$  group is

$$\begin{aligned} \mu(G|S, F) &= \frac{\mu(G|S)\Phi(F|G)}{\mu(G|S)\Phi(F|G) + \mu(B|S)\Phi(F|B)} \\ &= \gamma \end{aligned}$$

by the symmetry-in-beliefs assumption. Similarly,  $\mu(G|X, X) = \gamma$  since no updating has occurred for this group. Thus, the  $t = 2$  posteriors for the groups  $(S, F)$ ,  $(F, S)$ , and  $(X, X)$  are identical and these managers are viewed as having equal ability at  $t = 2$ .

The  $t = 2$  posterior for the  $(S, X)$  group is

$$\mu(G|S, X) = \mu(G|S) = \frac{\gamma\Phi(S|G)}{\gamma\Phi(S|G) + [1 - \gamma][1 - \Phi(F|B)]}$$

since no updating takes place at  $t = 2$ . The posterior for the  $(X, S)$  group is identical since this group began the  $t = 1$  project with a prior of  $\gamma$  and then succeeded. Thus, the groups  $(S, X)$  and  $(X, S)$  are viewed as identical, as are the groups  $(F, X)$  and  $(X, F)$  (by a similar argument). This leaves us with five observationally equivalent sets of managers:

$$\{(S, S), [(S, X) \cup (X, S)], [(S, F) \cup (F, S) \cup (X, X)], [(F, X) \cup (X, F)], (F, F)\}.$$

We have established that there are only  $5 = 2(t) + 1$  distinct groups at  $t = 2$ . Now consider moving on to  $t = 3$ . For the members of the five distinct  $t = 2$  groups previously listed, there are only three possible  $t = 3$  outcomes (no project, success, and failure). For example, members of the  $t = 2$  group  $(S, S)$  can be members of the  $t = 3$  groups  $(S, S, S)$ ,  $(S, S, F)$ , and  $(S, S, X)$ . It is easy to show that

$$\mu(G|S, S, F) = \mu(G|S, X) = \mu(G|X, S).$$

That is, the group of managers denoted  $(S, S, F)$  have the same  $t = 3$  posterior probability of being  $G$  as the group  $(X, S) \cup (S, X)$  at  $t = 2$ . Intuitively, the combination of one success and one failure is equivalent to one “no project”. Similarly,  $\mu(F|S, S, X) = \mu(G|S, S)$  since no updating occurs when no project is allocated. The group  $(S, S, S)$  has a  $t = 3$  posterior that is *strictly different* from any  $t = 2$  group. In fact,  $(S, S, S)$  and  $(F, F, F)$  are the *only*  $t = 3$  groups that have posterior probabilities that are not equal to some possible  $t = 2$  group. Thus, moving from  $t = 2$  to  $t = 3$  adds only 2 possible distinct groups of managers, and the number of distinct groups of  $t = 3$  is  $7 = 2(t) + 1$ . One can work forward to any period  $t'$  and show that, in moving from  $t = t' - 1$  to  $t = t'$ , only *two* “new” groups are created (where “new” at  $t'$  means having a posterior probability of being  $G$  different from that of any group at  $t' - 1$ ). Thus, the number of possible distinct groups of managers at any  $t$  is equal to  $2(t) + 1$ . ■

## 5.2 Proof of Lemma 2

Let  $n(t)$  be the difference between the number of successes and failures a manager has had up through  $t$ . First consider date  $t = 1$ . At  $t = 1$  there are 3 possible distinct groups of managers (by *Lemma 1*): those with one success ( $n(1) = 1$ ), those with one failure ( $n(1) = -1$ ), and those with no experience ( $n(1) = 0$ ). The  $t = 1$  posterior for the group with  $n(1) = 1$  is

$$\mu(G|S) = \mu(G|n(1) = 1) = \frac{\gamma\Phi(S|G)}{\gamma\Phi(S|G) + [1 - \gamma][1 - \Phi(F|B)]}$$

The posterior for the group with  $n(1) = 0$  is

$$\mu(G|\text{no project}) = \mu(G|n(1) = 0) = \gamma,$$

and the posterior for the group with  $n(1) = -1$  is

$$\mu(G|F) = \mu(G|n(1) = -1) = \frac{\gamma[1 - \Phi(S|G)]}{\gamma[1 - \Phi(S|G)] + (1 - \gamma)\Phi(F|B)}$$

Clearly, there are no other  $t = 1$  groups and

$$\mu(G|n(1) = 1) > \mu(G|n(1) = 0) > \mu(G|n(1) = -1).$$

Thus, the  $t = 1$  posterior is completely determined by  $n(1)$ , and the posterior is increasing in  $n(1)$ . By *Lemma 1*, there are  $2(t) + 1 = 5$  possible groups at  $t = 2$ . Utilizing the notation from *Lemma 1*, these groups are  $(S, S) \rightarrow n(2) = 2$ ;  $(S, X) \cup (X, S) \rightarrow n(2) = 1$ ;  $(S, F) \cup (F, S) \cup (X, X) \rightarrow n(2) = 0$ ;  $(F, X) \cup (X, F) \rightarrow n(2) = -1$ ; and  $(F, F) \rightarrow n(2) = -2$ . Again, it is obvious that

$$\mu(G|n(2) = 2) > \mu(G|n(2) = 1) > \mu(G|n(2) = 0) > \mu(G|n(2) = -1) > \mu(G|n(2) = -2)$$

and thus that the  $t = 2$  posterior is determined solely by  $n(2)$  and is increasing in  $n(2)$ .

Proceeding forward in time, at any time  $t$  there are  $2(t) + 1$  possible groups, each with a unique  $n(t) \in \{t, t - 1, t - 2, \dots, -t + 2, -t + 1, -t\}$  and it is easy to show that  $\mu(G|n(t)) > \mu(G|n(t) - 1)$  for all possible  $n(t)$  at  $t$ . ■

### 5.3 Proof of Lemma 3

For each posterior probability that a manager is  $G$ , there is an associated probability of success for that manager in the next period if given a project:

$$\Pr(\text{success}|\text{mgr history}) = \left\{ \begin{array}{l} \Phi(S|G) \times \mu(G|\text{mgr history}) \\ + \Phi(S|B) \times \mu(B|\text{mgr history}) \end{array} \right\}.$$

By *Lemma 2*,  $\mu(\text{success}|\text{mgr history})$  is solely a function of  $n(t)$ . We can then write the posterior as  $\mu(G|n(t))$  and the probability of success in the next period as  $\Pr(S|n(t))$ . Moreover by *Lemma 2* we know that

$$\mu(G|n(t)) > \mu(G|n(t) - 1)$$

$\forall n(t) \in \{-t + 1, \dots, t\}$ , i.e., the probability of success for a manager in the next period is increasing in  $n(t)$ . If all projects at  $t$  are identical, then the expected value of a project at  $t$  given a manager with history  $n(t)$  is:

$$\{\Pr(S|n(t)) \times S\} + \{\Pr(F|n(t)) \times F\},$$

which is clearly increasing in  $n(t)$ . Therefore, if managers do not capture the entire surplus from their higher posterior quality, then the firm will maximize its expected shareholder value by allocating projects to managers with the highest  $n(t)$  first. ■

### 5.4 Proof of Lemma 4

We prove each of the four parts of this lemma in order below.

**Part 1** We will first prove that the probability that the manager is assigned a project at date  $t$  is strictly increasing in  $n_i(t)$ . At any time  $t < T$ , the CEO allocates projects on a “top down” basis (as in *Lemma 3*). Thus, the probability that manager  $i$  receives a project at  $t$  depends on  $n_i(t)$  and is given by either:

i).  $\Pr(\text{project at } t | n_i(t) = t) =$

$$\left\{ \begin{array}{ll} 1 & \text{if } M \geq c(\mathcal{N}_i^*(t)) \\ \frac{M}{c(\mathcal{N}_i^*(t))} & \text{otherwise,} \end{array} \right.$$

ii).  $\Pr(\text{project at } t | -t < n(t) = j < t) =$

$$\begin{cases} 1 & \text{if } \left[ M - \sum_{k=j+1}^t c(\mathcal{N}_k^*(t)) \right] > c(\mathcal{N}_j^*(t)) \\ \frac{M - \sum_{k=j+1}^t c(\mathcal{N}_k^*(t))}{c(\mathcal{N}_t^*(t))} & \text{if } 0 < \left[ M - \sum_{k=j+1}^t c(\mathcal{N}_k^*(t)) \right] < c(\mathcal{N}_j^*(t)) \\ 0 & \text{otherwise,} \end{cases}$$

or

iii).  $\Pr(\text{project at } t | n(t) = -t) =$

$$\begin{cases} \frac{M - \sum_{k=j+1}^t c(\mathcal{N}_k^*(t))}{c(\mathcal{N}_t^*(t))} & \text{if } 0 < \left[ M - \sum_{k=t+1}^t c(\mathcal{N}_k^*(t)) \right] < c(\mathcal{N}_{-t}^*(t)) \\ 0 & \text{otherwise.} \end{cases}$$

It is easy to show, by comparing probabilities directly, that

$$\Pr(\text{project at } t | n(t)) \geq \Pr(\text{project at } t | n(t) - 1),$$

$\forall n(t) \in \{-t+1, \dots, t\}$ . For example, compare  $n(t) = t$  with  $n(t) = t-1$ . If  $M \geq c(\mathcal{N}_t^*(t))$ , then all managers in group  $\mathcal{N}_t^*(t)$  get a project with probability 1 while the probability that a manager in  $\mathcal{N}_{t-1}^*(t)$  gets a project is a function of  $c(\mathcal{N}_t^*(t))$  and  $c(\mathcal{N}_{t-1}^*(t))$ . On the other hand, if  $M < c(\mathcal{N}_t^*(t))$ , then each manager in  $\mathcal{N}_t^*(t)$  gets a project with positive probability, but no managers in  $\mathcal{N}_{t-1}^*(t)$  are assigned projects. All further comparisons work identically. If there are more than enough projects for all the managers in a particular group, then they will all get projects w.p. 1 and the remainder are allocated to the “next group down”.

If there are not enough projects to go around in a particular group, then all managers in “lower” groups have zero probability of receiving a project. Thus, there is a positive and monotonic relationship between the probability of receiving a project and  $n(t)$ .

We will now prove that the probability that manager  $i$  will never be assigned another project in the future is (weakly) greater if his project failed than if it succeeded. At  $t = T-1$ , the probability that manager  $i$  never receives another project is a function of  $n_i(T-1)$  and is simply one minus the probability of receiving a project at  $T-1$ . That is, it is either

i).  $\Pr(\text{no project at } T-1 | n(T-1) = T-1) =$

$$\begin{cases} 0 & \text{if } M > c(\mathcal{N}_{T-1}^*(T-1)) \\ 1 - \frac{M}{c(\mathcal{N}_{T-1}^*(T-1))} & \text{otherwise,} \end{cases}$$

ii).  $\Pr(\text{no project at } T-1 | -(T-1) < n(T-1) = j < T-1) =$

$$\begin{cases} 0 & \text{if } M - \sum_{k=j+1}^{T-1} c(\mathcal{N}_k^*(T-1)) > c(\mathcal{N}_j^*(T-1)) \\ 1 - \frac{M - \sum_{k=j+1}^{T-1} c(\mathcal{N}_k^*(T-1))}{c(\mathcal{N}_j^*(T-1))} & \text{if } 0 < \left[ M - \sum_{k=j+1}^{T-1} c(\mathcal{N}_k^*(T-1)) \right] < c(\mathcal{N}_j^*(T-1)) \\ 1 & \text{otherwise,} \end{cases}$$

or

iii).  $\Pr(\text{no project at } T-1 | n(T-1) = -(T-1)) =$

$$\begin{cases} 1 - \frac{M - \sum_{k=-T+2}^{T-1} c(\mathcal{N}_k^*(T-1))}{c(\mathcal{N}_{-(T-1)}^*(T-1))} & \text{if } 0 < \left[ M - \sum_{k=-T+2}^{T-1} c(\mathcal{N}_k^*(T-1)) \right] < c(\mathcal{N}_{-(T-1)}^*(T-1)) \\ 1 & \text{otherwise.} \end{cases}$$

Now, at  $t = T-2$ , the probability of *never* receiving another project again is

$$\Pr(\text{no project at } T-2 \text{ and } T-1 | n(T-2)) = \left\{ \begin{array}{l} \Pr(\text{no project at } T-2 | n(T-2)) \times \\ \Pr(\text{no project at } T-1 | n(T-1) = n(T-2)) \end{array} \right\},$$

where we know  $n(T-1) = n(T-2)$  since the manager did not receive a project at date  $T-2$ . We also know that

$$\Pr(\text{no project at } T-2 | n(T-2)) = 1 - \Pr(\text{project at } T-2 | n(T-2)),$$

and (by the earlier part of this proof) that  $\Pr(\text{project at } T-2 | n(T-2))$  is increasing in  $n(T-2)$ . This implies that the probability of *no* project at  $T-2$  is *decreasing* in  $n(T-2)$  and that

$$\Pr(\text{no project at } T-1 | n(T-1) = n(T-2))$$

is decreasing in  $n(T-2)$  as well.

Now move back to  $t = T-3$ . For manager  $i$  at  $t = T-3$  with  $n(T-3) = n'$  that *receives* a project at  $t = T-3$ , success in that project leads to  $n(T-2) = n' + 1$ , while failure leads to  $n(T-2) = n' - 1$ . Since  $\Pr(\text{no project at } T | n(t))$  is decreasing in  $n(t)$ ,

$$\Pr(\text{no project at } T-2 | n(T-2) = n' + 1) \leq \Pr(\text{no project at } T-2 | n(T-2) = n' - 1)$$

and

$$\Pr(\text{no project at } T-1 | n(T-1) = n' + 1) \leq \Pr(\text{no project at } T-1 | n(T-1) = n' - 1).$$

Thus,

$$\Pr(\text{no project at } T-2 \text{ and } T-1 | n(T-2) = n' + 1) \leq \Pr(\text{no project at } T-2 \text{ and } T-1 | n(T-2) = n' - 1).$$

One can then work backwards from  $T = T-4$  to show the same result, as well as inductively for *any*  $t$ . Therefore, given a project at any time  $t$ , the probability of never receiving another project (from  $t+1$  on through  $T-1$ ) is strictly higher if the date- $t$  project fails than if it succeeds. ■



**Part 2** This result proceeds exactly as in Part 1 above. ■

**Part 3** Define the “expected waiting time” as the expected number of periods until the next project arrives or until the game ends at  $T$ , whichever comes first. First, consider  $t = T - 1$ . The expected waiting time for manager  $i$  is

$$E[\text{waiting time at } T - 1 | n_i(T - 1)] = \left\{ \begin{array}{l} [0 \times \Pr(\text{project at } T - 1 | n_i(T - 1))] \\ + [1 \times \Pr(\text{no project at } T - 1 | n_i(T - 1))] \end{array} \right\}.$$

Here, if a manager receives a project at  $t = T - 1$ , then the waiting time is 0, and if no project arrives, then the waiting time is 1 since the game ends at  $t = T - 1$  (i.e., no more projects are allocated). Since  $\Pr(\text{no project at } t - 1 | n_i(T - 1))$  is monotonically decreasing in  $n_i(T - 1)$  (a direct application of Part 1 of *Lemma 4*), then at  $t = T - 1$  the expected waiting time is monotonically decreasing in  $n_i(T - 1)$ .

Moving back to  $t = T - 2$ , manager  $i$ 's expected waiting time is:

$$E[\text{waiting time at } T - 2 | n_i(T - 2)] = \left\{ \begin{array}{l} \{0 \times \Pr(\text{project at } T - 1 | n_i(T - 1) + 1)\} \\ + \left\{ \begin{array}{l} 1 \times \Pr(\text{no project at } T - 2 | n_i(T - 2)) \\ \times \Pr(\text{no project at } T - 1 | n_i(T - 1) = n_i(T - 2)) \end{array} \right\} \\ + \left\{ \begin{array}{l} 2 \times \Pr(\text{no project at } T - 2 | n_i(T - 2)) \\ \times \Pr(\text{no project at } T - 1 | n_i(T - 1) = n_i(T - 2)) \end{array} \right\} \end{array} \right\}.$$

Since the probability of getting a project in any *future* period is monotonically increasing in  $n(t)$  (by Part 2 of *Lemma 4*), and since  $\Pr(\text{no project at } T - 2 | n_i(T - 2))$  is decreasing in  $n(T - 2)$ , the expected waiting time at  $T - 2$  is monotonically decreasing in  $n_i(T - 2)$ . One can proceed inductively to show that at any point at date  $t$

$$E[\text{waiting time at } t | n_i(t)] = \{0 \times \Pr(\text{project at } t | n_i(t))\} \\ + \left\{ \begin{array}{l} \sum_{k=1}^{T-t} k \times \left( \prod_{j=t}^{t+k-1} \Pr(\text{no project at } j | n_i(j)) \right) \\ \times \Pr(\text{project at } t + k | n_i(T + k) = n_i(t)) \end{array} \right\}$$

is monotonically decreasing in  $n_i(t)$ . ■

**Part 4** The result follows directly from the proof of Part 3 above. ■

## 5.5 Proof of Proposition 1

We will first prove that manager  $i$ 's expected number of future project assignments is monotonically increasing in  $n_i(t)$  at  $t < T$ . Again, start at  $T - 1$ . Manager  $i$ 's expected number of future projects

is

$$E[\text{number of future assignments}|n_i(T-1)] = 1 \times \Pr(\text{project at } T-1|n_i(T-1)),$$

which is obviously increasing in  $n(T-1)$ . Moving back to  $T-2$ ,

$$E[\text{number of future assignments}|n_i(T-2)] = \left\{ \begin{array}{l} [1 \times \Pr(\text{project at } T-2|n_i(T-2))] \\ + [1 \times \Pr(\text{project at } T-1|n_i(T-2))] \end{array} \right\},$$

which again, using the results of Parts 1 and 2 of *Lemma 4*, is monotonically increasing in  $n_i(T-2)$ .

Again, one works inductively to show that at any  $t$

$$E[\text{number of futre assignments}|n_i(t)] = \sum_{k=1}^T [1 \times \Pr(\text{project at } k|n_i(t))]$$

is monotonically increasing in  $n_i(t)$ .

The rest of the proof follows that of Part 1 of *Lemma 4*. ■

## 5.6 Proof of Lemma 5

We know that the CEO's objective is to minimize  $W(L)$ , and from the I.C. constraints in (8) and (9), it is transparent that minimizing  $W(H)$  and  $W(0)$  permits  $W(L)$  to be minimized. Since the lower bound on managerial compensation is 0, this calls for  $W(H) = W(0) = 0$ . The I.R. constraint (7) should obviously hold tightly in an equilibrium involving  $\alpha = 1$ . This gives  $W(L) = U^{-1}(\bar{U} + D(1))$ . ■

## 5.7 Proof of Lemma 6

From the maximization program, it is clear that, conditional on choosing  $\alpha = 2$ , the manager faces a positive probability for each of the three states  $x = 0$ ,  $x = L$  and  $x = H$ . From the IC constraint (11) it follows that minimizing  $W(0)$  and  $W(L)$  relative to  $W(H)$  improves incentives to choose  $\alpha = 2$ . That is,  $W(H) > \max\{W(0), W(L)\}$ . Now, it is easy to see that the IR constraint (10) should hold tightly in equilibrium. The result that  $W(0)$  and  $W(L)$  are positive follows from the fact the  $U(\cdot)$  is concave and that, conditional on  $\alpha = 2$ , the probability of  $x = 0$  is positive as is the probability that  $x = L$ . ■

## 5.8 Proof of Lemma 7

The manager's probability of winning is maximized by maximizing the posterior probability that he will be perceived to be good ex post. From (12), (13) and (14), it is straightforward to show that

$$1 > \Pr(\text{mgr is } G|x = H, \alpha = 2) = \Pr(\text{mgr is } G|x = L, \alpha = 2) > \Pr(\text{mgr is } G|x = 0, \alpha = 2) > 0.$$

Thus, if the manager chooses  $\alpha = 2$ , then *ex ante*, he estimates the expected probability of being considered good as:

$$\begin{aligned}
& \Pr(\text{mgr is } G | \alpha = 2, \{\alpha = 2\}) \\
&= \left\{ \begin{aligned} & \Pr(x = H | \alpha = 2, \{\alpha = 2\}) \times \Pr(\text{mgr is } G | x = H, \alpha = 2) \\ & + \Pr(x = L | \alpha = 2, \{\alpha = 2\}) \times \Pr(\text{mgr is } G | x = L, \alpha = 2) \\ & + \Pr(x = 0 | \alpha = 2, \{\alpha = 2\}) \times \Pr(\text{mgr is } G | x = 0, \alpha = 2) \end{aligned} \right\} \\
&= \left\{ \begin{aligned} & p\bar{\Phi} \Pr(\text{mgr is } G | x = H, \alpha = 2) \\ & + [1 - p]\bar{\Phi} \Pr(\text{mgr is } G | x = L, \alpha = 2) \\ & + [1 - \bar{\Phi}] \Pr(\text{mgr is } G | x = 0, \alpha = 2) \end{aligned} \right\} \\
&< \Pr(\text{mgr is } G | x = L, \alpha = 2).
\end{aligned}$$

But if the manager chooses  $\alpha = 1$  when the CEO expects both managers to choose  $\alpha = 2$ , then  $L$  occurs with probability 1, and the (deviating) manager's *ex ante* expectation is:

$$\Pr(\text{mgr is } G | \alpha = 2, \{\alpha = 1\}) = \Pr(\text{mgr is } G | x = L, \alpha = 2) > \Pr(\text{mgr is } G | \{\alpha = 2\}).$$

Thus, the manager is better off choosing  $\alpha = 1$  when the CEO believes he has chosen  $\alpha = 2$ . Since the manager prefers  $\alpha = 1$ , the choice of  $\alpha = 2$  by both managers is not a Nash equilibrium. ■

## 5.9 Proof of Proposition 2

Suppose the CEO wants each manager to choose  $\alpha = 2$ . Now, given *Lemma 7*, each manager prefers to choose  $\alpha = 1$  because it leads to a higher probability of winning  $\Omega$ . Since  $x = L$  for sure when  $\alpha = 1$ , restoring incentive compatibility requires increasing  $W(H)$  and  $W(0)$  relative to  $W(L)$ . However, 0 is a lower bound for  $W(L)$ , and we also know from *Lemma 6* that the managerial risk aversion leads to  $W(L) > 0$ . Moreover, as  $\Omega$  increases, both  $W(H)$  and  $W(0)$  must necessarily result in higher expected managerial wages. For  $\Omega$  high enough, it is possible that eliciting  $\alpha = 2$  requires wages such that

$$\bar{\Phi} [p[H - W(H)] + [1 - p][L - W(L)]] - [1 - \bar{\Phi}]W(0) < L - U^{-1}(\bar{U} + D(1)).$$

The shareholders will now be better off with each manager choosing  $\alpha = 1$ . ■

## References

1. Amihud, Yakov, and Baruch Lev, 1981, "Risk Reduction as a Managerial Motive for Conglomerate Mergers", *Bell Journal of Economics* 12, 605-617.
2. Brander, J., and M. Poitevin, 1992, "Managerial Compensation and the Agency Costs of Debt Finance", *Managerial and Decision Economics* 13-1, 55-64.
3. Crystal, Graef, 1991, *In Search of Excess: the Overcompensation of American Executives*, W. W. Norton, New York.
4. Dybvig, Philip, and Jamie Zender, 1991, "Capital Structure and Dividend Irrelevance with Asymmetric Information", *Review of Financial Studies* 4, 201-218.
5. Ehrenberg, Ronald G., and Michael L. Bognanno, 1990, "Do Tournaments Have Incentive Effects?", *Journal of Political Economy* 98-6, 1307-1324.
6. Frank, Robert H, and Philip J. Cook, 1995, *The Winner-Take-All Society*, The Free Press, New York.
7. Green, Jerry R., and Nancy L. Stokey, 1983, "A Comparison of Tournaments and Contracts", *Journal of Political Economy* 91-3, 349-364.
8. Greenwald, Bruce C. and Joseph E. Stiglitz, 1990, "Asymmetric Information and the New Theory of the Firm: Financial Constraints and Risk Behavior", *American Economic Review* 80-2, 160-165.
9. Hirshleifer, David, and Yoon Suh, 1992, "Risk, Managerial Effort, and Project Choice", *Journal of Financial Intermediation* 2, 308-345.
10. Holmstrom, Bengt, and Joan Ricart i Costa, 1986, "Managerial Incentives and Capital Management", *Quarterly Journal of Economics* 101, 835-860.
11. Jaggia, Priscilla Butt, and Anjan V. Thakor, 1994, "Firm-Specific Human Capital and Optimal Capital Structure", *International Economic Review* 35, 283-308.
12. Jensen, Michael, and William Meckling, 1976, "Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure", *Journal of Financial Economics* 3, 305-360.
13. Lambert, Richard A., 1986, "Executive Effort and the Selection of Risky Projects", *Rand Journal of Economics* 17-1, 77-88.
14. Lazear, Edward P., and Sherwin Rosen, 1981, "Rank-Order Tournaments as Optimum Labor Contracts", *Journal of Political Economy* 89-5, 841-864.
15. MacLeod, W. and J. Malcolmson, 1988, "Reputation and Hierarchy in Dynamic Models of Employment", *Journal of Political Economy* 96, 832-854.
16. Mookherjee, D., 1984, "Optimal Incentive Schemes with Many Agents", *Review of Economic Studies* 51, 433-446.
17. Nalbantian, Haig R., and Andrew Schotter, 1997, "Productivity Under Group Incentives: An Experimental Study", *American Economic Review* 87-3, 314-341.
18. Nalebuff, B., and J. Stiglitz, 1983, "Prizes and Incentives: Towards a General Theory of Compensation and Competition", *Bell Journal of Economics* 13, 21-43.
19. Radner, R., 1985, "Repeated Principal-Agent Games with Discounting", *Econometrica* 53, 1173-1198.