Form of Compensation and Managerial Decision Horizon

Working Paper #9605-11

M. P. Narayanan
University of Michigan Business School

*Copyright rights retained by M. P. Narayanan
FORM OF COMPENSATION AND
MANAGERIAL DECISION HORIZON

M. P. Narayanan*

February 1994
(Revised: July 1996)

* The University of Michigan
School of Business Administration
Ann Arbor, MI 48109-1234
Ph: (313) 763-5936

I thank Elazar Berkovitch and Jonathan Paul for extensive discussions and suggestions and David Hirshleifer, Ronen Israel, Naveen Khanna, Kevin Murphy, Ram Natarajan, Steve Slezak, Jaime Zender and participants of the finance workshops at the University of Michigan, New York University, Columbia University, and the 1996 Western Finance Association meetings for their comments and suggestions on earlier versions. I also thank two anonymous referees of the journal for their extensive comments while retaining the responsibility for any remaining errors.
ABSTRACT

This paper investigates the relation between the form of compensation and the manager's decision horizon. It finds that while all-cash contracts induce managers to underinvest in the long-term, all-stock contracts induce overinvestment in the long-term. It shows that compensation contracts consisting of both cash and restricted stock can produce efficient investment, thereby providing a rationale for the existence of both cash and stock incentive schemes in executive compensation packages. This explains why the adoption of either type of incentive scheme results in a positive stock price reaction. In addition, the paper derives the following testable hypotheses: (i) the proportion of the stock compensation is decreasing in the precision of the manager's ability and increasing in the precision of the firm's cash flows; (ii) firms compensate their managers with proportionately more stock in profitable years and proportionately more cash in leaner years; (iii) the greater the growth opportunities the higher the proportion of stock compensation.
FORM OF COMPENSATION AND MANAGERIAL DECISION HORIZON

The perceived decline of American business competitiveness is currently a topic of intense debate. One of the reasons that is cited by critics of American business is the shortsightedness of management (see Jacobs (1991)). American managers have been accused of choosing projects with quicker payback at the expense of higher net present value projects whose yields are long-term. The issue of myopia has generated considerable interest in the academic literature. Existing theories on managerial myopia attempt to rationalize myopic behavior on the basis of information asymmetries and/or agency problems between the manager and the investors.¹ This naturally leads to the question of whether compensation contracts can be designed to better align the manager's incentives to those of investors.

It has been suggested (see Lambert and Larcker (1991), for example) that the problem of managerial myopia can be solved, or at least mitigated, by tying the manager's compensation to the market stock price. While it is possible that such contracts will solve the problem of myopic investment, it is not clear that mitigation or elimination of myopia is the desired goal. The objective should be efficient investment: that is, value maximizing investment regardless of the project's time horizon. It is not obvious that stock compensation alone can result in efficient investment. If it was the case, we should find managers' compensation packages being dominated by stock options, restricted stock and similar equity-based plans (also referred to as long-term compensation plans). While it is true that most of the firms have stock-based plans it is also true that about 90% of the firms have cash incentive plans (also referred to as short-term compensation plans).² In


²Arreglado (1992) reports that, in 1992, 93% of the firms had a cash incentive plan, 43% had a restricted-stock plan, and 77% had a stock option plan. For manufacturing companies, the median long-term
addition, investors seem to view the adoption of both long-term plans with stock-based compensation and short-term cash incentive plans favorably, as evidenced by the positive stock market reaction to the adoption of both types of plans (Brickley, Bhagat, and Lease (1985) and Tehranian and Waegelein (1985)). This suggests that cash compensation may play a role in inducing efficient investment.

This paper explains why stock-based compensation alone cannot produce efficient investment and demonstrates the need for both short-term (cash) and long-term (stock-based) compensation plans. We argue that while stock compensation might resolve the problem of myopic investment (underinvestment in long-term projects), it might create an incentive to overinvest in long-term projects. While the problem of myopic investment has received considerable attention, the problem of overinvestment in long-term projects has been relatively ignored (Bebchuk and Stole (1992) is one of the few exceptions). We show that in order to induce the manager to invest efficiently regardless of the decision horizon of the project we need to offer her a compensation package consisting of both cash and some form of equity instrument. The equity instrument that we focus on is restricted stock.

In our model, the firm's cash flows depend on investors' capital, the manager's ability, and her investment decisions. The manager's ability is unknown to everyone, including herself. The manager can choose the investment level and this choice is unobservable to investors. Investors only observe the net cash flow (or the free cash flow net of investments). The amount the manager invests in the future affects the current net cash flow of the firm. If she overinvests in long-term projects there will be higher cash flows in later years but current cash flows will be lower; if she underinvests in long-term projects, current cash flows will be higher but at the expense of future cash flows.

Performance awards in 1991 to the five highest paid executives as a group was 58% of their salary while the corresponding number for cash incentive awards was 56%. Jensen and Murphy (1990) report that 50% of the managers' incentive pay in their sample is in the form of cash bonuses.
The manager has a reservation wage that is a function of her ability as perceived by investors. The reservation wage changes over time depending on the perceived posterior ability of the manager. Investors' future perception of the manager's ability could possibly differ from the manager's own perception because of the informational asymmetry regarding the project choice. The manager is paid her reservation wage in each period. If the manager is paid partly in stock, the total market value of her compensation in each period must at least equal her reservation wage for that period.

In this setting, the first-best investment decision trades off one marginal dollar of current cash flow to one marginal (discounted) dollar of future cash flow. However, if the manager is offered a cash-only compensation contract, she underinvests in the long-term. By doing so, she boosts current cash flow in an attempt to fool investors into thinking she is more able than she really is, if they attribute the increased current cash flow to her ability. Since the increase in perceived ability lingers on to future periods, it takes more than the loss of one future dollar to offset the manager's gain in the earlier period from increased wages. Therefore, she is willing to trade off one current dollar for more than one future dollar at the margin; that is, she will underinvest in the future.

Similarly, if offered a stock-only compensation contract with the stock being restricted stock that cannot be traded immediately, the manager overinvests in the long-term. To see this, suppose investors believe that the manager invests efficiently. If the manager overinvests, investors mistakenly believe that lower cash flows during the earlier years are due to lower ability, thus undervaluing her ability and the wage to be paid. The stock used to pay the wage is also undervalued, for two reasons. First, since the firm value is a function of managerial ability, undervaluation of ability results in undervaluation of stock. Second, since investors believe that the manager has invested efficiently, they mistakenly expect lower cash flows in later years, further undervaluing the stock. By overinvesting in the long-term, the manager depreciates investors' perception of her ability and her wage, but gets paid in stock that is undervalued to an even greater extent,
resulting in her being overpaid relative to her true ability. Thus, the manager strictly prefers to overinvest in the long-term. Of course, in equilibrium, investors recognize this incentive and adjust their beliefs accordingly. While the standard explanation for not using exclusively stock-based compensation is the risk aversion of the manager, this paper argues that stock-based compensation also induces inefficient investment.

The key idea here is that while the manager's compensation is a function of her perceived ability alone, the stock price is a function of both her ability and her investment decision. When the investment decision is unknown to investors, they misvalue both the ability and the future incremental cash flows from the investment decision. It is this combined effect that creates an incentive for overinvestment in the long-term if the compensation is all-stock.\(^3\) By contrast, in the case of cash-only contracts, mispricing of the stock is irrelevant and the only critical issue is the misvaluation of the manager's ability; hence the manager has the incentive to underinvest in the long-term to boost the perceived ability above its true value. Such perverse incentives exist as long as the manager's compensation during intermediate periods is related to her perceived ability.

The above discussion suggests that a compensation contract consisting of both cash and stock might induce the manager to make efficient (that is, value maximizing) investment decisions. We show that this is indeed the case. We also show that even if the manager holds stock in the firm, the optimal contract contains both cash and stock in the firm.

The model explains several empirical facts and provides testable hypotheses linking the characteristics of the manager and the firm to the form of compensation. The first set of hypotheses relates the form of compensation to the precision with which the\(^3\)

\(^3\)The idea that managers might shift income or cash flows from the present to the future to maximize their compensation has generated considerable interest in the compensation literature (see Healy (1985) and Holthausen, Larcker, and Sloan (1995), for example). Unlike past papers on this topic that take the compensation contract as given, we derive the optimal contract to solve this problem.
manager's ability and the firm's cash flow are known. The lower the precision of the manager's ability or the higher the precision of the firm's cash flows, the greater the weight on the current output in updating the manager's perceived ability. Hence, any deviation from investors' belief will have a larger impact on investors' perception of ability. Therefore, the cash component of the compensation increases the incentive to underinvest in the long-term. The stock component is immune to misperceptions of ability since misvaluations of ability are completely offset by corresponding changes in the stock price. To counter this incentive to underinvest, the stock component must be increased. The second result is that the proportion of stock compensation is higher for managers of firms with greater growth opportunities. As future growth opportunities increase, the value of the stock per unit of managerial ability increases. Therefore, the manager receives fewer shares for the same perceived ability. Thus the incentive to overinvest in the long-term decreases, or equivalently, the incentive to underinvest increases. The cash compensation is unaffected, since it only depends on misperceptions of ability. To compensate, the proportion of the stock component must increase. The last result is that the proportion of stock component is increasing in output; the more the firm's profits the higher the stock component. As output increases, the manager's perceived ability increases, causing the optimal investment level to increase. Therefore, the value of the stock per unit of managerial ability increases. As in the previous result, this decreases the incentive to overinvest causing the optimal proportion of stock compensation to increase.

There is a growing body of research on the relation between compensation contracts and efficient investment.\(^4\) Hagerty, Ofer, and Siegel (1993) argue that the compensation contract that prevents the manager from suboptimally shifting resources to

\(^4\)There is also extensive literature on the level of compensation (see Jensen and Murphy (1990), for example).
the near-term exhibits convexity resembling stock options. Gibbons and Murphy (1992) show that, as managers get more experienced and their ability is known with better precision, contracts that put more weight on current performance relative to future performance induce optimal effort because such managers are less motivated by career concerns. Paul (1992) points out that compensation based on only stock price will result in sub-optimal allocation of managerial effort across different projects. Bizjak, Brickley, and Coles (1993) argue that compensation contracts that put relatively greater weight on long-term stock returns, as opposed to short-term returns, result in efficient investment. There is a stream of research (Bushman and Indjejikian (1993), Kim and Suh (1993), and Sloan (1993), for example) that investigates why managerial compensation should be linked to both accounting measures such as earnings and the stock price.

While this paper belongs to the same genre of papers listed above, it tackles a different problem. Most of the papers listed above (with the exception of Hagerty, Ofer, and Siegel (1993)) investigate the parameters on which managerial compensation should be based - the relative weights on long-term versus short term performance, accounting earnings versus stock price, etc. The focus of this paper is the manner in which the manager should be paid (cash versus stock-based compensation) and how the form of compensation impacts the manager's decision horizon. Similar to Paul (1992) we argue that, contrary to conventional wisdom, stock compensation (as opposed to stock ownership) does not necessarily align managers' interests to that of stockholders. While Hagerty, Ofer, and Siegel (1993) provide a rationale for executive stock options, we provide a rationale for both cash incentive plans and restricted stock compensation. The contribution of our paper is that it provides an explanation for the simultaneous existence of both cash incentive plans and stock compensation (in the language of the executive compensation literature, both short-term and long-term incentive plans) and shows how they mitigate the "horizon problem".
The paper is organized as follows. Section 1 describes the model. Section 2 explains how managerial ability is updated both under symmetric and asymmetric information. Section 3 poses the investors' problem in the principal-agent framework and section 4 provides the first-best solution. Section 5 analyzes managerial incentives under all-cash and all-stock compensation schemes and shows that a compensation package containing both cash and stock results in efficient investment. Section 6 studies the effect of the characteristics of the manager and the firm on the form of compensation and section 7 concludes.

1. The Model

Consider an all-equity firm in a three-period, four-date world run by a risk-neutral manager. The firm has one share outstanding. At date 0, risk-neutral investors (i.e., stockholders or owners) hire the manager to operate the firm. The manager produces cash flows at dates 1, 2, and 3 using her own expertise and the capital supplied by investors. The manager contributes to the firm's cash flows in two ways. First, the firm's cash flows depend partly on the manager's ability: a more skillful, or a more able, manager can produce greater cash flows with the same amount of capital. Second, she decides how much of the date 1 cash flows to reinvest in the firm. Cash flows so invested produce returns in the future. Therefore, the cash flow produced at date \( t \), \( y_t \), is a function of the manager's ability \( a \) (\( a > 0 \)), the capital \( K \), the manager's reinvestment decision \( d \), and a random noise \( \varepsilon_t \). For tractability, we specify the following production function:  \(^{6} \varepsilon_t \)

---

5 It is assumed that the manager does not have the wealth to buy the entire firm.

6 We have assumed that the production function is multiplicative in the manager's ability \( a \) and the capital \( K \). The fact that the output is a function of the product of \( a \) and \( K \) makes the stock price a function of the manager's ability, and is important for the papers' results. Such a specification is also intuitively appealing since it implies that both managerial ability and capital are required to produce output.
\begin{align*}
\gamma_1(d) &= aK - d + \epsilon_1 \\
\gamma_2(d) &= a[K + f(d)] + \epsilon_2 \\
\gamma_3(d) &= aK + \epsilon_3
\end{align*}

We describe below the production function and its components in more detail.

The output \( \gamma \) is gross of managerial compensation. Any cash that remains after paying managerial wages is distributed as dividends to investors. If in any period the output is less than the compensation that needs to be paid in that period, it is assumed that investors make up the shortfall.\(^7\) We use the following terminology to denote the various date 1 cash flows (or "outputs", a term used interchangeably with "cash flows"). Gross cash flow refers to \( \gamma_1 = aK + \epsilon_1 \) which is gross of both investment and managerial wage. Net cash flow refers to \( \gamma_1 \), as defined in equation (1), which is net of investment but gross of managerial wage. The term "dividend" is used to refer to cash flow net of both investment and wage.

When the manager is hired, her ability \( a \) is not known either to the investors or to the manager. The investors and the manager have the same priors on her ability, on the basis of information that is common knowledge, such as the manager's educational qualifications and performance in previous jobs. The manager's perceived ability at date 0 is distributed normally with mean \( \mu_0 \) and precision \( h_0 \).

Investors supply capital \( K \) at date 0.\(^8\) It is assumed that there is no need for further capital infusion by investors. Future capital needs are met by internal funds generated. At

\(^7\)If the value of the firm is positive, it is in the investors' interest to make up any shortfall. In practice, the firm may borrow on a short-term basis to pay the manager. In order to avoid the complexities that arise with the presence of debt, we assume that investors provide the necessary amount from their personal account. They may borrow on their personal account to do so.

\(^8\)Even though we state \( K \) as exogenously determined capital, we actually have the following explanation in mind about \( K \). Suppose the capital supplied by investors is denoted by \( k \) (note the lower case), not \( K \). Also, suppose the gross output \( y' \) (dropping the subscripts) is given by

\[ y' = aK(k) + \epsilon \]

where \( K \) is a transformation that converts capital to net output per unit of managerial ability. Investors then endogenously decide the optimal capital \( k^* \) to be supplied, by setting \( K'(k^*) = 0 \), assuming that \( K(.) \) satisfies the usual regularity conditions. The factor \( K \) in equation (1) is then \( K(k^*) \). For simplicity, we
date 1, the manager observes the gross cash flow produced and then decides how much to reinvest in future projects. Let $d > 0$ represent the amount so invested.\(^9\) The incremental cash flow from this investment at date 2 is $af(d)$, where $f' > 0$ and $f'' < 0$.\(^10\) In other words, the incremental cash flow from the date 1 investment depends both on the amount invested and the manager's ability. At date 3, there is no incremental cash flow from the investment. A manager who chooses a higher-than-optimal value of $d$ (the optimal value being the amount investors would have chosen) can be considered as overinvesting in the long-term while one who chooses a lower-than-optimal value of $d$ is sacrificing the firm's long-term interests for short-term gain. The net present value of the optimal decision is assumed to be positive.

It is assumed that the risk-free rate is zero. The random terms $\varepsilon_i$ are independent and distributed normally as follows.\(^11\)

$$
\begin{align*}
\varepsilon_1, \varepsilon_2 &\sim N\left(0, \frac{\sigma^2}{Kt}\right) \\
\varepsilon_2 &\sim N\left(0, \frac{\sigma^2}{(K+f(d))t}\right)
\end{align*}
$$

We now specify the information set of the various agents. At date 0, when the compensation contract is signed, there is symmetric information. At date 1, the manager decides how much to invest in future projects after observing the gross cash flow at date 1. The investment decision and the gross cash flow are not observable to investors.

---

\(^9\)The amount $d$ could be investment in research and development, preventive maintenance of existing facilities or assets, or training of human resources. In general, this investment increases future cash flow at the expense of current cash flow.

\(^10\)It is assumed that the regularity conditions, $f'(0) = \infty$ and $f''(\infty) = 0$, are satisfied so that equilibrium investment levels are positive and finite.

\(^11\)This specification of the precisions of $\varepsilon_i$ simplifies the updating rules as will be clear later. Intuitively, the specifications imply that standard deviations of cash flows are proportional to the output due to capital, namely $K$ and $f(d)$. 
They only observe the cash flow net of the amount invested (i.e., net cash flow). At date 2 the incremental cash flow from the date 1 investment decision is realized. At date 3 the firm is liquidated and the cash is distributed to investors and the manager according to the contract signed at date 0.12

The manager is paid in each period, starting from date 0. She has a reservation wage, i.e., a wage below which she is unwilling to work, that depends on her ability as perceived by the market for managerial labor. We assume that the reservation wage in each period is proportional to the manager’s perceived ability, with \( \alpha > 0 \) being the proportionality constant.13 The constant \( \alpha \) represents the manager’s bargaining power in the labor market. We assume that \( \alpha < K \) to ensure that the manager’s compensation in every period is less than the output in that period. Since her perceived ability varies over time depending on her past performance, the reservation wage also varies over time.

Managerial compensation may consist of cash and stock in the firm as long the market value of the package is at least equal to the reservation wage. If part of the manager’s compensation is in the form of shares in the firm, it is assumed that the shares are existing shares and not new ones, thus maintaining the number of shares outstanding constant: investors simply buy the shares in the open market and pay the manager.

Moreover, any stock compensation is in the form of restricted shares that can be traded only at date 2.14 If the stock can be traded immediately, it is effectively cash

---

12Note that the investment decision is not known to investors even at date 3, the date of liquidation.

13The results of the paper are robust to different specifications of the wage function. It is not difficult to show that if the manager does not appropriate all cash flows (an unlikely scenario since the output also depends on the capital contributed by investors), and if the reservation wage is a monotonic function of managerial ability, our results hold.

14Restricted shares are quite common in practice. Crystal (1989) and Aisenbrey (1989) report that more than 40% of the Fortune 100 companies use restricted shares in their compensation packages. In addition, Aisenbrey (1989) reports that restricted stock is the fastest growing long-term incentive grant type.
2. Updating Managerial Ability

The firm's investors and the manager update perceived managerial ability in each period based on the output produced by the manager up to (and including) that period. However, the manager and investors will update the manager's ability differently because investors observe only the net output date 1. The manager, who knows the gross output, calculates her mean perceived posterior ability, denoted by $\mu_t^*$, $t = 1, 2$, as follows:

$$\mu_t^* = \frac{1}{h_t} \left[ \mu_0 + \frac{\theta}{K} \{y_t(d) + d\} \right]$$  \hspace{1cm} (3)$$

where $h_t = h_{t-1} + \theta$. The manager uses the gross output, $y_t(d) + d$, to update her ability since it is the gross output that is attributable to the manager's ability. The posterior date 1 mean perceived ability is a weighted average of the prior mean perceived ability $\mu_0$ and $[y_1(d) + d]/K$, the date 1 gross output attributable to the manager's ability, the weights being related to the prior precision of the manager's ability and the precision of the firm's cash flows. The updating rule in equation (3) is a result of our assumption that the perceived ability at date 0 and the error terms $\epsilon_t$ are distributed normally (see DeGroot (1970), for example). Note that the gross output, $y_t(d) + d$, and hence $\mu_t^*$, are independent of $d$. The precision of the manager's ability increases deterministically over time.

Similarly, the manager calculates her date 2 perceived ability as follows:

---

15The only reason the manager would want to buy or sell shares (other than for consumption) is because she has inside information that the shares are mispriced. As this violates security laws, our assumption appears to be reasonable.

16Note that $\tilde{y}_t = \frac{y_t - d}{K} = a + \epsilon_t$, where $\epsilon_t = \epsilon_t / K \sim N(0, \theta)$. Then $\tilde{y}_t$ can be viewed as a sample from a normal distribution with unknown mean $a$ and known precision $\theta$. 
\[ \mu^*_2 = \frac{1}{h_2} \left[ h_1 \mu^*_1 + \frac{\theta}{K + f(d)} y_1(d) \right] \]

The date 2 posterior mean perceived ability is a weighted average of the date 1 mean perceived ability \( \mu^*_1 \) and \( y_2(d)/(K + f(d)) \), the date 2 output attributable to the manager's ability.\(^{17}\) It can be shown that the above expression is equivalent to

\[ \mu^*_2 = \frac{1}{h_2} \left[ h_0 \mu_0 + \frac{\theta}{K} \left( y_1(d) + d \right) + \frac{\theta}{K + f(d)} y_2(d) \right] \quad (4) \]

The interpretation of equation (4) is as follows. The mean perceived date 2 ability is the weighted average of the prior \( \mu_0 \) and the net output attributable to the manager's ability, which in turn equals \( [y_1(d) + d]/K \) at date 1 and \( y_2(d)/(K + f(d)) \) at date 2. As before, note that \( \mu^*_1 \) is independent of \( d \). It can be shown that \( \mu^*_1 \) follows a martingale process, i.e.,

\[ E_{t+1} \mu^*_{t+1} = \mu^*_{t}, \quad \forall t. \]

Note that \( \mu^*_1 \) is the full information date 1 mean perceived ability. For convenience we denote it the "true" mean perceived ability.

Investors use a similar procedure to update the manager's perceived ability.

However, since they do not know the investment decision, their updating is based on their belief about the level of investment at date 1. Suppose investors believe that the manager has invested the amount \( b \) at date 1. Let \( \mu^*_i(b, d) \) denote investors' perception of manager's mean ability at date \( t \) if investors' belief is \( b \) and the manager's decision is \( d \). Then the following equations describe investors' perception of the manager's mean ability.

\[ \mu^*_i(b, d) = \frac{1}{h_i} \left[ h_0 \mu_0 + \frac{\theta}{K} \left( y_1(d) + b \right) + \frac{\theta}{h_i K} (b - d) \right] \quad (5') \]

\[ = \mu^*_1 + \frac{\theta}{h_i K} (b - d), \quad \text{using equation (3)}. \quad (5) \]

\(^{17}\)Note that \( \tilde{y}_2 = \frac{y_2}{K + f(d)} = a + \epsilon_2, \) where \( \epsilon_2 = \epsilon_2/(K + f(d)) \sim N(0, \theta). \) Then \( \tilde{y}_2 \) can be viewed as a sample from a normal distribution with unknown mean \( a \) and known precision \( \theta. \)
Equation (5') implies that investors' perception of manager's date 1 mean ability is the weighted average of the date 0 common prior $\mu_o$ and $[y_1(d) + b]/K$, the gross output attributable to manager's ability according investors' belief. By comparing equations (5') and (3) we get equation (5) which indicates that if investors' belief of date 1 investment is greater (less) than the true investment, there is overvaluation (undervaluation) of manager's ability.

Investors' perception of the manager's mean ability at date 2 is given by

$$\mu_2^b(d) = \frac{1}{h_2} \left[ h_1 \mu_1^b(d) + \frac{\theta}{K + f(b)} y_2(d) \right]$$

That is, the mean date 2 ability is the weighted average of the date 1 prior $\mu_1^b(d)$ and $y_2(d)/(K + f(b))$, the output attributable to the manager's ability according to investors' belief. Equation (6') can be rewritten as

$$\mu_2^b(d) = \frac{1}{h_2} \left[ h_0 \mu_o + \frac{\theta}{K} [y_1(d) + b] + \frac{\theta}{K + f(b)} y_2(d) \right]$$

$$= \mu_2^* + \frac{\theta}{h_2} \left[ \frac{b - d}{K} + y_2(d) \left( \frac{1}{K + f(b)} - \frac{1}{K + f(d)} \right) \right], \text{ using equation (4).}$$

(6)

The first equation is intuitive. It shows that the date 2 perceived ability from investors' perspective is the weighted average of the prior precision $\mu_o$ and the total output (date 1 plus date 2 output) attributable to the manager's ability which is the sum of $[y_1(d) + b]/K$ and $y_2(d)/(K + f(b))$.

From the investors' perspective, $\mu_2^b(d)$ follows a martingale process, i.e.,

$$E_t[\mu_{t+1}^b(d)] = \mu_t^b(d), \forall t,$$

where the superscript on the expectation operator indicates that the expectation is taken by investors who do not observe $d$. This can be verified by taking expectations of equations (5') and (6') and noting that $E_t[a] = \mu_t^b(d), \forall t$. Finally, note that $\mu_t^b(b) = \mu_t^*, \quad t = 1, 2$; that is, if investors' beliefs coincide with the manager's decision, investors' perception of the mean ability is the true mean perceived ability. The
inferences made in the paper assume that the realizations of output are such that the posterior mean perceived abilities are positive.

3. The investors' problem

The investors' problem is to choose a compensation package consisting of cash and stock that maximizes the value of the firm net of managerial wages, subject to the constraints that the manager must be paid at least her reservation wage in each period and that the project choice is observable only to the manager. Investors must decide the wage \( w_t \) to be offered in each period, and the fraction \( \lambda_t \) of the wage that must be paid in stock. Therefore, a compensation contract can be characterized by the sequence \( \{ w_t, \lambda_t \}, t = 0, 1, 2.\)

It is important to note that \( w_t \) is the market value of the manager's date \( t \) compensation. When the compensation includes stock, the true value of the compensation could diverge from the market value if the stock is mispriced. The true value of the date \( t \) compensation will depend, in addition to the compensation contract, on the history of outputs till \( t \), denoted by \( y_t \), the market stock price and the true value of the stock. Since the market stock price and the true value of the stock depend on investors' belief and the manager's decision, the true value of the date \( t \) compensation depends on the compensation contract \( \{ w_t, \lambda_t \} \), the history of outputs \( y_t \), investors' belief \( b_t \), and the manager's decision \( d_t \). Let \( u_t((w_t, \lambda_t), y_t, b_t, d_t) \) denote the true value of the date \( t \) compensation at the time the manager makes the investment decision; this will be date 1, after the manager has observed the gross output but before making the investment.

---

\( w_t \) and \( \lambda_t \) are functions of the history of outputs up to date \( t \). We suppress this dependency for notational simplicity.
decision.\textsuperscript{19} Note that if the compensation is all-cash and contains no stock, the true and market values are equal, i.e., \( u_c(w, 0, y', b, d) = w_r \) for all values of \( b \) and \( d \).

The investors' problem can then be represented by the following program:

**Program A**

\[
\begin{align*}
\text{Max} & \quad E_0 \sum_{i=0}^{2} (y_i - u_i) \\
\text{such that} & \\
& w_i \geq \alpha u_i^d(d), \quad w_i \geq \alpha u_i^d(d), \text{ for all } d \text{ and for all histories } y' \\
& E_i \sum_{i=0}^{2} u_i((w, \lambda), y', d, d) \geq E_i \sum_{i=0}^{2} u_i((w, \lambda), y', d, d'), \text{ for all } d \neq d
\end{align*}
\]

**Remarks:**

1. All expectations are taken over joint distributions of \( y_i \) and the subscript on the expectation operator indicates the time at which the expectation is taken.

2. Note that the output in year 3, \( y_3 \), is not included in the objective function since it is not affected by the manager's decision. Similarly, \( w_0 \) is not included in the wage constraint since it is independent of the manager's decision.

3. The expectation in constraint (8) is taken by the manager just after observing the gross output at date 1 but before the investment decision.

The program above states that investors choose the compensation contract \( \{w, \lambda\} \) and the investment \( d \) to maximize the present value of the firm's cash flows net of the manager's wages. Constraint (7) specifies that the manager must be paid at least her reservation wage as perceived by investors in each period for all possible histories \( y' \). The reservation wage is the fraction \( \alpha \) of the expected output at each date. This implies the

\textsuperscript{19}Note that true value of compensation received even in years prior to the decision (year 0 in our model) is affected by the investment decision if part of the compensation is in stock. This is because the investment decision affects both the dividend received on previously acquired stock and the value of the stock.
existence of a competitive labor market for managers; if the manager is paid less than the market's estimate of her reservation wage, she could be bid away by another firm offering a slightly better compensation contract. By forcing the firm to pay the manager her reservation wage each period, constraint (7) also precludes the firm shifting compensation from earlier periods to future periods. Such intertemporal shifting effectively forces the manager to stay with the firm and is tantamount to bondage. Constraint (8) is the incentive compatibility constraint. It states that the manager is better off implementing the decision chosen by investors. Note that while the expectation is taken at date 1, the manager is concerned about $u_0$, the true value of date 0 compensation. This is because, as stated in footnote 19, the value of the stock component of $u_0$ will depend on the date 1 decision.

4. The first-best solution

We define the first-best scenario as one in which the manager’s decision $d$ is observable to investors. In this case, investors can maximize their wealth by instructing the manager to choose the investment that maximizes firm value which is characterized by the first-order condition $\mu_t^* f(\nu_t^*(y_{t+1})) = 1$, where $\nu_t^*(y_{t+1})$ is the first-best level of investment in the long-term project given the date 1 gross output $y_{t+1}$. Note that $\nu_t^*$ is a function of $\mu_t^*$ and hence a function of the date 1 gross output $y_{t+1}$. Since there is no informational asymmetry regarding the manager’s perceived ability, the manager’s and investors’ valuations of the stock will be identical. Therefore, $u_t = w_t$. The optimal contract is $w_t = \alpha \mu_t^*$, $t = 1, 2$, and it is easy to see that such a contract satisfies constraint (7). The form of payment is immaterial since the stock will always be correctly priced.

---

20 From now on, we drop the argument of $d^*$ for notational simplicity. It should be kept in mind that all date 1 equilibrium decisions are functions of $y_{t+1}$. 
5. Analysis under asymmetric information

Before deriving the compensation contract that results in efficient investment, we investigate the effect of all-cash and all-stock compensation packages on the investment decision when there is asymmetric information between the manager and investors. Such an analysis helps us understand the different incentives that are produced by cash and stock compensations and is also helpful in understanding the intuition behind the results.

a. All-cash compensation contracts

In this case, investors solve Program A, but with the contract restricted to \( \{w, 0\} \). The following proposition describes the investment decision if the compensation is all-cash. All proofs are given in the Appendix.

**Proposition 1:** If the manager is offered an all-cash compensation contract that satisfies the labor market constraint (7) with equality, she will underinvest in the long-term; that is, \( d_e < d^* \), where \( d_e \) is the equilibrium level of investment when the form of payment is restricted to cash.

By investing less than the optimum amount in the long-term and thereby producing more cash flows in the short-term, the manager hopes to fool investors at date 1 into thinking that she is a more able person than she actually is, and hence benefit from a higher wage at date 1. Suppose investors believe that the manager's decision is \( d^* \) (i.e., the first-best decision). Then the manager gains at date 1 by choosing \( d < d^* \). This will increase the net cash flow at date 1, overvaluing the manager's perceived ability and increasing her wage (equation (5)). Of course, her date 2 ability will be undervalued (equation (6)). Proposition 1 states that the decision \( d \) has be lower than \( d^* \) for this undervaluation to offset the overvaluation at date 1. To see this, consider the first-order condition from the Appendix when the form of compensation is only cash:
\[ \mu_1 f'(c_1) = \frac{K + f(d_c)}{K} \left(1 + \frac{h_2}{h_1}\right) > 1. \]

There are three reasons why it is optimal for the manager to underinvest at date 1. 1) By underinvesting she boosts her date 1 ability as perceived by investors. This effect lingers on at date 2. To offset this overvaluation of ability, and to compensate for the date 1 overpayment of wage, one unit of reduction in date 1 investment must result in more than one unit of reduction in date 2 output (that is, \(\mu_1 f'(d) > 1\), implying \(d < d^\ast\)). 2) Since the precision of the manager’s perceived ability increases with time (\(h_2 > h_1\)), the date 2 cash flow has less impact on perceived ability than the date 1 cash flow. Hence the marginal productivity must be greater than 1 to reverse the effect of date 1 overvaluation. 3) The output (and hence the wage) per unit of ability is higher at date 2 compared to date 1, because of the date 1 investment. Thus the manager is willing to trade a unit of overvaluation at date 1 for a unit of undervaluation at date 2. The first and second reasons are independent of the wage contract as long as reservation wages as monotonic functions of perceived ability; hence the underinvestment incentive exists for a wide variety of wage contracts.

Thus the manager has the incentive to underinvest in the long-term. Investors, knowing this, adjust their beliefs accordingly, that is, \(b = d_c\), and in equilibrium, the manager does not gain anything. Therefore, an all-cash contract results in underinvestment in the long-term.\(^{21}\)

\textbf{b. All-stock compensation contracts}

In this case, investors solve Program A, but with the contract restricted to \(\{w_t, 1\}\). At dates 1 and 2, the manager is paid from the cash flows generated by the firm. Investors

\(^{21}\)This result is not new and has been obtained by many researchers. See footnote 1 for references. We state it for completeness and to show that our model is consistent with the earlier ones.
simply purchase shares ex-dividend in the open market and pay the manager: so, there is no net change in the number of shares outstanding. At date 0, when the compensation contract is signed, it is assumed that investors supply the cash for the manager’s wage which is used to purchase shares to pay the manager. For an ongoing firm, this is equivalent to assuming that the date 0 wage is paid out of date 0 cash flows. The value of the equity of the firm is simply the present value of the firm’s cash flows, net of the manager’s wages. Since the information sets of investors and the manager differ, they may value the equity differently. We use the following notation:

- \( N_t \): number of shares the manager is paid at date \( t, t = 0, 1, 2 \).
- \( s_t^i(d) \): the symmetric information value of the stock at date \( t, t = 0, 1, 2 \), if the manager’s decision is \( d \).
- \( s_t^b(d) \): the market stock price (i.e., the stock price as perceived by investors) at date \( t, t = 1, 2 \), if investors’ belief of the level of date 1 investment is \( b \) and the manager’s decision is \( d \).
- \( v_t^b(d, \lambda_2) \): the true value of the stock at date 1 and date 2, respectively, if investors’ belief of the level of date 1 investment is \( b \), the manager’s decision is \( d \). The true value of the stock at date 1 depends on the fraction of date 2 compensation in stock. Since there is no compensation at date 3, the second argument is omitted for \( v_2 \).

All the stock prices are ex-dividend, and after the manager’s wage for that period is paid. The true value of the stock may differ from the market price because of two reasons. One, the investors’ expectation of future cash flows may differ from the true expected value due to both misvaluation of the manager’s ability as well as misjudgment of the level of investment she made at date 1. Two, the true value of the manager’s future wage may differ from that perceived by investors. The true value of the stock depends on the fraction of the manager’s future wages paid in stock. For example, suppose the
manager is overvalued. The amount of overpayment will clearly depend on the form of compensation. The market stock price does not depend on the form of compensation because, given investors’ beliefs, the value of the compensation is independent of how it is paid.

Note that \( s^d_t(d) = v^d_t(d) = s^*_t(d) \), \( t = 1, 2 \). That is, if investors' belief coincides with the manager's decision, the market price and the true value of the stock are both equal to the symmetric information stock price. We derive below expressions for \( s^*_t(d) \), \( s^b_t(d) \), and \( v_t \).

\[
\begin{align*}
  s^*_2(d) &= E_2(aK + \epsilon_2) = \mu_2^*K \\
  s^*_1(d) &= E_1(y_2(d) - w_2 + s^*_2(d)) = E_1(a[K + f(d)] + \epsilon_2 - \alpha \mu_2^* + \mu_2^*K) \\
  &= \mu_1^* \{2K - \alpha + f(d) \}.
\end{align*}
\]

The subscript in the expectation operator indicates the date at which the expectation is taken. The date 2 stock price is simply the expected date 3 cash flow since the manager is not paid at date 3. The date 1 stock price is the expected date 2 cash flow net of expected managerial wage plus the expected date 2 stock price. Note that \( w_t = \mu_t^*, \ t = 1, 2 \), under symmetric information.

The following expressions define market stock prices when investors' beliefs differ from the manager's decision.

\[
\begin{align*}
  s^b_2(d) &= E_2(aK + \epsilon_2) = \mu_2^b(d)K \\
\end{align*}
\]

\[22\text{The true value } v_t \text{ does not depend on current or past forms of compensation because of our assumption that any stock compensation the manager receives is from existing shares and the total number of shares outstanding is unchanged.}\]
\[ s_t^*(d) = E_1^t\left(y_2(b) - w_2 + s_2^*(d)\right) \]
\[ = E_1^t\left(a(K + f(b)) + \varepsilon_2 - \alpha \mu_1^*(d) + \mu_2^*(d)K\right) \]
\[ = \mu_1^*(d)\left(2K - \alpha + f(b)\right) \tag{10b} \]

The superscript on the expectation operator indicates that expectations are taken given the information available to investors at that date. As before, the market stock price under asymmetric information is the expected future cash flow net of managerial wages given investors' beliefs and their information. Equation (10b) follows from the fact (see Section 2) that \( \mu_1^*(d) \) follows a martingale process from the investors' perspective.

The following expressions define the true value of equity when investors' beliefs differ from the manager's decision.

\[ v_t^*(d) = E_2^t(aK + \varepsilon_2) = \mu_1^*K \tag{11a} \]

\[ v_t^*(d, \lambda_2) = E_1^t\left(a[K + f(d)] + \varepsilon_2 - \lambda_2\frac{\alpha \mu_2^*(d)}{s_2^*(d)} v_2^*(d) - (1 - \lambda_2) \alpha \mu_2^*(d) + v_2^*(d)\right) \]
\[ = \mu_1^*[2K - \alpha + f(d)] - (1 - \lambda_2) \frac{\alpha \theta}{h_2}\left[ \frac{b - d}{K} + \frac{\mu_1^*}{K + f(b)}(f(d) - f(b)) \right] \tag{11b} \]

Since we are computing true values, it can be viewed that expectations in expressions (11) are taken given the information set of the manager (who knows the true value). At date 2, the true value of equity depends only on expected true date 3 cash flows. At date 1, the true value of equity is the sum of the expected date 2 net cash flow after wages and the expected date 2 true value of the stock. The first three terms under the expectation operator in equation (11b) represent the gross cash flow at date 2; the fourth term is the true value of the stock component of the date 2 wage; the fifth term is value of the cash component of the wage; and the last term is the true value of the stock at

\[ ^{23}\text{For notational simplicity, expectation taken conditional on information that is available to the manager (that is, complete information) is denoted without any superscripts.} \]
date 2. If the fraction $\lambda_2$ of the date 2 compensation is in stock, the number of shares the manager receives at date 2 is given by $N_2 = \lambda_2 \alpha \mu_2^*(d) / s_2^*(d)$. The true value of this compensation is therefore $N_2 \times v_2^*(d) = \lambda_2 \alpha \mu_2^*$ (using equation (11a)). Substituting for $\mu_2^*$ from equation (6) and taking expectations we get the expression in (11b).

Since in this section we are considering only stock compensation ($\lambda_2 = 1$), equation (11b) reduces to

$$v_1^*(d, l) = \mu_1^*[2K - \alpha + f(d)]$$  \hspace{1cm} (11c)

Comparing expressions (10) and (11) by using equation (5) it can be seen that $v_1^*(d, l) > s_1^*(d)$ if $d > b$ while the reverse is true if $d < b$. That is, if the manager’s investment in the long-term is greater (lower) than investors’ belief the equity is undervalued (overvalued) by investors at date 1.

The next proposition describes the manager’s optimal investment strategy if the compensation is all-stock.

**Proposition 2:** If the manager is offered the stock-only compensation contract $(w_n, 1)$ that satisfies constraint (7) with equality, she will overinvest in the long-term; that is, $d_4 > d^*$, where $d_4$ is the equilibrium level of investment when the form of payment is restricted to stock.

The intuition behind Proposition 2 is best understood by investigating the manager’s incentives when $b = d^*$, that is, when investors believe that the first-best level of investment has been made at date 1. If the manager overinvests in the long-term ($d > d^*$), her perceived ability and the stock are both undervalued at date 1. The number of shares she receives at date 1, $N_1$, is given by

$$N_1 = \frac{\alpha \mu_1^*(d)}{s_1^*(d)} = \frac{\alpha \mu_1^*(d)}{\mu_1^*(d)[2K - \alpha + f(b)]} = \frac{\alpha}{2K - \alpha + f(b)}$$
Note that $N_t$ depends only on investor beliefs and is independent of the manager's decision. However, by overinvesting, each share she receives is worth more since the expected future cash flows are higher, i.e., $v^d_i(d, 1) > s^d_i(d)$ if $d > d^*$. Equivalently, overinvestment causes both the manager's ability and the stock price to be undervalued. However, the stock price is undervalued not only due to the undervaluation of her ability but also because investors' expectation of long-term cash flows is lower given their beliefs. This can be seen by comparing the expression for stock price $s^d_i(d)$ -- equation (10b) -- with that for the true value of stock $v^d_i(d, 1)$ --equation (11c). The cost of overinvestment is the loss in value of previously acquired shares $N_0$. By overinvesting, the manager is trading off the gain from current compensation against the loss in value of previously acquired shares. Therefore, there is some $d^*$ at which it does not pay the manager to overinvest any further. Of course, as before, investors anticipate the manager's behavior and adjust their beliefs accordingly. In equilibrium, while the manager overinvests in the long-term, she does not gain from it.\textsuperscript{24}

From equation (A1) in the Appendix, it can be seen that the equilibrium investment is determined by the ratio of the previously acquired shares to the number of shares received at date 1.\textsuperscript{25} This gives us a measure of the inefficiency that is likely to occur in practice if the form of compensation is limited to stock. Lambert and Larcker (1987) find that the median ratio of the value of stock ownership to annual cash

\textsuperscript{24}A similar result is obtained by Bebchuk and Stole (1993) who show that, if the compensation contract is a linear function of both short-term and long-term stock prices, the manager has an incentive to overinvest in the long-term project if the productivity is unobservable to investors. In their model, if the unobservable variable is investment (instead of productivity), there is underinvestment in the long-term project. By contrast, in our model whether there is under or overinvestment in the long-term project depends on whether the compensation is cash or stock.

\textsuperscript{25}It can be seen from equation (A1) in the Appendix that the equilibrium investment decision under all-stock compensation also depends on a factor that is function of $\alpha$, $K$, $\theta$, and $h_1$. This factor represents the fact that any reduction in the manager's current wage due to overinvestment increases the value of her previously acquired shares. This represents an additional benefit to overinvestment. This benefit is a function of the misperception of the manager's ability (measured by $K$, $\theta$, and $h_1$) and the wage factor $\alpha$. 
compensation is 4.4 for Forbes 500 chief executive officers. They cite research that indicates that the cash compensation is 80-90% of total compensation. In this case, we can assume that the median ratio of the value of stock ownership to annual total compensation is approximately 4. This implies that the first order condition is $\mu_t^* f^* (d_t) < 0.8$ which means that the manager will accept projects that lose 20¢ for every dollar invested.

c. Cash and stock contracts

The above proposition suggests that a compensation contract that is part cash and part stock might result in efficient investment. To show this, we use the following procedure. Suppose investors believe that the manager is using the first-best decision rule, that is, $b = d^*$. We consider the compensation contract where $w_t = \alpha \mu_t^* (d_t), \ t = 1, 2,$ which obviously satisfies the wage constraint (7), and then show that there exists a sequence of $\lambda_t$ such that the incentive compatibility constraint (8) is satisfied. The following proposition characterizes the optimal compensation contract with a mixture of cash and stock.

**Proposition 3:** There exists a compensation package $\{w^*_t, \lambda^*_t\}$ such that the manager's equilibrium decision rule is $d^*$. The contract is defined by

(i) The market value of the compensation is given by $w_t = \alpha \mu_t^* (d_t), \ t = 1, 2$.

(ii) All the date 2 compensation is in stock, that is, $\lambda^*_2 = 1$.

(iii) The fraction of the compensation in stock at date 1 is given by,

$$\lambda^*_1 (y^*_1) = \frac{(\beta \kappa (1 - N_0) - \theta}{\frac{h_t K}{2K - \alpha + f(d^* (y^*_1))}}$$

(iv) The form compensation at date 0 is irrelevant.
For a given $\alpha$, the expression for $\lambda^*_1$ can be interpreted as follows. Note that $\theta/h_1$ is the change in the date 1 perceived ability for a marginal change in the level of date 1 investment $d$ (equation (5)). Therefore, $\theta/h_1$ is the change in the cash compensation for a marginal change in the level of investment. The second term in the denominator of equation (12) is the number of date 1 shares received by the manager. In equilibrium, the change in the value of the firm for a marginal change in date 1 investment is 1. Therefore, the second term in the denominator of equation (12) represents the change in the stock compensation for a marginal change in the investment. Thus, $\lambda^*_1$ is the ratio of the change in cash compensation to the sum of the changes in cash and stock compensation for a marginal change in the level of date 1 investment.

There are several points worth noting here. First, $0 < \lambda^*_1 < 1$; this follows from the fact that $N_0$ is positive and less than or equal to one. Second, $\lambda^*_1$ depends on the date 1 perceived ability of the manager, and hence the date 1 output. We will explore this relation later in the paper. Investors can deduce $d^*$ (and the gross output $y_1'$) given the observed net output $y_1$. Hence, they can base $\lambda^*_1$ on $d^*$. Third, the number of previously acquired shares ($N_0$) affects the form of date 1 compensation: the greater the number of previously acquired shares the less the optimal stock component in the date 1 compensation. At $d = d^*$, the direct effect of any marginal changes in $d$ on the date 1 dividends received from previously acquired shares is exactly offset by the changes in the capital gains from these shares. Therefore, previously acquired shares have no direct effect on the form of financing. However, there is a second order effect: a change in $d$ also changes investors' perception of manager's ability and hence her wages. For example, an increase in $d$ reduces manager's ability as perceived by investors and hence her wages, thus raising the dividend to previously acquired shares. This second order

\[ \mu^*_1(d)f^*(b) = (h_1)^{1/2}[h_1\lambda_0 + \% (y_1 + b)]f^*(b) = 1. \]
effect on the income from previously acquired shares gives the manager an incentive to overinvest in the long-term and this effect increases with the number of previously acquired shares. To counter this, the proportion of stock compensation at date 1 needs to be reduced. Finally, by paying all of date 2 compensation is in stock, distortions from the symmetric information wage can be avoided. Any misvaluations in date 2 ability is completely offset by misvaluations of the stock at date 2 because the manager's decision does not affect date 3 cash flows.

It is difficult to directly test the results of propositions 1 through 3 since investment is assumed to be unobservable in our model. However, the available empirical evidence regarding observable investment suggests that cash only compensation results in lower long-term investment. Dechow and Sloan (1991) find that chief executive officers whose incentive compensation does not include stock or options tend to spend less on research and development in the final years of their office implying that cash compensation alone produces myopic investment behavior. By contrast, those who own stock and options are less likely to reduce discretionary expenditures prior to their departure implying that stock or other equity compensation increases the manager's incentive to undertake long-term projects. The result of proposition 3 that both cash and stock compensation are required for efficient investment is consistent with the evidence of positive stock price reaction to the adoption of both short-term and long-term compensation plans. Tehranian and Waegelein (1985) find that the adoption of short-term compensation plans (cash incentive plans) is viewed as positive news while Bhagat, Brickley, and Lease (1985) and DeFusco, Johnson, and Zorn (1990) find similar results for long-term plans (restricted stock, stock options, stock appreciation rights, etc.) and stock options, respectively. If we assume that these firms were using suboptimal compensation plans that either did not include any performance-based compensation or included only either short-term or long-term compensation, then our model implies that adoption of the other type of plan will be viewed positively by investors.
6. Effect of managerial and firm characteristics on the form of compensation

In this section we investigate the effects of the characteristics of the manager and the firm on the optimal proportion of stock and cash compensation. In particular, we study the effects of the prior precision of the manager’s ability \(h_0\), the precision of the firm’s cash flows \(\theta\), the extent of future growth opportunities relative to assets in place \((f(d^*/K))\), manager’s bargaining power \(\alpha\), and \(y_1\), the date 1 net cash flow.

**Proposition 4:** For a given date 1 perceived ability (that is, for a given \(h_0^*\)), the proportion of the date 1 compensation paid in stock is decreasing in the precision of the manager’s ability \(h_0\) and increasing in the precision of the firm’s cash flows \(\theta\).

The intuition behind these results can be understood by investigating how changes in \(\theta/h_0\) affect managerial incentives. As \(\theta/h_0\) increases, the misvaluation of ability for a given deviation from investor beliefs increases (see Equation (5)). Therefore, the cash component increases the manager’s incentive to underinvest in the long-term in order to show better short-term results. The stock component is immune to changes in \(\theta/h_0\) since any resulting change in perceived ability is completely offset by corresponding changes in the stock price. The net effect is to increase the incentive to underinvest to offset which the manager must be offered less cash and more stock.

There is some empirical evidence that is consistent with proposition 4. Gibbons and Murphy (1992) find that the closer the chief executive officers are to retirement, the higher the sensitivity of their cash compensation to firm performance. They interpret this result (based on their model) as evidence that more experienced managers are motivated less by career concerns and more by current compensation and hence the higher

---

27 This is mitigated by another effect due to changes in \(\theta/h_0\). A decrease in cash compensation due an increase in \(\theta/h_0\) increases the date 1 dividend to any shares previously acquired by the manager. However, this benefit does not fully offset the loss in wage since the manager owns only a fraction of the firm.
sensitivity of cash compensation to current performance. This result is also consistent
with our model where the optimal compensation package for more experienced managers
whose ability is known with more precision contains a higher proportion of cash. Both
models' results are driven by the increased precision of the manager's ability as she
becomes more experienced. However, to fully test the implication of proposition 4, it
needs to be verified whether there is a corresponding reduction in the proportion of
equity compensation as the manager nears retirement.

The following proposition relates the form of compensation to the value of
growth opportunities relative to assets in place.

Proposition 5: The greater the growth opportunities relative to assets in place, the
greater the proportion of stock compensation. That is, \( \lambda' \) is increasing in \( f(d')/K \).

As future growth opportunities increase relative to assets in place, the value of the
stock per unit of managerial ability increases. Therefore, the manager receives fewer
shares for the same perceived ability. This decreases the change in stock compensation
for a marginal change in date 1 investment level, decreasing the incentive to overinvest
for the long-term. Cash compensation is unaffected since it depends only on perceived
ability. To compensate, the proportion of the stock component must increase.\(^{28}\) There is
considerable evidence that is consistent with the notion that the proportion of stock-based
compensation is positively correlated to growth opportunities. Lewellen, Loderer, and
Martin (1987), Bizjak, Brickley, and Coles (1993), and Gaver and Gaver (1993) find that
firms with higher market to book ratio of equity (which is used a proxy for growth

\(^{28}\)A similar implication is obtained by Hagerty, Ofer, and Siegel (1993).
opportunities) put less weight on cash bonuses, while Smith and Watts (1992) obtain a similar result at the industry level.\textsuperscript{29}

Earlier we had noted that the proportion of the date 1 stock component depends on the date 1 output. The next proposition relates the proportion of stock compensation to output.

**Proposition 6:** The higher the date 1 output ($y_1$) the higher the fraction of stock compensation.

As date 1 output increases, the manager's perceived ability increases causing the optimal investment level ($d^*$), and hence $f(d^*)$, to increase. From the previous proposition we know that an increase in $f(d^*)$ results in an increase in the proportion of the stock component.

The next proposition relates the manager's bargaining power to the form of compensation.

**Proposition 7:** Managers with greater bargaining power are paid more in cash. That is, $\lambda^*_1$ is decreasing in $\alpha$.

As $\alpha$ increases, manager's wage for the a given perceived ability increases. This results in a decrease in date 1 stock price (which is net of managerial wage) for a given date 1 output. Therefore, the number of shares the manager receives per unit of stock compensation increases. As we know from Proposition 2, manager's incentive to overinvest depends on the ratio of date 1 shares ($N_1$) to the stock of previously acquired shares ($N_0$). Hence, an increase in the number of date 1 shares increases her incentive to overinvest, requiring her to be paid less in stock. Since this result depends on the impact

\textsuperscript{29}Bizjak, Brickley, and Coles (1993) also find that firms with high R & D expenditures relative to assets put less weight on cash bonuses. They interpret the results as supporting the hypothesis that firms with more asymmetric information put less weight on compensation tied to near-term stock performance.
of manager's compensation on stock value, it becomes important only in firms where the manager's compensation is a significant fraction of the output.

7. Conclusions

Both theoretical and empirical research suggest that cash compensation creates an incentive for the manager to sometimes underinvest in long-term projects. One suggestion that is often made to mitigate or eliminate such myopic investment decisions by managers is to include long-term incentive plans (i.e., stock-based compensation) in their compensation package. However, in practice we find that cash constitutes a significant proportion of the total compensation.

This paper provides a justification for incentive contracts that use both cash and stock compensation. Using a model of unknown managerial ability and privately observable investment decision, we show that while cash-only incentive contracts result in underinvestment in long-term projects, contracts that are exclusively stock-based (restricted stock, stock options, etc.) result in overinvestment in long-term projects. The key idea is that, even though the manager's ability will be undervalued in the near term if she overinvesting long-term projects, the stock used to pay her is overvalued to an even greater extent, resulting in the manager getting overpaid. Therefore, in order to induce efficient investment, we needs contracts that offer a mixture of cash and stock.

Besides explaining why the adoption of either short-term or long-term incentive plans result in positive stock returns, the paper derives several interesting testable hypotheses:

- The proportion of the stock compensation is decreasing in the precision of the manager's ability, and increasing in the precision of the firm's cash flows.
- The greater the output, the greater the proportion of stock compensation. Thus, in periods where a firm is highly profitable, most of the compensation would be in the form of stock.
• The greater the growth opportunities of the firm, the higher the proportion of stock compensation. Thus, in industries where the manager has to make decisions about investments in research and development and new technology, one would expect to see higher stock compensation.

• Managers with more bargaining power will be paid more in cash.

This paper explains some of the features of managerial compensation contracts and how the components of the contracts affect investment decisions. In the model presented here, restricted stock is the only stock-based compensation considered. The model, as it stands, does not distinguish between different equity instruments. For example, if managerial compensation consists entirely of stock options instead of stock, the manager still has the incentive to overinvest in long-term projects as long as the strike price of the option is set as a function of the stock price (in practice, the strike price is generally set at the prevailing stock price). It would be interesting to study why we see different equity instruments in executive compensation contracts.
Appendix

Proof of Proposition 1

Let \( b \) be the investors' belief about manager's choice of investment level.

Consider the compensation contract that satisfies constraint (7) with equality, i.e.,
\[
 w_t = \alpha \mu_t^*(d), \quad t = 1, 2. 
\]
Since the compensation is all-cash, the true value of the compensation package, \( u_t \), equals \( w_t \). Therefore,
\[
 u_t([w_t, 0], y_t, b, d) = \alpha \mu_t^*(d), \quad t = 1, 2. 
\]

Define
\[
 \pi_1([w_1, 0], b, d) = E_1\left[ \sum_{i=1}^2 u_i([w_i, 0], y_i, b, d) \right] 
\]

\( \pi_1 \) is the total expected payoff at date 1 to the manager if her decision is \( d \) and investors' choice (or belief) is \( b \), where the expectation is taken by the manager after she observes the first period gross cash flow but before making the investment decision. Note that date 0 wage is irrelevant since it was in the form of cash. From equations (5) and (6), we can write
\[
 \pi_1([w_1, 0], b, d) = \frac{\alpha \theta}{h_1 K} (b - d) + \frac{\alpha \theta}{h_2} \left( \frac{b - d}{K} + \frac{\mu_1^*}{K + f(b)} [f(d) - f(b)] \right) + 2 \alpha \mu_1^* 
\]

This follows from the fact that \( E_1 y_2(d) = \mu_1^* [K + f(d)] \). Note that since the manager has already observed the gross output, \( \mu_1^* \) is known to her. The first-order condition with respect to \( d \) is given by
\[
 \mu_1^* f(d) = \frac{K + f(b)}{K} \left( 1 + \frac{h_2}{h_1} \right) 
\]

Let \( d_c \) be the solution to this equation. In equilibrium, \( b = d = d_c \). Therefore, the first-order condition is
\[ \mu_* f'(d_*) = \frac{K + f(d_*)}{K} \left( 1 + \frac{h_2}{h_1} \right) > 1. \]

It can be verified that the second order condition is satisfied. From the concavity of \( f(d) \) it follows that \( d_* \) is less than \( d^* \). Therefore, there is underinvestment in the long-term project. Q.E.D.

**Proof of Proposition 2**

As in the proof of the previous proposition, let \( \pi_1(\{w_1,1\}, b, d) \) denote the manager's total expected payoff at date 1 under a stock-only contract if her decision is \( d \) and investors' belief is \( b \). Then

\[ \pi_1(\{w_1,1\}, b, d) = E_1 \sum_{t=0}^{2} u_t(\{w_1,1\}, y(d), b, d) \]

where \( u_t \) is the true value of the date \( t \) package at date 1. Note that \( u_0 \) is the value of the stock portfolio that the manager received as date 0 compensation and its value depends on her current decision. The true value of the date 2 compensation package is given by

\[ u_2(\{w_2,1\}, y^2(d), b, d) = \frac{\alpha \mu^*_{1}(d)}{s^2_{*}(d)} v^*_2(d) = \alpha \mu^*_{1} \]

using equations (10a) and (11a). The true value of the date 1 compensation package is given by

\[ u_1(\{w_1,1\}, y^1(d), b, d) = \frac{\alpha \mu^*_{1}(d)}{s^1_{*}(d)} v^*_1(d,1) = \alpha \frac{2K - \alpha + f(d)}{2K - \alpha + f(b)} \mu_* \]

using equations (10b) and (11c). Since the manager already owns \( N_o \) shares from her date 0 compensation she receives a dividend at date 1 equal to \( N_o(y_1' - d - w_1) \), where \( y_1' \) is the gross output at date 1 which the manager has already observed. In addition, these \( N_o \) shares have a true value of \( N_o v^*_1(d,1) \). Since \( w_1 = \alpha \mu^*_{1}(d) \) from the wage constraint, the true value of the date 0 compensation package is given by
\[ u_0 = N_o \{ y_1 - d - \alpha \mu_i^*(d) + \nu_i^*(d, 1) \} \]

Note that \( u_0 \) and \( u_1 \) are deterministic since the manager already knows the gross output. \( u_2 \) is a random variable which does not depend on \( d \). Therefore, we can effectively ignore the expectation operator in calculating the first order condition. From equations (5), (10b), and (11c) it can be seen that

\[
\frac{\partial \mu_i^*(d)}{\partial d} = -\frac{\theta}{h_i K}; \quad \frac{\partial s_i^*(d)}{\partial d} = -\left\{2K - \alpha + f(b) \right\} \frac{\theta}{h_i K}; \quad \text{and} \quad \frac{\partial \nu_i^*(d, 1)}{\partial d} = \mu_i^* f'(d). 
\]

Therefore, it follows that

\[
\frac{\partial u_0}{\partial d} = N_o \left( -1 + \frac{\alpha \theta}{h_i} + \mu_i^* f'(d) \right); 
\]

\[
\frac{\partial u_1}{\partial d} = \frac{\alpha}{2K - \alpha + f(b)} \mu_i^* f'(d) \quad \text{and} \quad \frac{\partial u_2}{\partial d} = 0. 
\]

The manager chooses \( d \) to satisfy the first-order condition \( \partial \pi_1/\partial d = 0 \). Let \( d_1 \) be the solution to the above equation. Then,

\[
\frac{\partial \pi_1}{\partial d} = N_o \left( -1 + \frac{\alpha \theta}{h_i} + \mu_i^* f'(d) \right) + \frac{\alpha \mu_i^* f'(d)}{2K - \alpha + f(b)} = 0 
\]

In equilibrium, \( b = d = d_1 \), and the first-order condition reduces to

\[
\mu_i^* f'(d_1) = \frac{N_o \left( 1 - \frac{\alpha \theta}{h_i K} \right)}{N_o + \frac{\alpha}{2K - \alpha + f(d_1)}} = \frac{N_o \left( 1 - \frac{\alpha \theta}{h_i K} \right)}{N_o + N_1} < 1. \quad (A1) 
\]

since the number of shares the manager receives at date 1, \( N_1 \), is given by

\[
N_1 = \frac{\alpha \mu_i^*}{s_i^*(d_1)} = \frac{\alpha \mu_i^*}{\mu_i^* \left[ 2K - \alpha + f(d_1) \right]} = \frac{\alpha}{2K - \alpha + f(d_1)} 
\]
It can be verified that the second order condition is satisfied at \( d = d^* \). From the concavity of \( f(d) \) it follows that \( d_s > d^* \).

Q.E.D.

**Proof of Proposition 3**

Consider a compensation contract \( \{w_i, \lambda_i\} \), where \( w_i = \alpha \mu_i^s(d), \quad t = 1, 2. \) This clearly satisfies constraint (7). For this contract to induce the first-best decision in equilibrium, it needs to be shown that there exist \( \lambda_i \) such that \( 0 \leq \lambda_i \leq 1, \quad t = 0, 1, 2, \) and the contract satisfies the incentive compatibility condition (8) when investors believe that the manager has chosen decision \( d^* \).

As before, let \( \pi_1(\{w_i, \lambda_i\}, b, d) \) represent the manager's total expected payoff at date 1 by choosing decision \( d \) when investors' belief is \( b \), given the compensation contract \( \{w_i, \lambda_i\} \). As in the proof of Proposition 2,

\[
\pi_1(\{w_i, \lambda_i\}, b, d) = E_i \sum_{i=0}^{2} u_i(\{w_i, \lambda_i\}, y_i(d), b, d),
\]

where

\[
u_0 = N_0\{y_1^*-d - \alpha \mu_0^s(d) + \nu_0^s(d, \lambda_2)\}
\]

\[
u_1 = \lambda_1 \frac{\alpha \mu_1^s(d)}{\alpha \lambda_1^s(d)} \nu_1^s(d, \lambda_2) + (1 - \lambda_1) \alpha \mu_1^s(d)
\]

\[
= \alpha \left[ \frac{\lambda_1 \nu_1^s(d, \lambda_2)}{2K - \alpha + f(b)} + (1 - \lambda_1) \mu_1^s(d) \right], \text{ using equation (10b).}
\]

\[
u_2 = \lambda_2 \frac{\alpha \mu_2^s(d)}{\alpha \lambda_2^s(d)} \nu_2^s(d) + (1 - \lambda_2) \alpha \mu_2^s(d)
\]

\[
= \alpha \left[ \mu_2^s + (1 - \lambda_2) \frac{b - d}{h_2} \left( \frac{1}{K + f(b)} - \frac{1}{K + f(d)} \right) \right],
\]

using equations (6), (10a), and (11a). Therefore,
\[ E_1 u_2 = \alpha \left[ \mu_{1}^* + (1 - \lambda_2) \frac{\theta}{h_2} \left( \frac{b - d}{K} + \frac{\mu_{1}^*}{K + f(b)} (f(d) - f(b)) \right) \right] \]

Note that only the value of the stock component of the date 0 wage is relevant to the manager's decision at date 1. That is why we omit the cash component in defining \( u_0 \).

For \( u_1 \) and \( u_2 \), both components are important.

The expression for \( E_1 u_2 \) may be interpreted as follows. The first term is the symmetric information expected wage of the manager and the second term is the distortion caused by the cash portion of the date 2 wage due to misvaluation of ability. The stock portion of the date 2 wage does not cause any distortion since the misvaluation of ability is completely offset by the misvaluation of the stock and since date 3 cash flows are independent of the manager's decision. Such distortion in date 2 compensation due to misvaluation of ability can be eliminated by setting \( \lambda_2 = 1 \), i.e., by paying all the compensation in stock. This in turn eliminates distortions to \( v_1(d, \Lambda) \), the date 1 true value of the firm (see equations (11b) and (11c)).

Setting \( \lambda_2 = 1 \) and differentiating \( u_0, u_1, \) and \( E_1 u_2 \) with respect to \( d \) we get (using the first derivatives calculated for the proof of Proposition 2),

\[
\frac{\partial u_0}{\partial d} = N_0 \left( -1 + \frac{\alpha \theta}{h_1 K} + \mu_1^* f(d) \right)
\]

\[
\frac{\partial u_1}{\partial d} = \alpha \left[ \frac{\lambda_1 \mu_1^* f(d)}{2K - \alpha + f(b)} - \left(1 - \lambda_1 \right) \frac{\theta}{h_1 K} \right]
\]

\[
\frac{\partial E_1 u_2}{\partial d} = 0
\]

Therefore,

\[
\frac{\partial \pi_1}{\partial d} = N_0 \left( -1 + \frac{\alpha \theta}{h_1 K} + \mu_1^* f(d) \right) + \alpha \left[ \frac{\lambda_1 \mu_1^* f(d)}{2K - \alpha + f(b)} - \left(1 - \lambda_1 \right) \frac{\theta}{h_1 K} \right]
\]
To derive the optimal contract, we calculate $\frac{\partial \pi_i}{\partial d}$ at $b = d = d^*$ and set it equal to zero (manager's first-order condition). It can be verified that the second order condition is satisfied. Recall that $\mu_i^* f'(d^*) = 1$.

$$\frac{\partial \pi_i}{\partial d} \bigg|_{d=b=d^*} = -(1 - N_0) \frac{\theta}{h_i K} + \lambda_i \left[ \frac{\theta}{h_i K} + \frac{1}{2K - \alpha + f(d^*)} \right] = 0$$

Solving the above equation, we get the equilibrium $\lambda_i^*$ as

$$\lambda_i^* = \frac{(1 - N_0) \frac{\theta}{h_i K} + \frac{1}{2K - \alpha + f(d^*)}}{\theta + \frac{1}{h_i K}}$$  \hspace{1cm} (A2)$$

Q.E.D.

Proofs of propositions 4, 5, 6, and 7

The proofs follow directly from equation (A2).  \hspace{1cm} Q.E.D.
References

*Compensation and Benefits Review* 21 (Issue 6), 34-46.


Crystal, G., 1989, "Incentive Pay that does not work," *Fortune* 120 (August 28), 101-104.


