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Emissions Control and Compliance Strategies: A Permit Auction-Based Model

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Abstract

Despite the growing prevalence of permit auctions, research has thus far not delineated firms' abatement, bidding, and production strategies in conjunction with permit auctioning. To the regulator, knowledge about the interrelationships among regulator levers for emissions control and firm levers for compliance is crucial given the goals of pollution control and a desired increasing level of stringency in the stipulation of pollution limits. From a firm's perspective, decisions such as the amount and type of investment in pollution abatement, permit bidding strategy, and production level need to be made given a policy stipulation, the accompanying cost of compliance with the policy, and the goal of profit-maximization. We model a three stage game in an oligopoly - investment in abatement, followed by a share auction for permits and finally, the production of output. We treat two scenarios in the end product market - Independent Demands and Cournot Competition. In both scenarios we find that reducing the number of available permits induces firms in a dirtier industry to a lesser extent than firms in a cleaner industry to engage in abatement. In addition, abatement levels taper off with increasing industry dirtiness levels. In the presence of competition, firms in a relatively clean industry can indeed benefit from a reduction in the number of available permits. The modeling framework employed is not limited to our auction format choice. It is general enough for the simultaneous assessment of emissions control and compliance strategies within the broad domain of permit auctions.

1 Introduction

"Emissions trading" refers to a market-based mechanism for emissions control that allows parties to buy and sell permits for emissions or credits for reductions in emissions of certain pollutants. It differs from a traditional regulatory approach that relies solely on an agency, usually the government, to issue standards and specific directives on the amount by which emitters must reduce their emissions, how they must do so, and the penalties for failure (National Round Table on the Environment and the Economy (NRTEE), 2003). In an emissions trading program, emitters are allocated or permitted a limited amount of emissions. The total number of permits corresponds to

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the overall emissions target of the covered sources. The fact that the target is less than "business as usual" emissions creates permit scarcity, resulting in a market price for permits. Emitters are responsible for ensuring that they hold sufficient permits to offset their emissions; they have the flexibility to cost-effectively administer compliance levers such as investment in abatement, procurement of permits, and adjustment of output level.[1] The concept of emissions trading has grown from a theoretical curiosity into a central idea in environmental regulation (Muller & Mestelman, 1998). The theory of emissions trading is well developed (Tietenberg, 2001) and attention has now shifted from whether tradable emissions schemes should be implemented to how they should be implemented (Muller & Mestelman, 1998).

Auctions for emissions permits are prevalent and are implemented in various formats and under diverse settings. As mandated by Title IV of the 1990 US Clean Air Act Amendments, the US EPA has been conducting SO_2 allowance auctions since 1993 in a phased program of restricting emissions from fossil fuel-fired power plants. Economy-wide participants in the UK DEFRA[2] greenhouse gas emissions trading scheme have voluntarily taken on a legally binding obligation to reduce their emissions. The DEFRA conducted the world's first multi-sector auction for CO_2 allowances in March 2002. The Chicago Climate Exchange (CCX) is a greenhouse gas emissions reduction and trading pilot program for emission sources and offset projects in the United States, and for offset projects undertaken in Brazil. The CCX is a voluntary, self-regulatory exchange designed and governed by its members. The CCX conducted the world's first multi-national multi-sector CO_2 allowance auctions in September 2003. The auction formats employed in the aforementioned programs are, however, distinct. The US EPA auctions for SO_2 permits are sealed-bid discriminatory price auctions, the UK DEFRA auction was in the descending clock format, and the CCX auctions were held in two formats - sealed-bid average price and sealed-bid discriminatory price.

Despite the growing prevalence of permit auctions[3], research has thus far not delineated firms' abatement, bidding, and production strategies in conjunction with permit auctioning. Within the ambit of permit auctions, the regulator has a number of levers which it can work with to ensure that emissions remain within desired limits and that firms engage in adequate levels of abatement. Levers include the auction format, the number of permits offered in the auction, penalties for non-compliance, subsidies or tax breaks for investments in abatement by firms, the treatment of unused allowances, and permit allocation across industries. Firms, on the other hand, can comply with stipulated regulations by investing in pollution abatement, selecting the appropriate type of investment (e.g., end-of-pipe or in-pipe, product or process based), varying production levels, or purchasing permits. To the regulator, knowledge about the interrelationships among the aforementioned levers is crucial given the goals of pollution control and a desired increasing level of stringency in the stipulation of pollution limits. From a firm's perspective, decisions such as the amount and type of investment in pollution abatement, permit bidding strategy, and production

[1]William Rogers, Technological Specialist - Environmental Strategies, Detroit Edison Co., corroborates that these indeed constitute the levers that firms have (*Personal correspondence, October 2003*).

[2]Department for Environment, Food, and Rural Affairs.

[3]Many other countries are considering auctioning as a means of allocating permits in their planned emissions trading programs (*http://www.nrtee-trnee.ca/EmissionsTrading/en/overview_countries.htm*).

level need to be made given a policy stipulation, the accompanying cost of compliance with the policy, and the goal of profit-maximization.

We model a three stage game in an oligopoly - investment in abatement, followed by an auction for permits and finally, the production of output. We model the auction for permits as a sealed-bid uniform price share auction where the bidders submit a schedule of prices for varying fractional shares of the block of permits being auctioned and receive shares at a sale price that equates the demand and supply of shares. We draw from the seminal work of Wilson (1979) for analysis of the share auction. Though permits are sold in discrete units, the representation of the auction for emissions permits as a share or divisible-good auction is apt since permits are homogeneous and the total number of auctioned permits is generally large[4]. Typically, governments use a sealed-bid auction to allocate multiple units of homogenous units such as treasury securities and emissions permits (Sunnevåg, 2001). However, there has been substantial debate on whether the discriminatory price format or the uniform price format is superior (Bikhchandani & Huang (1989, 1993), Back & Zender (1993), Daripa (2001), Wang & Zender (2002)). Our modeling choice of a uniform price auction is driven by reasons of analytical tractability.[5] The modeling framework employed is, however, not limited to our auction format choice. It is general enough for the simultaneous assessment of emissions control and compliance strategies within the broad domain of permit auctions.

We treat two scenarios in the end product market - Independent Demands and Cournot Competition. In the case of Independent Demands, firms do not compete for end customer demand (e.g., if the firms serve distinct geographic regions). In the case of competition, firms compete for end customer demand. In both cases, however, firms do compete for scarce emissions permits. In both scenarios we find that reducing the number of available permits induces firms in a "dirtier"[6] industry to a lesser extent than firms in a cleaner industry to engage in abatement. In addition, abatement levels taper off with increasing industry dirtiness levels. In the presence of competition, firms in a relatively clean industry can indeed benefit from a reduction in the number of available permits. Our findings are robust to changes in the modeling of abatement and the cost of production.

To the best of our knowledge, ours is the first framework that provides for a simultaneous assessment of emissions control and compliance strategies within the domain of permit auctions. Concomitantly, we also contribute to auction theory by deriving an equilibrium bidding strategy for a uniform-price share auction in which the (homogeneous) items have decreasing marginal value. Lyon (1986) examines equilibrium properties of a range of auctions and other procedures for allocating transferable permits. The focus is on transfer-neural[7] mechanisms that allocate permits efficiently. Firm profits are associated with firm "types" with no modeling of production relationships or abatement. Since early 1995, the MIT Center for Energy and Environmental Policy Research (CEEPR) has contributed greatly to public understanding of emissions trading through its

[4]125,000-150,000 in EPA auctions of SO_2 permits (*http://www.epa.gov/airmarkets/auctions/factsheet.html*).

[5]It is worth mentioning that, because of analytical intractability, there are scarcely any existing results on the characterization of equilibria in divisible good auctions (Hortaçsu, 2002).

[6]We quantitatively describe "dirty" and "clean" in subsequent sections.

[7]Implying that no net revenue is generated for the seller.

definitive study of the implementation of the U.S. Acid Rain Program, including the EPA auctions for SO_2 permits. Working papers published by the center are primarily empirical.[8] Laffont & Tirole (1996a) study the impact of spot and futures markets for tradable pollution permits on potential polluters' compliance decisions. Polluters can buy permits, invest in pollution abatement, or else stop production and source out. Stand alone spot markets induce excessive investment. The introduction of a futures market reduces this incentive to invest. They extend their analysis in a closely related paper (Laffont & Tirole, 1996b) with the revised assumption that innovation is a public good - invention of substitutes or pollution abatement devices can be used by all other agents. They find that options to pollute at a given striking price fare better than pollution allowances from a social welfare point of view. However, they treat the decision of investment in abatement, as well as the choice between investment in abatement and production as binary when, in fact, these exist as continuua. In addition, they do not model permit auctions. Unold & Requate (2001) propose a combination consisting of free permits and a menu of call options when there is imperfect information about aggregate abatement costs so that the regulator can approximate the marginal damage function. The authors, however, do not model permit auctions, nor do they explicitly model abatement and production relationships. Sunnevåg (2001) evaluates the pros and cons of two competing permit auction designs - the standard ascending-clock auction and an ascending-clock implementation of Vickrey pricing - where the allocation of permits has consequences on the level of production as well as on market shares. The paper is understandably limited in analytical findings and uses numerical approaches to provide insights.

The remainder of this paper is organized as follows. Section 2 describes the model. Sections 3 and 4 respectively treat the two end product market scenarios - Independent Demands and Cournot Competition. Section 5 extends the analysis to situations when investments in abatement affect the cost of production. Section 6 extends the analysis to a variation on the specification of the emissions function. A numerical example is presented in Section 7, and implications for regulators and firms are discussed in Section 8. We conclude and provide directions for future research in Section 9.

2 The Model

We model the problem as a three stage game. In the first stage, each of the $n \geq 2$ firms decides its abatement level D_i from an investment $g_i(D_i)$ in pollution mitigating innovations, where g_i (henceforth called the abatement function) is increasing in D_i. In the second stage, firms bid for emissions permits in a sealed-bid uniform price share auction. Each firm submits a sealed tender specifying a schedule of prices bid for varying fractional shares of the available pollution allowances. An alternative, equivalent format is a schedule that for each possible price, specifies the share requested. The regulator then selects the sales price such that the total share requested by the bidders matches the available supply (i.e., unity). As in Wilson (1979), we assume that the number of bidders is known beforehand by all participants to be n. Also, we only consider situations in which

[8] MIT CEEPR working papers are available at *http://web.mit.edu/ceepr/www/workingpapers.htm.*

the bidders are symmetric, and where the optimal strategy is a symmetric subgame perfect Nash equilibrium. The auction results in a share allocation vector $s = (s_1, s_2, \ldots, s_n)$, or equivalently a permit allocation vector $\beta = (\beta_1, \beta_2, \ldots, \beta_n)$. $B = \sum_{i=1}^{n} \beta_i$, is the total permissible pollution level, where $\beta_i = s_i \cdot B$. Each permit allows a firm to emit a unit of the pollutant. Each firm pays e per emissions permit. In the third stage, firm i produces output y_i which results in a pollution level $\theta_i(y_i, D_i)$, and firms redeem their available allowances against their pollution levels. The unit cost of production is c_i. We consider two distinct demand situations. In the independent demands case, each firm faces an independent, inverse demand function $p_i = h_i(y_i)$, where h_i is decreasing in y_i. In the case of competition, the n firms compete in a Cournot fashion and face an inverse total demand function $P = H(Y)$ where $Y = \sum_{i=1}^{n} y_i$, and H is decreasing in Y.

In practice, unused permits can be banked for future use or trade. Since we treat a single-period problem, we assume a terminal or salvage value of u_i to firm i per unused permit. This value could represent either the value of a permit in a secondary market or the net present value of benefits accruing from future use of a permit left over at the end of the first period. We assume that the penalty for not having the requisite number of permits to account for emissions is large enough so that non-compliance is deterred.[9] θ_i (henceforth called the emissions function) is assumed to be separable in y_i and D_i. In other words, θ_i can be represented as $\theta_i = f_i(y_i) - D_i$ where f_i (henceforth called the Production Emissions (PE) function) is increasing in y_i.[10] The optimization problem of each firm is: *Maximize* $\Pi_i = p_i y_i + (\beta_i - \theta_i)u_i - g_i(D_i) - e\beta_i - c_i y_i; \quad i = 1, \ldots, n$; *Subject to:* $\theta_i \leq \beta_i$, $y_i, D_i \geq 0$. The constraint implies that each firm must have sufficient permits to account for emissions. We proceed conventionally, by backward induction, to ascertain the subgame perfect equilibria in the different stages of the game.

For the remainder of this paper we assume a linear emissions function $\theta_i(y_i, D_i) = \alpha y_i - D_i$ and linear downward sloping demand functions; $p_i = a_i - b_i y_i$ in the independent demands case and $p = \hat{a} - \hat{b} \sum_{i=1}^{n} y_i$ in the case of competition $(\alpha, a_i, b_i, \hat{a}, \hat{b} > 0)$. We drop the subscript i where unambiguous, for notational and typographic convenience.

3 Independent Demands

We begin by analyzing the independent demands case. The optimization problem of a representative firm in the third stage of the game, given abatement levels and a permit allocation vector is: *Maximize* $_{\{y\}}$ $\Pi = (a - by)y + (\beta - (\alpha y - D))u - g(D) - e\beta - cy$; *Subject to:* $\theta \leq \beta$, $y \geq 0$.

[9]According to the EPA Acid Rain Program 2001 Progress Report, a source that does not hold enough allowances in its unit account to cover its annual SO_2 emissions has "excess emissions" and must pay a \$2000 (in 1990 dollars) per-ton penalty. The \$2000 penalty is adjusted annually for inflation, so the year 2001 penalty was \$2774. Only two firms were short by a total of 11 allowances to cover their emissions for the 2001 compliance year.

[10]In Section 6 we test the validity of our results with an emissions function in which the abatement level depends upon the production output. The structural attributes of our pertinent results remain unchanged.

3.1 Output Subgame

Concavity of the profit function permits us to arrive at the optimal production quantity in the third stage of the game. The profit-maximizing quantity is limited by imposition of the constraint that emissions cannot exceed the number of permits available. The following proposition states this formally.

Proposition 1 *Assume $u \leq \frac{a-c}{\alpha}$. In the third stage, given abatement levels and a permit allocation vector, the profit-maximizing quantity $y^* = \min\left\{\frac{a-c-\alpha u}{2b}, \frac{\beta+D}{\alpha}\right\}$.*

Proof: See Appendix A.

$(\beta+D)$ can be interpreted as a measure of permissible pollution as a result of an abatement level of D chosen in the first stage and β permits procured in the second stage. Since each production unit results in a pollution level of α, $\frac{(\beta+D)}{\alpha}$ represents the maximum production quantity that keeps emissions within the allowable limit. $(c + \alpha u)$ can be interpreted as the marginal cost per unit of production, which includes the variable cost c and the "opportunity cost" αu since salvage value equal to αu is lost as a result of producing one unit and emitting α units of pollutant. The unconstrained profit-maximizing quantity is $\frac{a-c-\alpha u}{2b}$ which should not exceed $\frac{(\beta+D)}{\alpha}$. Hence the resulting expression for y^*. The assumption $u \leq \frac{a-c}{\alpha}$ has a useful interpretation. The maximum incremental revenue that can be earned from the ownership of a permit is $\frac{a-c}{\alpha}$. If u exceeds this bound then there is no incentive to engage in production in the third stage and firms would trivially salvage pollution permits.

The constraint on θ can be re-written in terms of β since, when $\frac{\beta+D}{\alpha} < \frac{a-c-\alpha u}{2b}$, the constraint on θ is binding, and vice-versa. Denote $\bar{\beta} = \frac{\alpha(a-c-\alpha u)}{2b} - D$. Thus, when $\beta < \bar{\beta}$, the constraint on θ is binding. We now establish expressions for the optimal profit for the two cases viz., $\beta < \bar{\beta}$ and $\beta \geq \bar{\beta}$.

Case (i): $\beta < \bar{\beta}$ (or equivalently, $y^* = \frac{\beta+D}{\alpha}$)

$$\begin{aligned} \Pi_C^* &= \left[a - b(\frac{\beta+D}{\alpha})\right](\frac{\beta+D}{\alpha}) + \left[\beta - \left(\alpha(\frac{\beta+D}{\alpha}) - D\right)\right]u - g(D) - e\beta - c(\frac{\beta+D}{\alpha}) \\ &= -\frac{b}{\alpha^2}\beta^2 + \left[\frac{\alpha(a-c-\alpha e) - 2bD}{\alpha^2}\right]\beta + \left[\frac{\alpha[(a-c)D - \alpha g(D)] - bD^2}{\alpha^2}\right] \qquad (1) \end{aligned}$$

Case (ii): $\beta \geq \bar{\beta}$ (or equivalently, $y^* = \frac{a-c-\alpha u}{2b}$)

$$\begin{aligned} \Pi_U^* &= \left[a - b(\frac{a-c-\alpha u}{2b})\right](\frac{a-c-\alpha u}{2b}) + \left[\beta - \left(\alpha(\frac{a-c-\alpha u}{2b}) - D\right)\right]u - g(D) - e\beta \\ &\quad -c(\frac{a-c-\alpha u}{2b}) \\ &= \psi + \beta(u-e) \qquad (2) \end{aligned}$$

Π_C^* represents the subgame perfect equilibrium profit when the emissions constraint is binding, Π_U^* is the subgame perfect equilibrium profit when the emissions constraint is not binding, and $\psi = [a - b(\frac{a-c-\alpha u}{2b})](\frac{a-c-\alpha u}{2b}) - [\alpha(\frac{a-c-\alpha u}{2b}) - D]u - g(D) - c(\frac{a-c-\alpha u}{2b})$ is a constant, given the abatement level chosen in the first stage.

3.2 Auction Subgame

The assumption that firms are constrained in profit-maximization by the availability of emissions implies that the optimization problem of a representative firm is:
Maximize $_{\{y\}}$ $\Pi = (a - by)y + (\beta - (\alpha y - D))u - g(D) - e\beta - cy$; *Subject to:* $\theta = \beta$, $y = \frac{\beta+D}{\alpha}$.

Since the emissions constraint binds, the marginal value of a permit is the shadow price corresponding to the emissions constraint. We construct a "marginal value function" $v(\beta)$ using equations (1) and (2). $v(\beta)$ is the marginal revenue from a permit when β permits are held. When $\beta < \bar{\beta}$,

$$\begin{aligned} v(\beta) &= -\frac{2b}{\alpha^2}\beta + [\frac{\alpha(a-c) - 2bD}{\alpha^2}] \\ &= \sigma - \lambda\beta \end{aligned} \tag{3}$$

When $\beta \geq \bar{\beta}$,

$$v(\beta) = u \tag{4}$$

Where $\sigma = [\frac{\alpha(a-c)-2bD}{\alpha^2}]$ and $\lambda = \frac{2b}{\alpha^2}$ are constants, given the abatement level chosen in the first stage. Note that $\sigma - \lambda\bar{\beta} = u$, and when $\beta < \bar{\beta}$, $v(\beta) > u$. The case when $\beta \geq \bar{\beta}$, is trivial because the emissions constraint does not bind and firms can therefore achieve unconstrained profit-maximization. In other words, firms' production decisions are unaffected by the availability of permits. The value then placed on a permit is simply the salvage value which can be obtained in the third stage of the game. We therefore assume for the remainder of the paper, that firms operate in the region defined by $\beta_i < \bar{\beta}_i$ $\forall i$ and that their beliefs are also restricted to this region. Hence the value function is restricted to be $v(\beta) = \sigma - \lambda\beta$. We can rewrite $v(\beta) = [\frac{(a-c)}{\alpha} - \frac{2b}{\alpha^2}D] - \frac{2b}{\alpha^2}\beta$. $[\frac{(a-c)}{\alpha} - \frac{2b}{\alpha^2}D]$ is the marginal value of a permit when the abatement level is D and no permits are held. As the number of permits held (β) increases we get closer to the unconstrained profit-maximizing quantity. The marginal value of a permit therefore decreases; the decrease is captured by the term $\frac{2b}{\alpha^2}\beta$.

Since our focus is on the constrained emissions permits case, $\bar{\beta}$ can be interpreted as a measure of the extent to which emissions permits constrain profit maximization. A lower value of D drives β to be less than $\bar{\beta}$ and results in emissions permits being a stiffer constraint for profit maximization in the third stage of the game. The condition $\beta < \bar{\beta}$ can be translated into an equivalent condition for n. Lemma 1 shows that the subgame perfect equilibrium share is $\frac{B}{n}$. $\frac{B}{n} < \bar{\beta}$ implies $n > \frac{2bB}{\alpha(a-c-\alpha u)-2bD}$. In other words, the number of firms n is sufficiently large so that firms' profit maximization in the third stage of the game is constrained by the availability of permits.

Marginal Value Function

In auctions literature, the value placed by a bidder on the item(s) being auctioned is typically specified exogenously or is assumed to be drawn from an exogenous distribution, rather than derived explicitly from the eventual use of the item(s). In contrast, we derive a representative firm's marginal value function for permits from revenue and cost relationships and use it to establish a

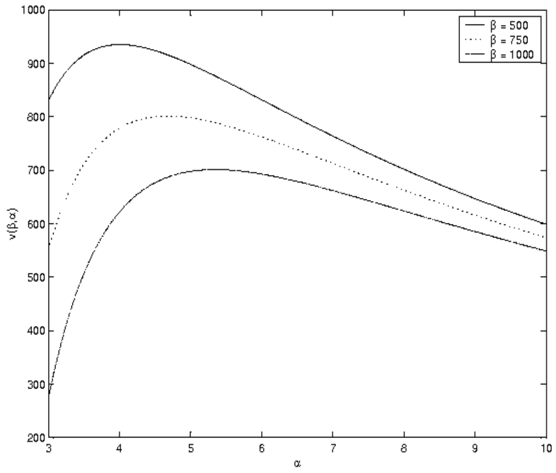

Figure 1: Marginal value of a permit, $v(\beta, \alpha)$, versus α

subgame perfect Nash equilibrium in the share auction. We elaborate on the properties of the marginal value function since our main results hinge on its specification and behavior.

$v(\beta, c)$ decreases in c which is the unit cost of production. $\frac{\partial[v(\beta,c)]}{\partial c} = -\frac{1}{\alpha} < 0$. For lower cost of production, the contribution per production unit is higher and hence the higher value per emissions permit. $v(\beta, D)$ decreases in D - the abatement level in the first stage. $\frac{\partial[v(\beta,D)]}{\partial D} = -\frac{2b}{\alpha^2} < 0$. For higher abatement levels, emissions are lower and hence the lower corresponding value per emissions permit. $v(\beta, b)$ decreases in b - the price sensitivity of demand. $\frac{\partial[v(\beta,b)]}{\partial b} = -\frac{2(\beta+D)}{\alpha^2} < 0$. For higher price sensitivity, the drop in price from an additional unit of production results in a lower value per emissions permit. The dependence on $v(\beta, \alpha)$ on α - the coefficient in the PE function - is not so straightforward. $\frac{\partial[v(\beta,\alpha)]}{\partial \alpha} = \frac{4b(\beta+D)}{\alpha^3} - \frac{(a-c)}{\alpha^2}$. This is > 0 when $4b(\beta + D) > \alpha(a - c)$, and < 0 when $4b(\beta + D) < \alpha(a - c)$. We know that when $\beta < \bar{\beta}$, $v(\beta) > u \geq 0$; i.e., $-\frac{2b}{\alpha^2}\beta + [\frac{\alpha(a-c)-2bD}{\alpha^2}] > 0$, or $\alpha(a - c) > 2b(\beta + D)$. Thus, when $\frac{2b(\beta+D)}{(a-c)} < \alpha < \frac{4b(\beta+D)}{(a-c)}$, we have $\frac{\partial[v(\beta,\alpha)]}{\partial \alpha} > 0$, and when $\alpha > \frac{4b(\beta+D)}{(a-c)}$, we have $\frac{\partial[v(\beta,\alpha)]}{\partial \alpha} < 0$.[11] Figure 1[12] shows the behavior of $v(\beta, \alpha)$ with respect to α. In the range $[\frac{2b(\beta+D)}{(a-c)}, \frac{4b(\beta+D)}{(a-c)}]$, the benefit of expanded production possibility from additional permits increases in α. But beyond this range, additional permits do not significantly expand the production possibility because emissions per production unit are relatively high; for $\alpha > \frac{4b(\beta+D)}{(a-c)}$, additional permits yield decreasing value.

As in Wilson (1979) we assume that no firm has any proprietary information about the demand, emissions, and cost functions, and therefore, about the derived marginal value functions. Symmetry implies that firms have a common marginal value function for permits. We can now work backwards to the second stage where firms participate in a sealed-bid uniform price share auction for pollution permits. Each firm submits a sealed tender specifying a schedule of prices bid for varying fractional shares of the block B of available pollution allowances. An alternative, equivalent format is a schedule that for each possible price, specifies the share requested. The regulator then selects the

[11] $\frac{\partial[v(\beta,\alpha)]}{\partial \alpha} = 0$ when $\alpha = \frac{4b(\beta+D)}{(a-c)}$.

[12] Parameter values were $a = 7,500$, $b = 5$, $c = 10$, $D = 1000$.

sales price such that the total share requested by the bidders matches the available supply. Lemma 1 provides a very elegant and useful result.

Lemma 1 *For the share auction in the second stage, it is an optimal strategy to submit a schedule such that at each price e, the requested number of permits is $\beta(e) = (\frac{1-2e/(n\sigma-\lambda B)}{n-1})B$.*[13] *The subgame perfect equilibrium price is $e^* = \frac{1}{2}(\sigma - \lambda\frac{B}{n})$, and the subgame perfect equilibrium number of permits received by each firm is $\beta^* = \frac{B}{n}$.*[14]

Proof: The proposed equilibrium satisfies $n \cdot \beta(e^*) = B$. We show that if the $(n-1)$ other bidders submit the schedule $\beta(e)$ then it is optimal for the remaining bidder to also submit the same schedule. Assume that the $(n-1)$ bidders submit the schedule $\beta(e)$ and the remaining bidder submits some schedule $\tau(e)$. The clearing price e^* satisfies

$$\tau(e^*) = B - (n-1)\beta(e^*)$$

and the remaining bidder's profit is

$$\begin{aligned}
\left[\int_0^{\tau(e^*)} (\sigma - \lambda z)\, dz\right] - e^* \cdot \tau(e^*) &= (\sigma z - \lambda\frac{z^2}{2})\Big|_0^{[\frac{2e^*B}{n\sigma-\lambda B}]} - e^* \cdot [\frac{2e^*B}{n\sigma - \lambda B}] \\
&= \sigma[\frac{2e^*B}{n\sigma - \lambda B}] - 2\lambda[\frac{e^*B}{n\sigma - \lambda B}]^2 - [\frac{2e^{*2}B}{n\sigma - \lambda B}] \qquad (5)
\end{aligned}$$

The function on the right hand side of equation (5) is concave in e^* and is maximized with respect to e^* when

$$2\sigma B(n\sigma - \lambda B) - 4\lambda B^2 e^* - 4B(n\sigma - \lambda B)e^* = 0; \ n\sigma \neq \lambda B$$

i.e., when $e^* = \frac{1}{2}(\sigma - \lambda\frac{B}{n})$. This is exactly the price that will result if the remaining bidder submits the schedule $\beta(e)$. Therefore the optimal strategy is to submit a schedule such that at each price e, the requested number of permits is $\beta(e) = (\frac{1-2e/(n\sigma-\lambda B)}{n-1})B$. The subgame perfect equilibrium price is $e^* = \frac{1}{2}(\sigma - \lambda\frac{B}{n})$, and the subgame perfect equilibrium number of permits received by each firm is $\beta^* = \frac{B}{n}$. ∎

We have, in fact, proved an important generalization of Wilson's (1979) result. In Wilson (1979), the value of a share is proportional to the share fraction. In other words, the value of a share is the value of the entire block multiplied by the share fraction. But, in practice, it is more likely that the marginal value of the item being auctioned decreases with the share fraction. Lemma 1 generalizes Wilson's result to the case when the marginal value of the item being auctioned decreases linearly with the share fraction.

3.3 Abatement Subgame

We can now work backwards to the first stage. Substituting the subgame perfect equilibrium permit price and permit share from Lemma 1 into the profit function in equation (1), we have

$$\Pi_C^* = -\frac{bB^2}{\alpha^2 n^2} + \left[\alpha\left(a - c - \alpha\left(\frac{\alpha\,(a-c) - 2\,bD}{2\alpha^2} - \frac{bB}{\alpha^2 n}\right)\right) - 2\,bD\right]\frac{B}{\alpha^2 n}$$

[13] Equivalently, the schedule for the fraction of total available permits is $s(e) = (\frac{1-2e/(n\sigma-\lambda B)}{n-1})$.

[14] Though this was the only subgame perfect equilibrium we could deduce, we do not guarantee uniqueness.

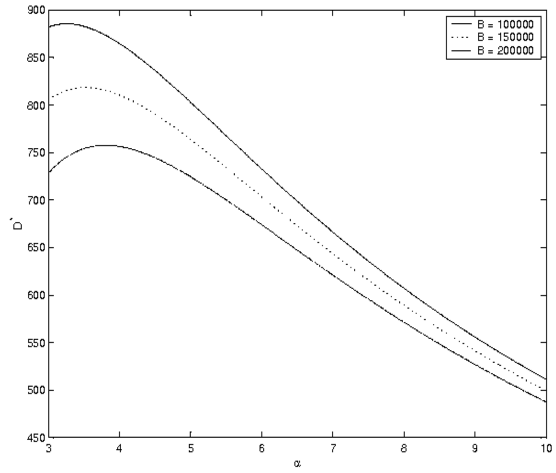

Figure 2: Equilibrium abatement (D^*) versus α

$$+\frac{\alpha\left[(a-c)\,D-\alpha g(D)\right]-bD^2}{\alpha^2} \tag{6}$$

$$= -\frac{bD^2}{\alpha^2}+\left(\frac{a}{\alpha}-\frac{c}{\alpha}-\frac{bB}{\alpha^2 n}\right)D-g(D)+\frac{B}{2\alpha n}(a-c)$$

Thus far we did not need to assume a specific form for the abatement function $g(D)$. For the remainder of this paper we assume an abatement function of the form $g(D)=\xi D^2$, where $\xi>0$. Such a specification implies that it is increasingly expensive to engage in abatement.[15]

Thus,

$$\Pi_C^* = -\frac{(b+\xi\alpha^2)}{\alpha^2}D^2+\left(\frac{a}{\alpha}-\frac{c}{\alpha}-\frac{bB}{\alpha^2 n}\right)D+\frac{B}{2\alpha n}(a-c) \tag{7}$$

Finally, we need to deduce the profit-maximizing abatement level. Proposition 2 shows that the profit function in the first stage of the game, after incorporating the results from the third and second stages, is concave in the abatement level.

Proposition 2 *The profit function Π_C^* in (7) is concave in D. $D^*=\frac{\alpha}{2(b+\xi\alpha^2)}(a-c-\frac{bB}{\alpha n})$ uniquely maximizes Π_C^*.*

Proof: See Appendix A.

Table 1 summarizes the subgame perfect equilibrium results for the independent demands case, in terms of the parameters of our model. The protagonists of our model are the number of permits B offered in the auction, the state of current technology (which defines the emissions α per production unit), and the subgame perfect equilibrium values of investment D^*, permit price e^*, output y^*, and profit Π_C^*. Proposition 3 formally describes pertinent relationships among these elements.

[15] Kennedy (2002), and Parry & Toman (2002) model similar abatement functions.

Table 1: Equilibrium Results: Independent Demands

	Symbol	Expression
Abatement	D^*	$\frac{\alpha}{2(b+\xi\alpha^2)}(a-c-\frac{bB}{\alpha n})$
Permit Price	e^*	$\frac{\xi\alpha}{2(b+\xi\alpha^2)}(a-c-\frac{bB}{\alpha n})-\frac{bB}{2\alpha^2 n}$
Permit Share	β^*	$\frac{B}{n}$
Output	y^*	$\frac{1}{2(b+\xi\alpha^2)}(a-c-\frac{bB}{\alpha n})+\frac{B}{\alpha n}$
Profit	Π_C^*	$\frac{1}{4(b+\xi\alpha^2)}(a-c-\frac{bB}{\alpha n})^2+\frac{B(a-c)}{2\alpha n}$

Proposition 3

a. D^* decreases linearly in B; e^* decreases linearly in B; y^* increases linearly in B; Π_C^* increases convexly in B.

b. $\exists\ \alpha_D$ such that D^* increases in α for $\alpha < \alpha_D$, and decreases in α for $\alpha > \alpha_D$; y^* decreases in α; Π_C^* decreases in α.[16]

c. $\frac{\partial^2 D^*}{\partial B\ \partial\alpha} > 0$; $\frac{\partial^2 e^*}{\partial B\ \partial\alpha} > 0$; $\frac{\partial^2 y^*}{\partial B\ \partial\alpha} < 0$; $\frac{\partial^2 \Pi_C^*}{\partial B\ \partial\alpha} < 0$ for $\alpha > \sqrt{\frac{b}{\xi}}$.[17]

Proof: See Appendix A.

The behavior of equilibrium abatement D^*, equilibrium permit price e^*, equilibrium output y^*, and equilibrium profit Π_C^* with respect to the total number of permits B, is intuitive. As the total number of permits is increased, the equilibrium number of permits secured in the auction by each firm increases - which expands the production possibility and results in an increase in equilibrium output. The equilibrium permit price drops since the marginal value placed by firms on permits decreases in the number of permits secured. And when more permits are secured, a greater level of emissions is allowed, which diminishes the incentive to engage in abatement and hence the equilibrium abatement drops.

Figures 2 and 3[18] depict the relationships proved in Proposition 3. Figure 2 shows how the equilibrium abatement varies with the PE function parameter (α) for different values of the total number of permits. The equilibrium abatement first increases in and thereafter tapers off with respect to α. In addition, the equilibrium abatement is decreasingly influenced by variations in the total number of permits. The behavior is counterintuitive for two reasons. Firstly, it might be expected that when emissions are excessive, the incentive to engage in abatement should be greater. Secondly, when emissions are excessive, reducing the number of available permits should induce at least as much or more abatement than in the case when emissions are lower. This seemingly

[16] $\alpha_D = \frac{b\xi B+\sqrt{(b\xi B)^2+b\xi n^2(a-c)^2}}{\xi n(a-c)}$. Numerical studies suggest that $\exists\ \alpha_e$ such that for $\alpha < \alpha_e$, $\frac{\partial e^*}{\partial\alpha} > 0$, and for $\alpha > \alpha_e$, $\frac{\partial e^*}{\partial\alpha} < 0$. α_e is, analytically, the positive real root of $(a-c)n\xi^2x^5 - 4bB\xi^2x^4 - (a-c)bn\xi x^3 - 4b^2B\xi x^2 - 2b^3B = 0$.

[17] The proposition as stated suffices to show that for large values of α the equilibrium profit is decreasingly influenced by variations in B.

[18] Parameter values were $a = 7,500$, $b = 5$, $c = 10$, $u = 200$, $\xi = 0.65$, $n = 150$.

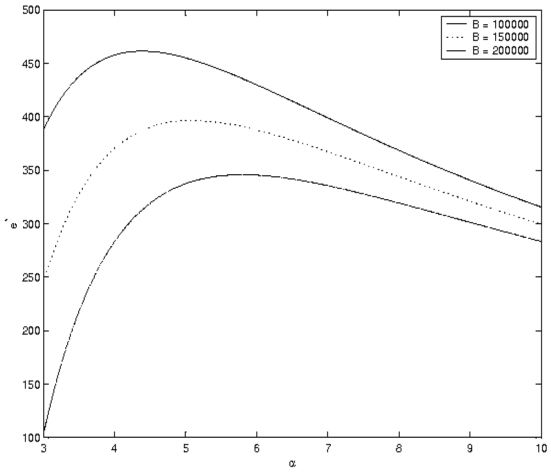

Figure 3: Equilibrium permit price (e^*) versus α

counterintuitive behavior is related to the valuation of permits described in Figure 1. For higher[19] values of α, the benefit from the expanded production possibility decreasingly outweighs the cost of permits. At an excessive emissions level, the production quantity is already limited. Expanding the number of available permits or investing in abatement at a very high emissions level does not yield as much benefit as when the emissions level is low. Knowledge about such behavior is crucial if the regulator desires that firms engage in sufficient levels of abatement to enable an increasing level of stringency in pollution norms.

Reducing the number of available permits induces firms in a dirtier industry to a lesser extent than firms in a cleaner industry to engage in abatement. In addition, abatement levels taper off with increasing industry dirtiness levels. Firms react to decreasing availability of permits by continually decreasing output since the benefit of expanded production possibility as a result of investment in abatement decreasingly outweighs the cost of abatement and permits.

The behavior of equilibrium permit price with respect to α is similar. At higher levels of α the drop in permit price as a result of an increase in the number of available permits is less than when α is low. And while equilibrium firm output and profit increase in the total number of permits, once again, the influence of altering the total number of permits on equilibrium permit price becomes decreasingly pronounced with increasing industry dirtiness levels.

4 Competition

In the case of competition, the optimization problem faced by a representative firm in the third stage of the game is: *Maximize* $_{\{y\}}$ $\Pi = (\hat{a} - \hat{b}Y)y + (\beta - (\alpha y - D))u - g(D) - e\beta - cy$; *Subject to:* $\theta \leq \beta$, $y \geq 0$.

[19] i.e., For $\alpha > \alpha_D$.

4.1 Output Subgame

Proposition 4 gives us the unconstrained subgame perfect equilibrium output when the firms compete in a Cournot fashion.

Proposition 4 *Assume* $u \leq \frac{\hat{a}-c}{\alpha}$. *The unconstrained subgame perfect equilibrium output in the third stage of the game when firms compete in a Cournot fashion, is* $\tilde{y} = \frac{\hat{a}-c-\alpha u}{\hat{b}(n+1)}$.

Proof: See Appendix A.

Again we choose to focus on the situation when the emissions constraint binds for all firms; i.e., when $\frac{(\beta+D)}{\alpha} < \frac{\hat{a}-c-\alpha u}{\hat{b}(n+1)}$ (or equivalently $\beta < \breve{\beta}$, where $\breve{\beta} = \alpha[\frac{\hat{a}-c-\alpha u}{\hat{b}(n+1)}] - D$), in which case Proposition 5 applies.

Proposition 5 *Assume that the emissions constraint binds for all firms. If the abatement level chosen by each firm in the investment stage is symmetrically* D, *and the number of permits secured by each firm in the auction stage is symmetrically* β, *the subgame perfect equilibrium output of each firm in the third stage of the game when firms compete in a Cournot fashion, is* $y^* = \frac{\beta+D}{\alpha}$.

Proof: See Appendix A.

We now establish expressions for the subgame perfect equilibrium profit for the two cases viz., $\beta < \breve{\beta}$ and $\beta \geq \breve{\beta}$. Let Z denote the total output of all other firms.
Case (i): $\beta < \breve{\beta}$ (or equivalently, $y^* = \frac{\beta+D}{\alpha}$)

$$\begin{aligned}
\hat{\Pi}_C^* &= \left[\hat{a} - \hat{b}(Z + \frac{\beta+D}{\alpha})\right](\frac{\beta+D}{\alpha}) + \left[\beta - \left(\alpha(\frac{\beta+D}{\alpha}) - D\right)\right]u - g(D) - e\beta - c(\frac{\beta+D}{\alpha}) \\
&= -\frac{\hat{b}}{\alpha^2}\beta^2 + \left[\frac{\alpha(\hat{a}-c-\alpha e) - \hat{b}(2D+\alpha Z)}{\alpha^2}\right]\beta \\
&\quad + \left[\frac{\alpha[(\hat{a}-c)D - \alpha g(D)] - \hat{b}D(D+\alpha Z)}{\alpha^2}\right] \qquad (8)
\end{aligned}$$

Case (ii): $\beta \geq \breve{\beta}$ (or equivalently, $y^* = \frac{\hat{a}-(c+\alpha u)}{\hat{b}(n+1)}$)

$$\begin{aligned}
\hat{\Pi}_U^* &= \left[\hat{a} - \hat{b}n[\frac{\hat{a}-c-\alpha u}{\hat{b}(n+1)}]\right][\frac{\hat{a}-c-\alpha u}{\hat{b}(n+1)}] + \left[\beta - \left(\alpha[\frac{\hat{a}-c-\alpha u}{\hat{b}(n+1)}] - D\right)\right]u - g(D) - e\beta \\
&\quad - c[\frac{\hat{a}-c-\alpha u}{\hat{b}(n+1)}] \\
&= \hat{\psi} + \beta(u-e) \qquad (9)
\end{aligned}$$

4.2 Auction Subgame

$\hat{\Pi}_C^*$ represents the subgame perfect equilibrium profit when the emissions constraint is binding, $\hat{\Pi}_U^*$ is the unconstrained subgame perfect equilibrium profit, and $\hat{\psi} = \left[\hat{a} - \hat{b}n[\frac{\hat{a}-c-\alpha u}{\hat{b}(n+1)}]\right][\frac{\hat{a}-c-\alpha u}{\hat{b}(n+1)}] -$

$\left(\alpha[\frac{\hat{a}-c-\alpha u}{\hat{b}(n+1)}]-D\right)u-g(D)-c[\frac{\hat{a}-c-\alpha u}{\hat{b}(n+1)}]$ is a constant, given the abatement level chosen in the first stage. The marginal value function $v(\beta)$ in the constrained competition case is:
When $\beta < \breve{\beta}$,

$$v(\beta) = -\frac{2\hat{b}}{\alpha^2}\beta + [\frac{\alpha(\hat{a}-c)-\hat{b}(2D+\alpha Z)}{\alpha^2}]$$

Substituting $Z=(n-1)(\frac{\beta+D}{\alpha})$ from symmetric equilibria in the investment and auction stages, we have

$$\begin{aligned} v(\beta) &= -\frac{2\hat{b}}{\alpha^2}\beta + [\frac{\alpha(\hat{a}-c)-\hat{b}[2D+\alpha(n-1)(\frac{\beta+D}{\alpha})]}{\alpha^2}] \\ &= -\frac{\hat{b}(n+1)}{\alpha^2}\beta + [\frac{\alpha(\hat{a}-c)-\hat{b}(n+1)D}{\alpha^2}] \\ &= \hat{\sigma}-\hat{\lambda}\beta \end{aligned} \tag{10}$$

When $\beta \geq \breve{\beta}$,

$$v(\beta) = u \tag{11}$$

Where $\hat{\sigma}=[\frac{\alpha(\hat{a}-c)-\hat{b}(n+1)D}{\alpha^2}]$ and $\hat{\lambda}=\frac{\hat{b}(n+1)}{\alpha^2}$ are constants, given the abatement level chosen in the first stage. Note that $\hat{\sigma}-\hat{\lambda}\breve{\beta}=u$, and when $\beta<\breve{\beta}$, $v(\beta)>u$. The marginal value function in the Cournot competition case includes terms in n, the number of firms. With competition in demands, the value of a permit decreases in the number of firms because the price, and hence the contribution per production unit decreases. We now have a representative firm's marginal value function before the auction is entered into.

Lemma 2 *For the share auction in the second stage, it is an optimal strategy to submit a schedule such that at each price e, the requested number of permits is* $\beta(e)=(\frac{1-2e/(n\hat{\sigma}-\hat{\lambda}B)}{n-1})B$. *The subgame perfect equilibrium price is* $\hat{e}^*=\frac{1}{2}(\hat{\sigma}-\hat{\lambda}\frac{B}{n})$*, and the subgame perfect equilibrium number of permits received by each firm is* $\hat{\beta}^*=\frac{B}{n}$.

Proof: See Appendix A.

Substituting the subgame perfect equilibrium permit price and permit share from Lemma 2 into the profit function in equation (8), we have

$$\hat{\Pi}_C^* = -\frac{(\hat{b}n+\xi\alpha^2)}{\alpha^2}D^2+\left(\frac{2\alpha n(\hat{a}-c)-\hat{b}B(3n-1)}{2\alpha^2 n}\right)D+\frac{B\alpha n(\hat{a}-c)-\hat{b}B^2(n-1)}{2\alpha^2n^2} \tag{12}$$

4.3 Abatement Subgame

We can now arrive at the profit-maximizing abatement level. Proposition 6 shows that the profit function in the first stage of the game, after incorporating the results from the third and second stages, is concave in the abatement level.

Proposition 6 *The profit function* $\hat{\Pi}_C^*$ *in (12) is concave in D.* $\hat{D}^*=\frac{2\alpha n(\hat{a}-c)-\hat{b}B(3n-1)}{4n(\hat{b}n+\xi\alpha^2)}$ *uniquely maximizes* $\hat{\Pi}_C^*$.

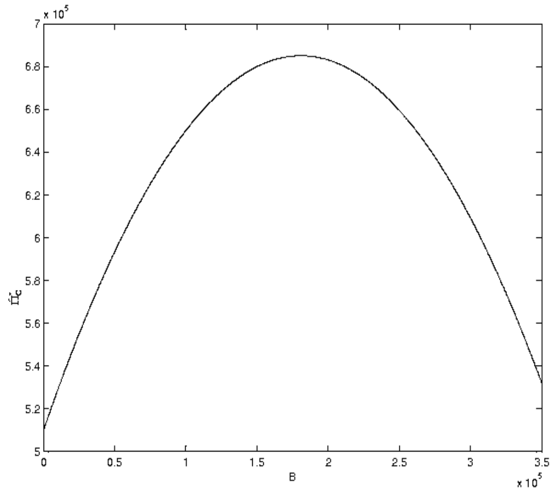

Figure 4: Equilibrium firm profit ($\hat{\Pi}_C^*$) versus B

Table 2: Equilibrium Results: Competition

	Symbol	Expression
Abatement	$\hat{D}^*$	$\frac{2\alpha n(\hat{a}-c)-\hat{b}B(3n-1)}{4n(\hat{b}n+\xi\alpha^2)}$
Permit Price	$\hat{e}^*$	$\frac{-\hat{b}^2B(n+1)^2+4\xi\alpha^2[\alpha n(\hat{a}-c)-\hat{b}B(n+1)]+2\hat{b}\alpha n(\hat{a}-c)(n-1)}{8\alpha^2 n(\hat{b}n+\xi\alpha^2)}$
Permit Share	$\hat{\beta}^*$	$\frac{B}{n}$
Output	$\hat{y}^*$	$\frac{2\alpha n(\hat{a}-c)-\hat{b}B(3n-1)}{4\alpha n(\hat{b}n+\xi\alpha^2)}+\frac{B}{\alpha n}$
Profit	$\hat{\Pi}_C^*$	$\frac{\hat{b}^2B^2(n+1)^2-4\alpha[n(\hat{a}-c)+2\xi\alpha B][\hat{b}B(n-1)-\alpha n(\hat{a}-c)]}{16\alpha^2n^2(\hat{b}n+\xi\alpha^2)}$

Proof: See Appendix A.

To ensure non-negative equilibrium values of abatement, permit price, output, and product price, we assume that $(\hat{a}-c) > \frac{\hat{b}B(3n-1)}{2\alpha n}$. Table 2 summarizes the subgame perfect equilibrium results for the case of competition, in terms of the parameters of our model. We again examine the interactions among the number of permits B offered in the auction, the coefficient α in the PE function, and the subgame perfect equilibrium values of investment $\hat{D}^*$, permit price $\hat{e}^*$, output $\hat{y}^*$, and profit $\hat{\Pi}_C^*$. Proposition 7 formally describes pertinent relationships among these elements.

Proposition 7

a. $\hat{D}^*$ decreases linearly in B; $\hat{e}^*$ decreases linearly in B; $\hat{y}^*$ increases linearly in B; $\exists\ B_{\hat{\Pi}_C}$ such that $\hat{\Pi}_C^*$ increases in B for $B < B_{\hat{\Pi}_C}$, and increases in B for $B > B_{\hat{\Pi}_C}$.[20]

b. $\exists\ \alpha_{\hat{D}}$ such that $\hat{D}^*$ increases in α for $\alpha < \alpha_{\hat{D}}$, and decreases in α for $\alpha > \alpha_{\hat{D}}$; $\hat{y}^*$ decreases in α.[21]

[20] $B_{\hat{\Pi}_C} = \frac{2\alpha n(\hat{a}-c)(\hat{b}n-\hat{b}-2\xi\alpha^2)}{\hat{b}[\hat{b}(n+1)^2-8\xi\alpha^2(n-1)]}$.

[21] $\alpha_{\hat{D}} = \frac{\hat{b}\xi B(3n-1)+\sqrt{[\hat{b}\xi B(3n-1)]^2+4\hat{b}\xi n^3(\hat{a}-c)^2}}{2\xi n(\hat{a}-c)}$. Numerical studies suggest that $\exists\ \alpha_{\hat{e}}$ such that for $\alpha < \alpha_{\hat{e}}$, $\frac{\partial \hat{e}^*}{\partial \alpha} > 0$,

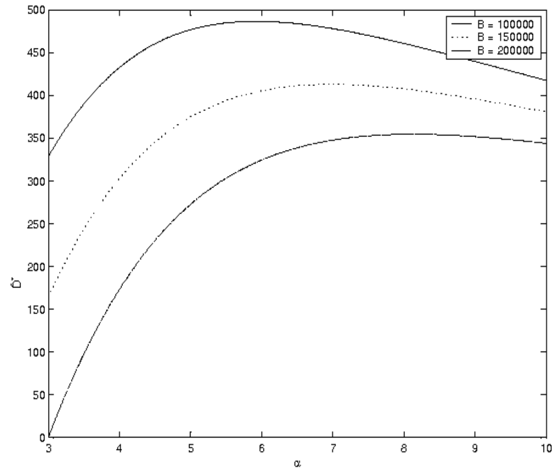

Figure 5: Equilibrium abatement ($\hat{D}^*$) versus α

c. $\frac{\partial^2 \hat{D}^*}{\partial B\ \partial\alpha} > 0$; $\frac{\partial^2 \hat{e}^*}{\partial B\ \partial\alpha} > 0$; $\frac{\partial^2 \hat{y}^*}{\partial B\ \partial\alpha} < 0$ for $\alpha > \sqrt{\frac{\hat{b}n}{4\xi}}$.[22]

Proof: See Appendix A.

Figures 4, 5, 6, and 7[23] visually describe the relationships proved in Proposition 7. The behavior of equilibrium abatement ($\hat{D}^*$), equilibrium permit price ($\hat{e}^*$), and equilibrium output ($\hat{y}^*$), with respect to the total number of permits (B) is intuitive and the explanation for the observed behavior is similar to that described in Section 3. However, the behavior of $\hat{\Pi}_C^*$ with respect to B is not so obvious. For low values of B, an increase in B results in an increase in equilibrium profit due to a drop in equilibrium permit price and decrease in equilibrium abatement. But, beyond a point[24] the profit decreases in the total number of available permits. When the total number of available permits is large, firms' production "capacities" in the third stage of the game are expanded. In the presence of competition, the price drops significantly, with an accompanying drop in profit. Figure 5 shows how the equilibrium abatement varies with the PE function parameter (α) for different values of the total number of permits. The equilibrium abatement first increases in and thereafter tapers off with respect to α. In addition, the equilibrium abatement is decreasingly influenced by variations in the total number of permits. The explanation for such counterintuitive behavior is similar to that detailed for the independent demands case in Section 3. As observable in Figure 6, the equilibrium permit price is significantly influenced by variations in the total number

and for $\alpha > \alpha_{\hat{e}}$, $\frac{\partial \hat{e}^*}{\partial \alpha} < 0$; and $\exists\ \alpha_{\hat{\Pi}}$ such that for $\alpha < \alpha_{\hat{\Pi}}$, $\frac{\partial \hat{\Pi}_C^*}{\partial \alpha} > 0$, and for $\alpha > \alpha_{\hat{\Pi}}$, $\frac{\partial \hat{\Pi}_C^*}{\partial \alpha} < 0$. $\alpha_{\hat{e}}$ is, analytically, the positive real root of $2(\hat{a}-c)\xi^2 n x^5 - 4\hat{b}B\xi^2(n+1)x^4 + (\hat{a}-c)\hat{b}\xi n(n-3)x^3 - 2\hat{b}^2 B\xi(n+1)^2x^2 + (\hat{a}-c)\hat{b}^2n^2(n-1)x - \hat{b}^3 Bn(n+1)^2 = 0$. $\alpha_{\hat{\Pi}}$ is, analytically, the positive real root of $4(\hat{a}-c)B\xi^2 n x^5 + [4(\hat{a}-c)^2\xi n^2 - 8\hat{b}B^2\xi^2(n-1)]x^4 - 2(\hat{a}-c)\hat{b}B\xi n(5n-3)x^3 + 2\hat{b}^2B^2\xi(n+1)^2x^2 - 2(\hat{a}-c)\hat{b}^2Bn^2(n-1)x + \hat{b}^3B^2n(n+1)^2 = 0$.

[22]The proposition as stated suffices to show that for large values of α the equilibrium output is decreasingly influenced by variations in B. The behavior of $\hat{\Pi_C}^*$ jointly with respect to B and α is analytically intractable and is therefore treated numerically in the discussion that follows.

[23]Parameter values for Figure 4 were $a = 7,500$, $\alpha = 5$, $b = 0.075$, $c = 10$, $u = 200$, $\xi = 0.65$, $n = 150$. Parameter values for Figures 5, 6, and 7 were $a = 7,500$, $b = 0.075$, $c = 10$, $u = 200$, $\xi = 0.65$, $n = 150$.

[24]i.e., For $B > B_{\hat{\Pi}_C}$.

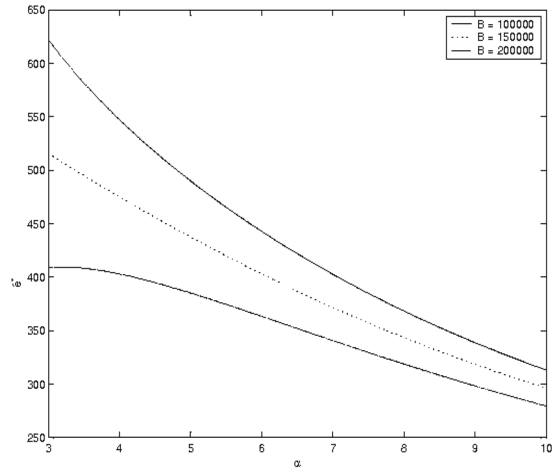

Figure 6: Equilibrium permit price ($\hat{e}^*$) versus α

permits for very low values of α. Firms' capacities vary significantly with changes in the number of available permits for very low values of α and, in the presence of competition, this translates into a pronounced influence on product price and revenue, and hence on the valuation of permits by firms.

The relationship of equilibrium profit with respect to α for varying values of the number of available permits is therefore intricate. Figure 7 shows that the profit first increases in α, with the curve corresponding to a smaller number of permits located above the curve corresponding to a larger number of permits. The curves initially converge with increase in α, reflecting a decreasing influence on equilibrium profit of altering the number of available permits. But with further increase in α the order of the curves gets reversed and they separate out, reflecting an increasing influence on equilibrium profit of varying the number of permits. When α is very low, firms' capacities are large; increasing the number of available permits further increases their capacities, which, in the presence of competition, results in a significant drop in product price and an accompanying drop in profit. As α increases, the effect of altering the number of available permits decreases and the curves therefore converge. But with further increase in α, additional permits compensate for shrinking capacity and begin to yield increasing benefit from the expanded production possibility; firm profit, however, decreases in α.

The above delineation of the effects of competition in the end product market on equilibrium outcomes is potentially insightful to both regulators as well as firms. In the presence of competition, cleaner firms can benefit from a reduction in the number of available permits. And in the case of competition too, reducing the number of available permits induces firms in a dirtier industry to a lesser extent than firms in a cleaner industry to engage in abatement. Firms react to decreasing availability of permits by continually decreasing output since the benefit of expanded production possibility as a result of investment in abatement decreasingly outweighs the cost of abatement and permits. Also, abatement levels taper off with increasing industry dirtiness levels.

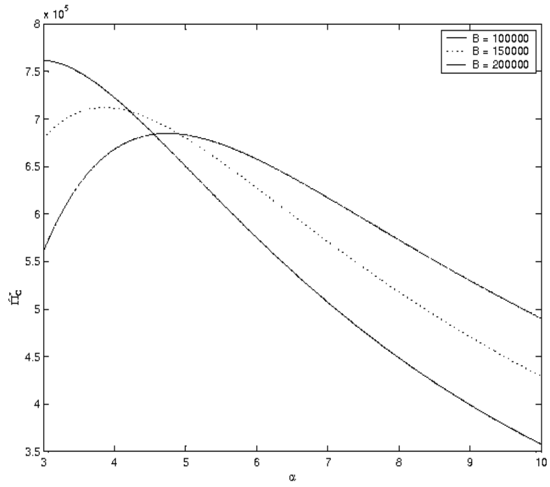

Figure 7: Equilibrium firm profit ($\hat{\Pi}_C^*$) versus α

5 Abatement and the Cost of Production

Emissions abatement efforts can influence the cost of production either positively or negatively. Pollution control R&D can result in the implementation of resource-efficient processing of materials, translating into a decrease in unit production cost. On the other hand, stipulations on emission levels could necessitate additional processing of materials to mitigate emissions, leading to an increase in unit production cost. We can incorporate such effects into our model by replacing the constant unit cost of production c with the function $c + \rho D$ where $\rho \in (-\frac{c}{D}, \infty)$ is a constant and D is the abatement level. Subsequent steps in the analysis are detailed in Appendix A. Subgame perfect equilibrium results are presented in Tables 3 and 4.

Table 3: Equilibrium Results: Independent Demands

	Symbol	Expression
Abatement	D^*	$\frac{\alpha}{2(b+\rho\alpha+\xi\alpha^2)}(a-c-\frac{bB}{\alpha n}-\frac{\rho B}{2n})$
Permit Price	e^*	$\frac{1}{2\alpha(b+\rho\alpha+\xi\alpha^2)}\left[a-c-(b+\frac{\rho\alpha}{2})(a-c-\frac{bB}{\alpha n}-\frac{\rho B}{2n})\right]$
Permit Share	β^*	$\frac{B}{n}$
Output	y^*	$\frac{1}{2(b+\rho\alpha+\xi\alpha^2)}(a-c-\frac{bB}{\alpha n}-\frac{\rho B}{2n})+\frac{B}{\alpha n}$
Profit	Π_C^*	$\frac{B\alpha^2 n(2\xi\alpha+\rho)(a-c)+\alpha^2 n^2(a-c)^2+B^2(b+\frac{\rho\alpha}{2})^2}{4\alpha^2 n^2(b+\rho\alpha+\xi\alpha^2)}$

Independent Demands

Figure 8[25] shows how the equilibrium abatement level varies with the coefficient α for different values of the total number of permits when abatement reduces the variable cost of production. We again encounter counterintuitive behavior. It might be expected that an increase in the number of

[25]Parameter values for Figure 8 were $a = 7,500$, $b = 5$, $c = 500$, $u = 200$, $\xi = 6.5$, $\rho = -2.25$, $n = 150$.

Table 4: Equilibrium Results: Competition

	Symbol	Expression
Abatement	$\hat{D}^*$	$\frac{2\alpha n(\hat{a}-c)-\rho\alpha B-\hat{b}B(3n-1)}{4n(\hat{b}n+\rho\alpha+\xi\alpha^2)}$
Permit Price	$\hat{e}^*$	$\frac{-\hat{b}^2B(n+1)^2+4\xi\alpha^2[\alpha n(\hat{a}-c)-\hat{b}B(n+1)]+2\hat{b}\alpha n(\hat{a}-c)(n-1)+\rho\alpha[2\alpha n(\hat{a}-c)+\rho\alpha B-4\hat{b}B]}{8\alpha^2 n(\hat{b}n+\rho\alpha+\xi\alpha^2)}$
Permit Share	$\hat{\beta}^*$	$\frac{B}{n}$
Output	$\hat{y}^*$	$\frac{2\alpha n(\hat{a}-c)-\rho\alpha B-\hat{b}B(3n-1)}{4\alpha n(\hat{b}n+\rho\alpha+\xi\alpha^2)}+\frac{B}{\alpha n}$
Profit	$\hat{\Pi}_C^*$	$\frac{\hat{b}^2B^2(n+1)^2-4\alpha[n(\hat{a}-c)+2\xi\alpha B][\hat{b}B(n-1)-\alpha n(\hat{a}-c)]+\rho\alpha B[\rho\alpha B+4\alpha n(\hat{a}-c)-2\hat{b}B(n-3)]}{16\alpha^2n^2(\hat{b}n+\rho\alpha+\xi\alpha^2)}$

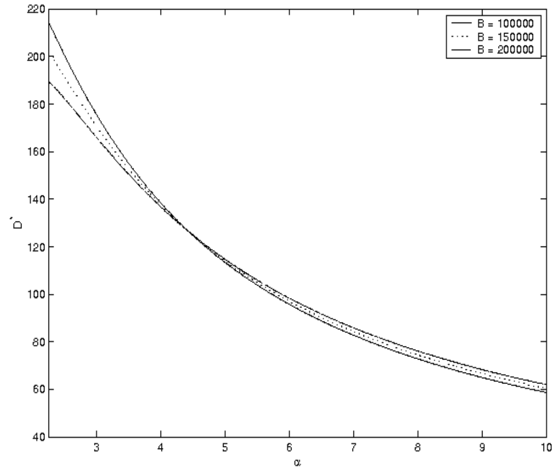

Figure 8: Equilibrium abatement (D^*) versus α

available permits would always decrease the incentive to engage in abatement. However, as observable in Figure 8, for large values of α the equilibrium abatement is greater when larger numbers of permits are offered in the auction. Proposition 8 formally states this. When abatement reduces the variable cost of production and α is large, there is a greater incentive to invest in abatement, reduce the cost of production, and hence better exploit the expanded production capacity resulting from a larger number of permits being secured in the auction.

Proposition 8 *D^* increases in B for $\alpha > \max\{\frac{2b}{|\rho|}, \frac{|\rho|}{\xi}\}$ when $\rho < 0$.*

Proof: See Appendix A.

Figure 9[26] demonstrates that the behavior of equilibrium permit price with respect to α for different values of ρ is rather unpredictable for low values of α. This is because of the interplay among a number of factors. For low α, the production capacity is high; thus the relatively lower value placed by firms on permits and the resulting low equilibrium permit price. Since production capacity is high, there is a greater incentive to invest in abatement when ρ is negative, to bring down

[26]Parameter values for Figure 9 were $a = 7,500$, $b = 5$, $c = 500$, $u = 200$, $\xi = 6.5$, $B = 150,000$, $n = 150$.

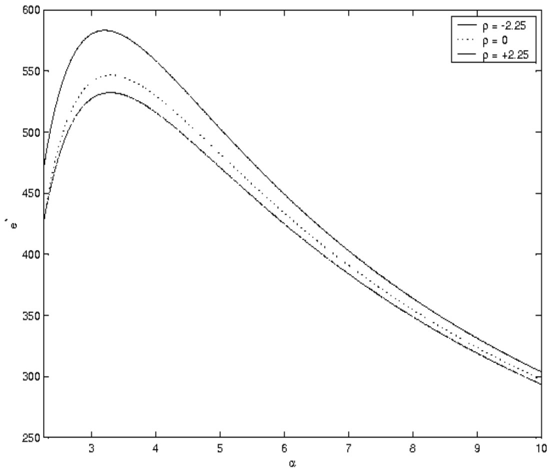

Figure 9: Equilibrium permit price (e^*) versus α for different values of ρ

the unit cost of production. But combined with the effect of firm output on revenue from sales, these effects translate into a behavior that is contingent on parameter values. As α increases, the ordering among the curves becomes more definite with equilibrium permit price increasing with decrease in ρ. Permits and investment in abatement are the two components which define production capacity. For large α, production capacity is low and well short of the (unconstrained) profit-maximizing level. Since the cost reduction from investment in abatement applies to all production units, both the incentive to invest as well as the value of permits as reflected in equilibrium permit price increase as ρ decreases. Figure 22 in Appendix B shows that the equilibrium abatement decreases convexly in ρ. The counterintuitive behaviors of equilibrium abatement and permit price tapering off in α, persist. The behaviors of equilibrium permit price, firm output and firm profit with respect to α for varying values of the total number of permits, are similar to those described in Section 3 (see Figures 23, 24, 25, and 26 in Appendix B).

Competition

In the case of competition too the incentive to engage in abatement increases in the number of available permits, for relatively large values of α (see Figure 27 in Appendix B). Proposition 9.a formally states this. The equilibrium permit price is significantly influenced by variations in the total number of permits for low values of α (see Figure 10[27]) due to two main effects on the valuation of permits by firms. Firstly, firms' capacities vary significantly with changes in the total number of permits for very low values of α; competition significantly influences product price and revenue from sales. Secondly, variations in the total number of permits for low values of α translate into significant differences in the abatement levels chosen to offset the downward pressure on price.

In certain situations equilibrium firm output behaves differently than what is observed in Section 4, running counter to the expectation that firm output would always decrease in α. Figure 29 in Appendix B depicts such a situation. For low values of α, firms' capacities are large and competition

[27]Parameter values for Figure 10 were $a = 7,500$, $b = 0.075$, $c = 500$, $u = 200$, $\xi = 6.5$, $\rho = -2.25$, $n = 150$.

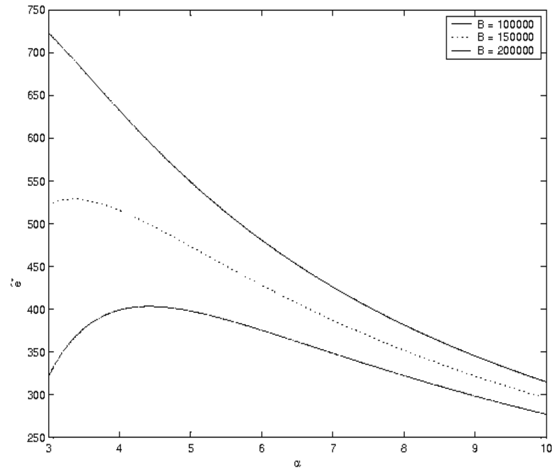

Figure 10: Equilibrium permit price ($\hat{e}^*$) versus α

exerts downward pressure on product price. This drives firms to engage in abatement in order to bring down the cost of production and exploit the large capacity. As α increases from a very low value, firms invest more in abatement to expand the shrinking capacity and to reduce the cost of production, which causes equilibrium firm output to increase in α. However, with further increase in α, firms' capacities are forced further downward, product price increases, and optimal investments in abatement no longer increase equilibrium output; equilibrium firm output decreases in α.

Proposition 9

a. $\hat{D}^*$ increases in B for $\alpha > \max\{\frac{\hat{b}(3n-1)}{|\rho|}, \frac{|\rho|}{\xi}\}$ when $\rho < 0$.

b. It is possible that $\exists\ \alpha_1, \alpha_2 \in \mathbf{R}^+$, such that when $\rho < 0$, $\hat{y}^*$ increases in α for $\alpha \in (\alpha_1, \alpha_2)$.[28]

Proof: See Appendix A.

The behavior of equilibrium permit price with respect to α for different values of ρ is again rather unpredictable for low values of α. For higher values of α the ordering among the curves corresponding to different values of ρ is similar to that in the independent demands case (see Figure 28 in Appendix B). Equilibrium firm profit (see Figure 30 in Appendix B) behaves in a manner similar to that described in Section 4.

6 Specification of the Emissions Function

It might be argued that the extent of abatement could depend upon the volume of production. It turns out that the structural attributes of our pertinent results hold even when abatement is modeled as a function of output. We test the validity of our results with an emissions function

[28]The proposition provides stronger than necessary conditions under which $\hat{y}^*$ increases in α. The proposition suffices to show that there exists situations in which $\hat{y}^*$ increases in α.

of the form $\theta = \frac{\alpha_0}{(1+\mu)}y$, where the abatement level μ is chosen in the first stage of the game and the cost of engaging in an abatement level of μ is $\xi\mu^2$ (where ξ is a constant), implying that it is increasingly expensive to engage in abatement. We denote $\alpha = \frac{\alpha_0}{(1+\mu)}$. Equilibrium results are presented in Tables[29] 5 and 6. Analysis leading to the results is outlined in Appendix A. Deductions of behaviors are analytically intractable and hence graphical depictions are provided in Appendix B. In the case of independent demands equilibrium permit price and output drop as the industry gets dirtier. Equilibrium firm output decreases in response to decreasing availability of permits. In Section 3, firm capacity was given by $\frac{B+D}{\alpha}$ where α had a fixed value and D was the abatement level chosen in the first stage of the game. A lower bound on firm capacity was therefore $\frac{B}{\alpha}$. Here, firm capacity is given by $\frac{B}{\alpha}$ where $\alpha = \frac{\alpha_0}{(1+\mu)}$ can be varied by altering μ. Larger levels of μ are optimal when greater numbers of permits are available because the benefits from additional capacity become increasingly significant and increasingly outweigh the costs of abatement. In the emissions function used in Section 3, abatement does not impact capacity as significantly as it does here. Equilibrium abatement, as before, tapers off as the industry gets dirtier.

In the case of competition, the behavior of equilibrium profit is similar to that observed in Section 4. An increase in the number of permits increases the capacities of the firms and exerts downward pressure on end product price and firm profitability. When the industry is clean, firm capacities are large and lower levels of abatement are optimal when larger numbers of permits are available, in contrast to the independent demands case. With competition, firms in a clean industry can indeed benefit from a reduction in the number of available permits. Equilibrium permit price and firm output show the same behavior as before. The points of discontinuity in the graphs in Figure 20 (Appendix B) correspond to zero abatement.

Table 5: Equilibrium Results: Independent Demands

	Symbol	Expression
Abatement	μ^*	$\frac{B(a-c)}{4\alpha_0\xi n}$
Permit Price	e^*	$\frac{(a-c)}{2\tilde{\alpha}} - \frac{bB}{\tilde{\alpha}^2 n}$
Permit Share	β^*	$\frac{B}{n}$
Output	y^*	$\frac{B}{\tilde{\alpha}n}$
Profit	Π_C^*	$\frac{B(a-c)}{2\tilde{\alpha}n} - \frac{B^2(a-c)^2}{16\xi{\alpha_0}^2n^2}$

7 A Numerical Example

We present a numerical example that elucidates pertinent results and corroborates our interpretations presented in previous sections. Parameter values chosen for the numerical example are listed in the tables below. Results for the two scenarios - Independent Demands and Competition are tabulated separately in Tables 7 and 8.

[29] In Table 5, $\tilde{\alpha} := \frac{\alpha_0}{1+\mu^*} = \frac{\alpha_0}{[1+\frac{B(a-c)}{4\alpha_0\xi n}]}$. In Table 6, $\tilde{\alpha} := \frac{\alpha_0}{1+\hat{\mu}^*} = \frac{\alpha_0}{[1+\frac{B\alpha_0 n(\hat{a}-c)-2\hat{b}B^2(n-1)}{2\hat{b}B^2(n-1)+4\xi{\alpha_0}^2n^2}]}$.

Table 6: Equilibrium Results: Competition

	Symbol	Expression
Abatement	$\hat{\mu}^*$	$\frac{B\alpha_0 n(\hat{a}-c)-2\hat{b}B^2(n-1)}{2\hat{b}B^2(n-1)+4\xi{\alpha_0}^2n^2}$
Permit Price	$\hat{e}^*$	$\frac{(\hat{a}-c)}{2\tilde{\alpha}}-\frac{(n+1)\hat{b}B}{2\tilde{\alpha}^2n}$
Permit Share	$\hat{\beta}^*$	$\frac{B}{n}$
Output	$\hat{y}^*$	$\frac{B}{\tilde{\alpha}n}$
Profit	$\hat{\Pi}_C^*$	$\frac{B(\hat{a}-c)}{2\tilde{\alpha}n}-\frac{(n-1)bB^2}{2\tilde{\alpha}^2n^2}-\xi[\frac{B\alpha_0 n(\hat{a}-c)-2\hat{b}B^2(n-1)}{2\hat{b}B^2(n-1)+4\xi{\alpha_0}^2n^2}]^2$

We first discuss the case when $\rho = 0$ (i.e., when investments in abatement does not affect the cost of production). Under both demand scenarios, the equilibrium abatement level decreases when α is increased from 2.5 to 12.5, running counter to the expectation that the abatement would be higher when firms pollute more, given that pollution norms need to be met with in either case. The output levels for $\alpha = 12.5$ are much lower than the output levels for $\alpha = 2.5$, providing numerical evidence that dirty firms decrease output instead of engaging in greater levels of abatement. Variations in the number of available permits result in smaller variations in equilibrium abatement when $\alpha = 12.5$ than when $\alpha = 2.5$, again running counter to the expectation that reductions in the number of available permits should result in greater changes in equilibrium abatement when α is larger. Additionally, in the case of competition, equilibrium profit can increase with a reduction in B; the equilibrium profit increases by about 3% when B is reduced from 200000 to 150000 when $\alpha = 2.5$. In both demand scenarios, equilibrium permit price decreases more rapidly with increase in B for $\alpha = 2.5$ than for $\alpha = 12.5$.

A negative value of ρ implies that investments in abatement cause a reduction in the cost of production. When $\rho = -2.25$, and $\alpha = 12.5$, equilibrium abatement, interestingly, decreases as the number of available permits B is decreased, in both demand scenarios. The counterintuitive behaviors of equilibrium abatement decreasing when α increases from 2.5 to 12.5, and of variations in the number of available permits having a decreasing influence on abatement and permit price as α increases, continue to be observed when $\rho < 0$. Equilibrium firm profit in the case of competition increases by about 4.5% with a reduction in B from 200000 to 150000 when $\alpha = 2.5$.

8 Implications

The regulator has a number of levers which it can work with to ensure that emissions remain within acceptable limits and that firms engage in adequate levels of abatement. Levers include penalties for non-compliance, the number of permits offered in the auction, emissions allowed per permit, subsidies or tax breaks for investments in abatement by firms, the treatment of unused allowances, and permit allocation across industries. Firms, on the other hand, can comply with stipulated regulations by investing in pollution abatement, selecting the appropriate type of investment (e.g., end-of-pipe or in-pipe, product or process based), varying production levels, or purchasing permits.

Table 7: Numerical Example: Independent Demands

(a=7500, b=5, c=500, u=200, ξ=6.5, n=150)						
$\rho = 0$						
α	B	D^*	e^*	y^*	p^*	Π_C^*
2.5	100000	155.25	742.47	328.77	5856.16	1109284.63
2.5	150000	136.99	490.41	454.79	5226.03	1536986.30
2.5	200000	118.72	238.36	580.82	4595.89	1969558.60
12.5	100000	41.23	257.35	56.63	7216.84	197772.06
12.5	150000	40.41	246.71	83.23	7083.83	290669.93
12.5	200000	39.60	236.07	109.83	6950.83	383576.51
$\rho =$ -2.25						
α	B	D^*	e^*	y^*	p^*	Π_C^*
2.5	100000	200.52	796.48	346.88	5765.63	1190668.40
2.5	150000	191.41	533.01	476.56	5117.19	1634473.00
2.5	200000	182.29	269.53	606.25	4468.75	2079340.28
12.5	100000	47.12	261.40	57.10	7214.48	200772.53
12.5	150000	48.64	250.82	83.89	7080.54	295031.64
12.5	200000	50.17	240.24	110.68	6946.60	389320.18

Given our modeling choice and assumptions we can elaborate on some of the aforementioned levers. The rest, though they merit attention, are beyond the scope of this paper and provide directions for future research.

The findings in this paper can aid regulators in the understanding of how observed abatement levels, permit prices, and industry output might result. The behaviors of these variables are intricate and often run counter to intuitive expectations. For instance, a drop in permit price cannot be simply associated with reductions in firms' costs of compliance. The EPA Acid Rain Program 2001 Progress Report states that emissions reductions continue to cost less (to firms) than what was anticipated when the Clean Air Act Amendments were enacted. The price of an allowance was initially estimated at \$500-1200/ton in 2001 dollars but actual prices have been significantly lower than predicted. During 2001, SO_2 allowances ranged in price from \$135 to \$210/ton. The EPA attributes the difference between estimated and actual prices to the reducing marginal cost of compliance - the cost of reducing the next ton of pollutant emitted from the industry sector - which is reflected in the price of an allowance. The model used in this paper is a fairly simple representation of investment, permit bidding and allocation, and production. Yet the interplay among the elements in the model renders the behavior of equilibrium outcomes intricate and often counterintuitive. Our model suggests that equilibrium permit price can be low in a relatively dirty

Table 8: Numerical Example: Competition

(a=7500, b=0.05, c=500, u=200, ξ=6.5, n=150)						
			$\rho = 0$			
α	B	$\hat{D}^*$	$\hat{e}^*$	$\hat{y}^*$	$\hat{p}^*$	$\hat{\Pi}_C^*$
2.5	100000	104.07	934.48	308.29	5187.79	751838.61
2.5	150000	65.19	756.62	426.08	4304.41	836727.79
2.5	200000	26.32	578.77	543.86	3421.04	812445.37
12.5	100000	39.10	262.95	56.46	7076.54	186083.83
12.5	150000	37.28	254.94	82.98	6877.63	265258.21
12.5	200000	35.45	246.93	109.50	6678.73	339178.59
			ρ = -2.25			
α	B	$\hat{D}^*$	$\hat{e}^*$	$\hat{y}^*$	$\hat{p}^*$	$\hat{\Pi}_C^*$
2.5	100000	139.90	975.79	322.63	5080.29	801537.84
2.5	150000	106.91	779.54	442.76	4179.26	881724.85
2.5	200000	73.92	583.28	562.90	3278.24	844269.02
12.5	100000	44.92	266.85	56.93	7073.05	188920.75
12.5	150000	45.40	258.29	83.63	6872.76	269283.00
12.5	200000	45.87	250.81	110.34	6672.48	344350.34

industry; because the benefit from expanded production possibility decreasingly outweighs the costs of production and permits, resulting in a decrease in the value placed on permits, a lower permit price in the auction, and lower production levels. The EPA's 2001 Progress Report also mentions that most SO_2 emissions occur in the Midwestern US - in the states of Ohio, Indiana, Pennsylvania, Illinois, Kentucky, Missouri, Tennessee, and West Virginia. Total power generation for these states, however, dropped from 82,279 MWh (MegaWatt-hour) in the year ending March 2002, to 81,930 MWh in the year ending March 2003.[30] While such anecdotal information is insufficient to make categorical inferences and excludes a host of other factors, it does point toward the need for a better theoretical understanding of equilibrium abatement, permit price, and industry output outcomes.

Given our modeling choice of a share auction, the regulator has at its discretion the choice of the total number of permits to be offered at the auction. It might seem straightforward that reducing the total number of permits will influence a dirtier industry to a greater extent than a cleaner one to engage in abatement. However we find that reducing the number of available permits decreasingly induces abatement for increasing industry dirtiness levels. Reductions in the number of available permits mean that firms in a relatively dirty industry are progressively driven to lower and lower output levels, and perhaps finally to extinction. While this augurs well for the environment, the

[30] *http://www.eia.doe.gov/cneaf/electricity /epm/table1_6_a.html.*

combined impact of decreases in output and pollution levels on consumer welfare also needs to be considered. Industry structure too influences equilibrium outcomes. Interestingly, in the case of competition in the end product market, reducing the number of permits offered in the auction can increase equilibrium firm profit if the industry is sufficiently clean. The regulator can then amicably enforce reductions in the number of available permits. This is favorable from the viewpoint of a regulator aiming to increase the level of stringency in pollution norms.

The paper presents a methodology by which firms can derive a marginal value function for permits and translate the value for permits into optimal bidding strategies. In addition the modeling effort provides a means for firms to deduce their optimal levels of abatement and output - principal levers for compliance. An industry comprising dirty firms risks low optimal output levels if stringent pollution norms are effected, given increasing marginal costs of abatement. Traditionally firms lobby to roll back pollution standards because of the lingering belief that stricter environmental regulations erode competitiveness (Michael Porter, 1995). But in a competitive setting, an excessive number of permits is detrimental to firm profit in a relatively clean industry, because of large capacities which bring down product price and firm profits.

Our model's predictions can aid a regulator attempting to control a set of pollutants via permit auctions, in choosing the pollutants to be controlled depending on the characteristics of the industry in which the pollutants are generated. For example, if the regulator desires that firm profitability should not be negatively affected as a result of stringency in pollution limits, it could target a pollutant in a competitive, relatively clean industry. On the other hand if the regulator intends to exclude a pollutant to the greatest extent possible (i.e., minimizing pollution) from discharge streams in a dirty industry, it can enforce stringent pollution caps which will drive down output levels. If a permit auctioning program is already in place, our results help in aiding a regulator in deciphering reactions from firms which are manifested in permit prices. In literature, low permit prices have been associated with low marginal cost of pollution control. But as per our model permit prices can be low in a dirty industry since output levels are low and hence the demand for pollution permits is low. If this is indeed the case and if the regulator desires that firm profitability should not be significantly affected by the imposition of emissions norms, then perhaps a mechanism other than the one modeled in our paper might be more appropriate. The applicability and effectiveness of various emissions control mechanisms (e.g., different permit allocation methods) to specific industry cleanliness levels and structures is beyond the scope of our paper but merits further research. In addition, our paper does not exhaustively prescribe effective pollution control maneuvers corresponding to different industry scenarios but we do provide a significant start.

9 Conclusion and Future Work

In this paper we modeled a three stage game in an oligopoly - investment in abatement, followed by a "share" auction for permits and finally, the production of output. We treated two scenarios in the end product market - Independent Demands and Cournot Competition. In both cases we found that reducing the number of available permits induces firms in a dirtier industry to a lesser extent than firms in a cleaner industry to engage in abatement. In addition, abatement levels taper off with increasing industry dirtiness levels. In the presence of competition, firms can benefit from a reduction in the number of available permits provided the industry is sufficiently clean. Our findings are potentially insightful to both regulators as well as firms, since many countries have chosen auctioning as a means of allocating permits in their planned emissions trading programs, and no research to date delineates the effects associated with permit auctioning and firms' abatement and output decisions.

Several extensions to this work merit consideration in future research. Uncertainty can be incorporated in end product demand and hence in the valuation of permits by firms. Uncertainty can also be introduced in beliefs about the cost structures of competitors or in the likelihood of abatement resulting from investment efforts. Our analysis assumes that no firm has any proprietary information on the valuation of permits. Even if no uncertainty is considered, a firm's valuation of permits could be private information and an extension to an auction with private values can be undertaken. In practice, permits can be banked for future use and investments in abatement yield emissions reductions across periods; an extension to multiple periods will therefore be very insightful. The assumption of symmetry could be relaxed but given the very limited results in divisible good auction theory due to analytical intractability, additional assumptions or modifications to the model are likely to be necessary. In practice, firms have grandfathered allowances in addition to the allowances secured in the auction; we believe that our results will continue to hold if the assumption that firms are constrained in profit-maximization by emissions stipulations is maintained. Emissions permits are also exchanged in markets outside of auctions through brokered trades, electronic screens, or direct trades between participants without an intermediary. Interactions between permit auctions and exchange mechanisms are worth exploring. Permit auctions are also conducted across industries and countries; additional intricacies in modeling and analysis will provide for interesting findings.

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Appendix A: Analysis

Proof of Proposition 1:

The optimization problem in the third stage is:

Maximize $_{\{y\}}$ $\Pi = (a-by)y + (\beta - (\alpha y - D))u - g(D) - e\beta - cy$; *Subject to:* $\theta \leq \beta$, $y \geq 0$.

Concavity of the profit function implies that $\tilde{y} = \{y : \frac{d\Pi}{dy} = 0\}$ is the unconstrained profit-maximizing quantity.

$$\frac{d\Pi}{dy} = a - 2by - \alpha u - c \Rightarrow \tilde{y} = \frac{a-c-\alpha u}{2b}$$

The constraint $\theta \leq \beta$ implies $\alpha y - D \leq \beta$; i.e., $y \leq \frac{\beta+D}{\alpha}$.

Concavity of Π and the assumption that $u \leq \frac{a-c}{\alpha}$ imply that $y^* = \min\left\{\frac{a-c-\alpha u}{2b}, \frac{\beta+D}{\alpha}\right\}$ is the profit-maximizing quantity. ∎

Proof of Proposition 2:

$$\frac{d\Pi_C^*}{dD} = -\frac{2(b+\xi\alpha^2)}{\alpha^2}D + \left(\frac{a}{\alpha} - \frac{c}{\alpha} - \frac{bB}{\alpha^2 n}\right) \tag{13}$$

$$\frac{d^2\Pi_C^*}{dD^2} = -\frac{2(b+\xi\alpha^2)}{\alpha^2} < 0 \tag{14}$$

Implying that Π_C^* is concave in D. $\frac{d\Pi_C^*}{dD} = 0$ yields $D^* = \frac{\alpha}{2(b+\xi\alpha^2)}(a - c - \frac{bB}{\alpha n})$, the profit-maximizing abatement level. ∎

Proof of Proposition 3.a:

i. $\dfrac{\partial D^*}{\partial B} = -\dfrac{\alpha b}{2\alpha n(b+\xi\alpha^2)} < 0$, $\dfrac{\partial^2 D^*}{\partial B^2} = 0$. ∎

ii. $\dfrac{\partial e^*}{\partial B} = -\dfrac{\xi b}{2n(b+\xi\alpha^2)} - \dfrac{b}{2\alpha^2 n} < 0$, $\dfrac{\partial^2 e^*}{\partial B^2} = 0$. ∎

iii. $\dfrac{\partial y^*}{\partial B} = -\dfrac{b}{2\alpha n(b+\xi\alpha^2)} + \dfrac{1}{\alpha n} = \dfrac{b+2\xi\alpha^2}{2\alpha n(b+\xi\alpha^2)} > 0$, $\dfrac{\partial^2 y^*}{\partial B^2} = 0$. ∎

iv. $\dfrac{\partial \Pi_C^*}{\partial B} = -(a - c - \dfrac{bB}{\alpha n})\dfrac{b}{2\alpha n(b+\xi\alpha^2)} + \dfrac{(a-c)}{2\alpha n} = \dfrac{(a-c)\xi\alpha^2 + \frac{b^2 B}{\alpha n}}{2\alpha n(b+\xi\alpha^2)} > 0$, since $a \geq c + \alpha u$;
$\dfrac{\partial^2 \Pi_C^*}{\partial B^2} = \dfrac{b^2}{2\alpha^2 n^2(b+\xi\alpha^2)} > 0$. ∎

Proof of Proposition 3.b:

i. $\dfrac{\partial D^*}{\partial \alpha} = \dfrac{1}{2n(b+\xi\alpha^2)^2}\left[-\alpha^2\xi n(a-c) + 2\xi\alpha bB + nb(a-c)\right]$

The numerator on the right hand side of the above equation is quadratic in α and has roots $\alpha = \frac{b\xi B \mp \sqrt{(b\xi B)^2 + b\xi n^2(a-c)^2}}{\xi n(a-c)}$.

The positive root is $\alpha = \frac{b\xi B + \sqrt{(b\xi B)^2 + b\xi n^2(a-c)^2}}{\xi n(a-c)}$.

Thus, we have $\dfrac{\partial D^*}{\partial \alpha} > 0$ for $\alpha < \frac{b\xi B + \sqrt{(b\xi B)^2 + b\xi n^2(a-c)^2}}{\xi n(a-c)}$, and $\dfrac{\partial D^*}{\partial \alpha} < 0$ for $\alpha > \frac{b\xi B + \sqrt{(b\xi B)^2 + b\xi n^2(a-c)^2}}{\xi n(a-c)}$. ∎

ii. $\dfrac{\partial y^*}{\partial \alpha} = \dfrac{bB}{2\alpha^2 n(b+\xi\alpha^2)} - \dfrac{\xi\alpha}{(b+\xi\alpha^2)^2}(a - c - \dfrac{bB}{\alpha n}) - \dfrac{B}{\alpha^2 n}$
$= -\dfrac{bB + 2B\xi\alpha^2}{2\alpha^2 n(b+\xi\alpha^2)} - \dfrac{\xi\alpha}{(b+\xi\alpha^2)^2}(a - c - \dfrac{bB}{\alpha n}) < 0$, since $(a-c) > \dfrac{bB}{\alpha n}$. ∎

iii. $\dfrac{\partial \Pi_C^*}{\partial \alpha} = \dfrac{bB}{2\alpha^2 n(b+\xi\alpha^2)}(a - c - \dfrac{bB}{\alpha n}) - \dfrac{\xi\alpha}{2(b+\xi\alpha^2)^2}(a - c - \dfrac{bB}{\alpha n})^2 - \dfrac{B(a-c)}{2\alpha^2 n}$
$= -\dfrac{\frac{b^2B^2}{\alpha n} + B(a-c)\xi\alpha^2}{2\alpha^2 n(b+\xi\alpha^2)} - \dfrac{\xi\alpha}{2(b+\xi\alpha^2)^2}(a - c - \dfrac{bB}{\alpha n})^2 < 0.$ ∎

Proof of Proposition 3.c:

i. $\dfrac{\partial^2 D^*}{\partial B\, \partial \alpha} = \dfrac{b\xi\alpha}{n(b+\xi\alpha^2)^2} > 0.$ ∎

ii. $\dfrac{\partial^2 e^*}{\partial B\, \partial \alpha} = \dfrac{b\xi^2\alpha}{n(b+\xi\alpha^2)^2} + \dfrac{b}{\alpha^3 n} > 0.$ ∎

iii. $\dfrac{\partial^2 y^*}{\partial B\, \partial \alpha} = -\dfrac{(b+2\xi\alpha^2)}{2\alpha n(b+\xi\alpha^2)^2}(b+3\xi\alpha^2) + \dfrac{4\xi\alpha}{2\alpha n(b+\xi\alpha^2)}$
$= -\dfrac{(b^2 + b\xi\alpha^2 + 2\xi^2\alpha^4)}{2\alpha^2 n(b+\xi\alpha^2)} < 0.$ ∎

iv. $\dfrac{\partial^2 \Pi_C^*}{\partial B\, \partial \alpha} = -\dfrac{(b+3\xi\alpha^2)}{2\alpha^2 n(b+\xi\alpha^2)^2}\left[(a-c)\xi\alpha^2 + \dfrac{b^2 B}{\alpha n}\right] + \dfrac{1}{2\alpha n(b+\xi\alpha^2)}\left[2\xi\alpha(a-c) - \dfrac{b^2 B}{\alpha^2 n}\right]$
$= -\dfrac{1}{2\alpha^2 n(b+\xi\alpha^2)^2}\left[(a-c)\xi\alpha^2(\xi\alpha^2 - b) + \dfrac{2b^3 B}{\alpha n} + \dfrac{4b^2 B\xi\alpha}{n}\right] < 0$ if $\alpha > \sqrt{\frac{b}{\xi}}$. ∎

Proof of Proposition 4:

We show that if the other $(n-1)$ firms choose $\tilde{y}$, then it is also optimal for the remaining firm to choose $\tilde{y}$. If the other $(n-1)$ firms choose $\tilde{y}$, the unconstrained optimization problem of the remaining firm is *Maximize* $_{\{y\}}\Pi = (\hat{a} - \hat{b}[(n-1)\tilde{y} + y])y + (\beta - (\alpha y - D))u - g(D) - e\beta - cy$.

$$\begin{aligned}\frac{d\Pi}{dy} &= \hat{a} - \hat{b}(n-1)\tilde{y} - 2\hat{b}y - \alpha u - c \\ \frac{d^2\Pi}{dy^2} &= -2\hat{b} < 0\end{aligned}$$

Implying that Π is concave in y and $\{y : \frac{d\Pi}{dy} = 0\}$ is the unique profit-maximizing output. But $\tilde{y} = \frac{\hat{a} - c - \alpha u}{\hat{b}(n+1)}$ satisfies $\frac{d\Pi}{dy} = 0$. Therefore, the unconstrained subgame perfect equilibrium output in the third stage of the game is $\tilde{y} = \frac{\hat{a} - c - \alpha u}{\hat{b}(n+1)}$. ∎

Proof of Proposition 5:

The factor $\frac{(\beta+D)}{\alpha}$ corresponds to the "capacity" constraint on a representative firm's production. In absence of the constraint the subgame perfect equilibrium output would be $\tilde{y}$. Since firm profit increases with output in the range $[0, \tilde{y}]$, each firm produces up to capacity in equilibrium; the resulting subgame perfect equilibrium output in the third stage is $y^* = \frac{\beta+D}{\alpha}$. ∎

Proof of Lemma 2:

The proposed equilibrium satisfies $n \cdot \beta(\hat{e}^*) = B$. We show that if the $(n-1)$ other bidders submit the schedule $\beta(e)$ then it is optimal for the remaining bidder to also submit the same schedule. Assume that the $(n-1)$ bidders submit the schedule $\beta(e)$ and the remaining bidder submits some schedule $\tau(e)$. The clearing price $\hat{e}^*$ satisfies

$$\tau(\hat{e}^*) = B - (n-1)\beta(\hat{e}^*)$$

and the remaining bidder's profit is

$$\begin{aligned}\left[\int_0^{\tau(\hat{e}^*)} (\hat{\sigma} - \hat{\lambda} z)\ dz\right] - \hat{e}^* \cdot \tau(\hat{e}^*) &= (\hat{\sigma} z - \hat{\lambda}\frac{z^2}{2})\Big|_0^{[\frac{2\hat{e}^* B}{n\hat{\sigma} - \hat{\lambda} B}]} - \hat{e}^* \cdot [\frac{2\hat{e}^* B}{n\hat{\sigma} - \hat{\lambda} B}] \\ &= \hat{\sigma}[\frac{2\hat{e}^* B}{n\hat{\sigma} - \hat{\lambda} B}] - 2\hat{\lambda}[\frac{\hat{e}^* B}{n\hat{\sigma} - \hat{\lambda} B}]^2 - [\frac{2\hat{e}*^2 B}{n\hat{\sigma} - \hat{\lambda} B}] \end{aligned} \tag{15}$$

The function on the right hand side of equation (15) is concave in $\hat{e}^*$ and is maximized with respect to $\hat{e}^*$ when

$$2\hat{\sigma}B(n\hat{\sigma} - \hat{\lambda}B) - 4\hat{\lambda}B^2\hat{e}^* - 4B(n\hat{\sigma} - \hat{\lambda}B)\hat{e}^* = 0;\ n\hat{\sigma} \neq \hat{\lambda}B$$

i.e., when $\hat{e}^* = \frac{1}{2}(\hat{\sigma} - \hat{\lambda}\frac{B}{n})$. This is exactly the price that will result if the remaining bidder submits the schedule $\beta(e)$. Therefore the optimal strategy is to submit a schedule such that at each price e, the requested number of permits is $\beta(e) = (\frac{1-2e/(n\hat{\sigma}-\hat{\lambda}B)}{n-1})B$. The subgame perfect equilibrium price is $\hat{e}^* = \frac{1}{2}(\hat{\sigma} - \hat{\lambda}\frac{B}{n})$, and the subgame perfect equilibrium number of permits received by each firm is $\hat{\beta}^* = \frac{B}{n}$. ∎

Proof of Proposition 6:

$$\frac{d\hat{\Pi}_C^*}{dD} = -\frac{2D(\hat{b}n + \xi\alpha^2)}{\alpha^2} + \frac{(2\alpha n(\hat{a} - c) - \hat{b}B(3n-1)}{2\alpha^2 n} \tag{16}$$

$$\frac{d^2\hat{\Pi}_C^*}{dD^2} = -\frac{2(\hat{b}n + \xi\alpha^2)}{\alpha^2} < 0 \tag{17}$$

Implying that $\hat{\Pi}_C^*$ is concave in D. $\frac{d\hat{\Pi}_C^*}{dD} = 0$ yields $\hat{D}^* = \frac{2\alpha n(\hat{a}-c) - \hat{b}B(3n-1)}{4n(\hat{b}n + \xi\alpha^2)}$, the profit-maximizing abatement level. ∎

Proof of Proposition 7.a:

i. $\frac{\partial \hat{D}^*}{\partial B} = -\frac{\hat{b}(3n-1)}{4n(\hat{b}n + \xi\alpha^2)} < 0,\ \frac{\partial^2 \hat{D}^*}{\partial B^2} = 0.$ ∎

ii. $\dfrac{\partial \hat{e}^*}{\partial B} = -\dfrac{\hat{b}^2(n+1)^2 + 4b\xi\alpha^2(n+1)}{8\alpha^2 n(\hat{b}n + \xi\alpha^2)} < 0, \ \dfrac{\partial^2 \hat{e}^*}{\partial B^2} = 0.$ ∎

iii. $\dfrac{\partial \hat{y}^*}{\partial B} = \dfrac{1}{\alpha n} - \dfrac{\hat{b}(3n-1)}{4\alpha n(\hat{b}n + \xi\alpha^2)} = \dfrac{\hat{b}n + 4\xi\alpha^2 + b}{4\alpha n(\hat{b}n + \xi\alpha^2)} > 0, \ \dfrac{\partial^2 \hat{y}^*}{\partial B^2} = 0.$ ∎

iv. $\dfrac{\partial \hat{\Pi}^*}{\partial B} = \dfrac{\hat{b}^2 B(n+1)^2 - 2\alpha[n\hat{b}(\hat{a}-c)(n-1) + 4\hat{b}B\xi\alpha(n-1) - 2\xi\alpha^2 n(\hat{a}-c)]}{8\alpha^2 n^2(\hat{b}n + \xi\alpha^2)}$

The right hand side of the above equation is > 0 if $B < \frac{2\alpha n(\hat{a}-c)(\hat{b}n-\hat{b}-2\xi\alpha^2)}{\hat{b}[\hat{b}(n+1)^2 - 8\xi\alpha^2(n-1)]}$, and < 0 if $B > \frac{2\alpha n(\hat{a}-c)(\hat{b}n-\hat{b}-2\xi\alpha^2)}{\hat{b}[\hat{b}(n+1)^2 - 8\xi\alpha^2(n-1)]}$. ∎

Proof of Proposition 7.b:

i. $\dfrac{\partial \hat{D}^*}{\partial \alpha} = \dfrac{1}{2n(\hat{b}n + \xi\alpha^2)}\left[-\xi\alpha^2 n(\hat{a}-c) + \hat{b}B\xi(3n-1)\alpha + \hat{b}n^2(\hat{a}-c)\right]$

The numerator in the right hand side of the above equation is quadratic in α and has roots $\alpha = \frac{\hat{b}\xi B(3n-1) \mp \sqrt{[\hat{b}\xi B(3n-1)]^2 + 4\hat{b}\xi n^3(\hat{a}-c)^2}}{2\xi n(\hat{a}-c)}$.

The positive root is $\alpha = \frac{\hat{b}\xi B(3n-1) + \sqrt{[\hat{b}\xi B(3n-1)]^2 + 4\hat{b}\xi n^3(\hat{a}-c)^2}}{2\xi n(\hat{a}-c)}$.

Thus, we have $\dfrac{\partial \hat{D}^*}{\partial \alpha} > 0$ for $\alpha < \frac{\hat{b}\xi B(3n-1) + \sqrt{[\hat{b}\xi B(3n-1)]^2 + 4\hat{b}\xi n^3(\hat{a}-c)^2}}{2\xi n(\hat{a}-c)}$, and $\dfrac{\partial \hat{D}^*}{\partial \alpha} < 0$ for $\alpha > \frac{\hat{b}\xi B(3n-1) + \sqrt{[\hat{b}\xi B(3n-1)]^2 + 4\hat{b}\xi n^3(\hat{a}-c)^2}}{2\xi n(\hat{a}-c)}$. ∎

ii. $\dfrac{\partial \hat{y}^*}{\partial \alpha} = \dfrac{-4B(\hat{b}n + \xi\alpha^2) - (\hat{b}n + 3\xi\alpha^2)[2\alpha n(\hat{a}-c) - \hat{b}B(3n-1)] + 2\alpha n(\hat{a}-c)(\hat{b}n + \xi\alpha^2)}{4\alpha^2 n(\hat{b}n + \xi\alpha^2)^2}$

$= \dfrac{-\hat{b}^2 Bn^2 - 4B\xi^2\alpha^4 - b^2 Bn - 3\hat{b}B\xi\alpha^2 - \xi\alpha^2 n[4\alpha(\hat{a}-c) - \hat{b}B]}{4\alpha^2 n(\hat{b}n + \xi\alpha^2)^2} < 0$ since $\alpha(\hat{a}-c) > \hat{b}B$. ∎

Proof of Proposition 7.c:

i. $\dfrac{\partial^2 \hat{D}^*}{\partial B\ \partial \alpha} = \dfrac{2\hat{b}\xi\alpha(3n-1)}{4n(\hat{b}n + \xi\alpha^2)^2} > 0.$ ∎

ii. $\dfrac{\partial^2 \hat{e}^*}{\partial B\ \partial \alpha} = \dfrac{\hat{b}^2(n+1)^2(\alpha\hat{b}n + 2\xi\alpha^3)}{4\alpha^4 n(\hat{b}n + \xi\alpha^2)^2} + \dfrac{b\xi^2\alpha(n+1)}{n(\hat{b}n + \xi\alpha^2)^2} > 0.$ ∎

iii. $\dfrac{\partial^2 \hat{y}^*}{\partial B\ \partial \alpha} = \dfrac{-\hat{b}^2 n^2 - b^2 n - 3\hat{b}\xi\alpha^2 - \xi\alpha^2(4\xi\alpha^2 - \hat{b}n)}{4\alpha^2 n(\hat{b}n - \xi\alpha^2)} < 0$ if $\alpha > \sqrt{\frac{\hat{b}n}{4\xi}}$. ∎

Proof of Proposition 8:
$\dfrac{\partial D^*}{\partial B} = -\frac{\alpha(\frac{b}{\alpha n} + \frac{\rho}{2n})}{2(b + \rho\alpha + \xi\alpha^2)}$. The right hand side is > 0 if for $\rho < 0$, $\frac{b}{\alpha n} - \frac{|\rho|}{2n} < 0$; i.e., if $\alpha > \frac{2b}{|\rho|}$. The condition in Proposition 16 can be rewritten as $\xi\alpha^2 - |\rho|\alpha + b \geq 0$; the left hand side is quadratic in α and is > 0 when $\alpha > \frac{|\rho| + \sqrt{\rho^2 - 4b\xi}}{2\xi}$ and hence, when $\alpha > \frac{|\rho|}{\xi}$. ∎

Proof of Proposition 9.a:
$\dfrac{\partial \hat{D}^*}{\partial B} = -\frac{\rho\alpha + \hat{b}(3n-1)}{4n(\hat{b}n + \rho\alpha + \xi\alpha^2)}$. The right hand side is > 0 if for $\rho < 0$, $|\rho|\alpha - \hat{b}(3n-1) > 0$; i.e., if

$\alpha > \frac{\hat{b}(3n-1)}{|\rho|}$. The condition in Proposition 19 can be rewritten as $\xi\alpha^2 - |\rho|\alpha + \hat{b}n \geq 0$; the left hand side is quadratic in α and is > 0 when $\alpha > \frac{|\rho|+\sqrt{\rho^2-4\hat{b}\xi n}}{2\xi}$ and hence, when $\alpha > \frac{|\rho|}{\xi}$. ∎

Proof of Proposition 9.b:
$\frac{\partial\hat{y}^*}{\partial\alpha} = -\frac{[4B\xi\alpha^2-(2n(\hat{a}-c)-5\rho B)\alpha+4\hat{b}Bn]}{4\alpha^2 n(\hat{b}n+\rho\alpha+\xi\alpha^2)} - \frac{[2\alpha n(\hat{a}-c)-\rho B\alpha-\hat{b}B(3n-1)](\hat{b}n+2\rho\alpha+3\xi\alpha^2)}{4\alpha^2 n(\hat{b}n+\rho\alpha+\xi\alpha^2)^2}$. When $\rho < 0$, $\hat{b}n - 2|\rho|\alpha+3\xi\alpha^2 > 0$ if $\alpha < \frac{|\rho|-\sqrt{\rho^2-3\hat{b}\xi n}}{3\xi}$. $2\alpha n(\hat{a}-c)+|\rho|B\alpha-\hat{b}B(3n-1) < 0$ if $\alpha < \frac{\hat{b}B(3n-1)}{2n(\hat{a}-c)+|\rho|B}$. Denote $\alpha_1 = \frac{(2n(\hat{a}-c)+5|\rho|B)-\sqrt{(2n(\hat{a}-c)+5|\rho|B)^2-64\hat{b}B^2\xi n}}{8B\xi}$, $\alpha_2 = \min\{\frac{|\rho|-\sqrt{\rho^2-3\hat{b}\xi n}}{3\xi}, \frac{\hat{b}B(3n-1)}{2n(\hat{a}-c)+|\rho|B}\}$, and $\alpha_3 = \frac{(2n(\hat{a}-c)+5|\rho|B)+\sqrt{(2n(\hat{a}-c)+5|\rho|B)^2-64\hat{b}B^2\xi n}}{8B\xi}$. If $\alpha_1, \alpha_3 \in \mathbf{R}^+$, $4B\xi\alpha^2 - (2n(\hat{a}-c)+5|\rho|B)\alpha + 4\hat{b}Bn < 0$ for $\alpha \in (\alpha_1, \alpha_3)$; and if $\alpha_1 < \alpha_2 < \alpha_3$ when $\rho < 0$, we have $\frac{\partial\hat{y}^*}{\partial\alpha} > 0$ for $\alpha \in (\alpha_1, \alpha_2)$. ∎

Analysis for Section 6 - Independent Demands:

The optimization problem in the third stage is:
Maximize $_{\{y\}}$ $\Pi = (a-by)y + (\beta-\alpha y)u - g(\mu) - e\beta - cy$; *Subject to:* $\theta \leq \beta$, $y \geq 0$.

Proposition 10 *Assume* $u \leq \frac{a-c}{\alpha}$. *In the third stage, given abatement levels and a permit allocation vector, the profit-maximizing quantity* $y^* = \min\left\{\frac{a-c-\alpha u}{2b}, \frac{\beta}{\alpha}\right\}$.

Proof: Identical to the proof of Proposition 1.

The constraint on θ can be re-written in terms of β since, when $\frac{\beta}{\alpha} < \frac{a-c-\alpha u}{2b}$, the constraint on θ is binding, and vice-versa. Denote $\bar{\beta}_0 = \frac{\alpha(a-c-\alpha u)}{2b}$. Thus, when $\beta < \bar{\beta}_0$, the constraint on θ is binding. We now establish expressions for the optimal profit for the two cases viz., $\beta < \bar{\beta}_0$ and $\beta \geq \bar{\beta}_0$.

Case (i): $\beta < \bar{\beta}_0$ (or equivalently, $y^* = \frac{\beta}{\alpha}$)

$$\begin{aligned}\Pi_C^* &= \left[a - b(\frac{\beta}{\alpha})\right](\frac{\beta}{\alpha}) + \left[\beta - \alpha(\frac{\beta}{\alpha})\right]u - g(\mu) - e\beta - c(\frac{\beta}{\alpha}) \\ &= -\frac{b}{\alpha^2}\beta^2 + \frac{(a-c-\alpha e)}{\alpha}\beta - g(\mu) \qquad (18)\end{aligned}$$

Case (ii): $\beta \geq \bar{\beta}_0$ (or equivalently, $y^* = \frac{a-c-\alpha u}{2b}$)

$$\begin{aligned}\Pi_U^* &= \left[a - b(\frac{a-c-\alpha u}{2b})\right](\frac{a-c-\alpha u}{2b}) + \left[\beta - \alpha(\frac{a-c-\alpha u}{2b})\right]u \\ &\quad -g(\mu) - e\beta - c(\frac{a-c-\alpha u}{2b}) \\ &= \psi_0 + \beta(u-e) \qquad (19)\end{aligned}$$

Π_C^* represents the subgame perfect equilibrium profit when the emissions constraint is binding, Π_U^* is the subgame perfect equilibrium profit when the emissions constraint is not binding, and $\psi_0 = [a - b(\frac{a-c-\alpha u}{2b})](\frac{a-c-\alpha u}{2b}) - \alpha u(\frac{a-c-\alpha u}{2b}) - g(\mu) - c(\frac{a-c-\alpha u}{2b})$ is a constant, given the abatement level chosen in the first stage. The"marginal value function" $v(\beta)$ is:

When $\beta < \bar{\beta}_0$,

$$\begin{aligned} v(\beta) &= -\frac{2b}{\alpha^2}\beta + \frac{(a-c)}{\alpha} \\ &= \sigma_0 - \lambda\beta \end{aligned} \tag{20}$$

When $\beta \geq \bar{\beta}_0$,

$$v(\beta) = u \tag{21}$$

Where $\sigma_0 = \frac{(a-c)}{\alpha}$ and $\lambda = \frac{2b}{\alpha^2}$ are constants, given the abatement level chosen in the first stage. Note that $\sigma_0 - \lambda\bar{\beta}_0 = u$, and when $\beta < \bar{\beta}_0$, $v(\beta) > u$. We now work backwards to the second stage where firms participate in a sealed-bid uniform price share auction for pollution permits.

Lemma 3 *For the share auction in the second stage, it is an optimal strategy to submit a schedule such that at each price e, the requested number of permits is $\beta(e) = (\frac{1-2e/(n\sigma_0-\lambda B)}{n-1})B$.*[31] *The subgame perfect equilibrium price is $e^* = \frac{1}{2}(\sigma_0 - \lambda\frac{B}{n})$, and the subgame perfect equilibrium number of permits received by each firm is $\beta^* = \frac{B}{n}$.*

Proof: Identical to the proof of Lemma 1.

Substituting the subgame perfect equilibrium permit price and permit share from Lemma 3 into the profit function in equation (18), we have

$$\Pi_C^* = \frac{B(a-c)(1+\mu)}{2\alpha_0 n} - \xi\mu^2 \tag{22}$$

Proposition 11 *The profit function Π_C^* in (22) is concave in μ and $\mu^* = \frac{B(a-c)}{4\alpha_0\xi n}$ uniquely maximizes Π_C^*.*

Proof:

$$\frac{d\Pi_C^*}{d\mu} = \frac{B(a-c)}{2\alpha_0 n} - 2\xi\mu \tag{23}$$

$$\frac{d^2\Pi_C^*}{dD^2} = -2\xi < 0 \tag{24}$$

Implying that Π_C^* is concave in μ. $\frac{d\Pi_C^*}{d\mu} = 0$ yields $\mu^* = \frac{B(a-c)}{4\alpha_0\xi n}$, the profit-maximizing abatement level. ∎

Analysis for Section 6 - Competition:

In the case of competition, each firm faces the following profit maximization problem in the third stage of the game: *Maximize* $_{\{y\}}$ $\Pi = (\hat{a} - \hat{b}Y)y + (\beta - \alpha y)u - g(\mu) - e\beta - cy$; *Subject to:* $\theta \leq \beta$, $y \geq 0$. Proposition 12 gives us the unconstrained subgame perfect equilibrium output when firms compete in a Cournot fashion.

[31] Equivalently, the schedule for the fraction of total available permits is $s(e) = (\frac{1-2e/(n\sigma_0-\lambda B)}{n-1})$.

Proposition 12 *Assume $u \leq \frac{\hat{a}-c}{\alpha}$. The unconstrained subgame perfect equilibrium output in the third stage of the game when firms compete in a Cournot fashion, is $\tilde{y} = \frac{\hat{a}-c-\alpha u}{\hat{b}(n+1)}$.*

Proof: Identical to the proof of Proposition 4.

Again we focus on the situation when the emissions constraint binds for all firms; i.e., when $\frac{\beta}{\alpha} < \frac{\hat{a}-c-\alpha u}{\hat{b}(n+1)}$ (or equivalently $\beta < \breve{\beta}_0$, where $\breve{\beta}_0 = \alpha[\frac{\hat{a}-c-\alpha u}{\hat{b}(n+1)}]$), in which case Proposition 13 applies.

Proposition 13 *Assume that the emissions constraint binds for all firms. If the abatement level chosen by each firm in the investment stage is symmetrically μ, and the number of permits secured by each firm in the auction stage is symmetrically β, the subgame perfect equilibrium output of each firm in the third stage of the game when firms compete in a Cournot fashion, is $y^* = \frac{\beta}{\alpha}$, where $\alpha = \frac{\alpha_0}{1+\mu}$.*

Proof: Identical to the proof of Proposition 5.

We now establish expressions for the subgame perfect equilibrium profit for the two cases viz., $\beta < \breve{\beta}_0$ and $\beta \geq \breve{\beta}_0$. Let Z denote the total output of all other firms.
Case (i): $\beta < \breve{\beta}_0$ (or equivalently, $y^* = \frac{\beta}{\alpha}$)

$$\begin{aligned}\hat{\Pi}^*_C &= \left[\hat{a} - \hat{b}(Z + \frac{\beta}{\alpha})\right](\frac{\beta}{\alpha}) + (\beta - \alpha(\frac{\beta}{\alpha}))u - g(\mu) - e\beta - c(\frac{\beta}{\alpha}) \\ &= -\frac{\hat{b}}{\alpha^2}\beta^2 + \left[\frac{(\hat{a} - c - \alpha e) - \hat{b}Z}{\alpha}\right]\beta - g(\mu) \qquad (25)\end{aligned}$$

Case (ii): $\beta \geq \breve{\beta}_0$ (or equivalently, $y^* = \frac{\hat{a}-(c+\rho D)-\alpha u}{\hat{b}(n+1)}$)

$$\begin{aligned}\hat{\Pi}^*_U &= \left[\hat{a} - \hat{b}n[\frac{\hat{a}-c-\alpha u}{\hat{b}(n+1)}]\right][\frac{\hat{a}-c-\alpha u}{\hat{b}(n+1)}] + \left[\beta - \alpha[\frac{\hat{a}-c-\alpha u}{\hat{b}(n+1)}]\right]u \\ &\quad -g(\mu) - e\beta - c[\frac{\hat{a}-c-\alpha u}{\hat{b}(n+1)}] \\ &= \hat{\psi}_0 + \beta(u-e) \qquad (26)\end{aligned}$$

$\hat{\Pi}^*_C$ represents the subgame perfect equilibrium profit when the emissions constraint is binding, $\hat{\Pi}^*_U$ is the unconstrained subgame perfect equilibrium profit, and $\hat{\psi}_0 = \left[\hat{a} - \hat{b}n[\frac{\hat{a}-c-\alpha u}{\hat{b}(n+1)}]\right][\frac{\hat{a}-c-\alpha u}{\hat{b}(n+1)}] - \alpha u[\frac{\hat{a}-c-\alpha u}{\hat{b}(n+1)}] - g(\mu) - c[\frac{\hat{a}-c-\alpha u}{\hat{b}(n+1)}]$ is a constant, given the abatement level chosen in the first stage. The marginal value function $v(\beta)$ in the constrained competition case is:
When $\beta < \breve{\beta}_0$,

$$v(\beta) = -\frac{2\hat{b}}{\alpha^2}\beta + \left[\frac{(\hat{a} - c - \alpha e) - \hat{b}Z}{\alpha}\right]$$

Substituting $Z = (n-1)(\frac{\beta}{\alpha})$ from symmetric equilibria in the investment and auction stages, we have

$$\begin{aligned}v(\beta) &= -\frac{\hat{b}(n+1)}{\alpha^2}\beta + \frac{(\hat{a} - c)}{\alpha} \\ &= \hat{\sigma}_0 - \hat{\lambda}\beta \qquad (27)\end{aligned}$$

When $\beta \geq \breve{\beta}_0$,

$$v(\beta) \quad = \quad u \tag{28}$$

Where $\hat{\sigma}_0 = \frac{(\hat{a}-c)}{\alpha}$ and $\hat{\lambda} = \frac{\hat{b}(n+1)}{\alpha^2}$ are constants, given the abatement level chosen in the first stage. Note that $\hat{\sigma}_0 - \hat{\lambda}\breve{\beta}_0 = u$, and when $\beta < \breve{\beta}_0$, $v(\beta) > u$. We now have a representative firm's marginal value function before the auction is entered into.

Lemma 4 *For the share auction in the second stage, it is an optimal strategy to submit a schedule such that at each price e, the requested number of permits is* $\beta(e) = (\frac{1-2e/(n\hat{\sigma}_0 - \hat{\lambda}B)}{n-1})B$. *The subgame perfect equilibrium price is* $\hat{e}^* = \frac{1}{2}(\hat{\sigma}_0 - \hat{\lambda}\frac{B}{n})$, *and the subgame perfect equilibrium number of permits received by each firm is* $\hat{\beta}^* = \frac{B}{n}$.

Proof: Identical to the proof of Lemma 2.

Substituting the subgame perfect equilibrium permit price and permit share from Lemma 4 into the profit function in equation (25), we have

$$\hat{\Pi}_C^* \quad = \quad \frac{B\alpha_0 n(\hat{a}-c)(1+\mu) - \hat{b}B^2(n-1)(1+\mu)^2}{2\alpha_0^2 n^2} - \xi\mu^2 \tag{29}$$

We can now arrive at the profit-maximizing abatement level. Proposition 14 shows that the profit function in the first stage of the game, after incorporating the results from the third and second stages, is concave in the abatement level.

Proposition 14 *The profit function* $\hat{\Pi}_C^*$ *in (29) is concave in* μ. $\hat{\mu}^* = \frac{B\alpha_0 n(\hat{a}-c) - 2\hat{b}B^2(n-1)}{2\hat{b}B^2(n-1) + 4\xi{\alpha_0}^2 n^2}$ *uniquely maximizes* $\hat{\Pi}_C^*$.

Proof:

$$\frac{d\hat{\Pi}_C^*}{d\mu} \quad = \quad \frac{B\alpha_0 n(\hat{a}-c) - 2\hat{b}B^2(n-1)(1+\mu)}{2\alpha_0^2 n^2} - 2\xi\mu \tag{30}$$

$$\frac{d^2\hat{\Pi}_C^*}{d\mu^2} \quad = \quad \frac{-\hat{b}B^2(n-1)}{\alpha_0^2 n^2} - 2\xi < 0 \tag{31}$$

Implying that $\hat{\Pi}_C^*$ is concave in μ. $\frac{d\hat{\Pi}_C^*}{d\mu} = 0$ yields $\hat{\mu}^* = \frac{B\alpha_0 n(\hat{a}-c) - 2\hat{b}B^2(n-1)}{2\hat{b}B^2(n-1) + 4\xi{\alpha_0}^2 n^2}$, the profit-maximizing abatement level. ∎

To ensure non-negative equilibrium values of abatement, permit price, output, and product price, we assume that $(\hat{a}-c) > \frac{2\hat{b}B}{\alpha_0}$.

Analysis for Section 5 - Independent Demands:

The optimization problem in the third stage is:

Maximize $_{\{y\}}$ $\Pi = (a-by)y + (\beta - (\alpha y - D))u - g(D) - e\beta - (c+\rho D)y$; *Subject to:* $\theta \leq \beta$, $y \geq 0$.

Proposition 15 *Assume* $u \leq \frac{a-(c+\rho D)}{\alpha}$. *In the third stage, given abatement levels and a permit allocation vector, the profit-maximizing quantity* $y^* = \min\left\{\frac{a-(c+\rho D)-\alpha u}{2b}, \frac{\beta+D}{\alpha}\right\}$.

Proof: Identical to the proof of Proposition 1.

The constraint on θ can be re-written in terms of β since, when $\frac{\beta+D}{\alpha} < \frac{a-(c+\rho D)-\alpha u}{2b}$, the constraint on θ is binding, and vice-versa. Denote $\bar{\beta}_0 = \frac{\alpha(a-(c+\rho D)-\alpha u)}{2b} - D$. Thus, when $\beta < \bar{\beta}_0$, the constraint on θ is binding. We now establish expressions for the optimal profit for the two cases viz., $\beta < \bar{\beta}_0$ and $\beta \geq \bar{\beta}_0$.
Case (i): $\beta < \bar{\beta}_0$ (or equivalently, $y^* = \frac{\beta+D}{\alpha}$)

$$\begin{aligned}
\Pi_C^* &= \left[a - b(\frac{\beta+D}{\alpha})\right](\frac{\beta+D}{\alpha}) + \left[\beta - \left(\alpha(\frac{\beta+D}{\alpha}) - D\right)\right]u - g(D) - e\beta - (c+\rho D)(\frac{\beta+D}{\alpha}) \\
&= -\frac{b}{\alpha^2}\beta^2 + \left[\frac{\alpha(a-(c+\rho D)-\alpha e) - 2bD}{\alpha^2}\right]\beta + \left[\frac{\alpha[(a-(c+\rho D))D - \alpha g(D)] - bD^2}{\alpha^2}\right] \quad (32)
\end{aligned}$$

Case (ii): $\beta \geq \bar{\beta}_0$ (or equivalently, $y^* = \frac{a-(c+\rho D)-\alpha u}{2b}$)

$$\begin{aligned}
\Pi_U^* &= \left[a - b(\frac{a-(c+\rho D)-\alpha u}{2b})\right](\frac{a-(c+\rho D)-\alpha u}{2b}) + \left[\beta - \left(\alpha(\frac{a-(c+\rho D)-\alpha u}{2b}) - D\right)\right]u \\
&\quad -g(D) - e\beta - (c+\rho D)(\frac{a-(c+\rho D)-\alpha u}{2b}) \\
&= \psi_0 + \beta(u-e) \quad (33)
\end{aligned}$$

Π_C^* represents the subgame perfect equilibrium profit when the emissions constraint is binding, Π_U^* is the subgame perfect equilibrium profit when the emissions constraint is not binding, and $\psi_0 = [a - b(\frac{a-(c+\rho D)-\alpha u}{2b})](\frac{a-(c+\rho D)-\alpha u}{2b}) - [\alpha(\frac{a-(c+\rho D)-\alpha u}{2b}) - D]u - g(D) - (c+\rho D)(\frac{a-(c+\rho D)-\alpha u}{2b})$ is a constant, given the abatement level chosen in the first stage. The"marginal value function" $v(\beta)$ is:
When $\beta < \bar{\beta}_0$,

$$\begin{aligned}
v(\beta) &= -\frac{2b}{\alpha^2}\beta + [\frac{\alpha(a-(c+\rho D)) - 2bD}{\alpha^2}] \\
&= \sigma_0 - \lambda\beta \quad (34)
\end{aligned}$$

When $\beta \geq \bar{\beta}_0$,

$$v(\beta) = u \quad (35)$$

Where $\sigma_0 = [\frac{\alpha(a-(c+\rho D))-2bD}{\alpha^2}]$ and $\lambda = \frac{2b}{\alpha^2}$ are constants, given the abatement level chosen in the first stage. Note that $\sigma_0 - \lambda\bar{\beta}_0 = u$, and when $\beta < \bar{\beta}_0$, $v(\beta) > u$. We now work backwards to the second stage where firms participate in a sealed-bid uniform price share auction for pollution permits.

Lemma 5 *For the share auction in the second stage, it is an optimal strategy to submit a schedule such that at each price e, the requested number of permits is* $\beta(e) = (\frac{1-2e/(n\sigma_0-\lambda B)}{n-1})B$.[32] *The subgame perfect equilibrium price is* $e^* = \frac{1}{2}(\sigma_0 - \lambda\frac{B}{n})$*, and the subgame perfect equilibrium number of permits received by each firm is* $\beta^* = \frac{B}{n}$.

Proof: Identical to the proof of Lemma 1.

Substituting the subgame perfect equilibrium permit price and permit share from Lemma 5 into the profit function in equation (32), we have

$$\begin{aligned}
\Pi_C^* &= -\frac{bB^2}{\alpha^2 n^2} + \left[\alpha\left(a-(c+\rho D)-\alpha\left(\frac{\alpha\left(a-(c+\rho D)\right)-2\,bD}{2\alpha^2}-\frac{bB}{\alpha^2 n}\right)\right)-2\,bD\right]\frac{B}{\alpha^2 n} \\
&\quad + \frac{\alpha\left[\left(a-(c+\rho D)\right)D-\alpha g(D)\right]-bD^2}{\alpha^2} \qquad (36) \\
&= -\frac{(b+\rho\alpha+\xi\alpha^2)}{\alpha^2}D^2 + \left(\frac{a}{\alpha}-\frac{c}{\alpha}-\frac{bB}{\alpha^2 n}-\frac{\rho B}{2\alpha n}\right)D + \frac{B}{2\alpha n}(a-c) \qquad (37)
\end{aligned}$$

Proposition 16 *Assume* $\rho \geq -\frac{(b+\xi\alpha^2)}{\alpha}$. *The profit function* Π_C^* *in (37) is concave in D and* $D^* = \frac{\alpha}{2(b+\rho\alpha+\xi\alpha^2)}(a-c-\frac{bB}{\alpha n}-\frac{\rho B}{2n})$ *uniquely maximizes* Π_C^*.

Proof:

$$\begin{aligned}
\frac{d\Pi_C^*}{dD} &= -\frac{2(b+\rho\alpha+\xi\alpha^2)}{\alpha^2}D + \left(\frac{a}{\alpha}-\frac{c}{\alpha}-\frac{bB}{\alpha^2 n}-\frac{\rho B}{2\alpha n}\right) \qquad (38) \\
\frac{d^2\Pi_C^*}{dD^2} &= -\frac{2(b+\rho\alpha+\xi\alpha^2)}{\alpha^2} \qquad (39)
\end{aligned}$$

Implying that Π_C^* is concave in D if $\rho \geq -\frac{(b+\xi\alpha^2)}{\alpha}$. $\frac{d\Pi_C^*}{dD} = 0$ yields $D^* = \frac{\alpha}{2(b+\rho\alpha+\xi\alpha^2)}(a-c-\frac{bB}{\alpha n}-\frac{\rho B}{2n})$, the profit-maximizing abatement level. ∎

To ensure non-negative equilibrium values of abatement, permit price, output, and product price when $\rho > 0$, we assume that $(a-c) > \frac{bB}{\alpha n} + \frac{\rho B}{2n}$.

Analysis for Section 5 - Competition:

In the case of competition, each firm faces the following profit maximization problem in the third stage of the game: *Maximize* $_{\{y\}}\ \Pi = (\hat{a}-\hat{b}Y)y + (\beta-(\alpha y-D))u - g(D) - e\beta - (c+\rho D)y$; *Subject to:* $\theta \leq \beta$, $y \geq 0$. Proposition 17 gives us the unconstrained subgame perfect equilibrium output when the firms compete in a Cournot fashion.

Proposition 17 *Assume* $u \leq \frac{\hat{a}-(c+\rho D)}{\alpha}$. *The unconstrained subgame perfect equilibrium output in the third stage of the game when firms compete in a Cournot fashion, is* $\tilde{y} = \frac{\hat{a}-(c+\rho D)-\alpha u}{\hat{b}(n+1)}$.

[32] Equivalently, the schedule for the fraction of total available permits is $s(e) = (\frac{1-2e/(n\sigma_0-\lambda B)}{n-1})$.

Proof: Identical to the proof of Proposition 4.

Again we focus on the situation when the emissions constraint binds for all firms; i.e., when $\frac{(\beta+D)}{\alpha} < \frac{\hat{a}-(c+\rho D)-\alpha u}{\hat{b}(n+1)}$ (or equivalently $\beta < \breve{\beta}_0$, where $\breve{\beta}_0 = \alpha[\frac{\hat{a}-(c+\rho D)-\alpha u}{\hat{b}(n+1)}] - D$), in which case Proposition 18 applies.

Proposition 18 *Assume that the emissions constraint binds for all firms. If the abatement level chosen by each firm in the investment stage is symmetrically D, and the number of permits secured by each firm in the auction stage is symmetrically β, the subgame perfect equilibrium output of each firm in the third stage of the game when firms compete in a Cournot fashion, is $y^* = \frac{\beta+D}{\alpha}$.*

Proof: Identical to the proof of Proposition 5.

We now establish expressions for the subgame perfect equilibrium profit for the two cases viz., $\beta < \breve{\beta}_0$ and $\beta \geq \breve{\beta}_0$. Let Z denote the total output of all other firms.
Case (i): $\beta < \breve{\beta}_0$ (or equivalently, $y^* = \frac{\beta+D}{\alpha}$)

$$\begin{aligned}\hat{\Pi}^*_C &= \left[\hat{a} - \hat{b}(Z + \frac{\beta+D}{\alpha})\right](\frac{\beta+D}{\alpha}) + \left[\beta - \left(\alpha(\frac{\beta+D}{\alpha}) - D\right)\right]u - g(D) - e\beta \\ &\quad -(c+\rho D)(\frac{\beta+D}{\alpha}) \\ &= -\frac{\hat{b}}{\alpha^2}\beta^2 + \left[\frac{\alpha(\hat{a}-(c+\rho D)-\alpha e) - \hat{b}(2D+\alpha Z)}{\alpha^2}\right]\beta \\ &\quad + \left[\frac{\alpha[(\hat{a}-(c+\rho D))D - \alpha g(D)] - \hat{b}D(D+\alpha Z)}{\alpha^2}\right] \end{aligned} \tag{40}$$

Case (ii): $\beta \geq \breve{\beta}_0$ (or equivalently, $y^* = \frac{\hat{a}-(c+\rho D)-\alpha u}{\hat{b}(n+1)}$)

$$\begin{aligned}\hat{\Pi}^*_U &= \left[\hat{a} - \hat{b}n[\frac{\hat{a}-(c+\rho D)-\alpha u}{\hat{b}(n+1)}]\right][\frac{\hat{a}-(c+\rho D)-\alpha u}{\hat{b}(n+1)}] + \left[\beta - \left(\alpha[\frac{\hat{a}-(c+\rho D)-\alpha u}{\hat{b}(n+1)}] - D\right)\right]u \\ &\quad -g(D) - e\beta - (c+\rho D)[\frac{\hat{a}-(c+\rho D)-\alpha u}{\hat{b}(n+1)}] \\ &= \hat{\psi}_0 + \beta(u-e) \end{aligned} \tag{41}$$

$\hat{\Pi}^*_C$ represents the subgame perfect equilibrium profit when the emissions constraint is binding, $\hat{\Pi}^*_U$ is the unconstrained subgame perfect equilibrium profit, and $\hat{\psi}_0 = \left[\hat{a} - \hat{b}n[\frac{\hat{a}-(c+\rho D)-\alpha u}{\hat{b}(n+1)}]\right][\frac{\hat{a}-(c+\rho D)-\alpha u}{\hat{b}(n+1)}] - \left(\alpha[\frac{\hat{a}-(c+\rho D)-\alpha u}{\hat{b}(n+1)}] - D\right)u - g(D) - (c+\rho D)[\frac{\hat{a}-(c+\rho D)-\alpha u}{\hat{b}(n+1)}]$ is a constant, given the abatement level chosen in the first stage. The marginal value function $v(\beta)$ in the constrained competition case is:
When $\beta < \breve{\beta}_0$,

$$v(\beta) = -\frac{2\hat{b}}{\alpha^2}\beta + [\frac{\alpha(\hat{a}-(c+\rho D)) - \hat{b}(2D+\alpha Z)}{\alpha^2}]$$

Substituting $Z = (n-1)(\frac{\beta+D}{\alpha})$ from symmetric equilibria in the investment and auction stages, we have

$$\begin{aligned} v(\beta) &= -\frac{2\hat{b}}{\alpha^2}\beta + [\frac{\alpha(\hat{a}-(c+\rho D)) - \hat{b}[2D + \alpha(n-1)(\frac{\beta+D}{\alpha})]}{\alpha^2}] \\ &= -\frac{\hat{b}(n+1)}{\alpha^2}\beta + [\frac{\alpha(\hat{a}-(c+\rho D)) - \hat{b}(n+1)D}{\alpha^2}] \\ &= \hat{\sigma}_0 - \hat{\lambda}\beta \end{aligned} \tag{42}$$

When $\beta \geq \breve{\beta}_0$,

$$v(\beta) = u \tag{43}$$

Where $\hat{\sigma}_0 = [\frac{\alpha(\hat{a}-(c+\rho D)) - \hat{b}(n+1)D}{\alpha^2}]$ and $\hat{\lambda} = \frac{\hat{b}(n+1)}{\alpha^2}$ are constants, given the abatement level chosen in the first stage. Note that $\hat{\sigma}_0 - \hat{\lambda}\breve{\beta}_0 = u$, and when $\beta < \breve{\beta}_0$, $v(\beta) > u$. We now have a representative firm's marginal value function before the auction is entered into.

Lemma 6 *For the share auction in the second stage, it is an optimal strategy to submit a schedule such that at each price e, the requested number of permits is* $\beta(e) = (\frac{1-2e/(n\hat{\sigma}_0 - \hat{\lambda}B)}{n-1})B$*. The subgame perfect equilibrium price is* $\hat{e}^* = \frac{1}{2}(\hat{\sigma}_0 - \hat{\lambda}\frac{B}{n})$*, and the subgame perfect equilibrium number of permits received by each firm is* $\hat{\beta}^* = \frac{B}{n}$*.*

Proof: Identical to the proof of Lemma 2.

Substituting the subgame perfect equilibrium permit price and permit share from Lemma 6 into the profit function in equation (40), we have

$$\begin{aligned} \hat{\Pi}_C^* &= -\frac{(\hat{b}n + \rho\alpha + \xi\alpha^2)}{\alpha^2}D^2 + \left(\frac{2\alpha n(\hat{a}-c) - \rho\alpha B - \hat{b}B(3n-1)}{2\alpha^2 n}\right)D \\ &\quad + \frac{B\alpha n(\hat{a}-c) - \hat{b}B^2(n-1)}{2\alpha^2 n^2} \end{aligned} \tag{44}$$

We can now arrive at the profit-maximizing abatement level. Proposition 19 shows that the profit function in the first stage of the game, after incorporating the results from the third and second stages, is concave in the abatement level.

Proposition 19 *Assume* $\rho \geq -\frac{(\hat{b}n+\xi\alpha^2)}{\alpha}$*. The profit function* $\hat{\Pi}_C^*$ *in (44) is concave in D.* $\hat{D}^* = \frac{2\alpha n(\hat{a}-c) - \rho\alpha B - \hat{b}B(3n-1)}{4n(\hat{b}n+\rho\alpha+\xi\alpha^2)}$ *uniquely maximizes* $\hat{\Pi}_C^*$*.*

Proof:

$$\frac{d\hat{\Pi}_C^*}{dD} = -\frac{2(\hat{b}n + \rho\alpha + \xi\alpha^2)}{\alpha^2}D + \left(\frac{2\alpha n(\hat{a}-c) - \rho\alpha B - \hat{b}B(3n-1)}{2\alpha^2 n}\right) \tag{45}$$

$$\frac{d^2\hat{\Pi}_C^*}{dD^2} = -\frac{2(\hat{b}n + \rho\alpha + \xi\alpha^2)}{\alpha^2} \tag{46}$$

Implying that $\hat{\Pi}_C^*$ is concave in D if $\rho \geq -\frac{(\hat{b}n+\xi\alpha^2)}{\alpha}$. $\frac{d\hat{\Pi}_C^*}{dD} = 0$ yields $\hat{D}^* = \frac{2\alpha n(\hat{a}-c)-\rho\alpha B-\hat{b}B(3n-1)}{4n(\hat{b}n+\rho\alpha+\xi\alpha^2)}$, the profit-maximizing abatement level. ∎

To ensure non-negative equilibrium values of abatement, permit price, output, and product price, we assume that $(\hat{a}-c) > \frac{\hat{b}B(3n-1)}{2\alpha n} + \frac{\rho B}{2n}$.

Appendix B: Figures

Section 3[33]

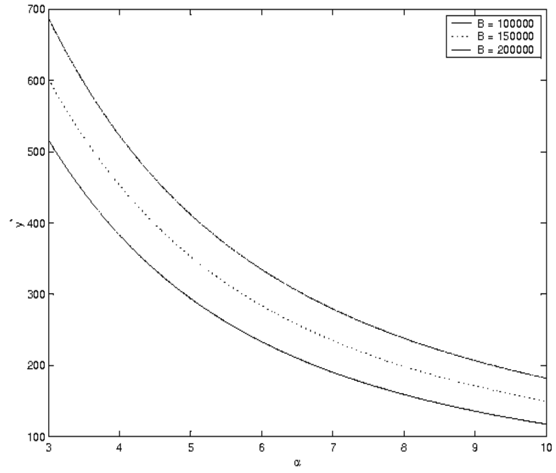

Figure 11: Equilibrium firm output (y^*) versus α

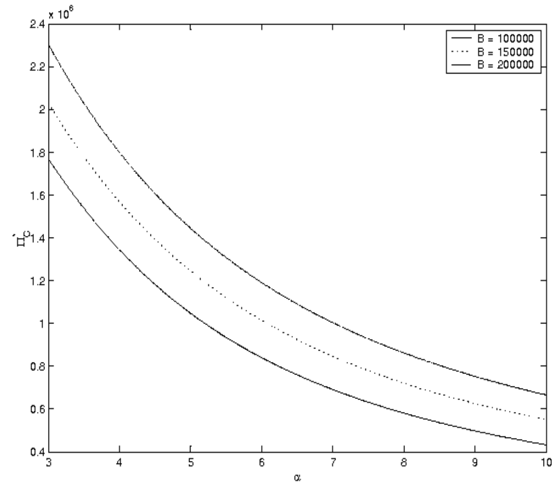

Figure 12: Equilibrium firm profit (Π_C^*) versus α

[33]Parameter values for Figures 11 and 12 were $a = 7,500$, $b = 5$, $c = 10$, $u = 200$, $\xi = 0.65$, $n = 150$.

Section 4[34]

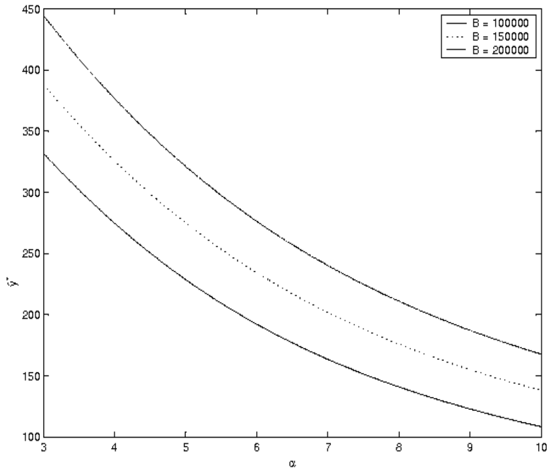

Figure 13: Equilibrium firm output ($\hat{y}^*$) versus α

[34]Parameter values for Figure 13 were $a = 7,500$, $b = 0.075$, $c = 10$, $u = 200$, $\xi = 0.65$, $n = 150$.

Section 6 - Independent Demands[35]

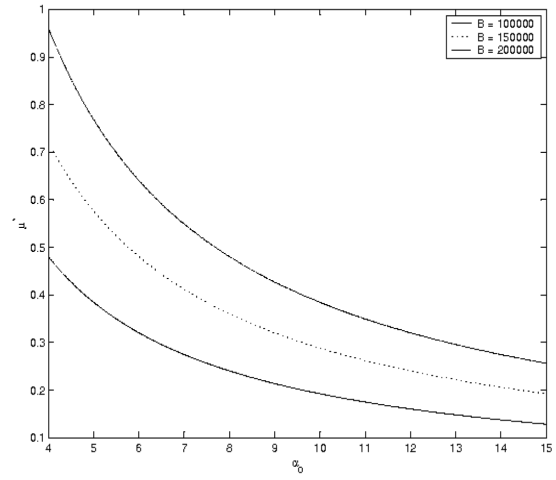

Figure 14: Equilibrium abatement (μ^*) versus α_0

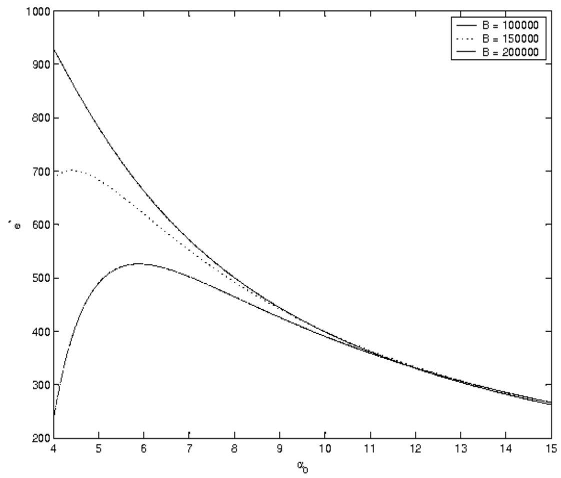

Figure 15: Equilibrium permit price (e^*) versus α_0

[35]Parameter values for Figures 14, 15, 16, and 17 were $a = 7,500$, $b = 5$, $c = 10$, $u = 200$, $\xi = 650000$, $n = 150$.

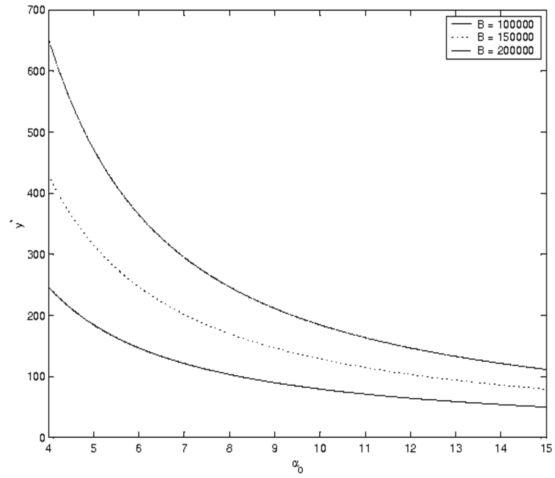

Figure 16: Equilibrium output (y^*) versus α_0

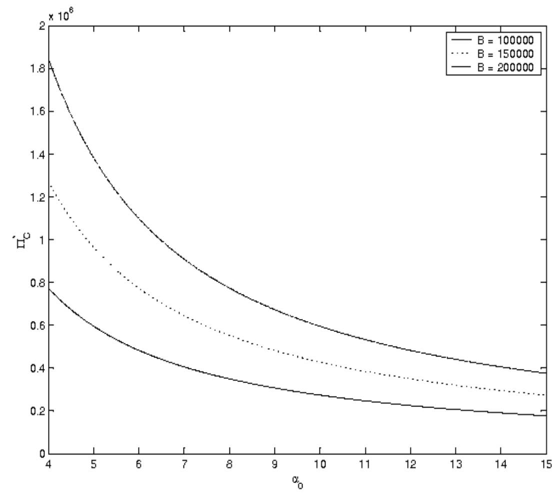

Figure 17: Equilibrium firm profit (Π_C^*) versus α_0

Section 6 - Competition[36]

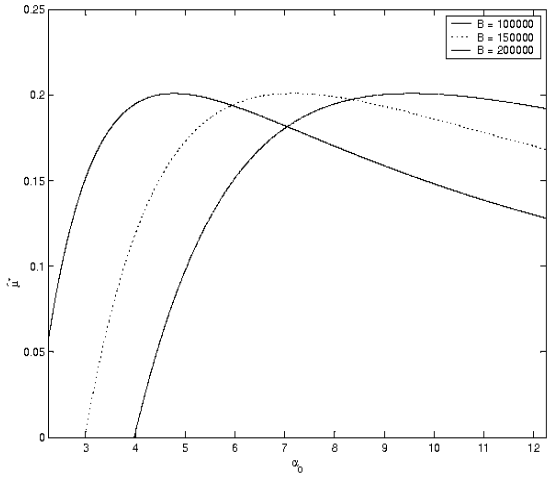

Figure 18: Equilibrium abatement ($\hat{\mu}^*$) versus α_0

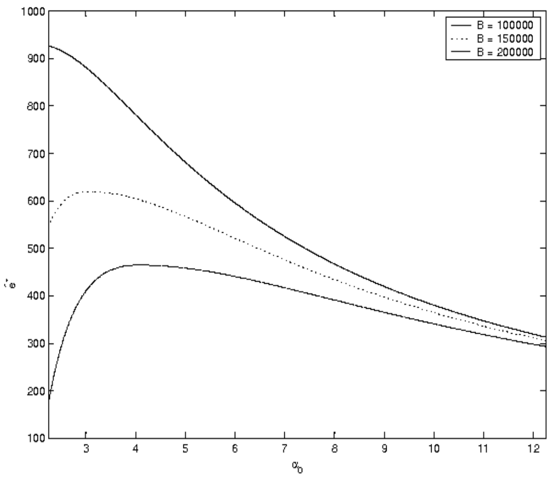

Figure 19: Equilibrium permit price ($\hat{e}^*$) versus α_0

[36]Parameter values for Figures 18, 19, 20, and 21 were $a = 7,500$, $b = 0.075$, $c = 10$, $u = 200$, $\xi = 650000$, $n = 150$.

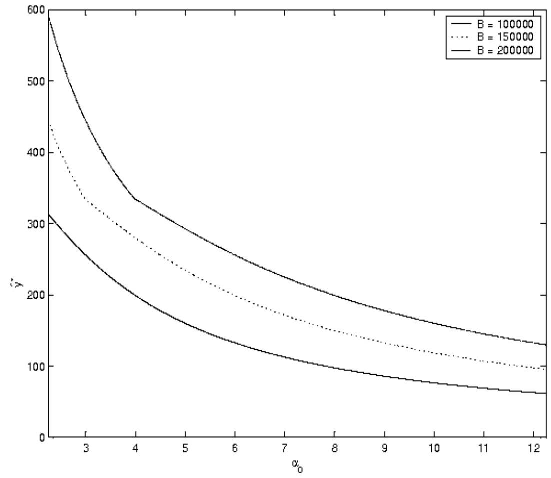

Figure 20: Equilibrium output ($\hat{y}^*$) versus α_0

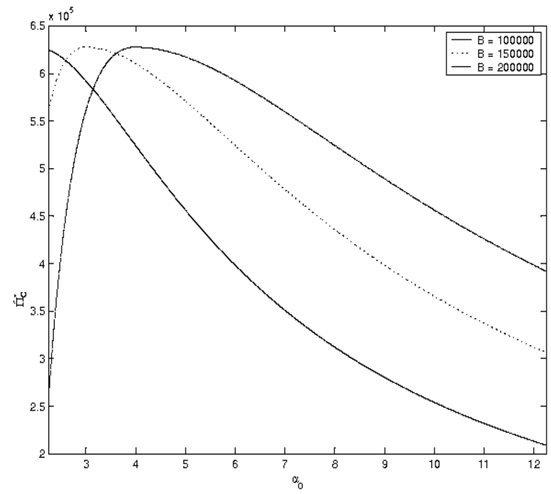

Figure 21: Equilibrium firm profit ($\hat{\Pi}^*_C$) versus α_0

Section 5 - Independent Demands[37]

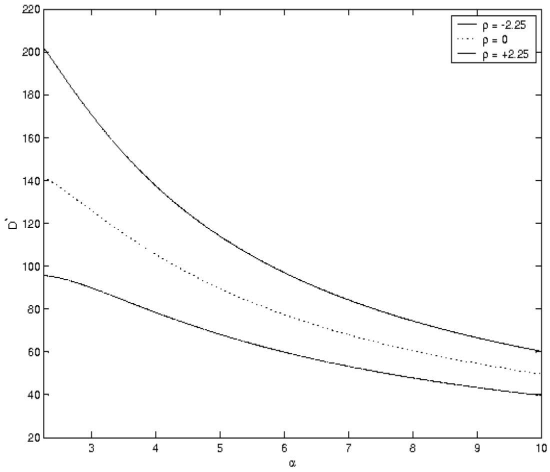

Figure 22: Equilibrium abatement (D^*) versus α for different values of ρ

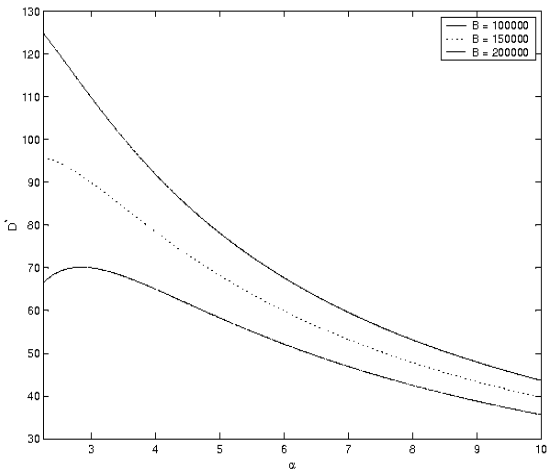

Figure 23: Equilibrium abatement (D^*) versus α when $\rho > 0$

[37]Parameter values for Figure 22 were $a = 7,500$, $b = 5$, $c = 500$, $u = 200$, $\xi = 6.5$, $B = 150,000$, $n = 150$. Parameter values for Figure 23 were $a = 7,500$, $b = 5$, $c = 500$, $u = 200$, $\xi = 6.5$, $\rho = +2.25$, $n = 150$. Parameter values for Figures 24, 25, and 26 were $a = 7,500$, $b = 5$, $c = 500$, $u = 200$, $\xi = 6.5$, $\rho = -2.25$, $n = 150$.

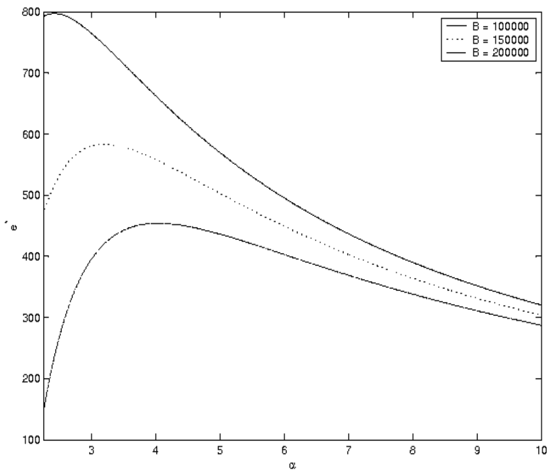

Figure 24: Equilibrium permit price (e^*) versus α

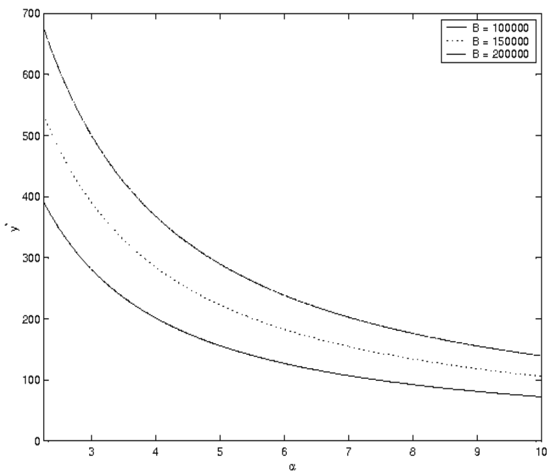

Figure 25: Equilibrium firm output (y^*) versus α

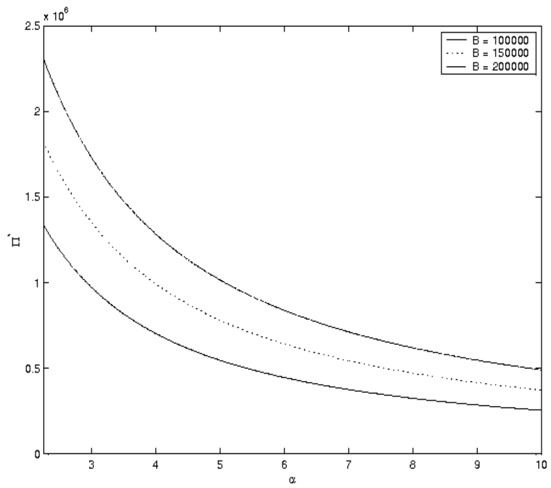

Figure 26: Equilibrium firm profit (Π_C^*) versus α

Section 5 - Competition[38]

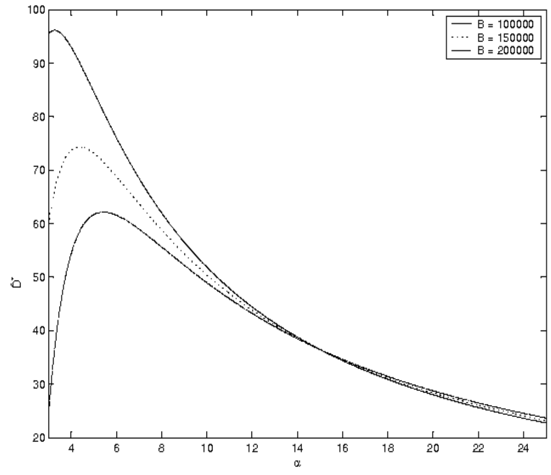

Figure 27: Equilibrium abatement ($\hat{D}^*$) versus α

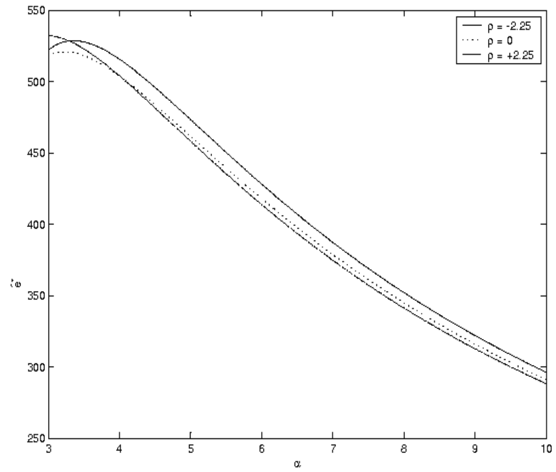

Figure 28: Equilibrium permit price ($\hat{e}^*$) versus α for different values of ρ

[38]Parameter values for Figures 27, 29, and 30 were $a = 7,500$, $b = 0.075$, $c = 500$, $u = 200$, $\xi = 6.5$, $\rho = -2.25$, $n = 150$. Parameter values for Figure 28 were $a = 7,500$, $b = 0.075$, $c = 500$, $u = 200$, $\xi = 6.5$, $B = 150,000$, $n = 150$.

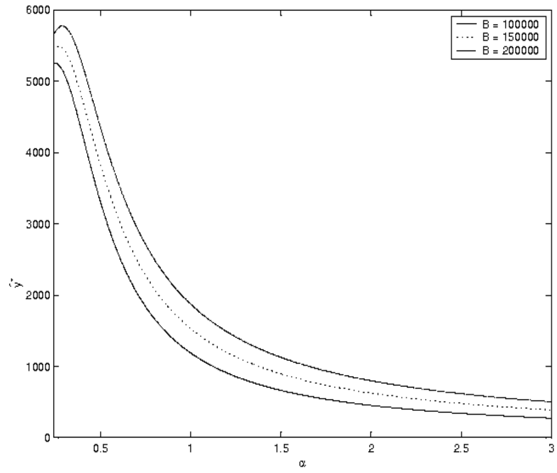

Figure 29: Equilibrium firm output ($\hat{y}^*$) versus α

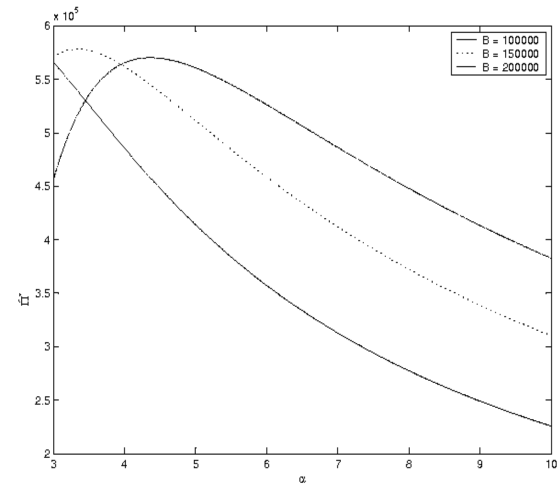

Figure 30: Equilibrium firm profit ($\hat{\Pi}_C^*$) versus α

Section 8[39]

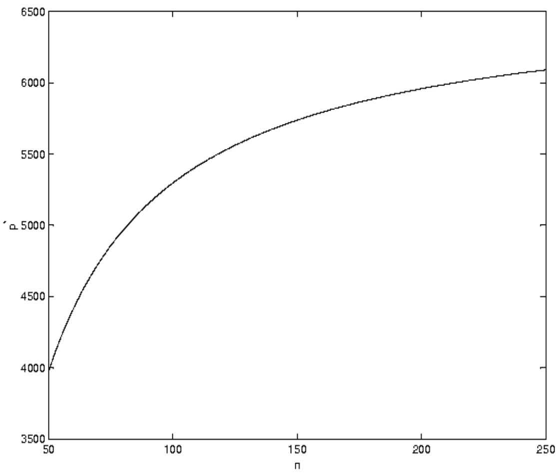

Figure 31: Equilibrium product price (p^*) versus n (Independent Demands)

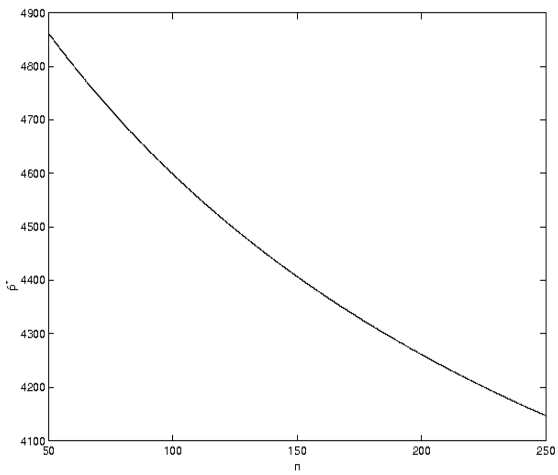

Figure 32: Equilibrium product price ($\hat{p}^*$) versus n (Competition)

[39] Parameter values for Figure 31 were $a = 7,500$, $b = 5$, $c = 10$, $u = 200$, $\xi = 0.65$, $\alpha = 5$, $B = 150,000$. Parameter values for Figure 32 were $a = 7,500$, $b = 0.075$, $c = 10$, $u = 200$, $\xi = 0.65$, $\alpha = 5$, $B = 150,000$.