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INCENTIVES FOR INFORMATION ACQUISITION

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Incentives for Information Acquisition

1.0 Introduction

In this paper we propose a one-period model which enables us to examine an investor's incentive for acquiring new information. The investor is one of many who are all homogeneous in their endowments, and all of whom have logarithmic utility functions. Before trading in the securities market, each investor must decide whether or not to buy new information from the information market, knowing that all other investors face the same decision, but without knowing what the others will do.

The problem of the information acquisition decision and the related question of the value of information are well-known in decision theory. The main underlying assumption in Savage [1972] and Hirshleifer and Riley [1979] is one of a constant market price. With costless information acquisition a constant market price implies an opportunity set that is unaffected by information acquisition. Therefore, it cannot be disadvantageous to an investor to acquire the information. Although possessing new information, the investor still has the option of realizing his prior investment-consumption decision. This result holds whatever other investors decide to do.

Hirshleifer [1971] and Hakansson, Kunkel, and Ohlson [1982] have developed models which include price changes. Price changes do not necessarily destroy an investor's incentive to acquire information. The effect on incentives depends on the effect of the new information on all investors' prior beliefs. With homogeneous prior and posterior beliefs and given aggregate claims, for example, information acquisition leads only to a redistribution of the claims. Then no investor has an incentive to acquire the information [Wahl 1983].

In the model below we introduce the possibility of heterogeneous beliefs and examine the effect on the decision to acquire new information. We do so under scenarios defined by whether information is costless or costly to produce, whether aggregate consumption is endogeneous or exogeneously given, and whether investors can or cannot costlessly cooperate. Cooperation between the informed and the uninformed investors means that they are costlessly able to enter into binding contracts with respect to the consequences of the information acquisition decision. In a market without cooperation all investors are assumed to act anonymously.

Section 2 establishes the model, how investors evaluate new information, and the definition of the stable acquisition decision. Section 3 examines the implications of the model for acquisition decisions under the scenarios just outlined above. Section 4 is a summary and conclusion.

2.0 A model of information acquisition

We begin by establishing the economic setting. Next, we determine how investors will assess the value of new information. We close the section with a definition of a condition under which all investors will choose to acquire new information before the securities market opens.

2.1 Assumptions, definitions, and foundations

We start with the following assumptions:

A.1. The capital market is perfect, competitive, and complete.

A.2. The exogeneous information market offers information at a competitive price to every investor but the information seller does not trade in the capital market. Furthermore, the information seller's price is such that he will with certainty exactly recover his marginal cost of producing the information. That is, the information seller knows the investors' demand functions and will not produce information if he cannot recover the cost for sure. Each investor has to decide whether or not to purchase the information before the capital market opens; no investor acts as an information seller.

As in Hellwig [1982], there is a lag between information acquisition and trading. Thus we avoid the Grossman and Stiglitz [1980] paradox.

A.3. The state space is finite and exogeneously given.

A.4. Every investor is a von Neumann-Morgenstern utility maximizer who optimizes the investment-consumption decision at time $\emptyset$. The state-independent utility function, U, is time-additive logarithmic with constant proportional risk aversion (Rubinstein 1977):

$$U = \rho_\emptyset \ln C_\emptyset + \rho_1 \ln C_1, \tag{1}$$

$\rho_t \equiv$ parameter of the utility function at time t ($t = \emptyset,1$;

$\rho_\emptyset + \rho_1 \equiv 1$, without loss of generality),

$C_t \equiv$ consumption at time t ($t = \emptyset,1$).

The expected utility of consumption is:

$$E[\tilde{U}] = \rho_\emptyset \ln C_\emptyset(\emptyset) + \rho_1 \Sigma_s f(s) \ln C_1(s), \tag{2}$$

$E[\cdot] \equiv$ expectation operator and the tilde denotes state-dependence,

$(\emptyset) \equiv$ state at time $\emptyset$, which occurs with probability 1,

$\Sigma_s \equiv$ denotes the sum over the s ($s = 1,2,\ldots,S$; $S<\infty$) mutually exclusive states at time 1,

$(s) \equiv$ state s at time 1 ($s = 1,2,\ldots S$),

$f(s) \equiv$ the investor's prior probability belief of the occurrence of state s ($s = 1,2,\ldots S$).

A.5. Investors are identical in their initial endowments and in the parameters (ρ_t) of their utility functions ($t = \emptyset,1$).

Let i ($i = 1,2,\ldots,I$) denote the investor index, and I, the number of investors in the market, and $\overline{C}$ the endowment in consumption claims. Then:

$$\overline{C}_{\emptyset i}(\emptyset) = \overline{C}_{\emptyset 1}(\emptyset) = (1/I)\ \Sigma_i\ \overline{C}_{\emptyset i}(\emptyset) \equiv (1/I)\ C_{\emptyset m}(\emptyset)\ (i = 1,2,\ldots,I), \tag{3}$$
$$\overline{C}_{1i}(s) = \overline{C}_{11}(s) = (1/I)\ \Sigma_i\ \overline{C}_{1i}(s) \equiv (1/I)\ C_{1m}(s)\ (i = 1,2,\ldots,I;\ s = 1,2,\ldots,S).$$

Every investor therefore shares (1/I) of the aggregate initial endowment in consumption claims, $C_{\emptyset m}(\emptyset)$ and $C_{1m}(s)$. Maximizing $E[\tilde{U}_i]$ with respect to $C_{\emptyset i}(\emptyset)$ and $C_{1i}(s)$ subject to the initial endowments $\overline{C}_{\emptyset i}(\emptyset)$ and $\overline{C}_{1i}(s)$ ($s = 1,2,\ldots,S$), gives the optimal investment-consumption decision.

Market equilibrium requires (a) an optimal investment-consumption decision by every investor, and (b) fulfillment of the clearing condition, i.e., $\Sigma_i \; C_{ti}(\cdot) = C_{tm}(\cdot)$ ($t=\emptyset,1$). Henceforth, we omit the time index t and the other indexes when they are superfluous for understanding.

Differentiating between exogeneous aggregate consumption (≡ without production), and endogenous aggregate consumption (≡ with production), will lead to interesting results. Therefore assume first:

A.6a. The aggregate consumption in every state, $C_m(\emptyset)$ and $C_m(s)$ ($s = 1,2,\ldots,S$), is exogenously given.

Lemma 1: Under assumptions A.1. to A.6a, the sharing rule in consumption claims is:

$$C_i(\emptyset) = (1/I)\; C_m(\emptyset),\; C_i(s) = (1/I)\; \frac{f_i(s)}{f_m(s)}\; C_m(s) \qquad (4)$$

$$(i = 1,2,\ldots,I;\; s = 1,2,\ldots,S)$$

and $f_m(s) \equiv \frac{1}{I}\,\Sigma_i\; f_i(s) \quad (s = 1,2,\ldots,S)$.

We will call $f_m(\cdot)$ the probability belief of the market, or the market forecast as it exhibits all the properties of a probability measure.

Proof: a). Eq. (3) shows that every investor possesses (1/I) of the corresponding aggregate consumption claims. Identical investors have no reason to change this endowment which meets the market clearing conditions.

b). Suppose that at least two investors differ in their probability assessment of at least two states of nature. Under assumptions A.1 to A.5, the Lagrangean procedure gives investor i's demand function for contingent consumption claims in state s:

$$C_i(s) = \rho_1\; W(\emptyset)\; f_i(s)/\pi(s) \quad (s = 1,2,\ldots,S) \qquad (4.1)$$

where $W(\emptyset)$ ≡ initial wealth of every investor, and $\pi(s)$ ≡ contingent price of a $1 payoff if and only if state s occurs, and $\pi(\emptyset) \equiv 1$. Summing (4.1) over i and inserting the result in (4.1) gives Eq. (4). ***

The aggregate consumption in every state is endogenous if the market as a whole can optimize consumption intertemporally. Therefore, to close the with-production model we make the following assumption:

A.6b. The risk-free marginal productivity of capital, $R_F > -1$, is exogenously given by technology.

This implies the following market clearing conditions:

$$\Sigma_i\ C_i(\emptyset) = C'_m(\emptyset) = (1-d)C_m(\emptyset) \quad (d<1),$$

$$\Sigma_i\ C_i(s) = C'_m(s) = [1+\beta(s)d]C_m(s) \qquad (s = 1,2,\ldots,S), \tag{5}$$

$C'_m(\cdot) \equiv$ optimal aggregate consumption in the corresponding state,

$d \equiv$ coefficient of production change,

$\beta(s) \equiv (1+R_F)C_m(\emptyset)/C_m(s)$ $(s = 1,2,\ldots,S)$.

One may interpret $C_m(\cdot)$ as a preliminary production plan for the economy which can be chosen arbitrarily as long as $C_m(\cdot) > 0$. The following implication of (5) clarifies the meaning of d:

$$C'_m(s) - C_m(s) = C_m(\emptyset)d(1+R_F). \tag{6}$$

That is, a positive (negative) d means that the preliminary aggregate consumption in state $\emptyset$ is reduced (augmented) by $C_m(\emptyset)d$ and that the preliminary aggregate consumption in all states s is augmented (reduced) by the same amount, times one plus the risk-free rate. The optimal d is given by:

Lemma 2: $\rho_\emptyset(1-d)^{-1} = \rho_1\ \Sigma_s\ f_m(s)\ \frac{\beta(s)}{1+\beta(s)d}$. (7)

Proof: $\underset{d}{\text{Max}}\ Z \equiv \rho_\emptyset \ln C'_m(\emptyset) + \rho_1\ \Sigma_s\ f_m(s) \ln C'_m(s)$ s. t. Eq. (5) [Wahl 1983]. ***

Lemma 3: Under assumptions A.1 to A.5, and A.6b, the sharing rule in consumption claims is:

$$C_i(\emptyset) = \frac{1}{I}(1-d)C_m(\emptyset),$$

$$C_i(s) = \frac{1}{I}\frac{f_i(s)}{f_m(s)}[1+\beta(s)d]C_m(s) \quad (i = 1,2,\ldots,I;\ s = 1,2,\ldots,S). \tag{8}$$

Proof: Lemma 3 follows immediately from Lemma 1 and the market clearing conditions (5). ***

Lemma 4: As before, let $\pi(s)$ denote the contingent price of a \$1 payoff if and only if state s occurs, and $\pi(\emptyset) \equiv 1$. Under assumptions A.1 to A.5, and A.6b, the equilibrium state-contingent price is:

$$\pi(s) = f_m(s) \frac{\rho_1 (1-d) C_m(\emptyset)}{\rho_\emptyset [1+\beta(s)d] C_m(s)} \quad (s = 1,2,\ldots,S). \qquad (9)$$

Proof: An equilibium must exhibit

$$\frac{\pi(s)}{\pi(\emptyset)} = \frac{\partial E[\tilde{U}]}{\partial C(s)} \frac{\partial C(\emptyset)}{\partial E[\tilde{U}]}$$

$$= \frac{\rho_1 f(s) C(s)^{-1}}{\rho_\emptyset C(\emptyset)^{-1}}$$ for every investor. Substituting in (8) gives (9). ***

With exogenously given aggregate consumption, i.e., d = 0, and homogeneous beliefs, i.e., $f_m(s) = f_i(s)$ for all i, Lemma 4 implies the well-known result:

$$\pi(s) = f(s) \frac{\rho_1 C_m(\emptyset)}{\rho_\emptyset C_m(s)} \qquad (s = 1,2,\ldots,S).$$

To obtain "improved" probability beliefs every investor may buy information in the information market. One may think of this "news" for example, as the annual report of a specific company. The information price is $p \geqq 0$. But buying the information means ex ante buying a probability distribution over information events. That is, the investor does not know for sure what the content of the news he receives will be. Let e denote a specific information event and $E < \infty$ be the number of mutually exclusive information events (e = 1,2,...,E).

A.7. Every investor knows the likelihood of observing information event e if state s were to occur, $f(e|s)$ (e = 1,2,...,E; s = 1,2,...,S).

Using A.4 and A.7 and Bayes' rule, it follows that every investor knows his posterior probability belief, $f(s|e) \propto f(e|s)f(s)$ for all e and s.

New information will in general change state-contingent prices in equilibrium. This implies that the investor's initial wealth $W(\emptyset)$ will depend on the information event that occurs, denoted by $W(\emptyset|e)$ $(e = 1,2,\ldots,E)$. The information price p causes an utility loss, K, because acquiring the information reduces the investor's wealth to $W(\emptyset|e) - p$, depending on the information event which occurs.

Lemma 5: $K = -\Sigma_e f(e) \ln(1 - \frac{p}{W(\emptyset|e)}) \geqq 0.$ (10)

Proof: Let $\lambda(e)$ denote the Lagrangean multiplier if event e occurs. Under assumptions A.1 to A.4 and A.7, $\lambda(e) = 1/W(\emptyset|e)$, and the conditional utility loss by information acquisition is approximately $p\ \lambda(e) = p/W(\emptyset|e)$. The latter expression is the linear approximation of $-\ln[1-p/W(\emptyset|e)]$, if $0 \leqq p < W(\emptyset|e)$ for all e. Ex ante, one must take the expectation to adjust for all possible information events. ***

2.2. The evaluation of new information

We follow the tradition in decision theory and evaluate information acquisition by the ex ante change in expected utility, Δ, it is able to generate.

Proposition 1: Under assumptions A.1 to A.5, A.6b and A.7,

$$\Delta = F + P - K , \qquad (11)$$

where $F \equiv \rho_1 \Sigma_e f(e) \Sigma_s f(s|e) \ln \frac{f(s|e)/f_m(s|e)}{f(s)/f_m(s)}$,

which we will call the forecast effect,

$$P \equiv \rho_\emptyset \Sigma_e f(e) \ln \frac{1-d(e)}{1-d} + \rho_1 \Sigma_e f(e) \Sigma_s f(s|e) \frac{1+\beta(s)d(e)}{1+\beta(s)d} ,$$

which we will call the production effect, and

$$K \equiv - \Sigma_e f(e) \ln \left(1 - \frac{p}{\rho_\emptyset[1-d(e)]\overline{C}(\emptyset)}\right) \geqq 0,$$

which we will call the information cost.

Proof: a). First note that initial endowments and therefore (preliminary) aggregate consumption does not depend on information acquisition.

b). Also note that in the new-information equilibrium the prices of state-contingent claims are independent of the information price p, i.e., $\pi(s|e;p) = \pi(s|e)$ for all s and e [Wahl 1983, pp. 133 and 157]. This is a consequence of the fact that with logarithmic utility, the information expense does not affect the marginal rate of substitution of optimal aggregate consumption between all pairs of states. Then

$$
\begin{aligned}
W(\emptyset|e) &= C(\emptyset|e) + \Sigma_s \pi(s|e)C(s|e) \\
&= (\rho_\emptyset/I)[1-d(e)]C_m(\emptyset) \qquad (12) \\
&= \rho_\emptyset[1-d(e)]\bar{C}(\emptyset)
\end{aligned}
$$

by using Eqs. (3), (8) and (9), properly adjusted for information event e.

c). Using Eqs. (2), (8), (10) and (12), it follows ex ante that

$$
\begin{aligned}
\Delta \equiv\ & \rho_\emptyset \Sigma_e f(e) \ln \frac{C(\emptyset|e)}{C(\emptyset)} + \rho_1 \Sigma_e f(e) \Sigma_s f(s|e) \ln \frac{C(s|e)}{C(s)} \\
& + \Sigma_e f(e) \ln \left(1 - \frac{p}{W(\emptyset|e)} \right) \\
=\ & \rho_\emptyset \Sigma_e f(e) \ln \frac{1-d(e)}{1-d} \\
& + \rho_1 \Sigma_e f(e) \Sigma_s f(s|e) \ln \left(\frac{f(s|e)/f_m(s|e)}{f(s)/f_m(s)} \cdot \frac{1+\beta(s)d(e)}{1+\beta(s)d} \right) \\
& + \Sigma_e f(e) \ln \left(1 - \frac{p}{\rho_\emptyset[1-d(e)]\bar{C}(\emptyset)} \right).
\end{aligned}
$$

***

Proposition 1 is the key to this article. Different assumptions will specify the sign of Δ by specifying the signs of the production and forecast effects. The information cost is nonnegative, and zero if and only if the information price is zero.

2.3 A definition of a stable information acquisition decision

We define the informed investor as one who trades using posterior beliefs after having acquired the information. The uninformed investor is one who does not acquire the information and therefore trades using prior beliefs. The ex ante change in an investor's expected utility from acquiring the information depends on whether other investors do or do not decide to buy the information. There will also be an ex ante change in the expected utility of an uninformed investor if some other investor should buy the information.

Let $\Delta_{a|\bar{a}}$ denote the ex ante change in expected utility of each of those investors (insiders) who acquire the information, subject to the condition that all others do not. Similarly, $\Delta_{\bar{a}|a}$ is the ex ante change in the expected utility of individual outsiders, those who do not acquire the information, given that there are investors who do. Thus informed investors have made the decision "a", and uninformed investors the decision "$\bar{a}$". Note that all insiders are informed, but informed investors are not necessarily insiders. When all investors acquire (do not acquire) there are no insiders (outsiders).

Definition: A stable acquisition decision is defined by the following vector ranking:

$$\begin{pmatrix} \Delta_{a|\bar{a}} \\ \Delta_{a|a} \end{pmatrix} \geq \begin{pmatrix} 0 \\ \Delta_{\bar{a}|a} \end{pmatrix} \tag{13}$$

in which "$\geq$" means that at least one inequality holds. Note that $\Delta_{\bar{a}|\bar{a}} \equiv 0$ because when no investor acquires the information there is no change from the benchmark of the market equilibrium under prior probability beliefs.

The definition simply means that each acquiring individual will always be better off no matter what the other investors do. Therefore the decision "a" is stable [see Cornell and Roll 1981, p. 202, and the references cited therein]. As we shall demonstrate, the standard assumption set we have used implies the stable acquisition decision.

3.0 The information acquisition decision

In order to simplify the paper, we make the following common assumption:

A.8. All investors share the same information set at time Ø which they will use completely, and all are identical in their information processing abilities.

Assumption A.8 yields two implications. First, it gives the traditional assumption of homogeneous prior beliefs, i.e., $f_i(s) = f_m(s)$ for all s and i. Every individual's forecast is identical with the market forecast before any individual acquires new information. Second, A.8 implies that for homogeneous posterior beliefs to pertain, i.e., $f_i(s|e) = f_m(s|e)$ for all i, s, and e, it is necessary and sufficient that <u>all</u> individuals acquire the information. If only some investors are informed, A.8 implies

<u>Lemma 6</u>: Let $\alpha \equiv I_a/I$ with I_a being the number of acquiring investors [$\alpha \in (0,1)$]. Then the probabilities fit one of the following inequalities:

$$f(s|e) > f_m(s|e) > f(s)$$

or (14)

$$f(s|e) < f_m(s|e) < f(s) \quad (s = 1,2,\ldots,S;\ e = 1,2,\ldots,E).$$

<u>Proof</u>: Eq. (4) shows $f_m(s|e) \equiv \frac{1}{I} \Sigma_i\ f_i(s|e) = \alpha\ f(s|e) + (1-\alpha)f(s)$ under A.8. But with $\alpha \in (0,1)$, $f_m(s|e)$ must be between $f(s|e)$ and $f(s)$ for all s and e. ***

Note that Lemma 6 also shows that if not all investors are informed, the informed will never push the market prices of the state-contingent claims to the levels they would reach if all investors were informed. Those levels are unreachable because investors are risk-averse and limit their portfolio changes.

In the sections below we discuss the implications of the model under several situations given by 1) the presence or absence of production, 2) the presence or absence of cooperation, and 3) the presence of costless (p=0) or costly (p>0) information.

3.1 Without production and without cooperation

We begin by considering the situation when new information is costless.

Proposition 2: Assume A.1 to A.5, A.6a, A.7, and A.8.

With exogenous aggregate consumption and costless information acquisition, the ex ante change in an investor's expected utility is

a) positive (negative) for insiders (outsiders), i.e.,

$\Delta_{a|\bar{a}}>0 \quad (\Delta_{\bar{a}|a}<0)$,

b) greater for being one of the informed when all are informed (homogeneous posterior beliefs) than for being an outsider, i.e., $\Delta_{a|a}>\Delta_{\bar{a}|a}$.

Proof: No production and costless information imply P = 0 because d = d(e) = 0 for all e, and K = 0 because p = 0. Then using Eq. (11) with assumption A.8, it follows that

a.1) $$\Delta_{a|\bar{a}} = F_{a|\bar{a}} \equiv \rho_1 \Sigma_e f(e) \Sigma_s f(s|e) \ln \frac{f(s|e)}{f_m(s|e)} > 0$$

by Jensen's inequality. $\Delta_{a|\bar{a}}$ is strictly positive because of Eq. (14).

a.2) $$\Delta_{\bar{a}|a} = F_{\bar{a}|a} \equiv \rho_1 \Sigma_e f(e) \Sigma_s f(s|e) \ln \frac{f(s)}{f_m(s|e)}$$

$$= -\rho_1 \Sigma_e f(e) \{\Sigma_s [f(s|e) - f_m(s|e)] \ln \frac{f_m(s|e)}{f(s)}$$

$$+ \Sigma_s f_m(s|e) \ln \frac{f_m(s|e)}{f(s)} \} < 0$$

because the first summand in the brackets is positive by Eq. (14), and the second is also positive by Jensen's inequality;

b) $\Delta_{a|a} = F_{a|a} \equiv \rho_1 \ln 1 = 0$ because $f_i(s|e) = f_m(s|e)$ for all i, e, s. ***

With Proposition 2, Eq. (13) holds. This immediately results in

Theorem 1: Under the assumptions of Proposition 2, the decision to acquire new information is stable. Every investor has an incentive to acquire new information.

To illuminate Theorem 1, let us examine the decisions of investors j and k, both having to decide independently whether or not to acquire new information. The signs of the ex ante changes in expected utilities are shown in the following table.

Table 1

j \ k	a	$\bar{a}$
a	(0,0)	(+,-)
$\bar{a}$	(-,+)	(0,0)

Clearly the decision to acquire dominates that of not to acquire the information. Note that if all investors acquire, none will have an advantage. However, the consequences for any investor j in the event that some investor k might (not) acquire the information when j does not (does), produces the requisite incentive. This result is independent of the probability, from the viewpoint of investor j, that any other investors may or may not acquire.

Suppose now that the price of new information is strictly positive (p>0). That is, that the information is costly to produce.

Let p^o and p^* be break-even prices of new information such that $\Delta_{a|\bar{a}}(p=p^o) \overset{!}{=} 0$ and $\Delta_{a|a}(p=p^*) \overset{!}{=} \Delta_{\bar{a}|a}(p=p^*)$. Then we have

<u>Lemma 7</u>: Assume the assumption set of Proposition 2. Then

a) $\Delta_{a|\bar{a}} \geqq 0$ if and only if $p \leqq p^o \equiv \rho_{\emptyset}\,\bar{C}(\emptyset)(1-e^{-F_{a|\bar{a}}})$;

b) $\Delta_{a|a} \geqq \Delta_{\bar{a}|a}$ if and only if $p \leqq p^* \equiv \rho_{\emptyset}\,\bar{C}(\emptyset)(1-e^{F_{\bar{a}|a}})$. (15)

<u>Proof</u>: The results follow from Eq. (11) taking into account that without production d = d(e) = 0 for all e and $\Delta_{a|\bar{a}} = F_{a|\bar{a}} - K(p=p^o)$ for a) above, and $\Delta_{a|a} = -K(p=p^*) = F_{\bar{a}|a} = \Delta_{\bar{a}|a}\,(p=p^*)$ for b). ***

Lemma 8: Let $c>o$ be the marginal cost of producing the information. Then with A.2, it follows:

$$c = pI_a \text{ implies } p \leqq \text{Min } \{p^o, p^* | p^o \neq p^*\}. \quad (16)$$

Proof: Suppose that either $p^o < p^*$ and $p > p^o$ or that $p^* < p^o$ and $p > p^*$ In both cases the price p may result in no investors acquiring the information. If $p \geqq \text{Max } \{p^o, p^* | p^o \neq p^*\}$, then clearly no investor will acquire the information; if p is between p^o and p^*, investors will not know what to do ex ante and so it is possible that none will acquire. Therefore, the information seller may not recover his production cost for sure and will not produce the information. This contradicts $p>0$. ***

Proposition 3: Assume A.1 to A.5, A.6a, A.7 and A.8. With exogenously given aggregate consumption and a positive information price, the ex ante change in an investor's expected utility is

a) nonnegative for insiders, i.e., $\Delta_{a|\bar{a}} \geqq 0$,

b) not smaller for being one of the informed when all are informed than for being an outsider, i.e., $\Delta_{a|a} \geqq \Delta_{\bar{a}|a}$.

Proof: Combining Lemmae 7 and 8 [Eqs. (15) and (16)], one can deduce that if $p \leqq \text{Min } \{p^o, p^* | p^o \neq p^*\}$, this implies that $\{p \leqq p^o < p^*\}$ or $\{p \leqq p^* < p^o\}$.

This in turn implies that either

$\{\Delta_{a|\bar{a}} \geqq 0$ and $\Delta_{a|a} > \Delta_{\bar{a}|a}\}$, or

$\{\Delta_{a|\bar{a}} > 0$ and $\Delta_{a|a} \geqq \Delta_{\bar{a}|a}\}$. ***

With Proposition 3, Eq. (13) holds. This immediately results in:

Theorem 2: Under the assumptions of Proposition 3, the decision to acquire new information is stable.

Theorem 2 shows that utility maximizing investors will acquire new information, even when each of them will bear a utility loss ex ante. Note that $\Delta_{a|a} = -K < 0$.

The problem facing each investor is how to minimize his losses. The underlying assumption set and Lemma 8 imply that for the information market to exist with certainty it is necessary for the information seller to set a price such that for every investor it is less expensive to lose wealth to the information seller than to insiders because of the decision to remain an outsider. One can infer that in equilibrium information would not be produced for which the production cost would result in an information cost exceeding the expected utility loss to an investor from being an outsider or the gain from being an insider, whichever is less.

3.2 Without production but with cooperation

In the absence of cooperation, every investor is driven to acquire new information by the desire to avoid the utility loss caused by redistributing consumption claims, even with costly information. Note from Eq. (12), that there is no redistribution of wealth by information acquisition because of a fixed aggregate consumption. Changes in the portfolio of state-contingent claims are offset by price changes, leaving wealth unchanged. This holds for every information event.

With cooperation, investors can avoid the utility loss. That is, they can make agreements before acquiring information to undo any redistribution of consumption claims which would result from trading after acquisition. Since, with cooperation there is no cost (gain) in being an outsider (insider), investors would be unwilling to pay anything for the information. Therefore no strictly positive information price is sustainable in equilibrium. A strictly positive information price would result in the decision not to acquire the information dominating the decision to acquire it. Therefore, we have:

<u>Theorem 3</u>: In the absence of production and with costly information, the information market can only exist if cooperation too is costly.

Information which cannot change aggregate consumption, for instance by affecting its intertemporal allocation, but which serves only to redistribute consumption claims among investors in each state of nature will command a zero price when investors can cooperate costlessly. This is consistent with the position in Hirshleifer [1971]. In the next sections we turn to the question of the impact of the existence of production, i.e., utility gains from intertemporal allocation of aggregate consumption, on the information price and the acquisition decision.

3.3 With production and without cooperation

Suppose p = 0, i.e., the new information is costless. The following lemma gives the sign of the production effect, P.

Lemma 9: Let $P_{(\cdot)}$ represent both cases, $P_{a|a}$ and $P_{\bar{a}|a}$. Then $P_{(\cdot)} > 0$. (17)

Proof: Define $\hat{P} \equiv \rho_{\emptyset} \ln(1-d) + \rho_1 \Sigma_s f(s)\ln[1+\beta(s)d]$, with d being the solution of Eq. (7) (see Lemma 2). Because $\left.\frac{\partial^2 Z}{\partial d^2}\right|_d < 0$, $\hat{P} = Z^{max}$. $\hat{P}$ is convex in the prior probability beliefs f(s) (s = 1,2,...,S), i.e.,

$$d^2\hat{P} = \Sigma_{q=1}^{S} \Sigma_{r=1}^{S} \frac{\partial^2\hat{P}}{\partial f(q)\partial f(r)} df(q)df(r) > 0.$$

The calculation is simplified by following a two-step procedure. First, calculate $d^2\hat{P}$ with respect to d and f(s) (s = 1,2,...,S), ignoring for the moment the fact that d is a function of the probabilities. Second, calculate the implicit derivative of Eq. (7), dd, and insert this in the result of the first step. Then

$$d^2\hat{P} = \rho_1^2 \frac{[\Sigma_s df(s) \frac{\beta(s)}{1+\beta(s)d}]^2}{\frac{\rho_{\emptyset}}{(1-d)^2} + \rho_1 \Sigma_s f(s)[\frac{\beta(s)}{1+\beta(s)d}]^2} > 0. \qquad (18)$$

Because $\Sigma_e f(e)f(s|e) = f(s)$ and by using the definition of the market forecast in Lemma 6, $\Sigma_e f(e)f_m(s|e) = \Sigma_e f(e)[\alpha f(s|e)+(1-\alpha)f(s)] = f(s)$ (s = 1,2,...,S), the convexity of $\hat{P}$ implies

$$P_{a|a} \equiv \Sigma_e\, f(e)\, \hat{P}_{a|a}(e) - \hat{P}_{a|a} > 0, \text{ and}$$

$$P_{\bar{a}|a} \equiv \Sigma_e\, f(e)\, \hat{P}_{\bar{a}|a}(e) - \hat{P}_{\bar{a}|a} > 0. \; ***$$

This leads to:

Lemma 10: $P_{a|a} > P_{\bar{a}|a} = P_{a|\bar{a}}$. (19)

Proof: Combine the convexity of $\hat{P}$ in the prior market forecast in Lemma 9 with the relative "spreading" of the posterior market forecast in Lemma 6. Then the inequality follows because $\Sigma_e\, f(e)f(s|e) = \Sigma_e\, f(e)f_m(s|e) = f(s)$, for all s. The equality follows because the market forecast is the same for insiders and outsiders. ***

Lemma 10 reveals that the greater the number of investors who are informed, the greater is the ex ante utility gain from production for both insiders and outsiders. The individual production effect is greatest with homogeneous posterior beliefs, for then the change in the prices of the state-contingent claims is the greatest for every information event.

Proposition 4: Assume A.1 to A.5, A.6b, and A.7 and A.8. With endogenous aggregate consumption as defined in A.6b and costless information acquisition, the ex ante change in an investor's expected utility is

a) positive for insiders, i.e., $\Delta_{a|\bar{a}} > 0$, and

b) greater for being one of the informed when all are informed than for being an outsider, i.e., $\Delta_{a|a} > \Delta_{\bar{a}|a}$.

Proof: By using Eq. (11) with p = 0, it follows that

a) $\Delta_{a|\bar{a}} = F_{a|\bar{a}} + P_{a|\bar{a}} > 0$ because $F_{a|\bar{a}} > 0$ from Proposition 2, and $P_{a|\bar{a}} = P_{\bar{a}|a} > 0$ from Lemmae 9 and 10; and

b) $\Delta_{a|a} - \Delta_{\bar{a}|a} = F_{a|a} + P_{a|a} - \left(F_{\bar{a}|a} + P_{\bar{a}|a}\right) > 0$ because $F_{a|a} = 0$, $F_{\bar{a}|a} < 0$ from Proposition 2, and $P_{a|a} - P_{\bar{a}|a} > 0$ from Lemma 10.***

With Proposition 4, Eq. (13) holds. This then gives:

Theorem 4: Under the assumptions of Proposition 4, the decision to acquire new information is stable.

With a positive price for new information, the information cost in general will depend on all information events and the market forecast because these influence the coefficient of production change. Looking at the break-even information prices for being an insider or not trading as an outsider, p^o and p^* respectively, such that $\Delta_{a|\bar{a}}(p=p^o) \overset{!}{=} 0$ and $\Delta_{a|a}(p=p^*) \overset{!}{=} \Delta_{\bar{a}|a}(p=p^*)$, Lemma 11 follows from Eq. (11).

Lemma 11: Given the assumption set of Proposition 4, then

a) $\Delta_{a|\bar{a}} \geqq 0$ if and only if $p \leqq p^o$ with p^o being determined by

$$F_{a|\bar{a}} + P_{a|\bar{a}} = -\Sigma_e f(e) \ln \left(1 - \frac{p^o}{\rho_\emptyset [1-d_{a|\bar{a}}(e)]\bar{C}_\emptyset} \right);$$

b) $\Delta_{a|a} \geqq \Delta_{\bar{a}|a}$ if and only if $p \leqq p^*$, with p^* being determined by

$$- F_{\bar{a}|a} + P_{a|a} - P_{a|\bar{a}} = -\Sigma_e f(e) \ln \left(1 - \frac{p^*}{\rho_\emptyset [1-d_{a|a}(e)]\bar{C}_\emptyset} \right).$$

Even with endogenous aggregate consumption, the upper bound for the information price given in Lemma 8 has to hold, for this is a necessary condition for an equilibrium. Then it follows:

Proposition 5: Assume A.1 to A.5, A.6b, and A.7 and A.8. With endogenous aggregate consumption and a positive information price, the ex ante change in an investor's expected utility is

a) nonnegative for insiders, i.e., $\Delta_{a|\bar{a}} \geqq 0$,

b) not smaller for being one of the informed when all are informed than for being an outsider, i.e., $\Delta_{a|a} \geqq \Delta_{\bar{a}|a}$.

Proof: Combine Lemmae 11 and 8, and use the logic of the proof of Proposition 3. ***

With Proposition 5 Eq. (13) holds. This yields

Theorem 5: Under the assumptions of Proposition 5 the decision to acquire new information is stable.

From Theorems 2 and 5 it is clear that if investors cannot cooperate, all will acquire the information, whether or not production decisions are possible. The opportunity to optimize market consumption intertemporally may change the price the information market can quote for information, but this does not affect the investors' decision to acquire. The existence of production shifts the bounds on the information price, leaving the decision unchanged.

3.4 With production and cooperation

In Section 3.2 above we showed that investors had an incentive to cooperate if the cost of cooperation was zero because by so doing they could drive down the price the information seller could charge. However, in the absence of production the upper bound on the information price was zero.

So far, cooperation has only involved undoing any redistribution of claims arising from the forecast effect. With endogeneous aggregate consumption a further aspect of cooperation arises, namely that investors may agree not to act as free riders. We deal with each of these aspects in turn.

3.4.1 Cooperation as a no redistribution contract only

Let us introduce production and omit (for the moment) the information cost. Outsiders will only cooperate when the cooperation contract requires that the undoing of the redistribution of consumption claims (induced by the forecast effect), take place. Then, if this cooperation contract is possible:

$$\begin{pmatrix} \Delta_{a|\bar{a}} \\ \Delta_{a|a} \end{pmatrix} = \begin{pmatrix} P_{a|\bar{a}} \\ P_{a|a} \end{pmatrix} \geq \begin{pmatrix} 0 \\ P_{\bar{a}|a} \end{pmatrix} = \begin{pmatrix} 0 \\ \Delta_{\bar{a}|a} \end{pmatrix} . \qquad (20)$$

Therefore, every investor has an incentive, before information costs, to acquire the information no matter what other investors do, as long as redistribution will be undone.

Now suppose that the information market wishes to charge a price equal to the individual's expected "information gain," i.e., $p = p'$ with p' being the solution of Eq. (21):

$$-\Sigma_e f(e) \ln \left(1 - \frac{p'}{\rho_\emptyset [1-d_{a|a}(e)]\overline{C}_\emptyset} \right) = P_{a|a} . \qquad (21)$$

An information price of p' is impossible. Let us divide the investors into two groups, j and k, and look at a representative investor in each group. Table 2 shows the signs of each investor's ex ante change in expected utility if $p = p' > 0$.

Table 2

j \ k	a	$\bar{a}$
a	(0,0)	(-,+)
$\bar{a}$	(+,-)	(0,0)

This implies that nonacquisition of the information dominates acquisition. Thus with a cooperation contract that requires only that redistribution be undone, $p = p'$ cannot be an equilibrium price because no investor will buy at that price.

<u>Lemma 12</u>: Assume the assumption set of Proposition 5 and costless cooperation with respect to the redistribution of claims. Then

a) $\Delta_{a|\bar{a}} \gtreqless 0$ if and only if $p \lesseqgtr p^o$ being the solution of

$$-\Sigma_e f(e) \ln \left(1 - \frac{p^o}{\rho_\emptyset [1-d_{a|\bar{a}}(e)]\overline{C}_\emptyset} \right) = P_{a|\bar{a}} ;$$

b) $\Delta_{a|a} \gtreqless \Delta_{\bar{a}|a}$ if and only if $p \lesseqgtr p^*$, being the solution of

$$-\Sigma_e f(e) \ln \left(1 - \frac{p^*}{\rho_\emptyset [1-d_{a|a}(e)]\overline{C}_\emptyset} \right) = P_{a|a} - P_{\bar{a}|a}.$$

Theorem 6: Under the assumption set of Proposition 5 and costless cooperation the decision to acquire new information is stable.

Proof: From Lemmae 9 and 10 we have that $P_{a|\bar{a}}$ and $(P_{a|a} - P_{\bar{a}|a})$ are positive. This implies a positive p^o and p^*. Then:

a). If p = 0, then with Lemma 12, Eq. (13) must hold.

b). If p > 0, then Lemma 8 must hold. Combining Lemmae 8 and 12, Eq. (13) again holds. ***

With production and a no redistribution contract, investors will decide to buy information which is costly to produce. The incentive, however, is the utility gain from production, not the desire to avoid losses to others, or to gain from them. While the information market will share in the gains from production, it does not exhaust them ($p<p'$). If it were to attempt to do so, every investor would have an incentive to be a free rider on the production gain which results from the acquisition of the new information by others. The result would be that none would acquire.

3.4.2 Cooperation as a no redistribution and no free riding contract

The extension of the cooperation contract to no free riding makes possible an information price, and hence a shared marginal cost of information production, in the interval $\text{Min}\,\{p^o, p^* | p^o \neq p^*\} < p = c/I_a < p'$. Investors agree to undo the distribution consequences of free riding. There is then no production gain from being an outsider, nor a net information cost to being an insider. Thus Table 2 converges to Table 3 below, where clearly acquisition dominates non-acquisition.

Table 3

j \ k	a	$\bar{a}$
a	(+,+)	(0,0)
$\bar{a}$	(0,0)	(0,0)

Note, agreeing to undo any redistribution of claims after trade is logically equivalent to agreeing not to free ride before trade.

Lemma 13: Assume the assumption set of Proposition 5 and costless cooperation to exclude redistribution of claims and free riding. Then

$\{\Delta_{a|\bar{a}} = 0$ and $\Delta_{a|a} > \Delta_{\bar{a}|a}\}$ if and only if $p<p'$.

Lemma 14: $c = p \cdot I_a$ implies $p<p'$.

The proofs of Lemmae 13 and 14 are obvious. It follows that Theorem 6 remains unchanged. That is, the decision to acquire is stable. The only change which has occurred is in the upper-bound on the information price which now is p' instead of the lower one from Lemma 8. However, the information seller cannot charge more than $p = {}^{c}/I_a = {}^{c}/I$ with given c because p is a competitive price. Therefore, the contractual exclusion of free riding enables the information seller to produce information which would otherwise not be acquired.

4.0 Summary and conclusion

In this paper we have developed a model with an information market, and investors who are homogeneous in their endowments, have logarithmic utility functions, and use Bayes' rule in updating their prior probability beliefs after acquiring new information. The model is a one-period one, with information acquisition having to take place before trading in securities occurs.

We examine the implications of the model under a variety of scenarios organized by exogenously or endogenously given aggregate consumption, costless or costly information, and with or without the possibility of cooperation. In all cases we find that all investors will acquire the information, provided the price the information market charges is less than endogenously determined bounds. In general, the upper bound is positive. Thus, a necessary condition for the existence of an information market, the presence of a demand for information, is satisfied.

Because all investors acquire the information and have the same information processing abilities, when they begin with homogeneous prior probability beliefs they also end with homogeneous posterior beliefs. Therefore when trading in claims opens, while trade may occur for other, unspecified reasons, it will not be a consequence of heterogeneous beliefs. Furthermore, universal acquisition of new information means that prices for claims will be informationally efficient in the sense that no investor can "beat the market" by acquiring the information.

One of the major implications of the model is that even when information is costly and aggregate consumption is given exogeneously, if investors cannot cooperate to undo the redistribution of claims which would result from trading with heterogeneous posterior beliefs, all will acquire the new information. This is despite the fact that all would be better off if no one acquired it. In essence, all investors will prefer to lose wealth to the information market when buying new information is less costly in terms of utility loss than to be uninformed and to trade with informed investors. This result is consistent with the existence of brokerage houses and other securities research sellers. These continue to sell their information to investors despite the fact that each purchaser, knowing that a large number of other investors have also purchased the same information, does not expect any information advantage.

This situation creates an incentive for investors to cooperate. If they can do so, the model implies that they will be unwilling to pay anything for information which could serve only to redistribute consumption claims on the existing, exogeneously given aggregate consumption. We thus prove one of the points Hirshleifer [1971] makes in his example. Furthermore, our result is consistent with the evidence in Ball, Torous, and Tschoegl [1985]. These authors argue that the common stockmarket convention of quoting prices in eigths is a way for market participants cooperatively to avoid investing in very precisely determining prices because the expected utility gain of increased precision is zero before information costs.

When aggregate consumption is determined endogeneously, the model further implies that even in the presence of costless cooperation, a positive price for new information is possible. Thus the information market could exist, depending on the cost of producing the information. However, the maximum price it can charge is such that even though all investors acquire the information at that price, the ex ante change in their utility is positive. Thus the information market shares in the utility gains from intertemporal optimization of aggregate consumption, but does not exhaust them.

Future extensions of the model might take several directions. One would loosen the assumption of homogeneous endowments. Another would explicitly model the information sellers, and thus the supply of new information. This would further enable one to derive implications for social welfare. Finally, it would be interesting to permit heterogeneous posterior beliefs. All of these possibilities, and others, remain as subjects for further research.

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