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EFFECT OF TRADE CREDIT ON ORDER QUANTITIES

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Abstract

It had been shown by some earlier researchers that even when the vendor permits delay in payment for the goods supplied, best order quantity is determined by the classical economical order quantity formula. Their analysis was based on the average cost analysis. We use the conceptually rigorous discounted cash flow approach to analyze this problem. Firstly, we show that the optimum order quantity increases as the delay time for payment increases. Also we show that the approach suggested by Chand and Ward yields an upper bound on the optimum order quantity. Though the classical economic order quantity formula disregards the timing information, surprisingly it yields better solutions under some normal circumstances than the formulation accounting for the delay in payment. We illustrate this anamoly with an example and provide analytical explanation for it.

Effect of Trade Credit on Order Quantities

Introduction

It is well known that the classical economical order quantity formula (or popularly known as the square root formula) provides optimal order quantities in deterministic demand environment when backlogging or shortages are not permitted. Further, it is assumed that the payment of the goods is made as soon as the items are received. Decision criterion used in these analyses is minimization of the average cost per period. However, a common industry practice is to provide a specific delay period for the payment after the goods are delivered. Some earlier researchers (Haley and Higgins [5], Chapman, Ward, Cooper and Page [1], and Goyal [3]) argued that when the cost of funds is the same as the return available on opportunities for the firm, order quantity is invariant with respect to the payment delay period. Further, they argued that the order quantity is exactly equal to the classical economic order quantity as determined by the square root formula. Intuitively, it is obvious that the availability of opportunity to delay the payment effectively reduces the cost of holding inventories and thus is likely to result in larger order quantities. Chand and Ward [2] analyzed the same problem and suggested an order quantity which increases as the delay time for payment increases. Their analysis is a hybrid approach in which they use discounting to take into consideration delay in payments and superimpose this effect on the classical economic order quantity formula. Thus we have two schools of thought--one arguing that the order quantity is invariant with respect to the delay time for payment and the other arguing for an increase in order quantity as the delay time for payment increases.

In this paper, we analyze the problem using the conceptually rigorous discounted cash flow analysis. In the next section, we briefly review the approaches used by the earlier researchers. Subsequently, we present our approach based on the discounted cash flow method. Further we show that the approach suggested by Chand and Ward yields an upperbound on the optimum order quantity. Later, we illustrate the procedures using a numerical example. Further, we will show why the classical economical order quantity formula, in spite of disregarding the delay in payment information, can provide superior results than Chand and Ward approach under some normal circumstances. In the final section, we conclude with discussion on the average cost analysis and provide future research directions.

Prior Research

In this paper, we assume that the demand for the product is deterministic and constant over infinite horizon. Further, it is assumed that no backlogging or shortages are permitted. The vendor permits some time delay in payment for the goods after their receipt by the customer. Also, it is assumed that the rate of return earned on funds equals the cost of funds. Notation used in the paper is defined in Table 1.

Haley and Higgins (HH), Chapman, Ward, Cooper and Page (CWWP), and Goyal used average cost analysis for analyzing the above problem. Essentially, their argument is as follows: benefits per period derived from the delay in payment is independent of the order size. If the order quantity is large, then we may get large benefits due to free use of funds (based on delay in payment), but the number of occasions on which such benefits is available decreases on per period basis. The reverse is true when order quantities are small. Overall, the benefit per period due to delay payment remains the same. Hence the order quantity is determined by other relevant costs. It is given by

TABLE 1

Notation

D	= Demand rate
h	= Holding cost per unit per period. This is exclusive of the capital charges
p	= Price of the item
r	= Discount rate or interest rate
s	= Set up cost or order cost
t	= Delay time permitted in paying for the goods after their receipt by the customer
p'	= $p/(1 + rt)$. For continuous discounting, it is given by pe^{-rt}.
Q(EOQ)	= Order quantity as determined by the classical economic order quantity formula
	$= \sqrt{2SD/(h + pr)}$
Q(CW)	= Order quantity determined using Chand and Ward approach
	$= \sqrt{\dfrac{2SD}{h + \dfrac{pr}{1 + rt}}}$
	$= \sqrt{2SD/(h + pre^{-rt})}$ (for continous discounting)
Q(OPT)	= Optimum order quantity determined using discounted cash flow analysis
T(EOQ), T(CW), T(OPT)	= Reorder intervals corresponding to Q(EOQ), Q(CW) and Q(OPT)

$$Q(EOQ) = \sqrt{2SD/(h + pr)} \qquad \text{I}$$

$$\text{Total cost per period} = \sqrt{2SD(h + pr)} - Dprt \qquad \text{II}$$

It can be easily seen that the above is same as the classical economic order quantity. Interested readers are directed to the original papers for further details (Haley and Higgins [1], Chapman et al. [2] and Goyal [3]). They is true irrespective of whether delay time (t) is greater or less than T(EOQ).

Chand and Ward analyzed the same problem from a slightly different perspective. They argued that providing a delay time in payment for the goods is essentially equal to lowering the price in such a way that the effective price is the time discounted value and it is given by

$$p/(1 + rt)$$

Substituting this (for continuous discounting, the above expression is substituted with pe^{-rt}) in the classical economic order quantity formula,

$$Q(CW) = \sqrt{\frac{2SD}{h + \frac{pr}{1 + rt}}} \qquad \text{IIIa}$$

$$= \sqrt{2SD/(h + pre^{-rt})} \quad \text{for continuous discounting} \qquad \text{IIIb}$$

It is clear from the above expression that as the payment delay period increases, order quantity increases. We conclude this secion with the following remark.

<u>Remark 1</u>: Classical order quantity, Q(EOQ), is always less than or equal to the order quantity determined using Chand and Ward approach, Q(CW).

<u>Proof</u>: The proof follows immediately from the definitions of Q(EOQ) and Q(CW). This is true as long as payments of goods is made at or after the delivery of goods.

Discounted Cash Flow Approach

It is well known that the average cost analysis is only an approximation to the conceptually rigorous discounted cash flow analysis (Hadley [5]). In order to overcome the conflicting conclusions derived by other researchers cited in the previous section, we analyze the problem using discounted cost flow approach. Suppose that the reorder interval is T. Net present value of all future costs is given by

$$NPV(T) = S + \int_0^T hD(T - \ell)e^{-r\ell}\, d\ell + DTpe^{-rt} + e^{-rT}\, NPV(T)$$

$$= \frac{S}{1 - e^{-rT}} + \frac{1}{1 - e^{-rT}} \int_0^T hD(T - \ell)e^{-r\ell}\, d\ell + \frac{DTpe^{-rt}}{1 - e^{-rT}} \qquad \text{IV}$$

The first term in the above expression relates to set up or order costs. Second term relates to the non-capital related holding charges such as warehousing etc. Third item relates the price paid for the items. Since we are using discounting, there is no need to state capital related holding charges explicitly. Above expression can be rewritten as

$$NPV(T) = \frac{S}{1 - e^{-rT}} + \frac{hD}{r^2} \frac{rT - 1 + e^{-rT}}{1 - e^{-rT}} + \frac{Dpe^{-rt}\, T}{1 - e^{-rT}} \qquad \text{V}$$

For our further analysis, we use annualized cost, ANN(T), instead of NPV(T). ANN(T) is that uniform cash stream over infinite time whose net present value is NPV(T). It is a surrogate for NPV(T). From the basic principles of discounting,

$$ANN(T) = r * NPV(T)$$

$$= \frac{Sr}{1 - e^{-rT}} + \frac{hD}{r} \frac{rT - 1 + e^{-rT}}{1 - e^{-rT}} + \frac{Drpe^{-rt}\, T}{1 - e^{-rT}} \qquad \text{VI}$$

Proposition 1: ANN(T) is convex in T.

Proof: Proof is similar to Rachamadugu [6]. Second derivative of ANN(T) in expression VI yields

$$\text{ANN}''(T) = \frac{Sr^3 e^{-rT}(1 + e^{-rT})}{(1 - e^{-rT})^3} + \frac{(h + pe^{-rt})Dre^{-rT}}{(1 - e^{-rT})^3}\left\{(2 + rT)e^{-rT} - (2 - rT)\right\} \qquad \text{VII}$$

It is clear that ANN"(T) is nonnegative if the term in curled parenthesis is nonnegative.

$$(2 + rT)e^{-rT} - (2 - rT) = \frac{1}{e^{rT}}\left\{\sum_{n=3}^{\infty} \frac{(rT)^n}{n!}(n - 2)\right\}$$
$$\geq 0$$

Since ANN"(T) $\geq$ 0, ANN(T) is convex in T.

■

Remark 2: Chand and Ward approach (IIIb) yields an order quantity which is an upper bound on the true optimum i.e., Q(CW) $\geq$ Q(OPT).

Proof: $$\text{ANN}(T) = \frac{Sr}{1 - e^{-rT}} + \frac{hD}{r}\frac{rT - 1 + e^{-rT}}{1 - e^{-rT}} + \frac{Drx\,T}{1 - e^{-rT}} \qquad \text{VIII}$$

where x is the discounted price of the item. Rachamadugu [6, page 5] has shown that an order quantity equal to $\sqrt{2SD/(h + xr)}$ is an upper bound on the true optimum for expression VIII. Substituting pe^{-rt} for x,

$$\sqrt{\frac{2SD}{h + pre^{-rt}}} \geq Q(OPT)$$

■

Details are omitted for the sake of brevity. Interested reader is referred to Rachamadugu [6].

Next we show that the optimal reorder interval (or order quantity) increases as the delay time increases.

<u>Proposition 2</u>: Q(OPT) and T(OPT) are increasing functions of delay time.

<u>Proof</u>: Earlier (Proposition 1) we had shown that ANN(T) is convex. Hence T(OPT) can be found by setting the first derivative of ANN(T) equal to zero.

$$ANN'(T) = -\frac{Sr^2\, e^{-rT}}{(1 - e^{-rT})^2} + (Dpe^{-rt}\, r + hD)\, \frac{1 - e^{-rT} - rTe^{-rT}}{(1 - e^{-rT})^2}$$

$$ANN'(T) = 0 \Rightarrow$$

$$e^{rT(OPT)} - rT(OPT) = \frac{Sr^2}{D(h + pre^{-rt})} + 1 \qquad \text{IX}$$

Using IX,

$$\frac{dT(OPT)}{dt} = \frac{Spr^3}{D} \cdot \frac{e^{-rt}}{h + pre^{-rt}} \cdot \frac{1}{e^{rT(OPT)} - 1}$$

Note that the right hand side of the above expression is always positive. Hence T(OPT) is an increasing function of delay time.

■

Above proposition contradicts the earlier claims made by Haley and Higgins, Chapman et al. and Goyal that, when the cost of funds are same as the returns available to the firm, the order quantity is equal to the classical order quantity and remains invariant with respect to the delay time.

Discussion

Our analysis in the previous section showed that the order lot size determined using Chand and Ward approach (IIIb) is an upper bound on the true optimum. Further, it is clear from Remark 1 that classical economic order quantity is less than the order lot size determined using CW approach. Also, we know that the NPV(T) and ANN(T) are convex. These results can lead to somewhat anamolous, though realistic, situations depicted in Figure 1.

Figure 1 illustrates a situation where the classical economic order quantity (which disregards the payment delay information) yields results superior to Chand and Ward approach (which appropriately takes into consideration delay payment information). We illustrate this using a simple numerical example.

Let S = \$200

r = 20% per dollar value per period

p = \$4 per unit

h = \$0.6 per unit per period

D = 200 units per period

Results for t = 0, 1/6 and 4 periods are shown in Figures 2, 3 and 4 respectively and also shown in Table 2. When t equals zero (Figure 2), both classical EOQ and Chand and Ward approach arrive at the same order quantity. Also, as per remark 2, the reorder interval is greater than the true optimum. This is shown in Figure 2. Arrows in Figure 2 indicate direction of change for various quantities as the payment delay period increases.

Figure 3 indicates the result for t = 1/6. Notice that in this case, Chand and Ward approach yields an order quantity greater than the classical EOQ. Further classical EOQ is greater than the true optimum. Thus, even though the classical EOQ disregards the delay payment information, it results in less annualized cost than CW approach. This may appear to be somewhat anamolous, but

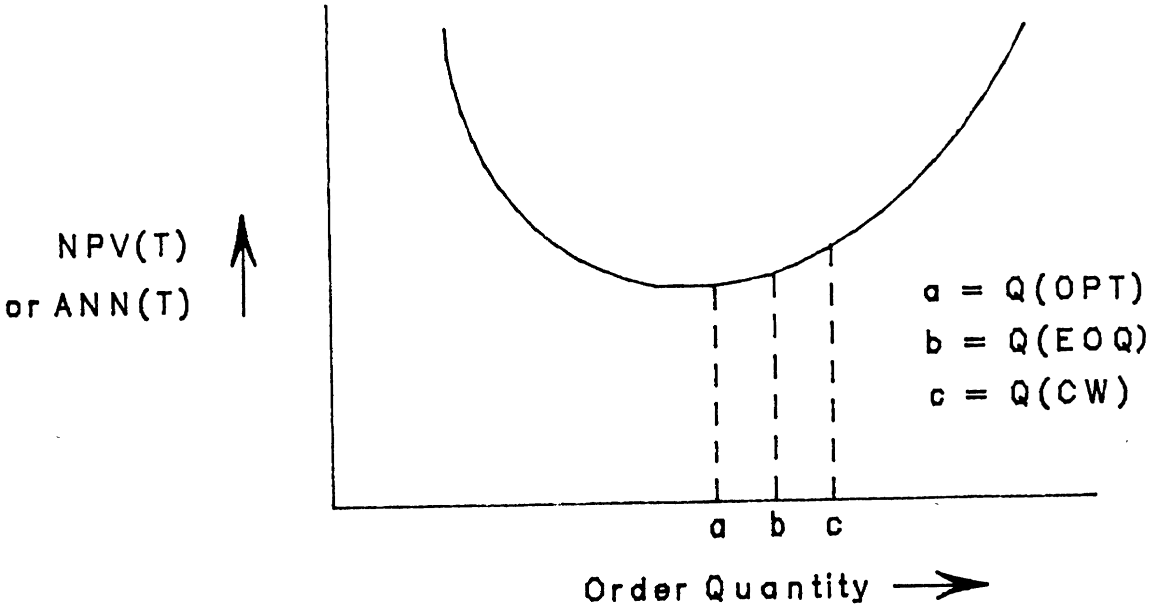

FIGURE 1

Delay Time = 0 Periods

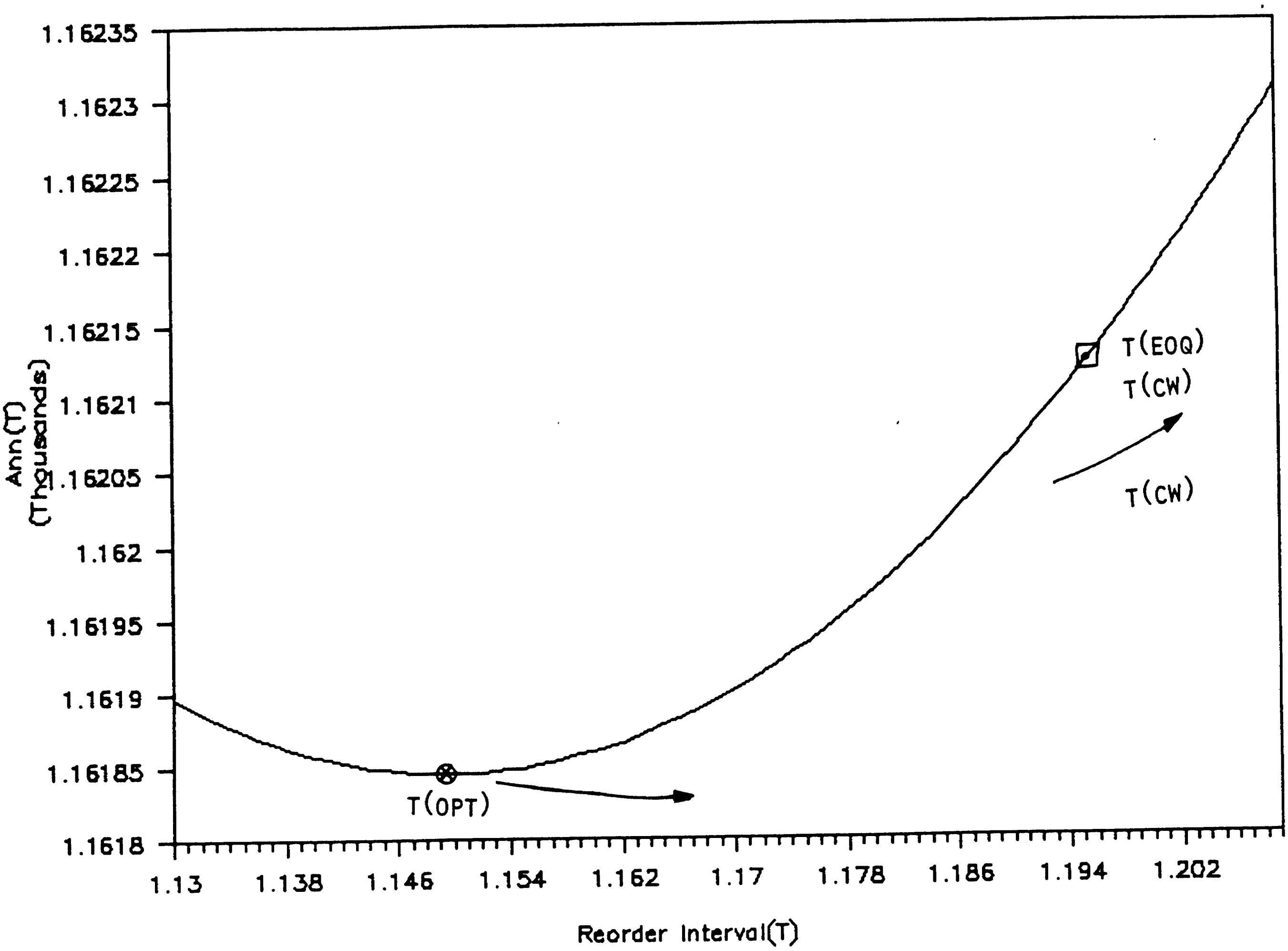

FIGURE 2

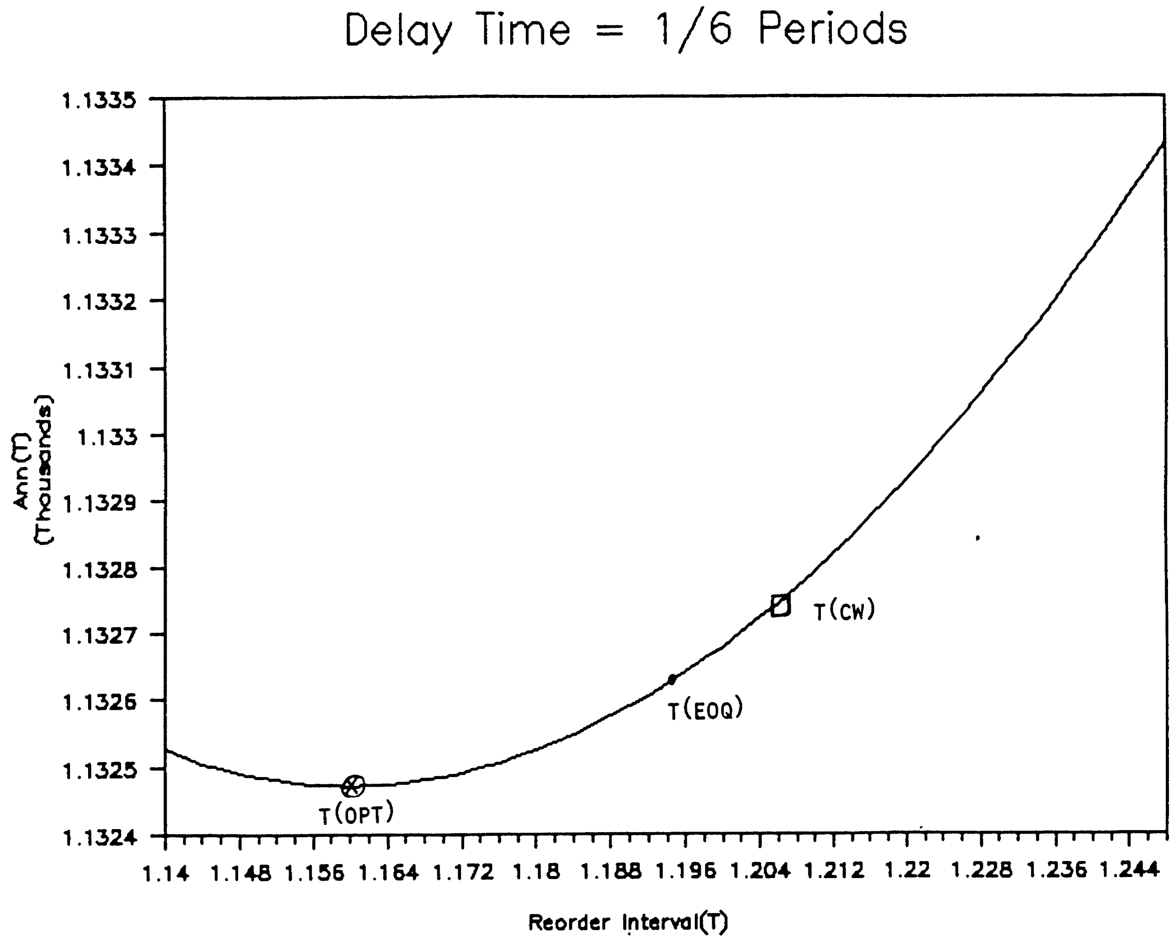

FIGURE 3

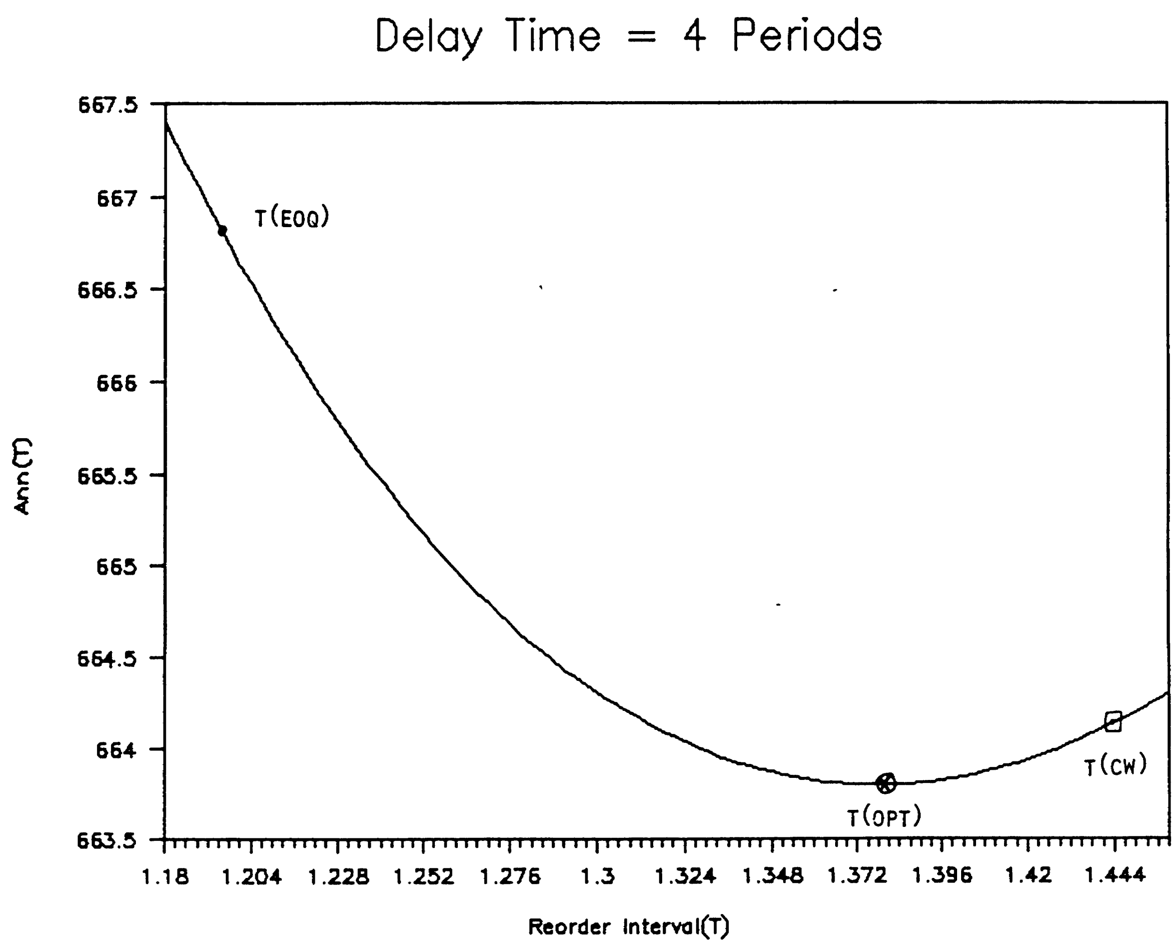

FIGURE 4

	t = 0		t = 1/6		t = 4	
	Reorder Interval	Annualized cost above optimum	Reorder Interval	Annualized cost above optimum	Reorder Interval	Annualized cost above optimum
Classical EOQ approach	1.195	0.2724	1.195	0.158	1.195	3.042
Chand and Ward approach	1.195	0.2724	1.206	0.271	1.444	0.335

TABLE 2

can be explained as follows: Classical EOQ is an upper bound on true optimum at t = 0. As t increases, true optimum increases, and classical EOQ remains the same. However, Chand and Ward approach yields an order quantity greater than the classical EOQ for all positive values of t. Since NPV function is convex, for small values of t, classical EOQ actually yields better results than CW approach. It can be verified that in the range t = 0 and 0.734 periods, as the value of t increases, relative error associated with the use of classical economic order quantity decreases. At t = 0.734 periods, classical economic order quantity yields optimal solution! (Details of te exact value of t at which the classical economic order quantity provides optimal solution are provided in the appendix, section I.) However as t further increases, true optimum will be greater than the classical EOQ. Further increases in t result in deteriorating performance of the classical EOQ. This is shown in Figure 4 for the case of t = 4. It is clear from this numerical example and the appendix (section I) that for small values of t, classical EOQ may yield better results than CW approach. However, CW approach is quite robust. In fact, an error bound on the annualized cost resulting from the use of CW approach can be derived exactly on same lines as given in Rachamadugu ([6], Figure 1). Analysis regarding the rubustness of CW approach are provided in the appendix, section II. Numerical investigations of the error bound (since no closed form evaluation is possible) indicates that the CW approach is quite robust. The interested reader is directed to the above source for further details.

Conclusion

In this paper, we analyzed two approaches to order size determination when delay in payment is permitted. One of them is invariant with respect to delay time. The other increases order size as delay time increases. Former approach is based on the average cost analysis. Latter approach is a hybrid using the average cost analysis and discounting. Since the average cost analysis is only an approximation to the conceptually rigorous discounted cash flow approach, we analyzed both procedures using the discounted cash flow approach. Firstly, we showed that, contrary to the claims made by earlier authors, optimal order quantity increases as payment delay time increases. Next, we showed that the Chand and Ward approach yields an order quantity which is an upperbound on the true optimum. This is helpful in determining the optimum order quantity should the need arise to determine the same. However, we showed that no single procedure dominates. The hybrid procedure suggested by Chand and Ward is robust.

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Appendix

I. Value of t at which classical EOQ yields optimal solution

From IX it is clear that at the optimum,

$$\frac{Sr^2}{D(h + pre^{-rt})} = e^{rT(OPT)} - 1 - rT(OPT) \qquad A(I)$$

$$\frac{r^2}{2} \cdot \frac{2S}{D(h + pr)} \cdot \frac{h + pr}{h + pre^{-rt}} = e^{rT(OPT)} - 1 - rT(OPT)$$

If classical EOQ should equal the optimum order quantity, then

$$\frac{(rT(EOQ))^2}{2} \cdot \frac{h + pr}{h + pre^{-rt}} = e^{rT(EOQ)} - 1 - rT(EOQ) \qquad A(II)$$

The value of t at which A(II) holds good determines the delay period for which classical EOQ yields optimal solution. For the numerical illustration in the paper, t equals 0.734 periods.

II. Relative error of CW approach

Relative error resulting from the use of Chand and Ward approach is given by

$$\frac{ANN(T(CW)) - ANN(T(OPT))}{ANN(T(OPT))} \qquad A(III)$$

At the optimum, ANN'(T(OPT)) = 0. This implies

$$Sr^2 = (Dpre^{-rt} + hD)(e^{rT(OPT)} - 1 - rT(OPT)) \qquad A(IV)$$

Using VIII and A(IV), with a little algebraic manipulation, A(III) can be re-written as

$$\frac{r(T(CW) - T(OPT)) - (e^{-rT(OPT)} - e^{-rT(CW)})}{(1 - e^{-rT(CW)})\left\{e^{rT(OPT)} - \left(\frac{h}{h + pre^{-rt}}\right)\right\}} \qquad A(V)$$

Numerical evaluation of A(V) was done by Rachamadugu [6] for different values of rT(CW) and $(h/h + pre^{-rt})$. Details can be found in [6, Figure 1]. For example, consider an extreme scenario--capital charges are 25% per year (excluding material handling, insurance etc.) and reorder interval is 1 year. It can easily be verified that as delay time and price tend to infinity, relative error resulting from the use of classical economic order quantity can be arbitrarily bad. Relative error for Chand and Ward approach can be no more than 3.75%. Thus Chand and Ward approach provides robust results for practical purposes.