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INNOVATIONS IN THE ECONOMICS OF PROJECT INVESTMENT

by

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This paper was prepared for presentation to the American Association of Cost Engineers, Great Lakes Chapter, November 18, 1970, Ann Arbor, Michigan. It is expected that much of the material presented will be included in a more ambitious capital investment manual. The purpose of such a manual will be to convey, through heavy use of practical illustrations, the significance of theoretical problems in investment analysis to the practicing financial analyst or cost engineer.

INNOVATIONS IN THE ECONOMICS OF PROJECT INVESTMENT

Introduction

The purpose of this paper is to explore some evolutionary developments in the evaluation of capital expenditures. The developments I wish to refer to are still evolving in that most of them are not yet explicitly applied by the engineers and financial analysts responsible for evaluating their firm's capital proposal. Before exploring these developments, I will briefly review some background fundamentals.

Part I reviews fundamental concepts of capital investment and present value mathematics and the application of internal rate of return and net present value yardsticks to the evaluation of capital projects. Part II discusses the differences in these yardsticks and the conditions under which one or the other is most appropriate. Part III presents an improvement in a serious deficiency of the internal rate of return yardstick used for measuring projects with net capital outlays for more than one year. Part IV considers the popular methods of adjusting project economics for risk. Finally, Part V presents an expanded method for evaluating projects which includes a quantified representation of risk.

I

Fundamental Concepts and Application

We may best orient our discussion by focusing on the characteristics of capital expenditures. A capital expenditure generally is expected to create an asset which will yield productive and profitable services over a period of years. Since a capital expenditure secures a stream of net income for several years in the future, a yearly return on the expenditure which is modest in comparison with the magnitude of the expenditure itself is usually acceptable. However, the conversion of substantial capital funds into a special purpose asset is often extremely difficult to undo. Ownership of such an asset creates great dependence on the sustained productivity of that asset throughout its economic life.

But sustained productivity of the asset depends, in turn, on the existence of many future conditions, such as sales or labor costs, which will remain favorable for such productivity. Unfortunately, the likelihood of such conditions remaining favorable is uncertain. The firm investing in a capital asset is a prisoner of future events because of the relative permanence or "undo-ability" of the capital asset. The firm's long-run profitability is thus dependent upon the proper allocation of capital resources to assets of sustainably high productivity and profitability.

Capital investment evaluation should provide a yardstick capable of making at least two kinds of assessments. First, the yardstick should assess the absolute benefit or desirability of the project. Second, it should aid the decision on whether or not the project is desirable in relation to competing projects.

Industry now generally uses one or more benefit yardsticks which recognize the time value of money -- that is, the concept that a dollar received next year is worth less than a dollar received today. The greater value of today's dollar stems from how it is used over the coming year. That is, possession of today's dollar entitles a person to its investment income or productivity over the year. By investing at 8 per cent, today's dollar will be worth $1.08 a year from now. Furthermore, after compounding, a dollar today will be worth $1.16 at the end of two years. Conversely, as shown in Table 1, the present value of $1.00 received a year from now and discounted at 8 per cent, is worth $0.926 today. Also, $1.00 two years from now has a present value of $0.857.

Ranking investment projects

Basically, there are two widely used techniques which use the time value of money to evaluate investment. One technique is known popularly as the DCF (for discounted cash flow) rate of return and known technically as the internal rate of return (IRR). This technique solves, via trial-and-error calculations, for the rate of interest that equates the present value of expected future receipts to the cost of the investment outlay.

The second technique is net present value (NPV). This technique may be defined as the present value of expected future cash inflows and cash outflows discounted at some minimum acceptable rate

Table 1

Present Value Calculations

A. $1.00 received in November, 1971 is worth today. $0.926

1. $0.926 invested in November 1970 @ 8%
2. Value in Nov., 1971
   0.926 + (0.926 x 0.08) = $1.00

B. $1.00 received in November, 1972 is worth today. $0.857

1. $0.857 invested in Nov., 1970 @ 8%
2. Value in Nov., 1971
   0.857 + (0.857 x 0.08) = $0.926
3. $0.926 invested in Nov. 1971 @ 8%
   0.926 + (0.926 x 0.80) = $1.00

of return. The minimum rate of return is sometimes called the "threshold" rate. In financial theory the threshold rate becomes the firm's cost of capital. Basically the concept of cost of capital recognizes the implicit, as well as explicit, costs of maintaining an optimum mix of capital sources (capital stock, bonds, and retained earnings). Theory posits that a firm invests in all projects which have returns in excess of the firm's cost of capital. In other words, all projects should be accepted which have a net present value greater than zero when calculated on the basis of a discount factor equal to the cost of capital.

Although most businessmen and students of business can intuitively grasp the techniques which use the time value of money as it is applied to project evaluation, it may be helpful to focus on a simple example. With this background, we can then delve into the innovative aspects of project evaluation.

Case I--semi-tractor and tank wagon

This case involves alternative means of providing a vehicle for transporting bulk liquid finished product to a customer who has recently signed a long-term contract to buy substantial quantities of the product. The alternative means of transporting the finished product to this customer have been narrowed down to two:

1. The firm can purchase its own semi-tractor and tank wagon.
2. The firm can employ a hired carrier.

These alternatives are described in Table 2. Basically the decision must be based on the economics of ownership versus employing a hired carrier.

Table 2

Semi-Tractor and Tank Wagon - alternative vehicles

1. Data on New Vehicle

Type	Diesel Tractor, Semi-Trailer
Capacity	8,000 gallons
Annual Miles	26,000
Annual Hours	2,900
Annual Volume	6,887,500
Investment	$30,690
Salvage in 8 years	$ 4,034

2. Data on Hired Carrier

Annual Cost	$16,332

The charge for the hired carrier is all-inclusive, covering the vehicle and all associated operating expenses. If such a carrier is employed, one problem which cannot be overlooked is that a company driver may retire or be terminated at a cost of $1,085. Another obscure economic element which should be considered is the likelihood of increased sales to the hired carrier which would yield a profit of $441 annually.

The cash flow for the proposed new vehicle consists of the investment at time zero, operating costs for items such as fuel, maintenance, and wages, and the tax benefits from writing off the new vehicle against the firm's operating income on double declining balance depreciation. The cash flows for the two alternatives are shown in Table 3.

The IRR on the purchase alternative may be developed by finding the difference between the cash flows of the two alternatives. Conventionally, if the rate developed exceeds the threshold rate, the vehicle would be purchased. If the rate falls below, the hired carrier would be used.

Table 4 presents the net cash flows of the two alternatives and the calculation of the IRR. The second of the two widely used time-adjusted yardsticks for project evaluation, NPV, is computed for the vehicle hire/purchase project in Table 5 by applying 12 per cent cost-of-capital discount factors. The calculation of NPV gives an absolute value which fails to reflect the amount of capital invested in the project and, therefore, is not an adequate measure of performance relative to competing projects.

Table 3

Cash Flows for Semi-Tractor Alternatives

Alternative 1 - Purchase

Year	Investment	Operating Costs	Tax Credit on Depr.	Salvage Value	Total
0	(30690)				(30690)
1		(8449)	1980		(6469)
2		(8730)	3465		(8265)
3		(8886)	2599		(6287)
4		(9094)	1949		(7145)
5		(9198)	1462		(7736)
6		(9302)	1096		(8206)
7		(9718)	822		(8896)
8		(9718)	617	4034	(5067)

Alternative 2 - Hired Carrier

Year	Cost	Terminations	Profits on Product Sales	Total
0				
1	(16332)	(1085)	441	(16976)
2	(16332)		441	(15891)
3	(16332)		441	(15891)
4	(16332)		441	(15891)
5	(16332)		441	(15891)
6	(16332)		441	(15891)
7	(16332)		441	(15891)
8	(16332)		441	(15891)

Table 4

Net Cash Flow and Internal

Rate of Return Calculation
- Semi-Tractor Case

Year	Purchase	Hired Carrier	Net Cash Flow	26% Discount Factor		DCF
0	(30690)		(30690)	1.000		(30690)
1	(6469)	(16976)	10507	.794		8343
2	(5265)	(15891)	10626	.630		6694
3	(6287)	(15891)	9604	.500		4802
4	(7145)	(15891)	8746	.397		3472
5	(7736)	(15891)	8155	.315		2569
6	(8206)	(15891)	7685	.250		1921
7	(8896)	(15891)	6995	.198		1385
8	(5067)	(15891)	10824	.187		2409
				Total	=	905

Internal Rate of Return on Purchasing = 26.2%

Table 5

Net Present Value Calculation

- Semi-Tractor Case

Year	Net Cash Flow	12% Cost of Capital Factors	Net Present Value
0	(30690)	1.000	(30690)
1	10507	.893	9383
2	10626	.797	8469
3	9604	.712	6838
4	8746	.636	5562
5	8155	.567	4624
6	7685	.507	3896
7	6995	.452	3162
8	10824	.404	4373

Net Present Value = $15617

Profitability Index

$$= \frac{\text{Investment in Year 0} + \text{NPV}}{\text{Investment in Year 0}}$$

$$= \frac{30690 + 15617}{30690}$$

$$= \frac{46307}{30690} = 1.51$$

An extension of NPV called the profitability index (PI) fulfills the task of comparing projects. As indicated in Table 5, it is the sum of NPV and the investment in the project divided by the investment in the project.

The NPV and IRR may be presented graphically in the "present value profile" of Figure 1. This graphic tool plots the present value of a project's (net) cash flows that correspond with each interest rate. The IRR may be found where the profile crosses the r axis. (Recall that P equals zero at the internal rate of return.) The NPV for the project is simply P evaluated at the interest rate of 12 per cent, the cost of capital. Finally, notice that the P axis intercept is just the sum of undiscounted project cash flows.

Present value mathematics

Both the IRR and the NPV calculations shown in Tables 4 and 5 may be represented mathematically by the expression:

$$P = C_0 + C_1 \frac{1}{1+r} + C_2 \left(\frac{1}{1+r}\right)^2 + \cdots\cdots + C_n \left(\frac{1}{1+r}\right)^{-n}$$

where P = net present value

C = cash flow

r = cost of capital for net present value

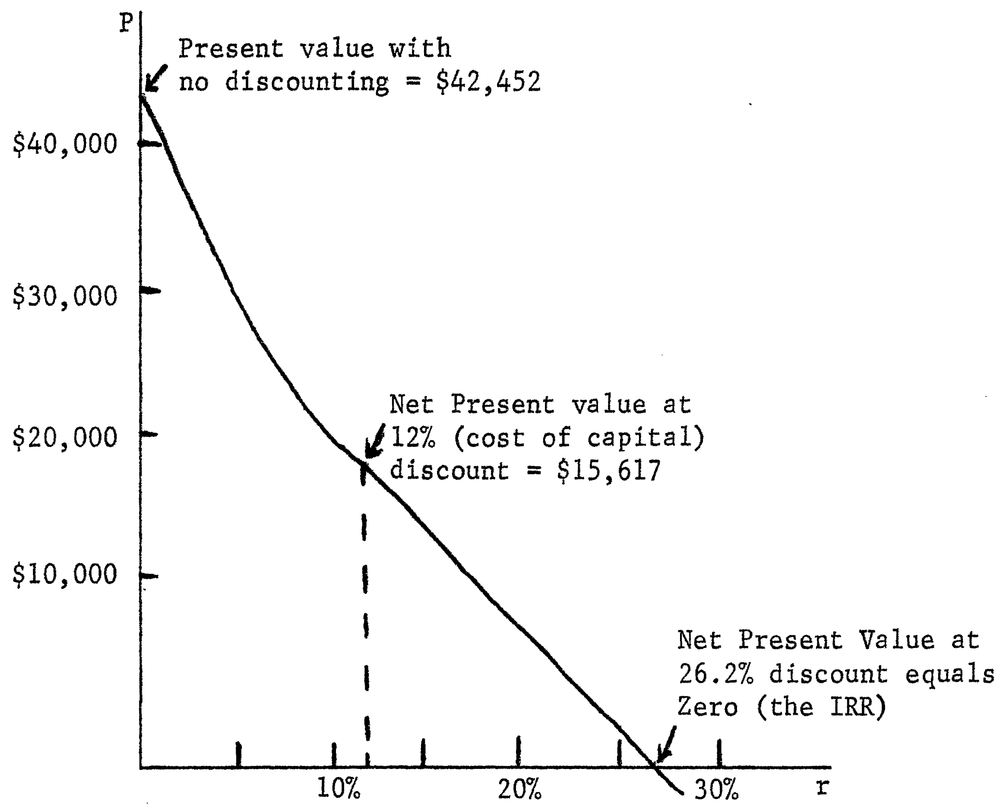
= internal rate of return for IRR

and the C subscripts are years.

The net present value yardstick is simply the value of P. However, by our earlier definition, to obtain the internal rate of return it is necessary to set P equal to zero and solve for r.

Figure 1

Case I - Present Value Profile



The decision rules for accepting projects are easily stated:

1. For NPV

   $\underline{P} > \underline{0}$

2. For DCF

   $\underline{r} > \underline{c}$

   where $\underline{c}$ is a specified threshold rate such as the cost of capital.

We will have occasion later to refer back to the mathematical expression for present value when we critically examine the theoretical soundness of NPV and IRR. It will be seen that neither rule applies universally to the capital investment problem of all firms and that the two rules differ in their assumptions and the conditions under which their use is appropriate. I will examine the appropriateness of these measures in the next section and then suggest means of expanding the use of time-adjusted project evaluation to overcome their usual limitations and to incorporate the considerations of risk and multiple possible project outcomes.

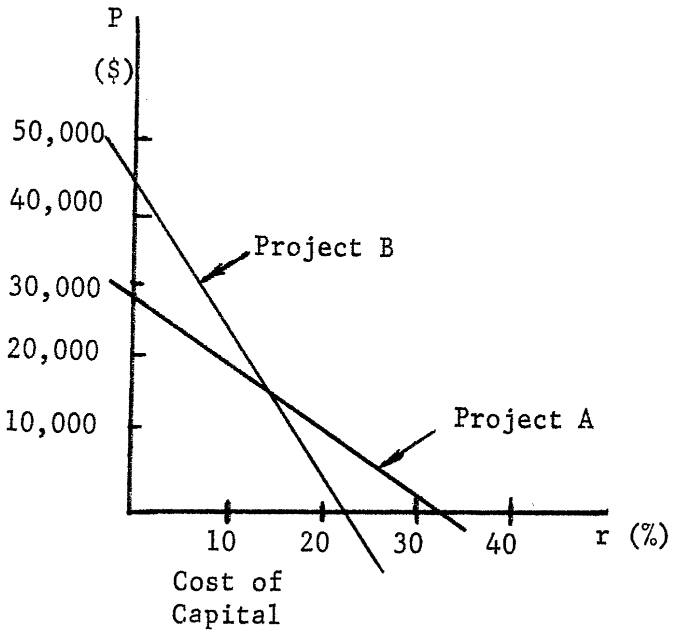
II

The Preferred Yardstick

The crucial question in the choice of a yardstick for investment evaluation is whether IRR or NPV is appropriate given the investment policies and opportunities of the individual firm. What makes the question crucial is the not-so-rare situation where the two yardsticks rank competing projects differently. For example, see

Figure 2

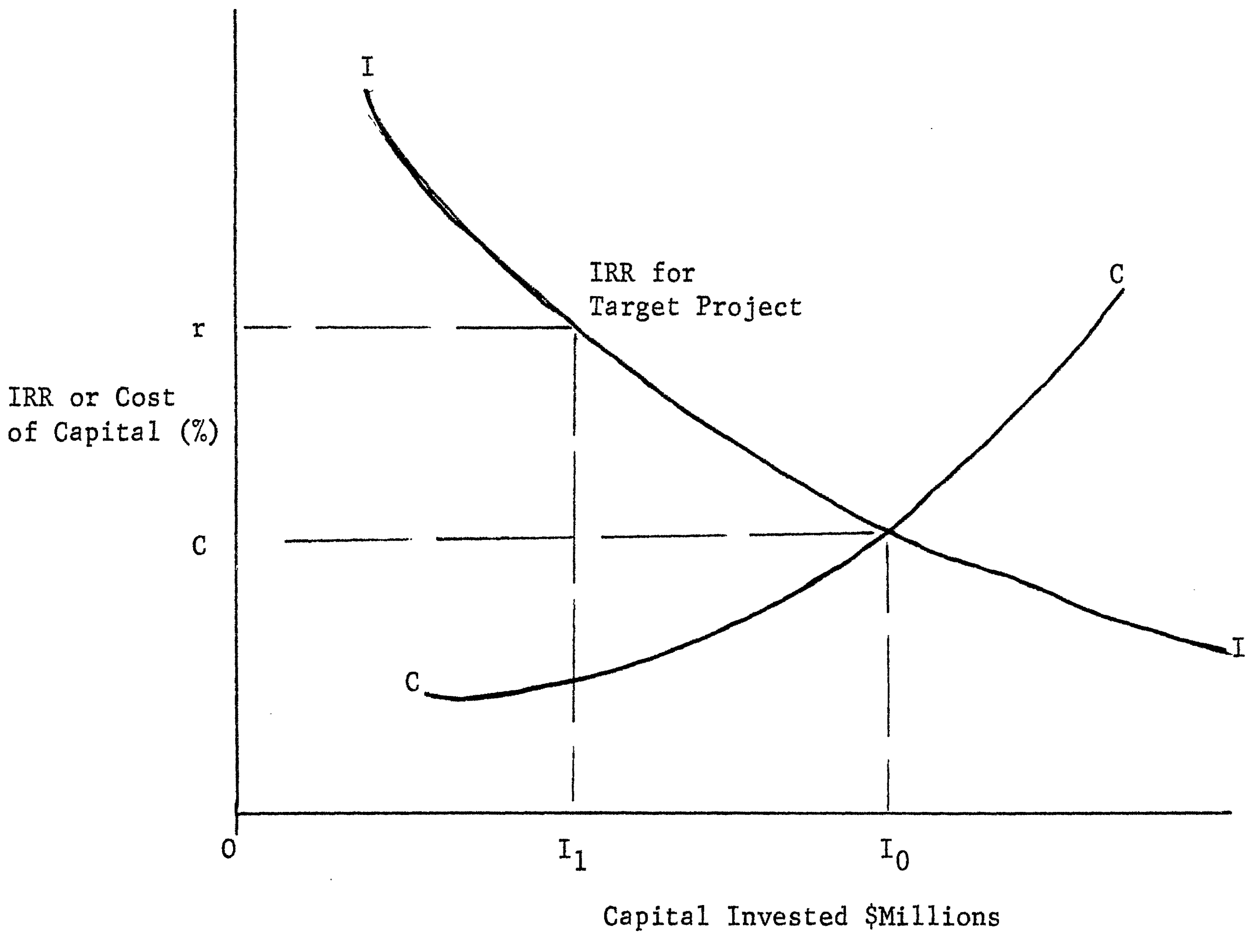
Conflicting Rankings of Projects



	NPV	IRR
Project A	$18,000	31.4%
Project B	$26,730	22.1%

Figure 3

Investment and Cost Relationships



rates of return when increasingly more capital is available for investment. Conversely, the upward sloping cost-of-capital schedule depicts ever-increasing cost as the firm attempts to provide increasingly greater amounts of capital to finance more and more projects.

In terms of Figure 3, the IRR yardstick implies that the firm invests only $\underline{OI_1}$, even though clearly profitable project opportunities exist which justify additional investment of $\underline{I_1 I_2}$. In other words, critics allege that the IRR yardstick implies that the firm's capital is rationed so that it cannot take full advantage of lucrative investment opportunities. To reiterate, the IRR yardstick implies this situation because it is based on the assumption that the cash inflows of the project being evaluated are reinvested at the project's internal rate of return.

In contrast, the NPV technique of discounting cash flows at the firm's computed cost of capital implies that the project's cash inflows are reinvested at the cost of capital. In terms of Figure 3, the NPV yardstick implies that the firm invests an amount $\underline{OI_o}$. In this case then, the firm exploits all of the investment opportunities available to it.

Determining the appropriate yardstick

The critics of IRR argue that the NPV is more realistic because many large firms do develop capital sources commensurate with their investment opportunities.[1/] These critics point out that large firms enjoy high credit standing and that these firms themselves determine

1/ Time-adjusted project evaluation still seems to be used mostly by large firms.

the volume of capital they actually raise rather than have lenders impose a credit limit. In other words, they can finance all of their identifiable opportunities.

But evidence exists that many firms actually do limit their profit-producing investments to an IRR well above the cost of capital. Part of the proof may be found by simply observing the capital investment decision-making process. Rarely is a genuine profit-making project submitted to the finance committee with the IRR approaching the cost of capital (or the NPV approaching zero) unless the project has a dominant obligatory dimension or a defensive, must-do-in-order-to-stay-in-business aspect. Management is sometimes unrealistic in lumping obligatory and defensive projects with profit-generating opportunities to make them minimally digestable on a rate of return basis. Thus projects having IRRs which approach the cost of capital often consist of smaller autonomous profit-generating projects with IRRs substantially higher than the cost of capital as well as discrete obligatory or defensive projects.

Perhaps the most important point in proving that many firms normally have reinvestment opportunities at IRRs well above the cost of capital is the so-called risk premium demanded in capital investment. Risk premium refers to management's instinct to cover risk with a safe profit margin above the breakeven point. I will treat this concept more formally in a later section. It is sufficient for now to point out that the uncertainty of returns compared to the greater certainty of capital costs induces management to reject profit-making projects long before their IRRs fall to the cost of capital.

Finally, the investment curve shown in Figure 3 does not adequately reflect the dynamic capital investment environment that management faces. Future investment opportunities are an evolutionary phenomenon not wholly apparent at the point in time represented by the static model in Figure 3. The arguments of critics of IRR imply that high-return projects are all exploited at one point in time and that cash flowing in from these projects in the future will have to be reinvested at cost-of-capital rates of return. This is almost certainly untrue for high-growth firms and, according to our arguments, probably equally untrue for many large normal-growth firms.

Realistically, then, we will assume that, given growth and a dynamic investment environment, the proportions of high-, medium-, and low-IRR projects for many firms is constant over time. We will proceed on the basis that the IRR yardstick is quite appropriate for a large contingent of firms with rather normal investment opportunities. Conversely, the NPV is inappropriate for such firms.

IRR and multiple results

One valid criticism of the IRR relates to the special circumstances under which more than one result may occur, i.e., the IRR calculation may yield two very different values of IRR. Generally, the circumstances require a major capital outflow during the middle of a project's life. On the surface such an outflow may appear atypical. Consider, however, the erratic cash flows associated with projects which begin with large-scale pilot operations or test marketing. Often a modest capital or promotional outlay occurs first as a more-or-less tentative commitment to the project and generates attractive cash flows shortly thereafter. Next, a massive capital outlay for permanent facilities may occur, followed by additional inflows. To illustrate how such a pattern of project cash flows may produce multiple IRRs, the motor oil additive case is analyzed below.

Case II--new motor oil additive

This case involves a firm's production of a new motor oil additive targeted for a large market. In the early stages of the project, two large petroleum firms contract with the supplier firm to purchase modest quantities of the new additive for evaluation and limited distribution. The supplier firm has decided to modify a reactor which is temporarily underutilized to produce the quantities needed initially. Since it is hoped that the superior additive will, in time, usurp the place of competitive products and result in growing volume, it has been decided that a substantial outlay for a high-capacity reactor and ancillary equipment may be justified two years from now. Unfortunately, although the product is superior, the succession of such products in the market is so rapid that management conservatively estimates a life of only two years after full-stream production begins.

Table 6 shows the cash flows associated with this project and Table 7 shows the IRR calculations. Notice the IRR method yields two results, 47.5 per cent and 100 per cent, both of which appear dramatically favorable. The project's present value profile features two crossings of the r axis as shown in Figure 4. On the surface it

Table 6

Case II - New Motor Oil Additive

Year	Capital Outlay	Cash Inflow	Net Cash Inflow
0	79,000	45,000	(34,000)
1	--	90,000	90,000
2	250,000	100,000	(150,000)
3	--	80,000	80,000
4	--	80,000	80,000

Table 7

Case II - Multiple IRR Rates of Return

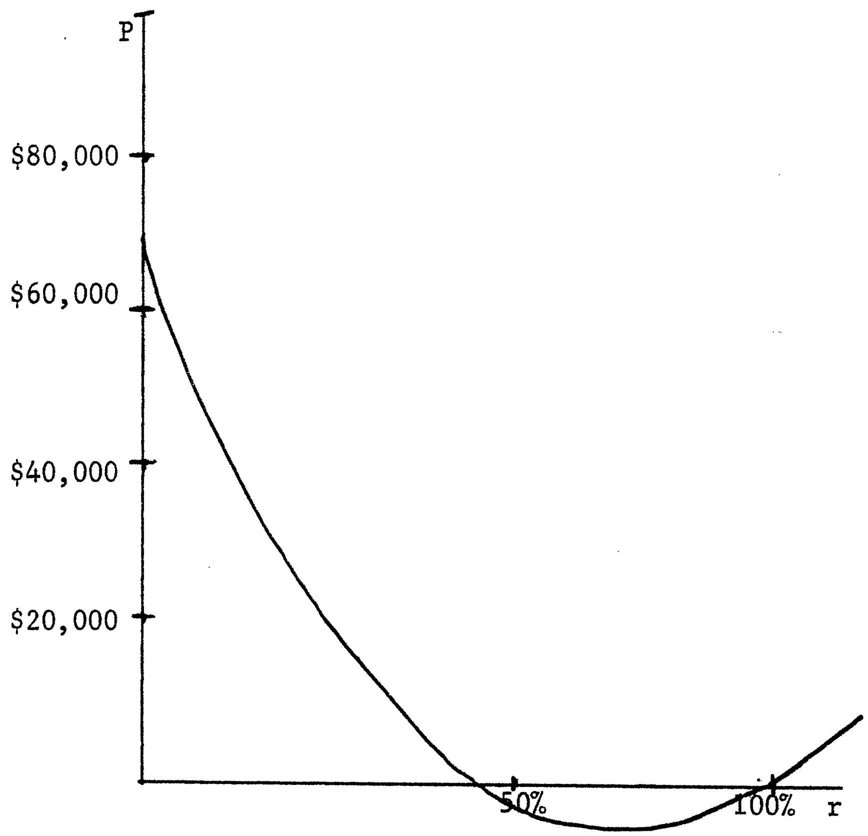
Year	Net Cash Flow	Discounted Cash Flow @ 45% Discount	Discounted Cash Flow @ 50% Discount	Discounted Cash Flow @ 100% Discount
0	(34,000)	-34,000	-34,000	-34,000
1	90,000	62,100	+60,000	+45,000
2	(150,000)	-71,300	-66,700	-18,750
3	80,000			
4	80,000			
		+ 1,100	(1,200)	(250)

IRR ≈ 47.5% (from the 45% and 50% columns)

IRR ≈ 100%

Figure 4

Case II - Present Value Profile



would appear that management should not hesitate to accept the project on the basis of either rate. However, because of the gross uncertainty of the product's acceptance, management believes this project must be classified as highly risky. As such, it will have to be compared with several other competing speculative projects having unusually high expected returns, and a decision will have to be made on which project to undertake. In such a case, which rate should management consider in comparing the oil additive project with competing projects? Alternatively, since this IRR calculation seems abnormal, should management place any faith at all in it? One possible answer is to modify the IRR method to derive an unambiguous and meaningful criterion. Before attempting this modification we must consider the cause of ambiguity related to the IRR. Then we will derive a means of altering the IRR yardstick to reduce this ambiguity.

III

The Two-Rate (TR) Method

Although the IRR yardstick is suitable for the conventional projects of growing firms, it is often not suitable for their projects requiring capital outlays after year zero. An improved yardstick is available which is more suitable for such projects and which, in addition, solves the dilemma of multiple results. We will call this improved yardstick the modified IRR or, more descriptively, the two-rate (TR) method.

Discounting future cash outflows

To develop the TR method we must examine a key mathematical assumption underlying the IRR calculation with respect to net capital expenditures occurring in a project after year zero. This assumption holds that the funds needed later are invested somewhere else in the interim, beginning with year zero, at a rate of return equal to the IRR of the project. Thus, the implication is that funds needed for a later capital outlay in the target project will be generated by alternative projects with IRRs equal to the target project. This allegedly justifies discounting later capital outlays with the often high IRR rate of discount of the target project. For example, in the oil additive project calculations shown in Table 7, the $150,000 needed in the second year has been discounted at 47.5 per cent or 100 per cent to a year-zero present value of $69,000 or $18,750.

This assumption is the converse of the assumption examined in Part II in which cash inflows of the target project were implicitly reinvested in alternative projects with the same IRR.

However, for firms with a growth rate greater then zero, the rationale in the case of discounting later cash outflows is not symmetrical with the cash inflow case. The source of funds for investment by growth firms in profit-generating projects includes substantial funds raised in the money and capital markets. In fact, in the case of rapid-growth firms, often only a small proportion of new investment capital is generated internally. Even in firms with small growth rates the long-term need to preserve an optimal debt-to-equity ratio assures some external fund raising.

Thus, for our purposes, the discount rate to be applied to cash outflows will represent the cost of obtaining the optimal mix of funds, i.e., the cost of capital.2/

Applying the TR method

The use of discounting future cash outflows at the cost of capital in the IRR calculation may be demonstrated with the motor oil additive case. Suppose our firm used 12 per cent as its cost of capital. The two-rate method (TR) calls for discounting project outflows at 12 per cent and solving for the modified internal rate of return. Referring again to the mathematical expression for present value given on p. 11 and recalling that $\underline{P}$ equals zero for internal rate of return, we obtain:

$$\underline{P} = 0 = -34{,}000 + 90{,}000\left(\frac{1}{1+\underline{r}}\right) \cdot -150{,}000\left(\frac{1}{1+.12}\right)^{2} + 80{,}000\left(\frac{1}{1+\underline{r}}\right)^{3} + 80{,}000\left(\frac{1}{1+\underline{r}}\right)^{4}$$

where $\underline{r}$ is the minimum rate of return required on investing \$90,000 in year 1 and \$80,000 in each of years 3 and 4 in alternate projects just as in the IRR case. The solution is:

$\underline{r}$ = TR = 22.2%

2/ In present value terms, because the funds needed for investment in a later year were not required in year zero, we are entitled to discount them at the rate they would have cost if we actually had held them. In a sense, we are rewarded for not holding them.

Of course the familiar trial-and-error solution used in the standard IRR calculation is the practical means of solving for r. The IRR format applied to the TR method is shown in Table 8.

Notice that except for the occurrenceof cash outflows after the zero year, the TR method is exactly the same as the old IRR method. The significance of this fact is that the TR method applied to projects with cash outflows beyond year zero may be directly compared with the accustomed IRR method applied to the more conventional projects with a cash outflow in year zero only. Thus our firm may legitimately compare our Case I, the semi-tractor and tank wagon case (negative outflow in zero year only), with Case II, the motor oil additive case (negative outflows in the second year as well as the zero year).

	Project	IRR or TR (Percentage)
I	Semi-tractor and tank wagon	26.2
II	Motor oil additive	22.2

Notice the marked effect of applying the TR yardstick to the motor oil additive case. Instead of the confusing and extravagent IRR results of either 47 per cent or 100 per cent internal rate of return, the TR method yields one result at a more ponderable level of 22 per cent. Given the high risk of this project, management might now approach it with far less trepidation. Comparatively, the motor oil additive project is no longer as appealing as the relatively certain semi-tractor and tank wagon project.

Table 8

Case II - Two-Rate Calculation

1. Year	2. Net Cash Flow	3. 12% Discount Factors	4. DCF @ 12% Discount	5. 22% Discount Factors	6. DCF @ % Discount
0	(34,000)	1.000	(34,000)	--	--
1	90,000	--	--	.8197	73,773
2	(150,000)	0.797	(119,550)	--	--
3	80,000	--	--	.5507	44,056
4	80,000	--	--	.4514	36,112
			(153,550)		153,941

Col. 6 - Col. 4 = 153,941 + (153,550) = 391

TR Return = 22.2%

The simple adjustment of discounting cash outflows with the cost of capital before applying the trial-and-error solution adds no new complications to this familiar procedure. On the contrary, the TR approach improves project evaluation in several ways:

1. It avoids multiple solutions.
2. It puts projects with negative cash flows beyond year zero on an equal basis with conventional projects for IRR comparisons.
3. It avoids taking unrealistically high credit for delaying cash outflows.

IV

Recognizing Risk

Our discussion thus far has omitted one crucial input into the evaluation of capital expenditure--the element of risk. Most analysts and managers have difficulty expressing their subjective grasp of the different "amount of risk" involved in different projects. The following are some of the means of incorporating this grasp of risk into an evaluation:

1. Use higher discount rates for riskier projects in computing NPV.
2. Raise the threshold rate for IRR.
3. Shorten the expected project life.
4. Progressively scale down the expected cash flows that are more distant in time.

Let us look at some examples to trace the effect of each of these methods of adjusting for risk. Tables 9, 10, 11, and 12 illustrate the effects of adjusting the discount rate and threshold rate, shortening the project life, and scaling down distant cash flows respectively for Case I.

In Table 9 a risk-adjusted discount rate of 16 per cent was assumed. The NPV and PI are still quite attractive, but fell from $15,617 and 1.51 in the original calculation using a 12 per cent discount rate (Table 5) to $10,056 and 1.33 after risk-adjustment using a 16 per cent discount rate. In the case of the IRR, of course, it is only necessary to note that the original IRR still comfortably exceeds the risk-adjusted threshold rate (Table 10).

The rationale behind this kind of risk adjustment is simple. Management recognizes that it has less control over the costs or revenues of some projects than it does over others. Thus, an electric power company may confidently estimate the costs and profits of running a new primary line to electrify an already wired new subdivision. Similarly the company can readily predict the characteristics of power usage by the new residents. On the other hand, installation of a new nuclear generating station by the same firm would be fraught with uncertainties including the lack of cost experience, growth patterns of the huge population to be served, and the ability to incorporate these factors into the company's rate structure in a reasonable period of time.

Table 9

Case I - Semi-Tractor and Tank Wagon
- Risk-Adjusted NPV Discount Rate

Year	Net Cash Flow	16% Discount Factors	Net Present Value
0	(30690)	1.000	(30690)
1	10507	0.862	9057
2	10626	0.743	7895
3	9604	0.641	6156
4	8746	0.552	4828
5.	8155	0.476	3882
6	7685	0.410	3151
7	6995	0.354	2476
8.	10824	0.305	3301

Net Present Value = 10,056

$$\text{Profitability Index} = \frac{10{,}056 + 30960}{30960} = 1.33$$

Table 10

Case I - Semi-Tractor and Tank Wagon

- Risk-Adjusted DCF Threshold Rate

Original:

DCF = 26.2% > 12% Threshold

Adjusted:

DCF = 26.2% > 16% Threshold

Table 11

Case I - Semi-Tractor and Tank Wagon

- Shortened Project Life

Year	Net Cash Flow	NPV 12% Discount	DCF 21%
0	(30690)	(30690)	(30690)
1	10507	9382	8682
2	10626	8471	7258
3	9604	6836	5421
4	20219*	12849	9432
		6848	103

New NPV = $6848   Original NPV = $15617

New PI = 1.22   Original PI = 1.51

New IRR = 27.1%   Original IRR = 26.2%

* Salvage value of $11,473 included.

Table 12

Case I - Semi-Tractor and Tank Wagon

- Scaled Down Cash Flows

Year	Net Cash Flow	Scale-Down Factors	Scaled-Down Cash Flows	NPV 12% Discount	DCF 23%
0	(30690)	-	(30690)	(30690)	(30690)
1	10507	-	10507	9382	8542
2	10626	-	10626	8471	7024
3	9604	-	9604	6836	5161
4	8746	0.90	7871	5002	3439
5	8155	0.85	6932	3933	2462
6	7685	0.80	6148	3115	1776
7	6995	0.70	4897	2215	1150
8	10824	0.55	7036	2842	1343
				11,106	213

New NPV = $11,106 — Original NPV = $15,617

New PI = 1.36 — Original PI = 1.51

New DCF = 23.3% — Original DCF = 26.2%

Obviously we should include in this risk-adjustment category those projects about which many project analysts have made such comments as, "The Finance Committee is going to want a 20 per cent IRR to accept this one." In this way an instinctive cushion against unfamiliar construction and operating costs or against indeterminate prospects for profitability is incorporated.

The two remaining risk-adjustment practices listed above are illustrated in Tables 11 and 12. Both of these practices, shortening project life and scaling down distant cash flows, imply that risk is a positive function of time, i.e., that risk increases for those less predictable later years of a project's life. To adjust for the increase in risk over time, these methods minimize the dependence of conventional NPV and IRR yardsticks on a project's cash inflows in the distant future.

Suppose in Case I that management feels there is a mild chance that its liquid-product tank wagon customer will acquire the know-how required to produce this product in house four years from now. Table 11 shows the effect on NPV and IRR of truncating the project cash flows at the end of four years. Notice that inclusion of the salvage value increases year four's cash flow substantially. In comparison to the more flexible option of hiring a carrier the decision to purchase a semi-tractor and tank wagon is less attractive in view of a shortened project life than in view of the original life. Certainly the project would look much different if the customer were believed capable of developing in-house production in two or three years.

Alternatively, suppose that the firm's transportation department believes maintenance on the purchased tractor in Case I could possibly increase rapidly in later years. Table 12 shows the effect on NPV and IRR of progressively scaling down the cash flow advantages of purchasing versus hiring in the last five years.

Finally, combinations of the above methods are used to insure doubly against risk. Thus a higher NPV discount rate as well as a truncated life might be applied in evaluating a risky project.

Assumptions behind risk recognition

Although it is important to incorporate some allowance for risk when evaluating projects in which management lacks complete confidence and experience, the above methods often resemble simple "fudge factors" instead of quantifiably justifiable modifications. Probably the most common method, raising the NPV discount factor or the DCF acceptance threshold, appears merely to erect a higher hurdle rather than to add meaningful information to the evaluation.

In reality, however, the upward adjustment of the NPV discount rate in particular has a specific mathematical meaning. It actually incorporates a geometric increase in risk over time. In this sense it is simply a distant cousin to the other methods of accounting for greater risk in the latter stages of a project's life, e.g., the shortened project life and scaled-down distant cash flow methods. Thus, risk-adjusted NPV distinctly accounts for risk at an increasing rate over the life of the project. The explanation for this lies in the compounding effect of the risk premium, defined as the cash flows discounted by the difference between the usual cost-of-capital rate and those discounted by the higher risk-adjusted rate. Because the compounding effect penalizes cash flows more and more over time, the contribution to NPV of distant cash flows is progressively minimized.

To illustrate, compare Table 5, in which Case I cash flows were discounted at the 12 per cent cost of capital, with Table 9, in which the same cash flows were discounted at a 16 per cent risk-adjusted rate. Note that in Table 13 the risk-adjusted discounted cash flows are a sharply decreasing percentage of cost-of-capital discounted cash flows over time. But, although the 4 per cent premium is constant, it implies that the riskiness of the project increases with time.

Analysts have defined the riskiness implied in the risk premium in terms of a risk index.[3/] In our example, the risk index can be expressed as

$$I = \frac{\text{cost of capital discount factors}}{\text{risk-adjusted discount factors}}$$

which, for year one, is

$$\frac{\frac{1}{1 + .12}}{\frac{1}{1 + .16}}$$

The risk index for eight years in Case I is shown in Table 14. In addition, a risk index for 20 per cent risk-adjusted discount rate is given. Notice that risk is positively related to both time and the risk premium.

3/ See John Weston and Eugene Brigham, Managerial Finance (3rd ed.; New York: Holt, Rinehart and Winston, 1969), pp. 245-49.

Table 13

Case I - Semi-Tractor and Tank Wagon

- Normal NPV vs. Risk-adjusted NPV

Year	1. Net Cash Flow	2. NPV 12%	3. NPV 16%	Col. 3 as Percent of Col. 2
0	(30690)	(30690)	(30690)	100.0
1	10507	9383	9057	96.5
2	10626	8469	7895	93.2
3	9604	6838	6156	90.0
4	8746	5562	4828	86.8
5	8155	4624	3882	84.0
6	7685	3896	3151	80.9
7	6995	3162	2476	78.3
8	10824	4373	3301	75.5

Table 14

Risk Index (Case I)

Years	Risk index using risk-adjusted discount factors of = * 16%	20%
0	1.000	1.000
1	1.036	1.072
2	1.073	1.148
3	1.111	1.230
4	1.153	1.318
5	1.190	1.412
6	1.236	1.513
7	1.277	1.621
8	1.325	1.737

* Cost of capital = 12%

A geometric increase in risk is very often not appropriate to project evaluation. For example, the electric power firm cited earlier may know with relative certainty the eventual population which it will serve in fifteen years with its new nuclear power plant. On the other hand, it may know far less about the pattern of population growth in the next few years. In this example, risk is greater for the immediate future and actually declines in later years.

V

Risk Measured by the Reliability of Information

Although management's assumptions underlying the usual approach to risk recognition are somewhat misguided, its instinct to be explicit about the uncertainty of the outcome of most projects is absolutely sound. This instinct resists placing full corporate faith in the highly precise calculation methods we discussed earlier. The sophisticated management knows that the estimates, forecasts, and educated guesses supporting the precise calculations are themselves not so precise.

In defense of the cost engineer and financial analyst, who make a career of striving to estimate or forecast expected costs or revenues, such information probably does provide the most likely outcome for the specific item being estimated. But the most likely cost for a structure or the most likely price and volume realized on a new product are often only one outcome among many possible outcomes.

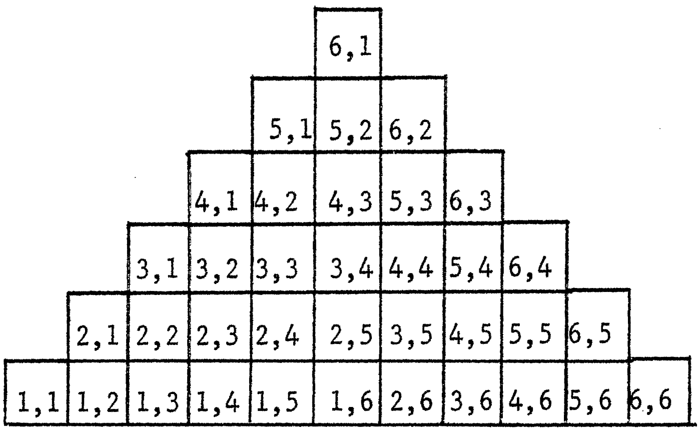
To illustrate, consider the rolling of a seven with a pair of dice.4/ Figure 5 shows that in thirty-six rolls, the probability of getting a seven is but one in six even though it is the most probable

4/ This illustration parallels that of David B. Hertz, "Risk Analysis in

Figure 5

Probability of Outcomes for Tossing

a Pair of Dice



Value of Toss:	2	3	4	5	6	7	8	9	10	11	12
Probability	1/32 .03	2/32 .06	3/32 .09	4/32 .13	5/32 .16	6/32 .19	5/32 .16	4/32 .13	3/32 .09	2/32 .06	1/32 .03

outcome. There is only a slightly smaller chance of rolling a six (for a little smaller payoff) or an eight (for a little higher payoff), values which are probably close enough in the context of estimating a project to keep the estimator on good terms with his boss. But the estimator lives in fear of calling a seven and having the toss of the dice come up a two, three, or four.

To carry the analogy a bit further, the estimator often knows in advance that there is a possibility, albeit remote (one chance in thirty-six), that a disastrous two will be the outcome.[5/] Normally, however, he is asked only for one most likely outcome or for his best guess. Thus, it seems unfair to place the entire blame on the estimator for the outcome of an occasional two.

Unfortunately, management too often considers only the most likely outcome and thereby suppresses information about the possibility that outcomes for values such as capital costs, operating costs, and product prices may vary considerably. In a real sense, then, management forfeits the opportunity to use possible variations in estimates as a representation of project risk.

Variability of outcomes as project risk

The reliability of estimates for any of the many factors that influence project profitability differs from project to project. For example, in the semi-tractor and tank wagon project we were able to

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5/ Presumably in some instances a twelve would be as fortuitous as the two is disastrous. However, a twelve estimate in competitive bidding might eliminate the estimator from competition.

estimate the truck driver's wages, fuel consumption, and, to some degree, maintenance costs with rather high accuracy. Better yet, we estimated with virtual certainty the volume of liquid product to be hauled because we knew a valid contract existed with the customer. On the other hand, in the motor oil additive project we may have made some wild guesses about new equipment costs, operating costs, and the size of the market because we were evaluating an unfamiliar product.

We can impute a subjective probability distribution of outcomes for each of the various factors entering these two projects. Figure 6 shows such distributions for driver's wages, fuel consumption, and maintenance in the semi-tractor case. Similarly, Figure 7 shows distributions for some of the critical factors affecting the oil additive case. These distributions may be imputed by estimating the probabilities of achieving certain discrete values such as the lowest, the most likely, the highest, and any intermediate value that seems clearly possible.

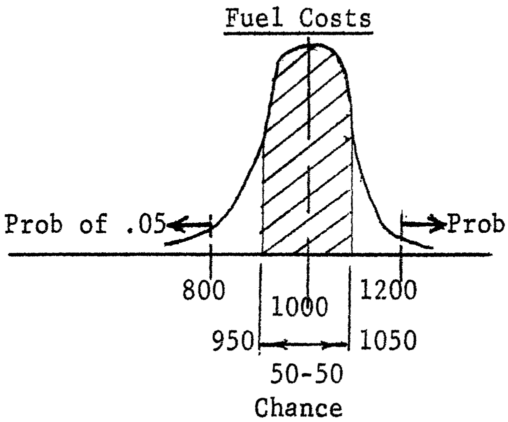
Suppose, for example, that the firm's transportation department calculated that fuel costs for the semi-tractor have a 50-50 chance of falling between $950 and $1050 per year. Furthermore, suppose they estimated that there is one chance in twenty of fuel costs being as low as $800 or as high as $1200. This probability distribution, shown in Figure 6, is tall and narrow and suggests that fuel costs may be determined with relative certainty.

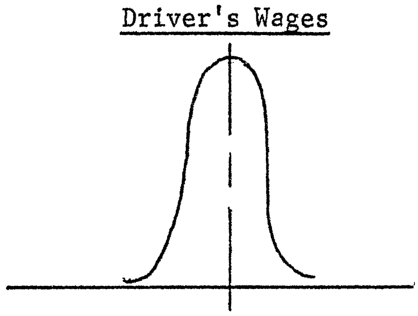
Contrast this distribution with the distribution imputed for pilot plant (first year only) production costs for the oil additive project in Figure 7. In this case, suppose plant management calculated

Figure 6

Probability Distributions for Some

Key Costs in Semi-Tractor Case





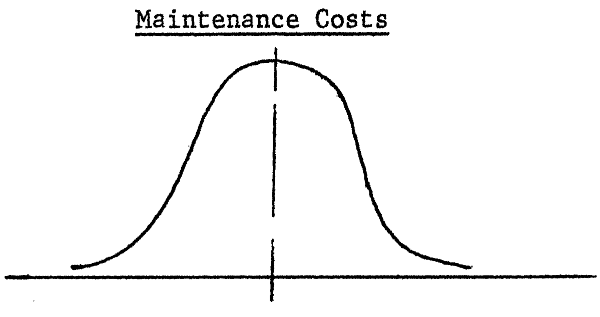
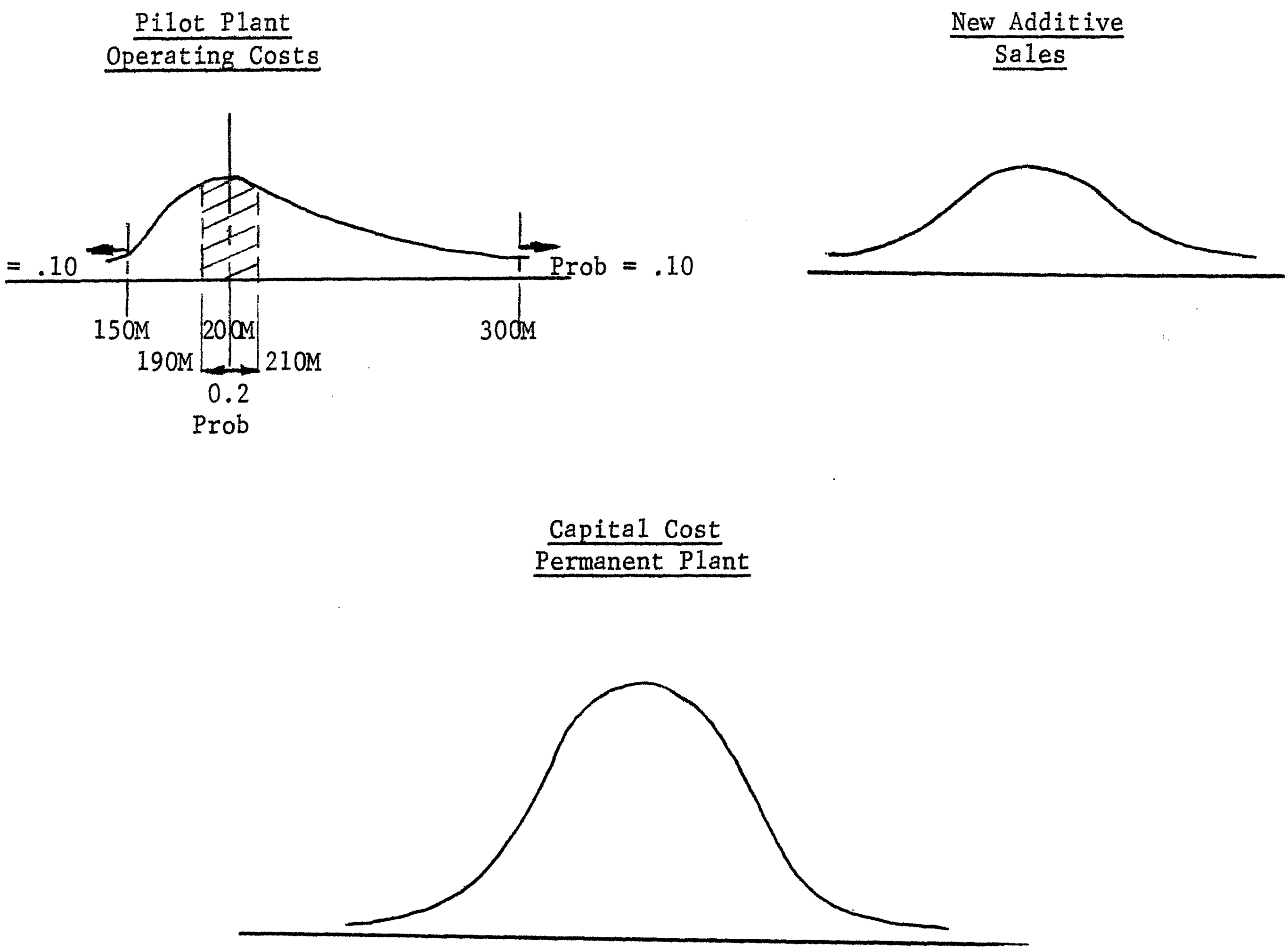


Figure 7

Probability Distributions for Some

Key Items in Oil Additive Case



that there are two chances in ten that production costs will fall between \$190,000 and \$210,000 for a year of pilot operations, and that there is one chance in ten that costs will be \$300,000 and one chance in ten that costs will be \$150,000. Obviously plant management believed that pilot plant operating costs could vary over a considerable range with a prospect that costs would be considerably higher than the most likely cost. Thus the probability distribution associated with pilot plant operating costs is flat and spread out.[6/] Similarly, the other factors affecting profitability for the motor oil additive project generally exhibit possible outcomes that are widely variable.

Ranking projects with two yardsticks

The probability distributions of factors affecting the cash flows of any project may be combined in a logical although somewhat complicated way as discussed in the final section of this paper. This combination produces two measures or yardsticks of expected project performance instead of the customary single measure. The first is the familiar expected benefit yardstick (TR, IRR, or NPV). The second is a measure of the expected variability of the benefit yardstick. The latter measure potentially goes a long way toward representing management's instinct for risk.

6/ In addition to the flattened spread, this particular distribution is skewed or asymmetrical with possible high-side outcomes being much larger than low-side outcomes. The skewness property could be important if it appeared consistently in project factors. For our purposes, we will assume essential symmetry throughout to avoid complicating our two-measure proposal with a third measure representing skewness.

The addition of variability to represent risk thus induces us to extend the usual single-measure analysis of capital investment economics to two measures. First, however, we need a simple parameter for expressing variability.

The basic statistical measure called variance, which measures the spread or dispersion of a probability distribution, is such a parameter.[7/] Its positive square root, called the standard deviation, perhaps has a more intuitively appealing meaning.

To illustrate the meaning of standard deviation let us refer to Figure 8A, which is a probability distribution of the modified internal rate of return (TR) developed by combining information on the variability and expected values of the various factors entering the oil additive project. The expected or most probable TR (or modified IRR) is 22.2 per cent, shown as point A. Standard deviation essentially gives us the horizontal distance of the center of gravity of the areas under the probability curve on either side of point A. This is distance AB in Figure 8A. Thus, this measure of the spread of the probability distribution tells us how close we can expect the actual TR to be to the most likely TR.

Compare the standard deviation shown in Figure 8A for the oil additive project with the standard deviation for the IRR of the semi-tractor project in Figure 8B. Statistical analysis tells us that the IRR (modified to TR) for the oil additive project has one chance in six of

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7/ Variance is the squared deviation of all possible outcomes about the mean (most likely outcome) weighted by the probabilities of the possible outcomes. We must deal with this second moment about the mean instead of the first moment. The latter is simply the sum of the positive and negative deviations about the mean and thus is equal to zero.

Figure 8A

Probability Distribution of Modified IRR (TR)

for Oil Additive Project

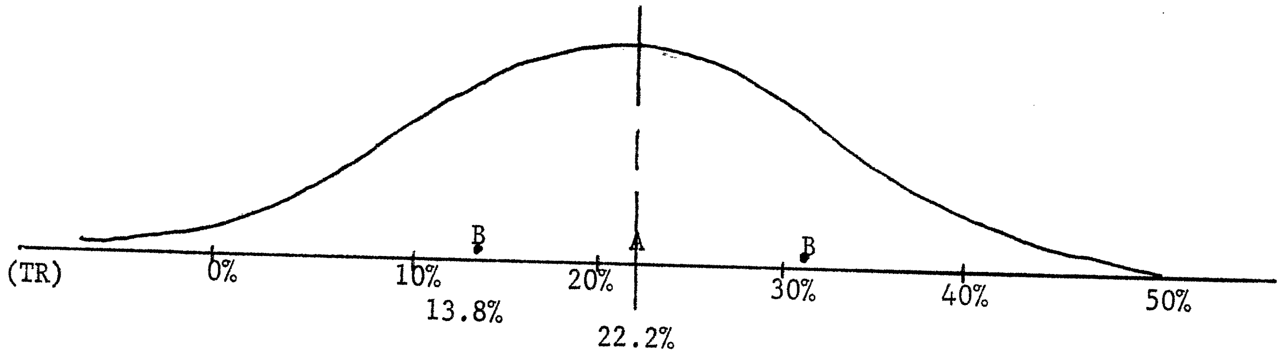
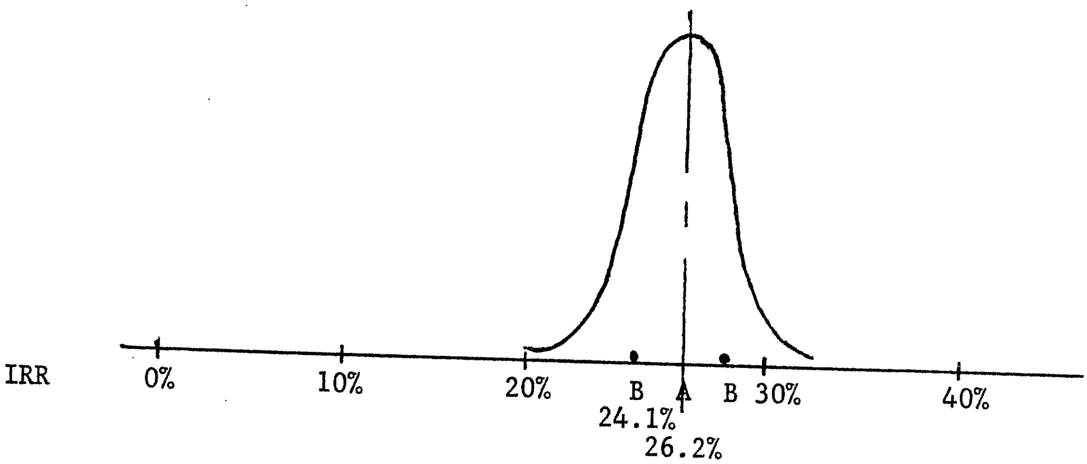


Figure 8B

Probability Distribution of IRR for

Semi-Tractor Project



falling below about 14 per cent. On the other hand, the IRR for the semi-tractor case has one chance in six of falling to only 24 per cent. It is precisely such a comparison that permits us to make quantified statements about the relative riskiness of these projects.

For example, suppose we calculate the two measures for our Projects I and II as follows:

Example A

Project	Most Probable IRR or TR (Percentage)	Variability (Risk) (Percentage)
I	26.2	8
II	22.2	38

Project I unambiguously dominates Project II, offering a higher most probable IRR or TR and a lower variability. But suppose the following were the case:

Example B

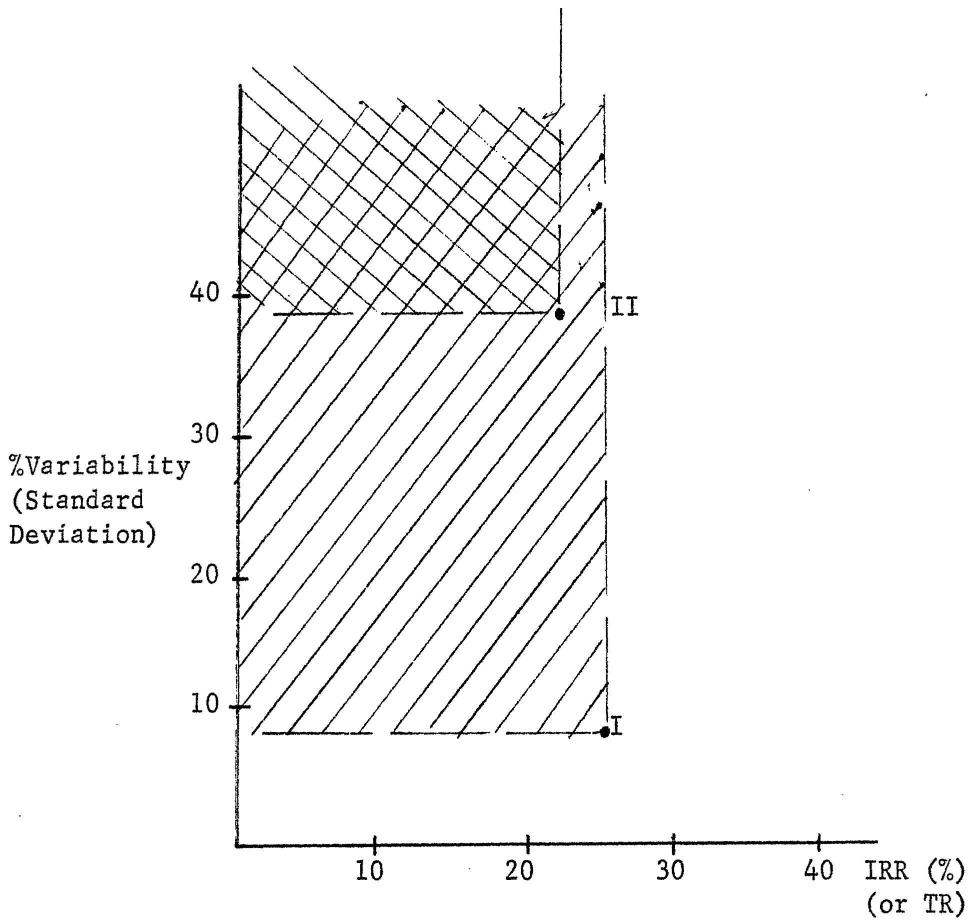
Project	Most Probable IRR or TR (Percentage)	Variability (Risk) (Percentage)
I	17.6	8
II	22.2	38

The decision as to which is the better project is no longer an unambiguous problem. It is now possible that Project II would be preferred by managements which are relatively insensitive to risk.

We can illustrate the two-measure evaluation problem graphically. In Figures 9 and 10 variability of outcome is plotted on the vertical axis against most probable outcome which is on the horizontal axis. From the criterion we stated above we can assert that any project on the graph dominates all projects to its northwest. Thus, the shaded area in

Figure 9

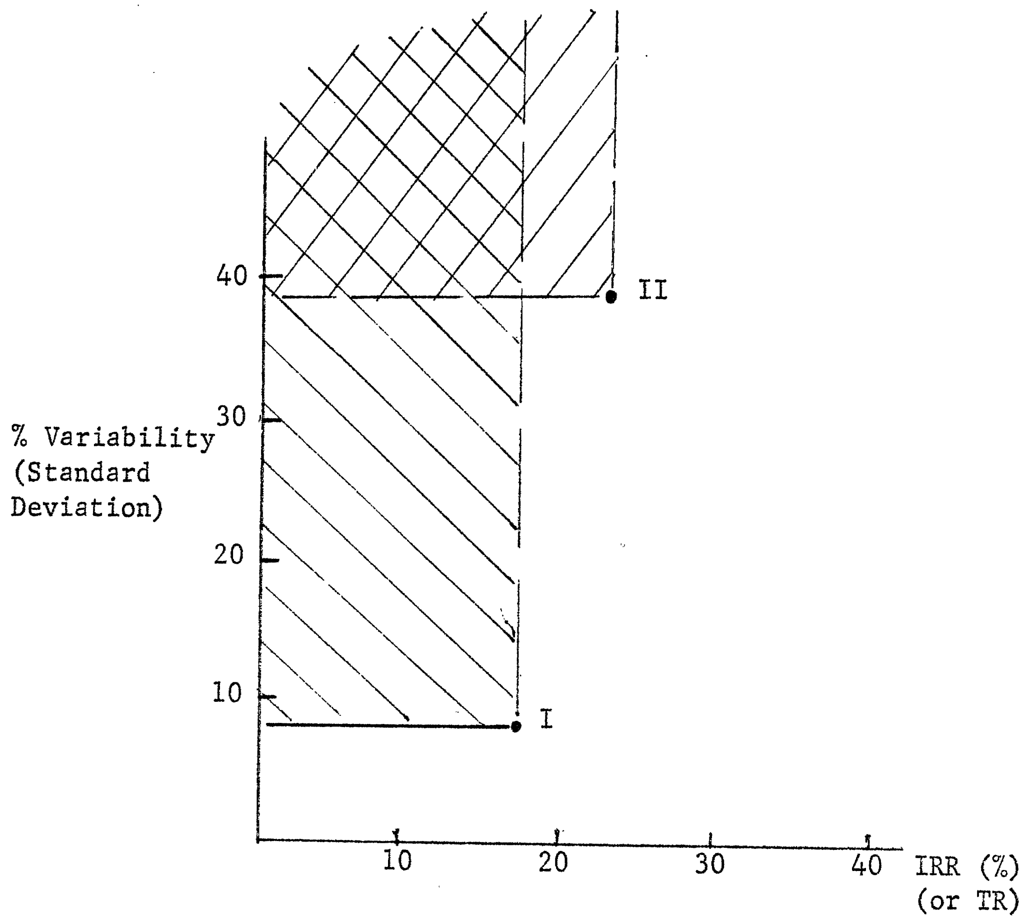
Comparing Projects in 2-Measures



Example A

Figure 10

Comparing Projects in 2-Measures



Example B

Figure 9 shows that Project 1 dominates Project II in the first example cited. However, the shaded areas in Figure 10 show that neither Project I nor Project II clearly dominate the other.

Conclusion

The significant advantage of the two-measure project evaluation over the single-measure approach is that in the former a representation of risk is introduced which reflects all of the available information about the project's uncertainty in a quantitatively correct manner. Although the common use of "fudge factors" in the single-measure approach to compensate for uncertainty reflects a commendable instinct for recognizing risk, these methods are often methodologically corrupt. More specifically, the degree of reliability of information and estimates related to the project is formally acknowledged in the two-measure method by expressing the spread of possible outcomes. But the single-measure approach either does not distinguish between the comparative reliability of information from project to project or else management distorts the information already embodied in the IRR or NPV calculation with unwarranted methods of adjusting for risk.

The omission of the degree of reliability of information when computing expected project returns can be costly. Management may unwisely favor a project with a highly uncertain outcome over a competing project with a quite certain outcome but slightly lower most probable return. Many managements, if not most, would willingly give up several IRR points to gain greater reliability.

A technical point must be clarified with respect to the derivation of a probability distribution of IRR, TR, NPV, or PI outcomes. As mentioned earlier, some means must be available to combine the probabilities of various factors affecting a project, e.g., probability distributions for sales, prices, capital costs, operating costs, and so forth.[8/] Fortunately, computers make it practical to generate a probability distribution of outcomes of a benefit measure such as IRR or NPV. Computer programs can randomly combine different possible outcomes of the distribution functions for the various factors entering the project. A probability distribution for IRR or NPV thus is generated by repeatedly computing the IRR or NPV outcomes associated with probabilistic combinations of factor outcomes. The two relevant measures of project performance, most probable outcome and variability of outcome, may be obtained directly from this distribution.

Finally, the rigor with which methods of project evaluation calculation are carried out often clouds the fact that the calculations themselves are only supported by probabilistic estimates and not by accomplished facts. It is hoped that the two-measure method will induce us all to discriminate among estimates according to their reliability.

8/ Notice, in addition, that some of these factors are interdependent, further complicating the analysis. For example, volume of sales probably depends largely on prices charged.

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