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Abstract

An analysis of two-dimensional scattering from a narrow groove in an impedance plane is presented. The groove is represented by a impedance surface and hence the problem reduces to that of scattering from an impedance strip in an otherwise uniform impedance plane. On the basis of this model, appropriate integral equations are constructed using a form of the impedance plane Green's functions involving rapidly convergent integrals. The integral equations are solved by introducing a single basis representation of the equivalent current on the narrow impedance insert. Both transverse electric (TE) and transverse magnetic (TM) polarizations are treated. The resulting solution is validated by comparison with results from the standard boundary integral method (BIM) and a high frequency solution. It is found that the presented solution for narrow impedance inserts can be used in conjunction with the high frequency solution for the characterization of impedance inserts of any given width.

1 Introduction

A topic of some concern in radar cross section studies is the scattering from gaps or cracks that may exist where two component parts of a target come together. Even if the crack is entirely or partially filled with some material, it can still provide a significant contribution to the overall scattering pattern of the target, and it is therefore necessary to develop methods for predicting its scattering.

The scattering by a groove or an impedance insert in a ground plane has already been considered by a variety of techniques. Integral equation [1]-[3] and finite element-boundary integral solutions [4]-[7] have been effectively used in this respect. Also, in the case of narrow grooves or impedance inserts in a ground plane, closed form solutions have been obtained which were found quite accurate for widths 0.15λ or less [8]-[10]. In this paper we present a similar solution to the scattering from narrow grooves in an otherwise uniform impedance plane. The solution is useful for computing the scattering by grooves in coated ground planes and is intended to supplement high frequency solutions which are suitable for large width impedance inserts [11],[12]. As in the solutions given in [13], the groove is simulated by an impedance insert, thus, forming a three-part impedance plane (see Figure 1). The solution of the scattering by the illustrated three-part impedance surface is obtained by introducing appropriate equivalent currents over the extent of the middle impedance strip and then constructing an integral equation for the solution of these currents. This integral equation is obtained by making use of the impedance plane Green's functions which are conveniently expressed in terms of rapidly converging semi-infinite integrals. An important aspect of our solution is the introduction of a one-term basis expansion for the equivalent current, appropriate for small width impedance inserts. The coefficient of the single basis function is found in closed form involving integrals which are functions of the insert's width but independent of the impedance characterizing the insert. Unfortunately, these integrals are also functions of the background impedance, thus, precluding their tabulation for different insert widths.

In the following sections we first proceed with the construction of the integral equations for the three-part impedance plane for both TE and TM incidences. The solution of the integral equations are then considered on the assumption of a small width impedance insert. The validity of the results are examined by comparison with a closely related BIM solution [14] and a third order high frequency solution. We conclude by establishing the bounds of the proposed small-width approximations.

2 Formulation

Consider a narrow filled groove of width w and depth d situated in an otherwise uniform impedance plane as shown in Figure 1. The groove is illuminated by the plane wave

$$\mathbf{E}^i(or\mathbf{H}^i) = \hat{z}e^{jk_0(x\cos\phi_0+y\sin\phi_0)} \tag{1}$$

for E- (or H) polarization, where $k_0 = 2\pi/\lambda_0$ is the free space wavenumber and ϕ_0 is the angle of incidence. We shall consider the E- and H-polarizations separately.

2.1 E-Polarization

For TM-incidence, the rectangular groove can be approximated by a strip of impedance [8]

$$\eta = \frac{jZ_0\mu'}{p}\tanh(pk_0d) \tag{2}$$

in which Z_0 is the free space intrinsic impedance,

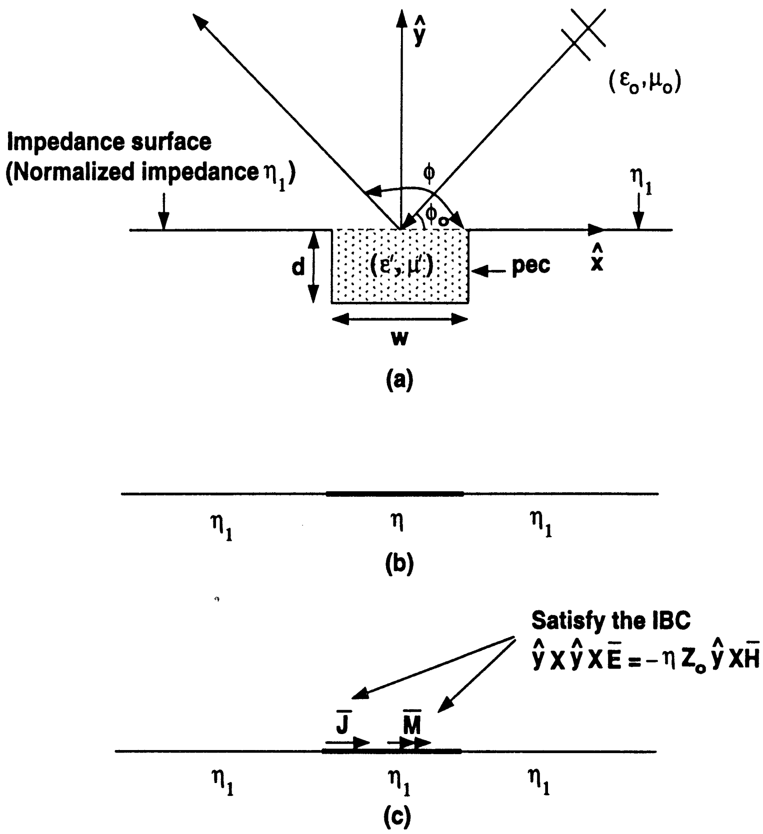
$$p = \sqrt{\left(\frac{\lambda_0}{2w}\right)^2 - \epsilon'\mu'} \tag{3}$$

d denotes the groove's depth, w is the groove's width, whereas ϵ' and μ' are the relative permeability and permittivity of the material filling the groove. The scattering by the subject impedance insert can be represented by introducing the equivalent electric and magnetic currents

$$\mathbf{J} = \hat{y}\times\mathbf{H} \qquad \mathbf{M} = \mathbf{E}\times\hat{y} \tag{4}$$

over the extent of the groove. These currents are assumed to radiate in the presence of a uniform impedance plane satisfying the boundary condition

$$\hat{y}\times\hat{y}\times\mathbf{H} = \eta_1 Z_0\hat{y}\times\mathbf{E} \tag{5}$$



1: (a) Geometry of the groove in an impedance plane (b) Impedance (c) Equivalent scattering problem

where η_1 is the associated normalized impedance (see Figure 1) and $(\mathbf{E}, \mathbf{H})$ denote total fields. One can think of (5) as being imposed at $y = 0^-$, but at $y = 0^+$ $\mathbf{E}$ and $\mathbf{H}$ must satisfy the boundary condition

$$\hat{y} \times \hat{y} \times \mathbf{H} = \eta Z_0 \hat{y} \times \mathbf{E} \quad -w/2 < x < w/2, y = 0^+ \tag{6}$$

over the extent of the groove as dictated by the original problem. Using (4) in (6) we obtain that (assuming $\mathbf{E} = \hat{z} E_z$)

$$J_z = -\eta Z_0 M_x, \quad J_z = \eta Z_0 E_z \tag{7}$$

i.e, $\mathbf{J}$ and $\mathbf{M}$ are linearly related over the impedance insert, and thus we need only consider the solution of one of these, say J_z.

To solve for J_z, we construct an integral equation by enforcing (6). First we decompose $E_z(E_z = \hat{z} \cdot \mathbf{E})$ as

$$E_z = E_z^i + E_z^s \tag{8}$$

where E_z^s denotes the scattered field and is equal to that radiated by $\mathbf{J}$ and $\mathbf{M}$ in the presence of the uniform impedance plane (see Figure 1(c)). On using (7), E_z^s can be expressed as

$$\begin{aligned} E_z^s &= -jk_0 Z_0 \int_{-w/2}^{w/2} J_z(x') G_{IE}(x, y/x', y') dx' \\ &\quad -\frac{Y_0}{\eta} \frac{\partial}{\partial y} \int_{-w/2}^{w/2} J_z(x') G_{IE}(x, y/x', y') dx' \end{aligned} \tag{9}$$

where G_{IE} is the electric Green's function of the uniform impedance plane (of normalized impedance η_1). It is given by [15]

$$G_{IE}(x, y/x', y') = -\frac{j}{4}\left\{ H_0^{(2)}\left(k_0\sqrt{(x-x')^2 + (y-y')^2}\right) + Q_E \right\} \tag{10}$$

where

$$\begin{aligned} Q_E &= H_0^{(2)}\left(k_0\sqrt{(x-x')^2 + (y+y')^2}\right) \\ &\quad -2\int_0^{\infty} \alpha' e^{-\alpha' \nu} H_0^{(2)}\left(k_0\sqrt{(x-x')^2 + (y+y'-j\nu)^2}\right) d\nu \end{aligned} \tag{11}$$

and $\alpha' = \frac{k_0}{\eta_1}$. On substituting (9) into (8) and then into the second of (7) we obtain the integral equation

$$\begin{aligned} \eta e^{jk_0 x \cos\phi_0} &= Y_0 J_z(x) + jk_0 Z_0 \eta \int_{-w/2}^{w/2} J_z(x') G_{IE}(x, y \to 0^+/x', 0) dx' \\ &\quad + Y_0 \frac{\partial}{\partial y} \int_{-w/2}^{w/2} J_z(x') G_{IE}(x, y \to 0^+/x', 0) dx' \end{aligned} \tag{12}$$

to be enforced at $y = 0^+$ for the solution of $J_z(x)$.

2.2 H-Polarization

For TE-incidence, the rectangular gap can be approximated by a strip of normalized impedance [8]

$$\eta = \frac{jZ_0 p}{\epsilon'} \tan(pk_0 d) \tag{13}$$

where

$$p = \sqrt{\epsilon' \mu'} \tag{14}$$

As before, equivalent currents are introduced and the application of the impedance boundary condition to be satisfied over the groove yields the relations (assuming $\mathbf{H} = \hat{z} H_z$)

$$M_z = \eta Z_0 J_x, \quad E_x = \eta Z_0 J_x \tag{15}$$

where

$$E_x = E_x^i + E_x^s \tag{16}$$

The corresponding tangential scattered $\mathbf{E}$ field is given by

$$\begin{aligned} E_x^s &= -\frac{jZ_0}{k_0}\left(k_0^2 + \frac{\partial^2}{\partial x^2}\right)\int_{-w/2}^{w/2} J_x(x') G_{IH}(x, y/x', y') dx' \\ &\quad -\eta Z_0 \frac{\partial}{\partial y}\int_{-w/2}^{w/2} J_x(x') G_{IH}(x, y/x', y') dx' \end{aligned} \tag{17}$$

where G_{IH} is the Green's function of the uniform impedance surface (of normalized surface impedance η_1). It is given by [15]

$$G_{IH}(x, y/x', y') = -\frac{j}{4}\left\{ H_0^{(2)}\left(k_0\sqrt{(x-x')^2 + (y-y')^2}\right) - Q_H \right\} \tag{18}$$

where

$$\begin{aligned} Q_H &= H_0^{(2)}\left(k_0\sqrt{(x-x')^2 + (y+y')^2}\right) \\ &\quad -2\int_0^\infty \beta' e^{-\beta'\nu} H_0^{(2)}\left(k_0\sqrt{(x-x')^2 + (y+y'-j\nu)^2}\right) d\nu \end{aligned} \tag{19}$$

and $\beta' = k_0\eta_1$. Substituting (17) into the second of (15) yields the integral equation

$$\begin{aligned} \sin\phi_0 e^{jk_0 x \cos\phi_0} &= \frac{j}{k_0}\left(k_0^2 + \frac{\partial^2}{\partial x^2}\right)\int_{-w/2}^{w/2} J_x(x') G_{IH}(x, y \to 0^+/x', 0) dx' \\ &\quad +\eta \frac{\partial}{\partial y}\int_{-w/2}^{w/2} J_x(x') G_{IH}(x, y \to 0^+/x', 0) dx' \\ &\quad +\eta J_x(x) \end{aligned} \tag{20}$$

for the solution of $J_x(x)$.

3 Integral Equation Solution

Typically, the solution of (12) and (20) can be accomplished numerically using standard techniques. Such a numerical solution is usually the only alternative for moderately sized grooves, but if the groove is small ($k_0 w \ll 1$) then certain simplifications are possible. Once again, we shall consider the E- and H-polarizations separately.

3.1 E-Polarization

It can be shown [16] that H_x, and consequently J_z, is of $O(1)$ near each of the impedance junctions (see Figure 1(b)) provided neither η_1 nor η are zero. Thus, for $k_0 w \ll 1$, we can assume that H_x is nearly constant at $y = 0^+$ over the extent of the groove. Based on this, we set

$$J_z(x) = \chi_e \tag{21}$$

where χ_e is a constant to be found. Substituting (21) into (12) and point matching at $x = 0$ yields

$$\chi_e = \frac{\eta}{Y_0 + \eta(I_1 + I_2) + I_3} \tag{22}$$

where the integrals I_1, I_2 and I_3 are given by

$$\begin{aligned} I_1 &= \frac{k_0 Z_0}{2} \int_{-w/2}^{w/2} H_0^{(2)}(k_0|x'|)\, dx' \\ &\approx \frac{k_0 Z_0}{2} w \left\{ 1 - \frac{j2}{\pi} \left[\ln\left(\frac{k_0 \gamma w}{4}\right) - 1 \right] \right\} \end{aligned} \tag{23}$$

$$I_2 = \frac{-k_0 Z_0}{2} \int_{-w/2}^{w/2} G'_{IE}(0, 0/x', 0) dx' \tag{24}$$

$$I_3 = \frac{jY_0}{2} \frac{\partial}{\partial y} \int_{-w/2}^{w/2} G'_{IE}(0, y/x', 0)\Big|_{y \to 0} dx' \tag{25}$$

where $\gamma = 1.781$ and

$$\begin{aligned} G'_{IE}(x, y/x', y') = &\int_0^\infty \alpha' e^{-\alpha' \nu} \\ &H_0^{(2)}\left(k_0 \sqrt{(x - x')^2 + (y + y' - j\nu)^2}\right) d\nu \end{aligned} \tag{26}$$

In carrying out the integrals in (24) and (25), G'_{IE} is computed numerically. Because of the exponential factor $e^{-\alpha'\nu}$, the infinite integral defining G'_{IE} converges very rapidly. For our implementation G'_{IE}, was evaluated by subdividing the range of integration in 6 intervals and employing 4 point Gaussian quadrature integration in each interval. When the argument of the Hankel function is small, to avoid numerical difficulties, the integration from 0 to 0.1 is carried out analytically by introducing the small argument expansion of the Hankel function [17].

3.2 H-Polarization

From diffraction theory, it can again be shown that H_z is of $O(1)$ and thus we can approximate J_x by

$$J_x(\xi) = \chi_h \tag{27}$$

where χ_h is a constant to be found. Substituting (27) into (20) and point matching at $x = 0$ yields

$$\chi_h = \frac{\sin\phi_0}{I_1 + \eta\,(I_2 + 1)} \tag{28}$$

The integrals I_1 and I_2 are given by

$$I_1 = \frac{1}{2k_0}\left(k_0^2 + \frac{\partial^2}{\partial x^2}\right)\int_{-w/2}^{w/2} G'_{IH}(x, 0/x', 0)\Big|_{x\to 0}\, dx' \tag{29}$$

$$I_2 = \frac{\partial}{\partial y}\int_{-w/2}^{w/2} G'_{IH}(0, y/x', 0)\Big|_{y\to 0}\, dx' \tag{30}$$

where

$$\begin{aligned} G'_{IH}(x, y/x', y') = &\int_0^\infty \beta' e^{-\beta'\nu} \\ &H_0^{(2)}\left(k_0\sqrt{(x-x')^2 + (y+y'-j\nu)^2}\right)\, d\nu \end{aligned} \tag{31}$$

These can be evaluated numerically without difficulty since G'_{IH} converges rapidly due to the presence of the exponential factor $e^{-\beta'\nu}$. Again, it is necessary to introduce the small argument expansion of the Hankel function and carry out the integration from 0 to 0.1 analytically as was done in [17].

4 Far Zone Scattered Fields and Echowidth

The far zone scattered fields are computed from (12) and (20) on using the large argument approximation of the Hankel function. By evaluating the integrals via the stationary phase method, we obtain the simple formulae

$$E_z^s = -wZ_0\chi_e\sqrt{\frac{k_0}{2\pi\rho}}e^{-j(k_0\rho-\frac{\pi}{4})}\frac{\eta_1\sin\phi}{1+\eta_1\sin\phi} \tag{32}$$

$$H_z^s = wZ_0\chi_h\sqrt{\frac{k_0}{2\pi\rho}}e^{-j(k_0\rho-\frac{\pi}{4})}\frac{\eta_1\sin^2\phi}{1+\eta_1\sin\phi} \tag{33}$$

The corresponding echowidth is given by

$$\sigma = k_0\left(\frac{Z_0\eta_1 w}{1+\eta_1\sin\phi}\right)^2\begin{cases}(\chi_e\sin\phi)^2 & \text{E-pol}\\ (\chi_h\sin^2\phi)^2 & \text{H-pol}\end{cases} \tag{34}$$

(35)

5 Validation of Results

The derived echowidth expressions are based on a low frequency solution of the exact integral equation. They are, thus, expected to be valid for small groove widths and it is, therefore, of interest to examine their accuracy limitations as the width of the groove increases. For this validation, we used the solution based on the BIM and a high frequency solution. The BIM reference solution was that presented by Moore and Ling [14] and applies to an isolated conductor-backed dielectric gap, such as that shown in Figure 2. To compare with this solution the dielectric coated conductor was approximated by the normalized impedance

$$\eta_1 = j\sqrt{\frac{\mu_r}{\epsilon_r}}\tan\left(k_0\sqrt{\epsilon_r\mu_r}d\right) \tag{36}$$

where ϵ_r and μ_r are the permittivity and permeability of the dielectric layer and d is the thickness of the dielectric. Also the gap impedances are computed from (2) and (13).

In Figures 3 and 4 we compare our small width approximation with the BIM data given in [14] for E and H polarizations, respectively. Each figure

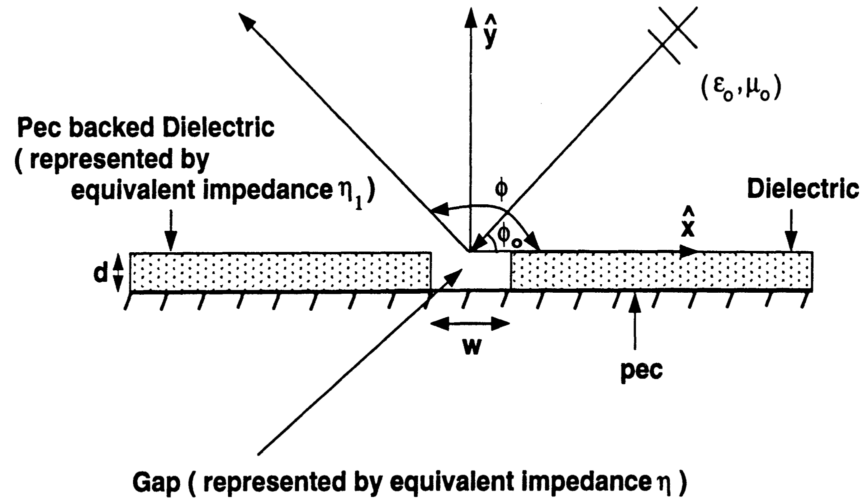


Figure 2: Geometry of the gap at the junction of two semi-infinite dielectric coatings

displays two curves, one corresponding to a gap of width $0.1\lambda_0$ and the other to a gap of width $w = 0.2\lambda_0$. The other gap parameters are given in the figures. As can be expected, the agreement between the two solutions is good for those gaps in the smaller thickness coatings. However, for the $0.2\lambda_0$ gap in a $d = 0.4\lambda_0$ thick coating, our small width and impedance approximations are no longer expected to be valid. Not surprisingly, the H-polarization echowidth for this geometry as computed via the small width approximation is 2-3 dB off from the BIM solution. The greater disagreement near grazing is due to the inaccurate simulation of the dielectric coating by an impedance surface.

Having validated our solution we now proceed with an assessment of its accuracy range and limitation. Figures 5-8 show the backscatter echowidth as a function of the insert's width for different values of η_1 and η. The small width and high frequency solutions[1] are compared in these figures. Of course, by its derivation, the high frequency solution becomes more accurate as w increases, whereas the presented small width approximation does the same as w decreases. Consequently, the two solutions will have a certain intermediate

[1]The reference high frequency solution is a modification of that given in [11] using the impedance junction diffraction coefficients [16]

range of widths where agreement is likely to be expected. Figures 5 and 6 show E-polarization curves at some oblique (45^0) incidence. Similarly, Figures 7 and 8 display some H-polarization curves at oblique and normal incidences. From these figures, we observe that the presented approximate solutions are indeed in good agreement for $0.15\lambda_0 < w < 0.3\lambda_0$, and in many cases even outside this range. The conclusion that can be reached from Figures 5-8 is that the combination of the simple closed form high frequency and small width approximations provide an accurate evaluation of the scattering by the three part impedance plane for all insert widths. Specifically, the high frequency solution can be used for $w > 0.25\lambda_0$, whereas the small width approximation is suitable for smaller w.

6 Summary

Integral equations for the analysis of the scattering by a groove in an impedance plane were constructed by representing the groove as an impedance surface. The integral equations for the three-part impedance plane were solved on the assumption of a small width impedance insert. The coefficient of the single basis function for the equivalent current across the insert was explicitly given in terms of integrals which were functions of the insert's width and the background impedance. The accuracy and limitations of the derived small width approximations were evaluated by comparison with the boundary integral method and a high frequency solution. From these comparisons, we inferred that the presented solution was accurate for insert widths up to $0.25\lambda_0$ and could be extended up to $0.4\lambda_0$ for near normal incidences. Thus, the presented approximate small-width approximation and the high frequency solution complement each other for the evaluation of the scattering by a three part impedance plane for all insert widths.

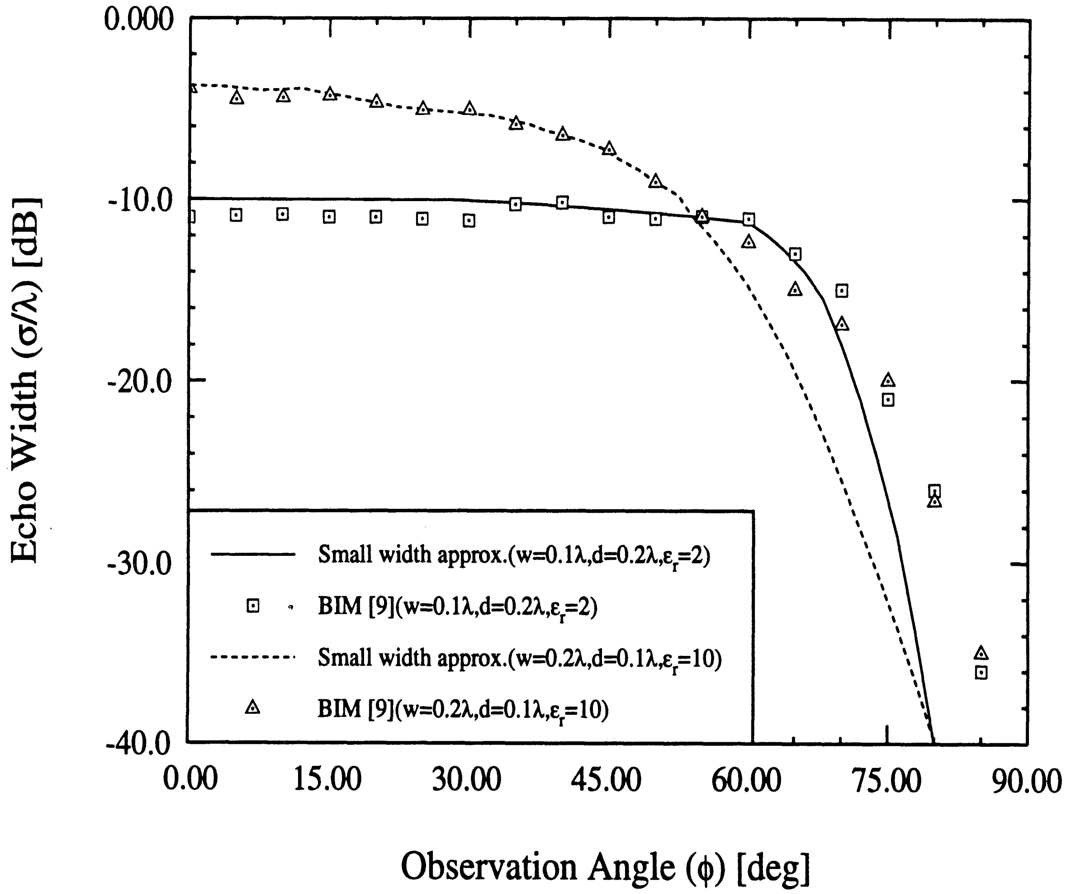


Figure 3: E-polarization backscatter echowidth for a narrow gap in a grounded dielectric layer as in Figure 2. (a) w=0.1λ, d=0.2λ and $\epsilon_r = 2$ (b)w=0.2λ, d=0.1λ and $\epsilon_r = 10$

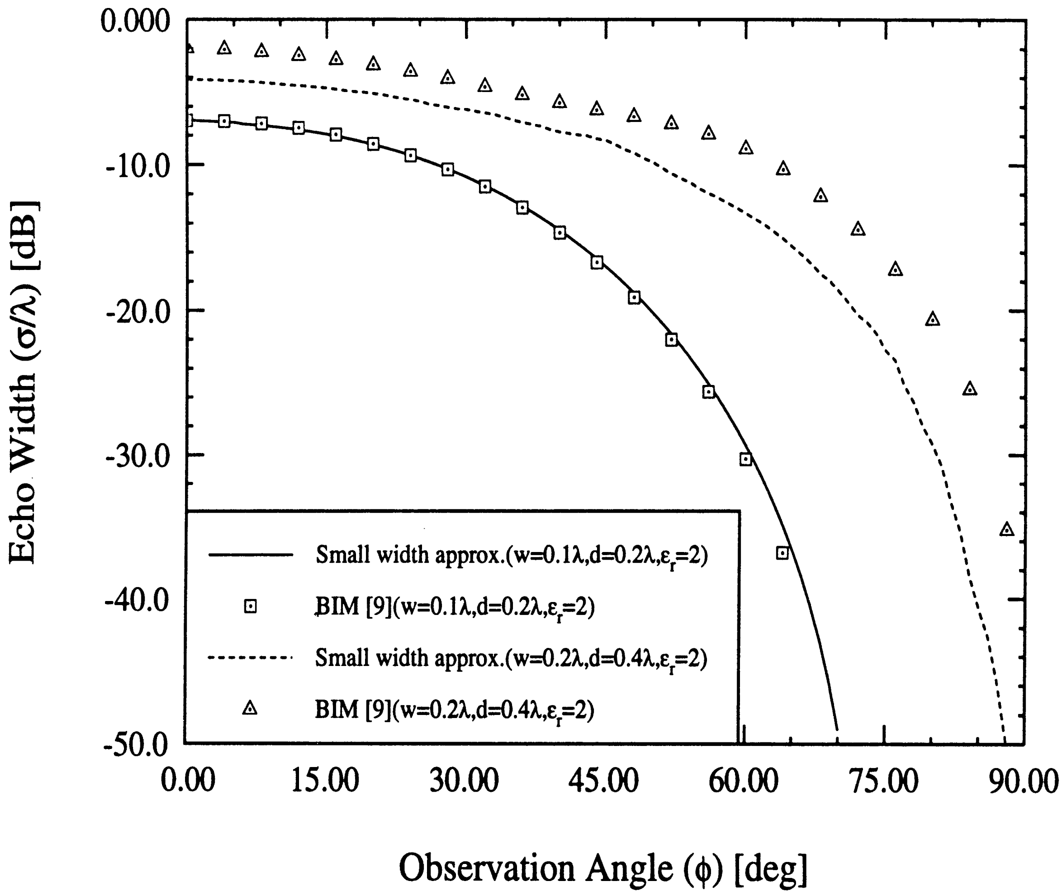


Figure 4: H-polarization backscatter echowidth for a narrow gap in a grounded dilectric layer as in Figure 2. (a) w=0.1λ, d=0.2λ and ϵ_r = 2 and (b) w=0.2λ, d=0.4λ and $\epsilon_r = 2$

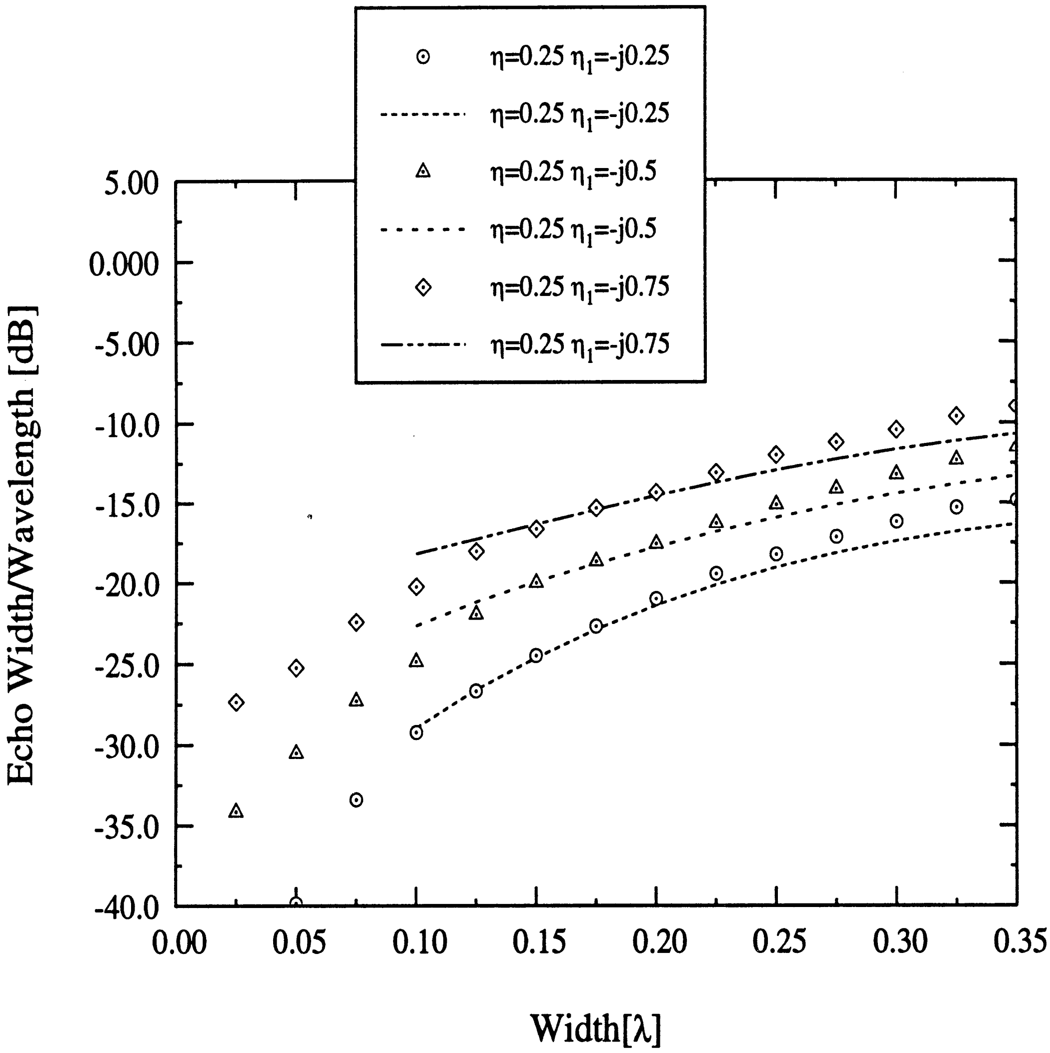


Figure 5: E-polarization backscatter echowidth at oblique incidence (45^0) for an impedance insert of normalized impedance $\eta = 0.25$ in various impedance planes as a function of the insert's width (symbols: small width approx, lines: high freq solution)

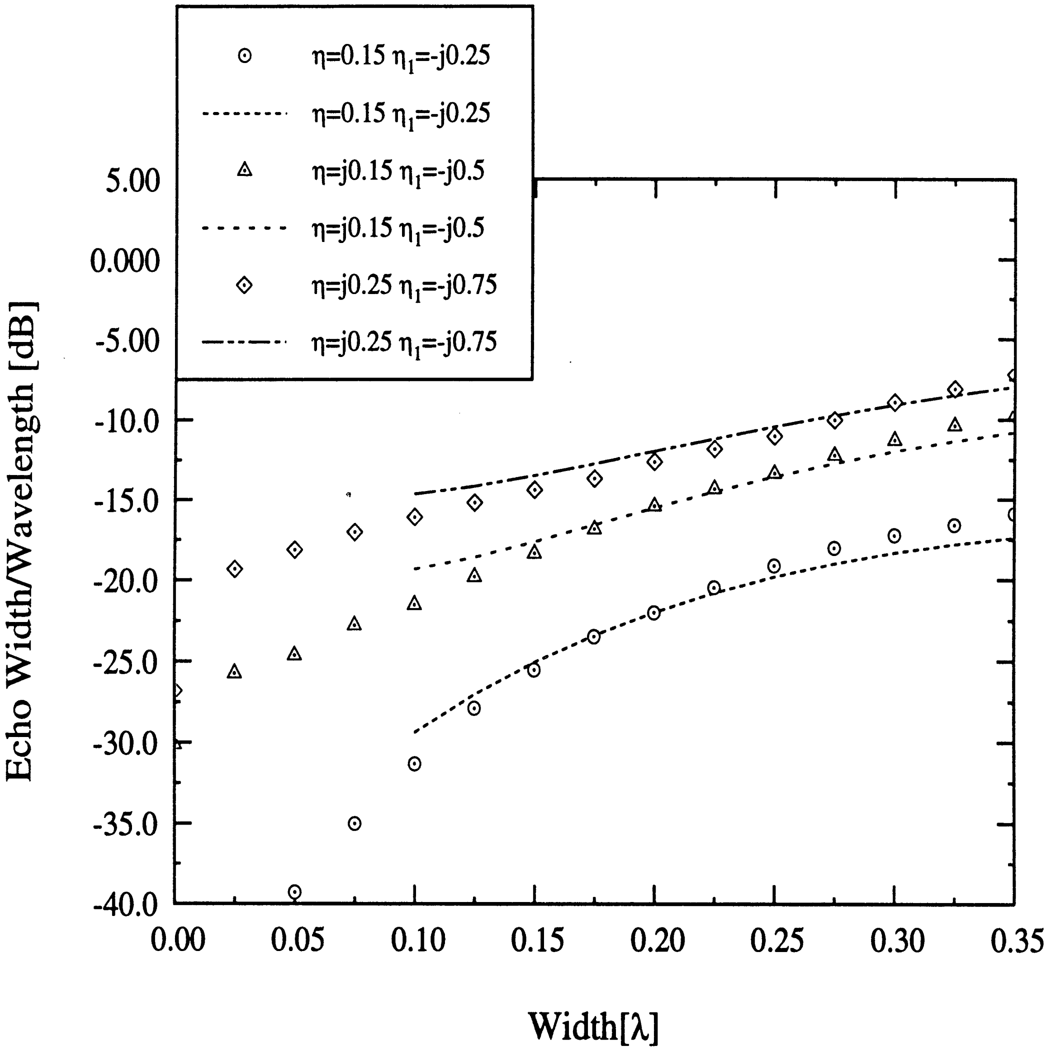


Figure 6: E-polarization backscatter echowidth at oblique incidence (45^0) for various three-part impedance planes as a function of the insert's width (symbols: small width approx, lines: high freq solution)

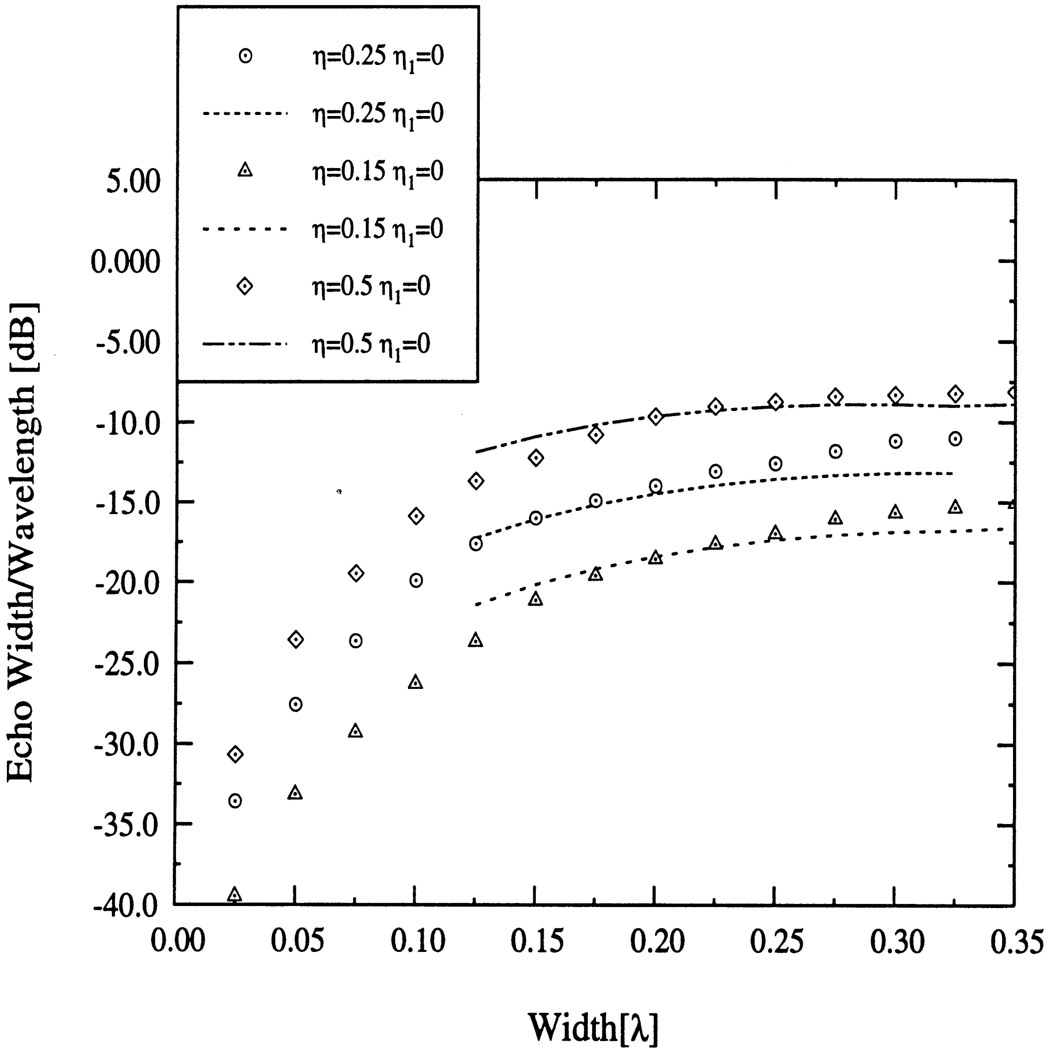


Figure 7: H-polarization backscatter echowidth at oblique incidence (45^0) for various impedance inserts in a metallic plane as a function of the insert's width (symbols: small width approx, lines: high freq solution)

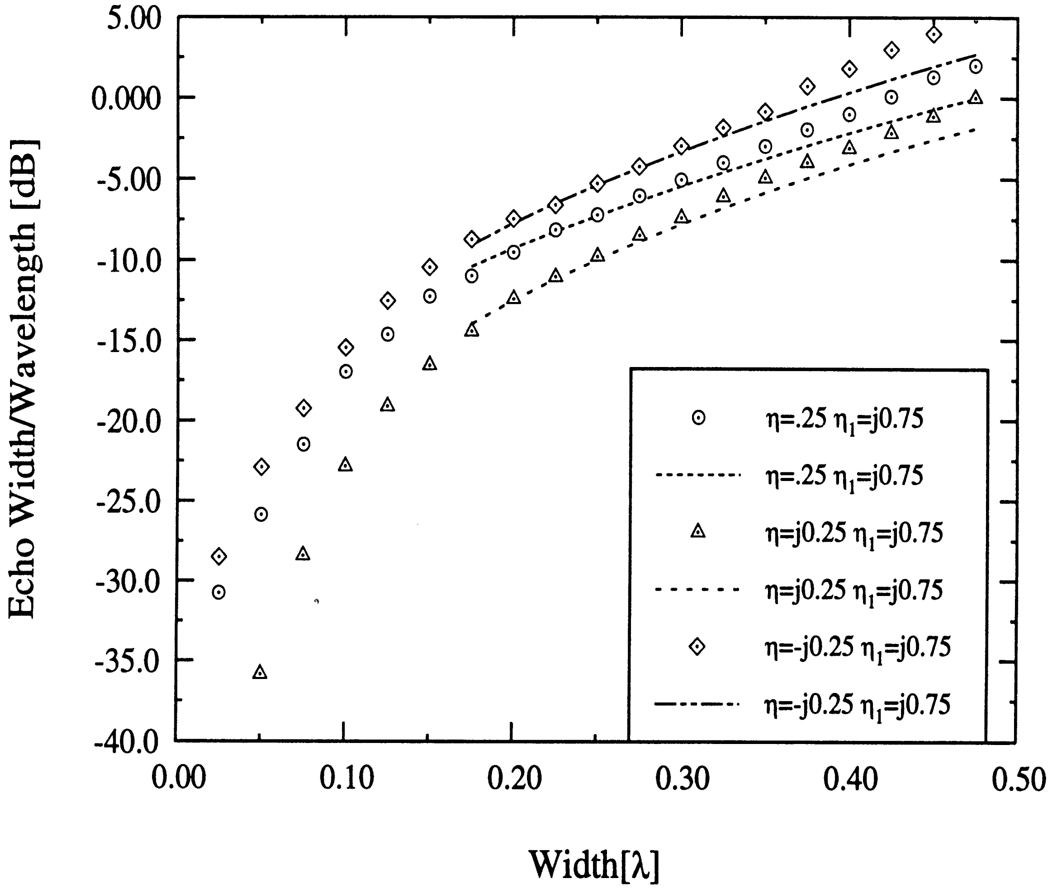


Figure 8: H-polarization backscatter echowidth at normal incidence for various impedance inserts in an impedance plane of normalized impedance $\eta_1 = j0.75$ as a function of the insert's width (symbols: small width approx, lines: high freq solution)

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