Division of Research
Graduate School of Business Administration
The University of Michigan

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THE PREDICTION OF SYSTEMATIC RISK USING EQUIVALENT RISK CLASSES

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Robert K. Rayner

The University of Michigan

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I. INTRODUCTION

The beta coefficient of the market model is widely used as a measure of risk for individual securities and portfolios. In applications, ex post estimates of beta are used to measure ex ante (future) risk, and so the problem of accurately forecasting beta is important and has been extensively investigated. Previous research [1,9,18] has shown that beta forecsts may be improved substantially when beta adjustment techniques (Blume [1], Vasichek [26]) are used, or portfolio size is increased.[1]

The primary purpose of this paper is to propose a new beta adjustment procedure--random coefficient regression.[2] In part, the motivation for this approach is a result due to Kadiyala and Oberhelman [16]. They have shown that under the assumptions of the random coefficient regression (RCR) model, the best linear unbiased predictor of the coefficients associated with a particular firm (unit, individual) is a weighted average of two estimators: the ordinary least squares estimator of the firm's coefficients, and the estimator of the population mean coefficients. Their results may be applied to the market model to get beta coefficient forecasts. Morever, it is shown here that while the RCR and Vasichek procedures have similar assumptions, the RCR procedure may use better estimates of the mean and variance of the cross-sectional distribution of betas. The RCR approach separates out specific risk as a component of the estimate of cross-sectional variance.

In this paper, an attempt is made to highlight the relationships of the random coefficient regression adjustment procedure to the Blume and Vasichek approaches. Particular attention is paid to model assumptions. In addition, the forecasting performance of adjustment procedures is investigated empirically under different risk class assumptions. Specifically, adjustment procedures are compared using systematic risk classes defined by (1) industries, (2) ordered-beta risk classes, and (3) groups formed by cluster analysis on variables which have been shown in the literature to be theoretically related to betas. As a standard of comparison, analyses are also done assuming all stocks belong to the same risk class.

This study uses the notion of systematic risk class as the starting point for beta adjustment techniques. As section III of the paper shows, random coefficient regression and the Blume and Vasichek procedures all implicitly assume that firms have been grouped into systematic risk classes. Some potential methods of constructing these classes are examined next.

II. EQUIVALENT RISK CLASSES

Because the concept of risk class is important to the discussion here of beta adjustment techniques, it may be useful to give a brief overview of some of the work that has been done in this area. The idea of risk class came into prominence following Miller and Modigliani's [20] landmark paper in which they established their propositions on capital structure assuming the existence of business risk classes.[3] Later, Elton and Gruber [7] made clear the importance of using homogeneous groups for forecasting and testing of hypotheses. They and others [13,19] have used cluster analysis as a methodological approach for grouping. Concurrently, there has been an investigation of the suitability of industry groupings as business risk classes, with mixed conclusions; some authors [14,21] have found that industries are reasonable approximations of business risk classes, while others [19,28] have reported opposite results. (This is an important question, because industry groupings are often used in the testing of economic hypotheses and by security analysts.)

In this paper, systematic risk classes are of interest. Systematic or nondiversifiable risk is synonymous with beta, and has two traditional components: business (or operating) risk and financial risk. Business risk is associated with the firm's investment decisions and hence, is thought to be affected by variables such as the company's cost structure and competitive position, and product demand characteristics. Financial risk, on the hand, is related to the financing decisions of the firm. A systematic risk class, then, might be defined as a group of firms which are "similar" in the characteristics that determine business and financial risk.

The usefulness of this concept to portfolio theory and security analysis depends on whether or not there are a relatively small number of risk classes that have stable compositions over time. As section IV on empirical results shows, this requirement is not very well met by risk classes constructed by ordering betas. Although true betas may be relatively stable over time, estimates of the individual security betas are not, which in fact is the motivation for beta adjustment techniques. Industry groupings, on the other hand, may be acceptable systematic risk classes, and that possibility is investigated further in section IV. We also examine risk classes formed by cluster analysis on variables which have been shown in the Subrahmanyam-Thomadakis [24] and Hamada [15] papers to be theoretically related to business and financial risk. Subrahmanyam and Thomadakis have recently provided a theoretical relationship between beta and business risk. They have shown that systematic risk is functionally related to firm-specific variables, namely, monopoly power and labor-capital ratio. These variables are approximately constant over short time periods, and so this procedure promises to produce stable systematic risk classes. Of course, the usefulness of a risk class definition depends too on how well it works in combination with beta adjustment procedures.

III. BETA ADJUSTMENT TECHNIQUES

Throughout the paper, the estimate b_i of an individual security's beta β_i refers to that which may be calculated from the market model:

$$R_{it} = \alpha_i + \beta_i R_{Mt} + \varepsilon_{it} \tag{1}$$

where R_{it} is the return in month t on the i^{th} security, R_{MT} is the return on the market in time t, and ε_{it} is a mean-zero, homoskedastic error term with variance σ_{ii}. Equation (1) may be rewritten as

$$R_i = R \begin{pmatrix} \alpha_i \\ \beta_i \end{pmatrix} + \varepsilon_i = R\gamma_i + \varepsilon_i \tag{2}$$

where R_i and ε_i are Tx1 vectors and R is a Tx2 matrix with 1's in the first column and market returns in the second column.

The Blume Procedure

Adjustment procedures have been studied in an attempt to improve beta forecasts. Because he noticed that estimated beta coefficients from one time period exhibited a significant regression tendency in the following period, that is, that beta estimates tended in the next time period to be closer to the overall mean of 1.0, Blume proposed a cross-sectional regression procedure to adjust betas. In this approach beta estimates $b_{i,p-1}$ and $b_{i,p}$ from two adjacent nonoverlapping time periods p-1 and p are assumed to be related by the model

$$b_{i,p} = \delta_1 + \delta_2 b_{i,p-1} + \varepsilon_{i,p} \tag{3}$$

where i refers security (or portfolio) i, and δ_1, δ_2 are regression coefficients to be estimated. Then assuming the same relationship applies from period p to period p+1, beta predictions for period p+1 are obtained using $b_{i,p}$ and the estimated regression coefficients. Blume found that predictions of systematic risk are improved considerably using this approach.[4]

Bayesian Method

Vasichek [23] has proposed a Bayesian approach for adjusting betas. Information on the prior or historical cross-sectional distribution of betas is combined with beta estimates calculated from period p to give beta forecasts $b'_{i,p+1}$ computed as follows:

$$b'_{i,p+1} = \frac{\bar{\beta}_p \,/\, \sigma^2_{\beta_p} + b_{i,p}/S^2_{b_{i,p}}}{1/\sigma^2_{\beta_p} + 1/S^2_{b_{i,p}}} \qquad (4)$$

where $\bar{\beta}_p$ and $\sigma^2_{\beta_p}$ are the mean and variance, respectively, of the cross-sectional distribution of betas in period p, and $S^2_{b_{i,p}}$ is the estimated variance of $b_{i,p}$. The forecast $b'_{i,p+1}$ is the mean of the posterior distribution of beta for security i. As can be seen from (4), the extent to which $b_{i,p}$ is adjusted toward the mean $\bar{\beta}_p$ depends on how precise (measured by the reciprocals of estimated variance) the estimates $b_{i,p}$ and $\bar{\beta}_p$ are. In practice, $\bar{\beta}_p$ and $\sigma^2_{\beta_p}$ are usually estimated from a sample of betas calculated from period p.

A reasonable cross-sectional distribution to use with the Vasichek procedure is the distribution on betas from a systematic risk class defined as in the previous section. It is clear from his paper that Vasichek recognized the potential usefulness of adjusting betas in this way. The Blume procedure might also be applied to risk classes, which leads to the interesting question of whether in fact betas regress toward their systematic risk class means as well as toward the overall mean of 1.0. For the rest of the discussion in this section, it is assumed that firms have been previously grouped into risk classes; adjustment procedures are discussed in the context of a given risk class.

Random Coefficient Regression

The concept of systematic risk class leads naturally to another adjustment procedure--random coefficient regression. RCR is one of several methodologies for dealing with pooled cross-sectional time series data, that is, data describing each of a number of firms (individuals, units) over a sequence of time periods.[5] In the random coefficient regression model, the vector of coefficients in the time series regression for each firm is assumed to be a random drawing from a common multivariate distribution. The approach may be thought of as lying somewhere between the extremes: (1) each firm has its own vector of coefficients and (2) all firms have the same coefficient vector. It is this "different but similar" aspect of random coefficient regression that is important here. Each firm has its own vector of coefficients (response), but responses are assumed to come from a population with a certain mean and covariance. When the beta coefficient is of concern, as it is here, it is reasonable to label this homogeneous population a systematic risk class.

Much of the work with the RCR model has been concerned with testing hypotheses on the population vector of means, or hypotheses on the covariance matrix. Focusing on a different problem, Kadiyala and Oberhelmen [16] have derived some results on predicting the coefficients associated with a particular firm. Their results are developed formally now, in the context of predicting the beta coefficient of the firm. For a more thorough discussion of the general RCR model, see Swamy [23, pp. 97-111].

For firms ($i=1,2,\ldots,N$) in a risk class, the following assumptions are made concerning the coefficients of the market model, equation (2):

(a) The vector $\gamma_i = (\alpha_i, \beta_i)'$ of coefficients is a random draw from a bivariate distribution with mean $\bar{\gamma} = (\bar{\alpha}, \bar{\beta})'$ and covariance matrix Δ, and

(b) $\gamma_1, \ldots, \gamma_N, \varepsilon_1, \ldots, \varepsilon_N$ are mutually independent.

Then, letting $C_i = \Delta + \sigma_{ii}(R'R)^{-1}$, the random coefficient regression predictor of γ_i is given by

$$g'_{i,p+1} = \Delta C_i^{-1} g_i + \sigma_{ii}(R'R)^{-1} C_i^{-1} \bar{g} \tag{5}$$

where all of the components of the right hand side of (5) are estimated from period p; specifically, the 2x1 vector g_i is the OLS estimator of γ_i calculated from the market model over period p, and $\bar{g}$, the estimator of the population mean $\bar{\gamma}$, is given by

$$\bar{g} = \left(\sum_{i=1}^{N} C_i^{-1}\right)^{-1} \sum_{i=1}^{N} C_i^{-1} g_i \ . \tag{6}$$

The beta coefficient forecast, $b'_{i,p+1}$, is given by the second component of $g'_{i,p+1}$. In practice, of course, the parameters Δ and σ_{ii} are not known and must be estimated.[6]

Comparison of Beta Adjustment Techniques

Adjustment procedures are similar in some respects, but differ in their underlying assumptions. As we will show, the principal differences depend on whether betas are assumed to be stationary (constant) over time.

It is useful to begin with the approach Blume [2] used to derive his cross-sectional regression procedure for adjusting betas. Assume for the moment that in any period p,

(1) betas for individual securities, $\beta_{i,p}$ can be thought of as drawings from a normal distribution with mean $\bar{\beta}_p$ and variance $\sigma^2_{\beta p}$,

(2) the error term $(b_{i,p}-\beta_{i,p})$ is distributed as a mean-zero, normal variate independent of $\beta_{i,p}$, and

(3) $\beta_{i,p}$ and $\beta_{i,p-1}$ are distributed as bivariate normal variables.

then $b_{i,p}$ and $b_{i,p-1}$ are bivariate normal variates, which implies the following regression equation:

$$E(\beta_{i,p}/b_{i,p-1}) = \frac{\bar{\beta}_p + \rho_{p,p-1}\sigma_{\beta_p}\sigma_{\beta_{p-1}}}{(\sigma^2_{\beta_p}+\sigma^2_{b_p})}(b_{i,p-1}-\bar{\beta}_{p-1}) \quad (7)$$

where $\rho_{p,p-1}$ is the correlation coefficient between true betas in periods p and p-1. This equation is closely related to equation (3), because

$$\begin{aligned} E(b_{i,p}/b_{i,p-1}) &= E(\beta_{i,p}/b_{i,p-1}) \\ &= \delta_1 + \delta_2 b_{i,p-1}. \end{aligned} \quad (8)$$

It is clear that (7) and (8) are equivalent if δ_1 and δ_2 are defined appropriately.

With these preliminaries we now examine the beta adjustment procedures in some detail. First, we note that the Bayesian (Vasichek) adjustment, equation (4), implicitly assumes that betas are constant over time. In contrast, the Blume procedure ((7),(8)) does not require constant betas. But it is interesting that if that assumption is made in the Blume model, i.e., if $\beta_{i,p} = \beta_{i,p-1}$, then equation (7) becomes

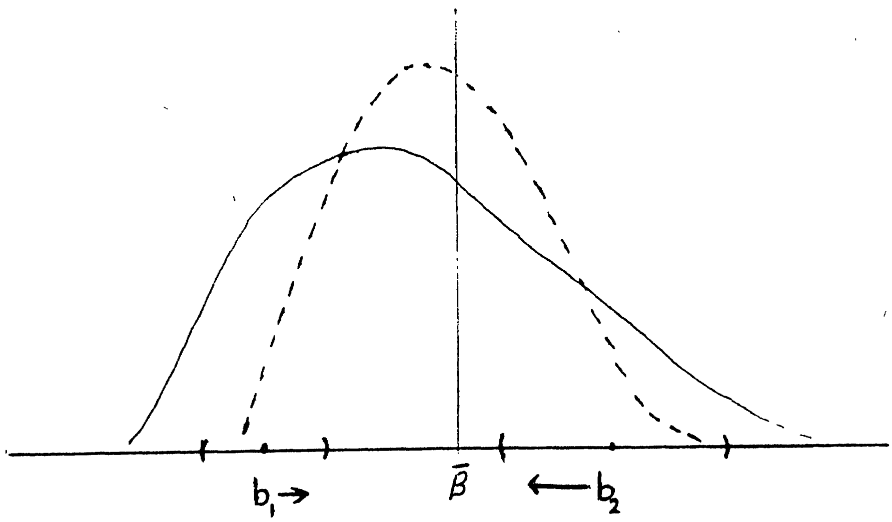
$$\begin{aligned} E(\beta_{i,p}/b_{i,p-1}) &= \bar{\beta}_p + \left(\frac{\sigma^2_{\beta_p}}{\sigma^2_{\beta_p}+\sigma^2_{b_p}}\right)(b_{i,p-1}-\bar{\beta}_p) \\ &= \frac{\bar{\beta}_p/\sigma^2_{b_p} + b_{i,p}/\sigma^2_{b,p}}{1/\sigma^2_{\beta p} + 1/\sigma^2_{\beta_p}}. \end{aligned} \quad (9)$$

Thus, if betas are constant over time the Blume adjustment is similar to Vasichek's. It is important to note, however, that the two approaches would not be identical because equation (4) allows for differences in the variances of the individual beta estimates, whereas (9) does not. In equation (9), $\sigma^2_{b_{i,p}} = \sigma^2_{b_{j,p}} = \sigma^2_{b_p}$.

The RCR and Bayesian approaches are closely related too. For example, equation (5) has an intuitive interpretaion that is very much like the one that was given for the Vasichek formula (equation (4)). Intuitively, in (5) the individual beta coefficient is adjusted toward the population mean of beta coefficients depending on the relative sizes of the estimated variances of these two quantities. Figure 1 may help to make this clear.

Figure 1

The RCR Adjustment Procedure



In Figure 1, two beta coefficient confidence intervals are given. The one associated with b_2 is wider than that for b_1, which accords with empirical evidence (see Fama [12]). Now if no other information were available, b_1 would be the best estimate of β_1. But if we have additional information and believe, for example, that the cross-sectional distribution of betas is as in the solid curve in Figure 1, we might expect that the time beta, β_1, is closer toward the mean, that is, that it is more likely that b_1 has been underestimated than overestimated. Similar logic could be applied to the b_2 estimate, but because b_2 has a wider confidence inteval, it might be reasonable to adjust the b_2 value more than b_1 toward the mean. Moreover, if the cross-sectional distribution of betas has smaller variance, as in the dotted curve, it is reasonable to adjust the b_1 and b_2 values still more towards the mean of betas. This is, in fact, what the RCR and Bayesian procedures do. The extent to which an individual beta coefficient b_i is adjusted toward the mean depends on the relative magnitudes of the cross-sectional variance, and the estimated variance of b_i.

The RCR and Varichek approaches are similar in their assumptions, and in particular, both assume that betas are constant over time. However, there is an important difference in the approaches. The Vasichek procedure assumes that a prior distribution on betas is available. Now in practice the parameters of this distribution (the mean and variance, $\overline{\beta}$ and σ_β^2) are estimated, and at least as it is usually implemented, the Vasichek predictor makes use of estimates of $\overline{\beta}$ and σ_β^2 which are likely to be not as good as the ones RCR uses. The reason is that the sample betas, the b_i's, that are used to estimate $\overline{\beta}$ and σ_β^2 are measured with error, and have two components of variance: the variance of b_i as an estimate of β_i, and the variance of β_i around $\overline{\beta}$. Thus, the sample variance of the b_i's will tend to be an over estimate of σ_β^2, with the result that in equation (4) more weight is put on the individual firm beta (the particular b_i of interest) than should be. Also, a simple arithmetic average of the b_i (as an estimate of $\overline{\beta}$) does not take into consideration the fact that the β_i's are measured with varying degrees of precision; a weighted average of the b_i would be more appropriate.

Random coefficient regression addresses these problems. It incorporates specific risk into the adjustment procedure. Moreover, RCR may use better estimates of $\overline{\beta}$ and σ_β^2, the mean and variance of the cross-sectional distribution of betas.

IV. EMPIRICAL RESULTS

This section on empirical results is divided into three subsections. The first gives the scope of the study. The next subsection deals with various ways of constructing risk classes, and with the stability of composition over time of these classes. Specifically, the types of risk class that are investigated are: industry, cluster group, ordered-beta, and all-in-one. As noted previously, the idea of risk class is important because beta adjustment procedures assume that firms have been grouped previously in this way. The third subsection presents the forecasting results for the RCR, Blume and Vasichek adjustment procedures. The level of analysis here is a risk class, i.e., adjustment techniques are applied to each risk class separately. For example, in the by-industry portion of the tables, each industry was assumed to be a systematic risk class, and beta adjustment techniques were applied to each industry.

Scope

The data for this study were taken from the CRSP and Compustat data tapes. Because one of the objectives of the paper is to test the suitability of industries as systematic risk classes, a random sample of 22 industries was chosen based on the four-digit Compustat industrial codes. Industries were selected at random except for the electric utilities group, which was included because it has been used in numerous cost of capital studies.

The period 1967-1979 is covered, and to be included a firm from a selected industry had to have complete data over that period. This requirement was imposed in order to facilitate comparisons and admittedly, there may be some "survivorship bias" as a result. In total, 197 firms were used. The industries and number of firms in them are given in Table 1.

Insert Table 1 here

The period 1967-1979 was used to provide five overlapping nine year periods: 67-75, 68-76, 69-77, 70-78, 71-79. Each of these nine year periods was subdivided further into three non-overlapping three year periods (the Blume procedure requires three such periods). For example, relative to the period 1967-75, the subscript "p-1" from the previous section of the paper refers to the period 67-69, "p" to 70-72, and "p+1" to 73-75. Five separate sets of analyses were performed, one for each of the five periods.

Beta coefficients were calculated according to the market model, equation (1), using monthly CRSP data. Because Fama [12] and others have shown empirically that continuously compounded returns measured by

$$R_{it} = \ln(1+r_{it})$$
$$R_{Mt} = \ln(1+r_{Mt}) \tag{7}$$

resemble more closely a normal distribution (they are less skewed to the right) than are simple returns r_{it} and r_{Mt}, returns used in equation (1) were calculated assuming continuous compounding for period t.

Systematic Risk Classes

Previous studies concerned with beta adjustment techniques have constructed risk classes by first ordering betas and then dividing them sequentially, into deciles for example. Table 2 presents some empirical evidence

TABLE 1

Description of Sample

Industry Code	Industry Name	Number of Sample Firms
1311	Crude Petroleum & Natural Gas	4
2000	Food & Kindred Products	12
2065	Candy & Other Confectionary	3
2085	Distilled Rectif Blend Beverages	4
2086	Bottled - Canned Soft Drinks	7
2400	Lumber & Wood Products	4
2649	Convert Paper - Paperbd Pd NEC	3
2911	Petroleum Refining	24
3000	Rubber & Misc Plastics Prods	9
3210	Flat Glass	3
3241	Cement Hydraulic	5
3290	Abrasive Asbestos and Misco Min	4
3310	Blast Furnaces & Steel Works	22
3540	Metalworking Machinery & Equip	6
3573	Electronic Computing Equip	5
3679	Electronic Components NEC	4
3841	Surg & Med Instruments & App	6
4011	Railroads-Line Haul Operating	9
4811	Telephone Communication	10
4911	Electric Services	44
5199	Whsl-Nondurable Goods NEC	6
7810	Serv-Motion Picture Production	3

to suggest that risk classes formed in this manner may be unstable over time. The table was constructed by grouping ordered betas into quintiles in each of the periods 1970-72, 71-73, and 74-76. For each ordered beta class (low through high) from 70-72, Table 2 gives the fraction of firms that were later grouped into other classes, first for period 71-73 and then for 74-76. For example, 15% of the firms that were in class "low" during the period 1970-72 were in class 3 in 71-73.

Insert Table 2 here

Table 2 suggests that instability is more pronounced over longer time periods, as might have been expected.

Another possible method of forming risk classes is by clustering on variables which are known to be related to beta.[7] Generally, the objective of cluster analysis is to separate a total sample of entities into groups which are "similar" within groups, with the groups themselves being "dissimilar." For this paper it was desired to construct risk classes for each of the five middle-three-year periods (that is, each of the five p periods).

The choice of variables is an important consideration that should reflect the purpose of the classification. The variables that were used in this study were debt ratio, market share (as a proxy for monopoloy power), and operating leverage (as a proxy for labor-capital ratio). Measures of these variables were developed from annual Compustat data. A firm's debt ratio was measured as the average of three annual ratios calculated by: long term debt/total assets. Operating leverage for a firm was calculated as the average of the three annual ratios: fixed assets/total assets. Finally, market share for firm i in an industry was taken to be the average

TABLE 2

Stability of Ordered Beta Risk Classes

Ordered Beta Classes 1970-72	Ordered Beta Classes, 1971-73				
	Low	2	3	4	High
Low	0.550	0.250	0.150	0.025	0.025
2	0.300	0.225	0.300	0.050	0.125
3	0.150	0.275	0.200	0.250	0.125
4	0.0	0.175	0.200	0.450	0.175
High	0.0	0.081	0.162	0.243	0.514

Ordered Beta Classes 1970-72	Ordered Beta Classes, 1974-76				
	Low	2	3	4	High
Low	0.325	0.225	0.200	0.175	0.075
2	0.175	0.300	0.250	0.125	0.150
3	0.100	0.150	0.200	0.325	0.225
4	0.250	0.100	0.175	0.225	0.250
High	0.162	0.243	0.189	0.162	0.243

of three annual values given by: $S_i / \sum_{j=1}^{n} S_j$, where S_i is yearly sales for firm i, and n is the number of firms in its industry.[8] Will respect to this last measure, market share, firms in the electric and telephone utility industries were treated differently. Because each firm in these industries may be thought of as a local monopoly, market share was taken to be 1.0 for each firm in those industries.

The clustering algorithm that was used is based on Ward's [27] method, a hierarchical procedure that starts by assuming each firm is a group.[9] Then, at each stage the number of groups is reduced by combining the two groups that result in the smallest increase in within-group variation; an approximate indication of the appropriate number of groups, a sort of stopping criterion, is given by an F statistic.[10]

The cluster algorithm was run on each of the five "p" periods using variables in each period defined as above. In each case, a statistically significant increase at the one percent level in the within-groups variation occurred when the number of groups was reduced from two to one, indicating that two groups are appropriate. For each of the p periods, the resultant groups were defined by (1) telephone and electric utilities, codes 4811 and 4911, in one group and (2) all other stocks in the sample in the other group. One might expect that this result is due at least in part to the way in which the market share variable was defined. However, the grouping results proved to be surprisingly robust. Even with this variable left out, telephone and electric utilities grouped together, but in addition, there were a few railroad firms in the group. This stability of risk classes constructed by clustering is in marked contrast to the lack of stability of ordered beta risk classes. This stability over time in the composition of the cluster groups follows from the lack of time series variation in the variables that were used.

While stability of composition is a desirable characteristic, it alone does not guarantee that the risk classes will be useful with beta adjustment techniques. The usefulness of beta risk classes constructed by various techniques is examined next.

Forecasting Results

The criterion that was used to judge beta forecasting accuracy is mean square forecast error, defined by:

$$MSE = \frac{1}{N} \sum_{i=1}^{N} (b_{i,p+1} - b'_{i,p+1})^2$$

where $b_{i,p+1}$ is the estimated beta coefficient for security i in period p+1, $b'_{i,p+1}$ is the beta forecast for security i made in period p, and N is the number of firms in the risk class. "Unadj β" refers to the forecasting procedure of using the unadjusted beta estimate from period p as the forecast of beta in period p+1.

The prediction results for the various adjustment procedures are given in Table 3, by risk class type.

Insert Table 3

Looking at industry groupings first, it is clear from the table that the betas of firms in some industries are more stable than those of firms in other industries. Firms in industry 4911, the telephone utilities, appear to have the most stable betas. In contrast, the electric computing equipment group, 3573, has very unstable betas. However, conclusions on industry groupings must be tempered by the fact that several industries have a small number of sample firms.

TABLE 3

Mean Square Errors of Beta Forecasts by Risk Class Type

Industry	No. Firms	Unadj β	RCR β	Blume β	Bayes β
1311	4	0.37247	0.31151	0.53724	0.33211
2000	12	0.16260	0.13510	0.14232	0.13643
2065	3	0.09929	0.11620	0.11093	0.10152
2085	4	0.57427	0.42539	0.44680	0.47596
2086	7	0.41375	0.38702	0.39219	0.35621
2400	4	0.13354	0.09876	0.14074	0.10880
2649	3	0.43758	0.28983	0.37297	0.34370
2911	24	0.17938	0.12292	0.11837	0.13305
3000	9	0.21772	0.17633	0.20440	0.18394
3210	3	0.32032	0.21523	0.34275	0.27002
3241	5	0.23225	0.23205	0.29672	0.21722
3290	4	0.16991	0.09031	0.19820	0.11832
3310	22	0.43312	0.28994	0.43980	0.33447
3540	6	0.37191	0.24256	0.28992	0.27762
3573	5	0.92015	1.00167	1.37629	0.91210
3679	4	0.60417	0.36047	3.14691	0.43802
3841	6	0.22750	0.20398	0.49075	0.20789
4011	9	0.16046	0.16142	0.16728	0.15316
4811	10	0.09798	0.06974	0.07993	0.07710
4911	44	0.12829	0.09724	0.12011	0.10118
5199	6	0.31488	0.26805	0.40840	0.27223
7810	3	0.42719	0.45677	0.84941	0.43345

Cluster Group	No. Firms	Unadj β	RCR β	Blume β	Bayes β
1	143	0.31191	0.22826	0.26207	0.24947
2	54	0.12263	0.08605	0.10079	0.09350

Ordered-Beta Class	No. Firms.	Unadj β	RCR β	Blume β	Bayes β
Low	40	0.19233	0.17846	0.18866	0.17876
2	40	0.17463	0.17285	0.17165	0.17196
3	40	0.14848	0.14992	0.14930	0.14859
4	40	0.25174	0.25253	0.25040	0.25120
High	37	0.55516	0.47958	0.70139	0.51619

The cluster analysis groups in Table 3 are defined as follows: group two contains the 54 utilities (telephone and electric), and group one the remainder of the total sample of 197 firms. Not surprisingly, the table shows that utility stock betas can be forecast much more precisely than can betas from other manufacturing companies.

From Table 3, class 3 ordered betas appear to be the most accurately forecast. This group includes betas around 1.0. We also note that forecast errors increase away from these central beta values, with the group "high" betas having, on average, the largest forecast errors.

On average, each of the adjustment procedures provides a substantial improvement over unadjusted beta predictions, regardless of grouping assumptions. The Blume procedure, however, does not do well when industry size is small. For example, forecast errors for industry 3679 are very large; in such cases, the problem may be due to the difficulty in estimating a regression model precisely from a small number of points.

The period by period performance of the beta adjustment techniques by risk class type is presented in Table 4. The figures in the by-industry portion of the table are average squared forecast errors for all 197 firms in the sample, in the case where adjustment procedures were conducted at the industry level. For the table segment labelled "cluster groups" adjustment procedures were calculated using the risk classes defined above. Mean square forecast errors also are shown for each of the adjustment procedures assuming all stocks belong to just one risk class, and for ordered beta classes.

Insert Table 4

From Table 4, none of the forecast procedures is uniformly better or worse

TABLE 4

Mean Square Errors of Beta Forecasts In Each Period by Risk Class Type

	Period	Unadj β	RCR β	Blume β	Bayes β
Industry	1	0.23769	0.16709	0.23172	0.18256
	2	0.37850	0.25938	0.67027	0.29615
	3	0.22147	0.17240	0.23457	0.18014
	4	0.22410	0.21430	0.26937	0.20785
	5	0.23844	0.21503	0.23890	0.21124
	Average	0.26004	0.20564	0.32897	0.21559
Cluster Groups	1	0.23769	0.13230	0.14197	0.16014
	2	0.37850	0.22208	0.32288	0.26158
	3	0.22147	0.17503	0.19141	0.18748
	4	0.22410	0.20610	0.21278	0.21114
	5	0.23844	0.21090	0.22025	0.21324
	Average	0.26004	0.18928	0.21786	0.20672
All Stocks In One Class	1	0.23769	0.12974	0.14715	0.15602
	2	0.37850	0.20758	0.31362	0.24763
	3	0.22147	0.18781	0.17754	0.19104
	4	0.22410	0.23214	0.23685	0.22723
	5	0.23844	0.23285	0.23931	0.22654
	Average	0.26004	0.19803	0.22289	0.20971
Ordered-Beta Classes	Low	.23769	.20683	.24755	.21934
	2	.37850	.34293	.51423	.36213
	3	.22147	.21370	.21442	.21311
	4	.22410	.21923	.21867	.21840
	High	.23844	.23291	.23539	.23372
	Average	0.26004	0.24312	0.28605	0.24934

than the others. On average, however, all of the adjustment procedures are considerable improvements over unadjusted betas. (Unadj β performed relatively well only in periods 4 and 5 under "all stocks in one class" grouping.)

Looking at average forecast errors over the five periods, random coefficent regression appears to be an improvement over the Blume and Vasichek approaches. Concerning the methods of constructing risk classes, classes formed by cluster analysis generally resulted in lower forecast errors than did industry classifications, or the all-in-one classification. Ordered-beta classes performed the worst.

V. SUMMARY AND CONCLUSIONS

This paper has two main concerns: (1) the definition and construction of equivalent systematic risk classes and their relationship to beta adjustment techniques, and (2) the development of a new beta adjustment procedures: random coefficient regression.

The concept of systematic risk class is central to the discussion here because all of the adjustment procedures that are investigated assume that firms have been grouped into homogeneous classes. In this paper, several methods for constructing risk classes are considered, namely, by industry, cluster analysis, and grouping by ordered betas. For comparison purposes, analyses are also done assuming all firms belong to one risk class.

To be useful in conjunction with adjustment procedures, a method for constructing risk classes should result in groups that have stable composition of firms over time, and should lead to lower mean square errors of beta forecasts. Concerning the first criterion, stability of group composition, industry groupings are relatively stable over time, and as this paper shows, so are groups formed by cluster analysis. In contrast, ordered beta risk classes seem to have highly changing compositions of member firms over time, and this effect is more pronounced as the length of time between grouping increases.

Compared to other grouping procedures, cluster analysis resulted on average in the lowest mean square forecast errors for betas, for all adjustment techniques. With clustering, only two groups--utilities, and all other stocks in the sample--emerged. Not surprisingly, utility stock betas can be forecast much more precisely than can betas for other firms.

Unfortunately, the evidence is inconclusive as to whether industries are good risk classes. This inconclusiveness may be due in part to the small sample sizes (as small as three) of some industries in this study.

Using mean square errors of beta forecasts as the criterion, random coefficient regression, on average, seems to be superior to other adjustment procedures in the literature. However, no adjustment procedure was uniformly better than the others.

FOOTNOTES

[1]Eubank and Zumwalt [9] have shown that forecast errors are also reduced considerably if the estimation time period (the period used to calculate ex post betas to be used as estimates of ex ante betas) is increased, or the time period used to calculate realized betas in the prediction period is increased.

[2]The term "random coefficient" is applied to several kinds of models. For example, Fabozzi and Francis [11] and Sunder [25] have used models in which coeffficents are allowed to vary over time. Alternatively, Swamy [23] has developed a model which assumes that coefficients are fixed over time but vary across firms. This is the model that is investigated here. This model is finding increasing applications in finance (see for example, Boness and Frankfurter [3] and Dielman, Nantell and Wright [6]).

[3]Hamada [15] subsequently was able to show that this assumption is nt necessary to establish their propositions I and II.

[4]Merrill Lynch, Pierce, Fenner & Smith, Inc. (MLPFS) use a similar adjustment procedure. Klemkosky and Martin [16] found that forecast errors using the MLPFS adjustment were very nearly the same as those associated with the Blume procedure.

[5]Dielman [6] gives a good review of pooled cross-sectional time series methodologies and he includes a comprehensive list of references.

[6]For each i, consistent estimates of σ_{ii} may be obtained from ordinary least squares estimation of equation (2). Concerning the Δ parameters, Swamy [23, p. 107] derives an unbiased consistent estimator which uses the OLS estimates g_i of γ_i. This estimate of Δ may be used in a generalized least squares formula for estimating $\bar{\gamma}$. Alternatively, maximum likelihood estimates of Δ and $\bar{\gamma}$ (conditional on estimates of σ_{ii}) may be obtained by maximizing the log likelihood function, equation 4.3.28 in Swamy, using a program such as the IMSL subroutine ZXMIN. The latter approach was followed in this paper, but in either case the computer programming is straightforward.

[7]Cluster analysis is a methodology for separating data into groups or clusters. Several methods are available, and the selection of method depends in part on how the resulting classes will be used. For a readable discussion of the theory and some of the issues that arise in applications, see [10].

[8]There are measurement problems with each of these variables. Ideally, debt ratios should be calculated using market values of debt and equity instead of book value. As an indication of the volatility of prefinancing earnings due to fixed versus variable costs of production, operating leverage measured by available historical data may be somewhat deficient. Finally, the variable "market share" is difficult to operationalize because most firms manufacture more than one product; industry groupings are only rough approximations of groups of firms that have similar product lines. Nevertheless, the operational variables are probably reasonable approximations of the theoretical variables.

[9]In any application of cluster analysis, the user must decide whether to standardize data to mean zero and unit variance. The decision was made here to standardize because it is known that grouping algorithms are senitive to units of measurement [10]. On the other hand, it was decided not to use principal components because that method is useful primarily as a data reduction technique, and does not necessarily reduce the problem of correlated variables. In any case, the three variables used in this study do not appear to be highly correlated.

[10]Everitt [10, pp. 59-60] gives the equation for this F statistic. For a discussion of Ward's method in a finance application, see Martin et al. [19].

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