EVALUATION OF INPUT-OUTPUT ANALYSIS AS A TOOL
FOR THE STUDY OF ECONOMIC STRUCTURE OF
THE GRAND TRAVERSE BAY AREA

Working Paper No. 36

by

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BACKGROUND OF THIS PAPER

This paper is part of the research conducted by the Bureau of Business Research, Graduate School of Business Administration, on the industrial development of the Grand Traverse Bay region in Michigan. The research has been supported by the Regional Economics and Water Resource Management project of the Sea Grant Program at the University of Michigan.
In the past two decades an increasing number of input-output studies have been carried out at a regional level. Phillip J. Bourque and Millicent Cox account for at least 123 such studies in their recent paper.\footnote{Phillip J. Bourque and Millicent Cox, "An Inventory of Regional Input-Output Studies in the United States," Occasional Paper No. 22 (Seattle, Washington: Graduate School of Business Administration, University of Washington, 1970).}  The enthusiastic use of the input-output method points to the fact that new light is being shed on the subject of economic structure in those regions covered by such studies. For a better understanding of the input-output technique the first part of this paper will be devoted to a description of the method and interpretation of the results.

For simplicity let us consider an economy which is composed of only two producing sectors--agriculture and manufacturing--and one final demand sector--the household. Each of these sectors interacts economically with every other sector. For example, agriculture and manufacturing draw their labor requirements from the household sector and, in return, pay wages to the household sector. The agricultural sector may use the output of the manufacturing sector in the form of fertilizers and farm implements. Similarly agricultural products may serve as inputs to the manufacturing sector for further
processing as in the case of meat, meat products, vegetables, fruits, and so on.

If the transactions between the above sectors were recorded and sorted out by sector and by purchase vs. sales category we would have the following six piles of transaction records:

- Purchased by Household
- Sold by Household
- Purchased by Agriculture
- Sold by Agriculture
- Purchased by Manufacturing
- Sold by Manufacturing

Fig. 1.

The most cursory study of the above bookkeeping system reveals that we really do not have all the relevant information about our economic structure. Namely, every pile of transactions affords us an aggregate sales and purchase figure but does not say anything about where the purchases were made and who absorbed the sales made by any sector. How, then, can we devise an economic accounting system that does not bury the identity of the buyers and sellers of individual transactions? A short reflection upon the nature of transactions will give us the answer.

Most any invoice system has the essential elements illustrated in the following diagram:

![Diagram](https://via.placeholder.com/150)

Fig. 2.

During a given period (usually one year) our economic system accumulates a large number of transactions such as the one shown
in Figure 2. Note that if we sort our numerous transactions not by
the buyer or seller, but rather by the buyer-seller combination we
achieve our goal of preserving the identity of buying and selling sectors.
The number of different piles of transactions that we are going to have
depends on how many different buyers and sellers there are in a given
economy. The most elementary exercise in combination indicates that,
for our simplified economy with three different sellers and three different
buyers, we need nine piles of transactions. This allows for buyers
and sellers being from the same economic sector as in the case where
the farmer consumes part of his own crop.

It is not uncommon for a real economy to have as many as 400
sectors which would require 160,000 different piles of transactions.
The input-output table, by using the transactions matrix, can contribute
to the physical handling of such a vast number of different transactions.
At this point the contribution of the input-output method is nothing more
than a systematic way of arranging the aforementioned transaction
piles to keep the piles manageable. For our hypothetical economy
consider a bin designed in the form shown in Figure 3.

Such a system makes bookkeeping simple. Every time a transaction
takes place in our economy we place the transaction record in the proper
pigeonhole. At the end of the year we total all the records which have
fallen into each pigeonhole and record them in a layout like that in
Figure 3. We call this layout the transactions matrix or the trans-
actions table. Have we lost the information about total purchases
and sales which were shown in Figure 1? The answer is no. Even though our transaction matrix does not yet illustrate totals, we can easily add up the numbers in each row to obtain total sales by each sector, and, similarly, sums of columns will provide us with the total purchases of each sector. In fact our accounting system in Figure 1 would only inform us of the row and column totals of the transactions matrix without any information about the numbers inside the transaction matrix. Those familiar with the national income and product accounts will readily identify them as the set of accounts introduced in Figure 1, or simply row and column totals of the transactions matrix. Note that Figure 4 assumes that necessary allowances are made for capital consumption in the form of individual transactions.
PURCHASED BY

<table>
<thead>
<tr>
<th></th>
<th>Agr.</th>
<th>Mfg.</th>
<th>Household</th>
<th>Total Sales</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Manufacturing</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Household</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Purchases</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 4.

The value of a transactions table would be marginal at best if we did not reap some benefit from it above and beyond the national product and income accounts. We do obtain additional benefits by observing that over a relatively small number of accounting periods (5 to 10), the ratio between each cell of the transactions matrix and the corresponding column total remains constant for all practical purposes. The row total could be used in a dual development of the input-output table. This indicates that, if we go through the trouble of sorting out transactions and preparing a transactions table for a given accounting period, we can use the constant ratios to come up with transactions
tables for another accounting period for which we only have income and product accounts. But why would we want to create a transactions matrix at all? Our real interest does not lie within the transactions table itself but rather in particular derivatives of the table. To explain the usefulness to the transactions table, it will be necessary to continue with our discussion of what can be derived from a transactions matrix.

Before we go on, however, we should make sure that we can properly interpret transaction matrices. Figure 5 illustrates a hypothetical transactions matrix with three producing sectors and a final demand sector. Each cell in the transition matrix depicts the amount

<table>
<thead>
<tr>
<th>PURCHASING SECTORS</th>
<th>Agr.</th>
<th>Mfg.</th>
<th>Svcs.</th>
<th>Final Demand</th>
<th>Total Output</th>
</tr>
</thead>
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<tr>
<td>Agriculture</td>
<td>4</td>
<td>8</td>
<td>2</td>
<td>16</td>
<td>30</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>7</td>
<td>15</td>
<td>6</td>
<td>22</td>
<td>50</td>
</tr>
<tr>
<td>Services</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>10</td>
<td>25</td>
</tr>
<tr>
<td>Final payments</td>
<td>13</td>
<td>22</td>
<td>13</td>
<td>0</td>
<td>48</td>
</tr>
<tr>
<td>Total input</td>
<td>30</td>
<td>50</td>
<td>25</td>
<td>48</td>
<td>153</td>
</tr>
</tbody>
</table>

Fig. 5.

of goods or services produced by the sectors named on the left-hand side and purchased by the sector named at the top of the matrix. For example, in this matrix the services sector purchased $6.00 worth of output produced by the manufacturing sector. Similarly consumers
purchased $10.00 worth of services and $16.00 worth of agricultural products. Care must be exercised in interpreting the totals. The totals in the right-hand column are the total sales of the sectors on the left of the matrix, while the totals in the bottom row are the total purchases of the sectors named at the top of the matrix.

We have already suggested that the ratio by which the output or purchases of a given sector is divided can be viewed as a constant within relatively short periods. Therefore, we can normalize the transactions matrix by assuming that the total purchases are equal to unity in each sector, and we can scale the elements of the transition matrix accordingly. It is customary to normalize by dividing every entry in a given column by the column total, since normalization by rows would require a totally different interpretation. In Figure 6 we depict the normalized version of the transactions matrix in Figure 5. Note that the two right-

<table>
<thead>
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<tr>
<td>Agriculture</td>
</tr>
<tr>
<td>Manufacturing</td>
</tr>
<tr>
<td>Services</td>
</tr>
<tr>
<td>Final payments</td>
</tr>
<tr>
<td>Total</td>
</tr>
</tbody>
</table>

Fig. 6.
hand columns have been omitted and, in fact, the bottom row can also be omitted since our interest lies in how much output from every other sector is required as input to enable a given sector to produce an additional unit of output. For example, in our hypothetical economy $.13 input from agriculture, $.23 input from manufacturing, $.20 input of services, and $.44 input of final payments are necessary to enable the agricultural sector to produce an additional unit of output. At this stage our table is called the direct requirements table.

At this point it is very natural to ask how we can assume, given that the sectors are all interrelated, that the manufacturing sector would be able to come up with $.23 without making further demands on the remaining sectors? Students of economics will immediately recognize this situation as the multiplier effect. In fact, to input $.23 to enable the agricultural sector to produce an additional unit of output the manufacturing sector would require ($ .23) X (.16) from agriculture, ($ .23) X (.30) from itself, ($ .23) X (.10) from services, and ($ .23) X (.44) from final payments. Very quickly we realize that there is no end to this chain of computations and that this direct method of attacking the problem is hopeless.

Those who have already identified the input-output system as a system of linear simultaneous equations do not wonder that we run into trouble trying to solve a total system of linear equations through successive iterations in lieu of solving the system in its entirety. The transactions table is actually a set of constraint equations that we must
satisfy simultaneously so that the sum of purchases made from any producing sector equals its total output. In the year the transactions table is derived this relationship is satisfied by definition as is shown in Figure 5. Stating Figure 5 in symbolic form:

\[
\begin{array}{cccc}
C_{11} & C_{12} & C_{13} & C_{14} \\
C_{21} & C_{22} & C_{23} & C_{24} \\
C_{31} & C_{32} & C_{33} & C_{34} \\
C_{41} & C_{42} & C_{43} & C_{44} \\
\end{array}
\]

where \( C_{ij} \) is the content of the cell in the \( i \)th row and the \( j \)th column and \( C_j \) is the total of column \( j \). It should be mentioned here that the marginal row and column totals will be equal, since the value of input to a sector from all factors of production equals the value of the sector's output. The transition from Figure 5 to Figure 6 can be depicted symbolically as follows:

\[
\begin{array}{cccc}
C_{11}/C_1 & C_{12}/C_2 & C_{13}/C_3 \\
C_{21}/C_1 & C_{22}/C_2 & C_{23}/C_3 \\
C_{31}/C_1 & C_{32}/C_2 & C_{33}/C_3 \\
C_{41}/C_1 & C_{42}/C_2 & C_{43}/C_3 \\
\end{array}
\]
Note that the numbers above will be considered fixed constants once they are calculated for the year the input-output table is derived.

Assume now that $C_{14}^*$, $C_{24}^*$, $C_{34}^*$ have been forecasted for some future year. The following simultaneous equation system obtains:

$$C_{11}^* C_{1} + C_{12}^* C_{2} + C_{13}^* C_{3} + C_{14}^* = C_{1}^*$$

$$C_{21}^* C_{1} + C_{22}^* C_{2} + C_{23}^* C_{3} + C_{24}^* = C_{2}^*$$

$$C_{31}^* C_{1} + C_{32}^* C_{2} + C_{33}^* C_{3} + C_{34}^* = C_{3}^*$$

Denoting $\frac{C_{ij}}{C_j}$ by $t_{ij}$ and rearranging we obtain:

$$(1-t_{11}) C_{1}^* - t_{12} C_{2}^* - t_{13} C_{3}^* = C_{14}^*$$

$$-t_{21} C_{1}^* + (1-t_{22}) C_{2}^* - t_{23} C_{3}^* = C_{24}^*$$

$$-t_{31} C_{1}^* - t_{32} C_{2}^* + (1-t_{33}) C_{3}^* = C_{34}^*$$

or in matrix notation:

$$(I-T) C^* = c_{14}^*$$

The solution to such a system is well known to be:

$$C^* = (I-T)^{-1} c_{14}^*$$

Let us pause a moment and see what the last equation means. The matrix $(I-T)^{-1}$ is called the direct and indirect requirements matrix


\[2/\] An asterisk indicates an estimate.
and is made up entirely of constants which appear in the direct requirements matrix. The vector $c^*_14$ is the projected final demand by sectors and $c^*_1$ is the vector of output production by sectors.

Since the entire economic system is working to satisfy the needs of its constituents, is it not reasonable to schedule output production according to the breakdown in final demand? The answer to this question is no. To find out how much output is demanded by a final consumer we must add up all his requirements. But others must consume to be able to produce what he requires, so we must also include intermediate consumption by those who produced for the final consumer. To illuminate what drastic errors in production scheduling could be made if we only relied on the demand of final consumers, consider the output of the steel industry. With minor exceptions the final consumer has no direct need for the output of the steel industry. You can imagine what would happen if we scheduled steel production at zero. This is the great contribution of the input-output method. By knowing the structure of the economy as represented by the input-output table and an estimate of what the final consuming sector will want in some future period, we are able to schedule production in various sectors to just meet the final demand requirements.

Not only is the input-output method an effective way to schedule future production in various sectors, it is also an effective planning tool for regional industrial development. We propose to use it in this capacity in the development planning of the Grand Traverse Bay area.
Our proposed research plan follows.

Research Plan

We will obtain primary data about employment and household and retail activity in the region since these factors will be most critical in a developing region. This should take the form of a limited survey of the largest firms in the area and should include as many of the smaller firms as funds and time permit. The public relations aspect of the direct questioning method will cause greater community involvement which may result in a greater degree of cooperation. The small firms in the area should be notified that such research is being conducted so that there will not be any feelings of alienation. The primary data will provide the backbone of an input-output table for the region. The next step will be combining the primary data together with secondary data on water resource requirements, transportation, and other sector data to complete the input-output table for the region. From this point on the table will be used for the evaluation of alternative future developments. As a starting point for evaluation, the impact of the industries recommended by the Battelle study should be evaluated. 3/

3/ Battelle Memorial Institute, "Industries suited for the Upper Great Lakes Region," a report by David C. Sweet, John M. Griffin, and Hal S. Maggied, Columbus, Ohio, 1970.
The introduction of a new company into the region corresponds to changing the technical coefficients (direct requirement coefficients) in the appropriate row of the table. Assuming that the purchase requirements of other sectors will remain the same, the numerical values of all the direct requirements coefficients in the row belonging to the newly expanded industry will be decreased with the exception of net export, which will be increased. The increasing emphasis on net export results because the production of the new company will not be required to meet demand from other sectors in the area. However, even assuming static demand in other sectors, the new company will increase the labor requirements of the area and, with that, the demand for services supporting businesses and households. Only after the transient effect of a disturbance such as the introduction of a new company has subsided should a second company be introduced into the model. After the introduction of each new company, the labor requirements of the area should be studied so that enough time is provided for laborers to migrate to this area if necessary. This method is comparable to a simulation model in which the initial economic structure of the region is perturbed by consecutive industrial developments. The model can be programmed to wait until virtual equilibrium in the economy has been reached before introducing another perturbation. The model can also be applied to alternative industrial developments to single out the few that look most promising.

An alternative would be to use an aggregate approach such as estimating the rate of change of the technical coefficients. This
approach would be useful in providing the necessary output of various
sectors to support some future level of final demand if certain
 technological and production capabilities of the region grow at the
estimated rate.

In our opinion the simulation approach which uses the input-
output model as the basic structure is the more viable and preferred
method. No matter which method is used for evaluating development
alternatives, a knowledge of the existing economic structure (the input-
output table) is a necessary starting point.

Inasmuch as the greatest problems in preparing an input-output
table are those connected with the accumulation of data—especially
secondary data—the remainder of this paper will be devoted to a dis-
cussion of data and related problems.

Differences in various input-output studies are primarily in the
choice of the economic sectors to be studied and the estimation pro-
cedures to be used when secondary data are unavailable. The data
are usually the critical aspect of such analyses both in terms of
availability and timing. Bourque and Cox allude to the data problem:

Most implementation problems in the construction
of the input-output tables can be traced to data
difficulties. This is especially true for regional
studies conducted as one-shot investigations for
which even secondary data must be "discovered."
Pertinent data are published in quantity for regions,
states, counties, and cities; in fact, the abundance
of data, collected by all levels of government,
commissions, trade associations, business advisory
services, and the like can be overwhelming. One
problem is to acquire, evaluate, and assimilate the
available information. Perhaps an evaluation, industry
by industry, of the data which are fairly universally
available for states should be made; this could take the form of an inventory and an appraisal of the appropriateness of area data as control totals or for their value in describing the structure of inputs or outputs. However, because economic journals are not prone, and perhaps not a suitable vehicle to publish quite narrowly specialized reports, there would be a problem of how to get this distributed. There is no "information central" to assume that distribution role. Moreover, this would presume that there are some agreements in principle on the concepts to be measured. We believe this to be so, although we are not so certain that the rationale behind areas of agreement have yet been articulated.  

Data gathering procedures in some of the regional studies have relied on surveys of industries within the region under consideration. Authors of such studies warn that this approach is very time consuming. We would propose a survey of limited scope to reduce both the timing requirements and the cost. We believe we can obtain satisfactory accuracy with a limited survey. In fact, the important aspect of inter-industry relations is perhaps not the accuracy of the input-output table---with the possible exception of data such as employment, households, retail, etc.---but the ability to experiment with various structural and developmental alternatives and see the effects of changes in one sector on the rest of the region's economy.

It is safe to say that there are as many different methods of estimation as there are input-output studies. Which method of estimation offers the statistically most desirable estimates is not

4/ Bourque and Cox, "An Inventory of Regional Input-Output Studies."
covered well in the literature. The estimation procedures generally fall into two broad categories: (1) the bottom-up approach and (2) the top-down approach. In the first approach secondary sources are scanned on a regional basis for data that will either constitute part of the input-output table or enable the analyst to estimate values using such data. Once the regional data are exhausted the remaining portion of the table is estimated from national data. The top-down approach, on the other hand, starts with the national input-output table as the first approximation for the regional tables. Needless to say, the national table will not satisfy the export-import relationship of the region, so the coefficients in the national table are adjusted to move toward a balance of the region's import and export figures.

National coefficients can be adjusted to conform to the actual regional totals. This is the same as varying several numbers while keeping their sum constant. It is obvious that this can be accomplished in more ways than one. Similarly the national input-output table can be adjusted in almost an infinite number of ways to conform to regional totals. Some authors have devised mechanical procedures and formulae which automatically adjust the national table and make it conform to the regional totals. It is this writer's opinion that a great deal of judgment must be incorporated into the final table regardless of how

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sophisticated a computer program is available for doing the mechanical manipulation. Judgment must be exercised in making adjustments to preserve the character of the regional economy, i.e., to preserve as many known ratios and relationships as possible and to make certain that the calculated interindustry relationships coincide with the subjective and intuitive grasp which the regional planners have of the area.

The lack of sufficient documentation of the estimation techniques used is also a problem. This renders any meaningful evaluation impossible. There are only a few studies which are well documented, and such studies form the main basis of our preliminary evaluation of the input-output method as a method for the regional analysis of the Grand Traverse Bay area. A bibliography of the studies that have had influence upon our evaluations are attached.

It is our opinion that an input-output study of the Grand Traverse Bay area which uses a combination of primary and secondary data will substantially increase insight into the existing structural relationships among the economic sectors and enhance our capabilities of evaluating alternative paths for the development of the region. However, we also feel that sufficient emphasis must be placed on the water resources of the region. As J. Hughes— has suggested, incorporating natural

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resources into regional input-output studies does not require a new arsenal of analytic techniques, but only the use of existing techniques to examine industry requirements in terms of the outputs of other industries and natural resources. A well-documented study of this nature has been carried out for the state of California,\(^7\) where fresh water is treated as an economic good. In this study water-use coefficients per unit of output are developed for every sector in the input-output table through a simple graphical method and they are applied to a projected schedule of state output to determine future requirements for water. Using a linear programming formulation, where value added is taken to be the objective function, optimum solutions are obtained at various levels of water availability. The dual solution offers the very useful shadow prices of water.

A possible drawback of the California study is that attention is not given to the problem of waste water or the possibility of recycling the water. We fully recognize that this is a difficult problem, but still hope eventually to incorporate such considerations in the Grand Traverse Bay study. In a recent paper W.A. Spivey\(^8\) has taken

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a step toward the successful incorporation of water quality into economic analysis. Spivey uses a modular linear programming approach to minimize the cost of waste removal over an entire river basin under the constraint of the minimum acceptable index of water quality. The quality index was a function of the dissolved oxygen content and the amount of solid waste per gallon. We are hopeful that this and similar undertakings will afford us the ability to evaluate alternative development plans while maintaining the quality of the water resources of the region.

We are confident that the input-output analysis of the Grand Traverse Bay area, augmented with water resources and water quality requirements, will be a valuable tool for systematically evaluating alternatives for planning a prosperous and healthy future for the region.

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Milroy, John, and Copp, E. Anthony. "Economic Impact Analysis of Texas Marine Resources and Industries." Industrial Economics Research Division, Texas Engineering Experiment Station, Texas A & M University, College Station, Texas, June, 1970.


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