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CONSUMERS WITH DIFFERING REACTION SPEEDS, SCALE ADVANTAGES, AND INDUSTRY STRUCTURE

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The development of an industry is seen as the outcome of a differential game in which market shares flow in response to price differences. It is hypothesized that while all consumers eventually react to price differences, some are slower than others. If scale advantages have some influence on marginal costs around average market share, then, under reasonable conditions, only skewed size distributions can be locally stable steady states. In addition, and contrary to common belief, the largest firm will not eventually monopolize the industry.

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The profit-maximizing price under monopolistic competition is given by the marginal cost plus a mark up determined by the firm-specific price sensitivity of demand. The simple intuition exploited in this paper is that if both marginal costs and firm-specific price sensitivity of demand decline with market share, then the two effects could balance each other out and produce similar prices for firms of very different sizes.

This situation is found as a locally stable steady state of a noncooperative differential game intended to model a mature oligopoly. The model is very simple and traditional in the sense that each firm uses an open-loop pricing strategy to maximize profits, which are discounted at a common rate. In addition, the cost function is noncontroversial, though untraditional, exhibiting returns to scale.

The one novel feature, which is responsible for the unusual result, is the dynamic constraint, which describes the flow of market shares between firms as a function of price differences. Contrary to the ordinary assumption, then, a firm does not lose all all of its sales by charging an above-average price for a limited period of time. In particular, it is assumed that consumers react with differing speed to limited price differences. Intuitively, this will cause a big firm to refrain from trying to capture the last little piece of the market, since doing so would involve extensive and prolonged price cutting on the rest of its already big market share. Conversely, a small firm can relatively easily take marginal share from a bigger firm. So an asymmetric, nonmonopolistic industry structure is a locally stable steady state. The symmetric solution is unstable under the assumptions of the paper, according to which the cost advantage of a marginally bigger firm dominates the difference in firm-specific price sensitivity of demand, if both firms have approximately average market shares.

The model is similar to those analyzed by the author elsewhere, [Wernerfelt forthcoming, 1982, Ch. 7], although the more complex mechanisms at work early in the product life cycle are disregarded here. The main contribution of this paper is to offer a deterministic, demand-oriented theory of industry structure, which hopefully can complement the stochastic (e.g., Ijiri and Simon [1974]; Nelson and Winter [1978] and cost-oriented [Flaherty 1980; Hjalmarsson 1974] theories in attempts to explain the compelling empirical evidence in the area. (A recent contribution is Buzzell [1980)]. If the mechanism at work in this paper is the dominant one, it should have policy implications for both regulators and managers. In addition, the paper may be seen as making a theoretical point, since it demonstrates a new approach to the theory of monopolistic competition. (See Wernerfelt [forthcoming, 1982, Ch. 8] for a model of a closed economy along these lines.)

In Section 1, I will present and discuss the basic model. The existence and stability properties of the symmetric steady state are established in section 2, while the assymmetric steady states are investigated in section 3.

1. A NO-NONSENSE DIFFERENTIAL GAME

In this section, I will present a noncooperative differential game intended to model the competitive process in a mature oligopoly.

The industry is assumed to consist of $n \geq 2$ firms, which differ only in their initial market shares, $s_{io} > 0$, $i \in \{1,...,n\} \equiv N$. (Of course, $\Sigma_{i=1}^{n} s_{io} = 1$.). Each firm selects a pricing strategy, $P_i(t)$, from the set of piecewise continuous functions from $[0,\infty]$ to $[0,K]$, where K is some big number. In choosing $P_i(t)$, firm i will take the strategies of the other firms as given and maximize its infinite horizon profits, discounted at rate $\rho > 0$.

If firm i charges the price p_{it} and has market share s_{it}, its sales volume will be $y(p_{it})s_{it}$, where $y(\cdot)$ is a declining C^2 function from $[0,K]$ to $[0,\infty]$. Similarly, its unit costs will be $C(y(p_{it})s_{it})$, where $C(\cdot)$ is a declining C^2 function from $[0,\infty]$ to $[0,\infty]$. So the objective of firm $i \in N$ is

$$(1) \quad \underset{P_i(t)}{\text{MAX}} \int_0^\infty e^{-\rho t} \, y(p_{it})s_{it} \, (p_{it} - C(y(p_{it})s_{it}))dt,$$

subject to various constraints, in particular the dynamic constraint on market shares. Of course, it is assumed that $y(\cdot)$ and $C(\cdot)$ are such that the maxim and is concave in p_{it}.

1.1 The Flow of Market Shares

While it could be rationalized in many other ways, the dynamic constraint is inspired by the view of consumers developed by the author elsewhere [Wernerfelt forthcoming, 1982 Ch. 3] and derives from the following intuitive ideas. Adopt the Lancaster view of consumption as the production of experiences from products. Suppose now that consumers are uncertain about the degree of differentiation between the products/(brands) of different sellers. A first-time buyer, with limited time and information at his disposal, will perform a rough market search and then purchase a brand, which he will learn to use better as he consumes it. When he comes back to the market as a second-time buyer, the brand bought earlier will offer some advantages over other brands, because the user skills have already been acquired. The induced switching cost will vary among consumers and cause some degree of brand loyalty to emerge as the market matures, although some consumers will be more conservative than others. Note that this effect will occur as long as consumers think that products might be heterogeneous, even though in fact they are homogeneous. In order to minimize the number of unusual features in this model, I will here assume that all consumers eventually react to price differences, but that some of them take longer than others to do so.

I will write the dynamic constraints as

$$(2) \quad \dot{s}_{it} = g(s_{it}, s_t^i, p_{it}, p_t^i); \qquad (i \in N),$$

where $s_t^i \equiv (s_{it},\ldots, s_{i-1t}, s_{i+1t},\ldots s_{ut})$ and p_t^i is defined analogously. In addition to some more special conditions to be stated later, the C^2 function $g(\cdot)$ is assumed to be declining and concave in p_{it} and $-p_t^i$. Also, using the shorthand g_i for $g(s_{it}, s_t^i, p_{it}, p_t^i)$, it follows from the above commentary that $\Sigma_{i=1}^n g_i = 0$ and that $g(\cdot) = 0$ when $p_{it} = p_t^i$. If only absolute price differences are allowed to count, we furthermore have

$$(A) \quad \frac{\partial g_i}{\partial p_{jt}} = -\sum_{k\neq j}^{n} \frac{\partial g_i}{\partial p_{kt}} = -\sum_{k\neq i}^{n} \frac{\partial g_k}{\partial p_{jt}} \qquad (i \in N).$$

The assumption that some consumers are slow to shift is modelled as

$$(B) \quad \frac{1}{s_{it}} \frac{\partial g_i}{\partial p_{it}} \to 0 \text{ for } s_{it} \to 1 \qquad (i \in N),$$

and

(C) $\frac{1}{s_{it}} \frac{\partial g_i}{\partial p_{it}} \to -\infty$ for $s_{it} \to 0$ $\qquad (i \in N)$.

(The function $\dot{s}_{it} = s_{it}^{\alpha} \Sigma_{j\neq i}^{n} s_{jt}^{\alpha} (p_{jt} - p_{it})$, $0<\alpha<1$, satisfies all assumptions made on $g_i(\cdot)$ alone in this paper.)

1.2 Necessary Conditions for Nash Equilibria

The current-value Hamiltonian for the control problem of the i^{th} firm is given by

$$H(s_{it}, s_t^i, \lambda_{it}, t; P_i(t), P^i(t)) =$$

$$y(p_{it})s_{it}(p_{it} - C(y(p_{it})s_{it})) + \sum_{j=1}^{n} \lambda_{ijt}\, g(s_{jt}, s_t^j, p_{jt}, p_t^j),$$

where the costate variables with values $\lambda_{it} \equiv (\lambda_{i1t},\ldots,\lambda_{int})$ correspond to the n constraints in (2) and $P^i(t)$ denote the strategies of the (n-1) other firms. I will now assume the game to be at a Nash equilibrium path and keep the symbols p_{it}, p_t^i, s_{it}, and s_t^i for the values of the corresponding functions along this Nash path.[1] Along this path, the following conditions should hold for all firms

$$(3) \quad s_{it}(y_i'(p_{it} - C_i - s_{it}\, y_i C_i') + y_i) + \sum_{j=1}^{n} \lambda_{ijt} \frac{\partial g}{\partial p_{it}} = 0 \qquad (i \in N),$$

$$(4) \quad \dot{\lambda}_{iit} = \rho\, \lambda_{iit} - y_i(p_{it} - C_i - s_{it}\, y_i C_i') - \sum_{j=1}^{n} \lambda_{ijt} \frac{\partial g_j}{\partial s_{it}} \qquad (i \in N),$$

$$(5) \quad \dot{\lambda}_{ijt} = \rho\, \lambda_{ijt} - \sum_{k=1}^{n} \lambda_{ikt} \frac{\partial g_k}{\partial s_{jt}} \qquad j \neq i \; (i \in N),$$

$$e^{-\rho t} \lambda_{ijt}\, s_{jt} \to 0 \text{ for } t \to \infty \qquad (i,j \in N),$$

where y_i and c_i denote the values of $y(p_{it})$ and $C(y(P_{it})s_{it})$, and y_i' and C_i' denote the derivatives, evaluated at p_{it}, s_{it}.

2. SYMMETRIC STEADY STATES

A steady state of the system represented by equations (2) through (5) is characterized by $\dot{s}_{it} = \dot{\lambda}_{ijt} = 0$, $(i,j \in N)$, and thus by $p_{it} = p_t^i$ and $\partial g_k/\partial s_{jt} = 0$, $(i,j,k \in N)$. Solving (3) and (4), and dropping time subscripts, we get the steady state price p_i^0:

$$(6) \quad p_i^0 = C_i + s_i y_i C_i' - \frac{y_i}{y_i' + \frac{y_i}{\rho}\frac{1}{s_i}\frac{\partial g_i}{\partial p_i}} \qquad (i \in N).$$

This differs from the traditional formula through the second part of the denominator, which corrects for the distributional effects of price competition.

2.1 Existence of Symmetric Steady States

In a symmetric steady state, where each firm has $1/n$ of the market, (6) reduces to

$$(7) \quad p^0 = C(\tfrac{y}{n}) + \frac{y}{n} C' - \frac{y}{y' + \frac{y}{\rho} n \frac{\partial g}{\partial p}},$$

where $\partial g/\partial p = \partial g_i/\partial p_i$, $(i \in N)$.

Since the right-hand side of (7) is a continuous function from $[0,K]$ to itself $\big($assume $(C_i(y/n) + C'y/n - y/(y' + (y/\rho)(\partial g/\partial p)n)) \in [0,K]\big)$, we arrive at Bouwer's fixed-point theorem:

THEOREM 1: There exists a symmetric steady-state industry structure.

2.2 Stability Properties of Steady States

In order to investigate the local stability of the system represented by equations (2) through (5) in the neighborhood of a steady state, we expand (2), using that $\partial g_i/\partial p_j = 0$, $(i,j \in N)$, at steady states.[2]

$$\dot{s}_{it} \simeq \sum_{j=1}^{n} \frac{\partial g_i}{\partial s_{jt}} = \sum_{j=1}^{n}\sum_{k=1}^{n} \frac{\partial g_i}{\partial p_k}\frac{\partial p_k^0}{\partial s_{jt}} \qquad (i,j \in N).$$

So we want to look at the characteristic roots of the matrix, call M, with typical element $\sum_{k=1}^{n} (\partial g_i/\partial p_k)(\partial p_k^0/\partial s_{jt})$, $(i,j \in N)$.

An important property of M can be brought out by using (A) on a diagonal element:

$$\frac{\partial g_i}{\partial s_{it}} = \sum_{j\neq i}^{n} \frac{\partial g_i}{\partial p_j}\frac{\partial p_j^0}{\partial s_{it}} + \frac{\partial g_i}{\partial p_i}\frac{\partial p_i}{\partial s_{it}} =$$

$$-\sum_{j\neq i}^{n}\sum_{k\neq i}^{n} \frac{\partial g_k}{\partial p_j}\frac{\partial p_j^0}{\partial s_{it}} - \sum_{k\neq i}^{n} \frac{\partial g_k}{\partial p_i}\frac{\partial p_i^0}{\partial s_{it}} =$$

$$-\sum_{j\neq i}^{n}\sum_{k\neq i}^{n} \frac{\partial g_j}{\partial p_k}\frac{\partial p_k^0}{\partial s_{it}} + \frac{\partial g_j}{\partial p_i}\frac{\partial p_i^0}{\partial s_{it}} = -\sum_{k\neq i}^{n} \frac{\partial g_j}{\partial s_{it}} \qquad (i \in N).$$

So all column sums of M are 0.

The fact that M has less than full rank follows, of course, from the properties of g(•) induced by (A). If all market shares have to add up to 1 then only (n-1) dimensions are free. Accordingly, we shall perform our stability analysis in (n-1) dimensions, using the n'th firm as a buffer to pick up the hypothetical shocks to one of the other market shares.

2.3 Instability of Symmetric Steady State

As a first step towards proving this property, I will now make the following key assumption

$$\text{(D)} \quad \left.\frac{\partial p_i}{\partial s_i}\right|_{s=1/n} = 2y_i\, C_i' + \frac{1}{n}\, y_i^2\, C_i'' + \left.\frac{\frac{y_i^2}{\rho}\, n\, \frac{\partial^2 g_i}{\partial p_i \partial s_i} - \frac{\partial g_i}{\partial p_i}\, n}{\left(y_i' + \frac{y_i}{\rho}\, n\, \frac{\partial g_i}{\partial p_i}\right)^2}\right|_{s=1/n} < 0 \qquad (i \in N)$$

where s = 1/n is used to denote that all derivatives are evaluated at the symmetric steady state. So I assume that a (hypothetical) small unilateral increase in s_i, from the symmetric steady state, will cause firm i, ceteris paribus, to want to lower its price. The mechanism underlying this phenomenon is that the resulting decline in marginal costs will dominate the decrease in the firm-specific price sensitivity of demand. This is a likely outcome if minimum efficient scale is large relative to average firm size.

At the symmetric steady state, $\partial g_i/\partial s_{jt} = \partial g_k/\partial s_{\ell t}$, $i \neq j$ when and only when $k \neq \ell$, $(i,j,k,\ell \in N)$, such that M is symmetric with identical off diagonal elements. Any (n - 1) x (n - 1) matrix, formed by removing one row and the corresponding column from M, will, of course, have the same properties. Therefore,

$$\left.\frac{\partial g_i}{\partial s_{it}} - \frac{\partial g_i}{\partial s_{jt}}\right|_{s=1/n} = \left.\frac{\partial g_i}{\partial p_i} - \frac{\partial g_i}{\partial p_j}\right|_{s=1/n} \left.\frac{\partial p_i^0}{\partial s_{it}} - \frac{\partial p_j^0}{\partial s_{it}}\right|_{s=1/n} \qquad (i,j \in N),$$

is a root of any such (n - 1) x (n - 1) matrix. This root is positive if

$$\text{(E)} \quad \left.\frac{\partial p_i^0}{\partial s_{it}} - \frac{\partial p_j^0}{\partial s_{it}}\right|_{s=1/n} = 2y_i C_i' + \frac{1}{n} y_i^2 C_i'' + \left.\frac{\frac{y_i^2}{\rho}\, n \frac{\partial^2 g_i}{\partial p_i \partial s_{it}} - \frac{\partial g_i}{\partial p_i}\, n - \frac{\partial^2 g_i}{\partial p_i \partial s_{jt}}}{\left(y_i' + \frac{y_i}{\rho}\, n \frac{\partial g_i}{\partial p_i}\right)^2}\right|_{s=1/n} < 0 \qquad (i,j \in N).$$

This amounts to the assumption that no other firm will react by a larger price cut than that made by firm i, in the event of a small increase in s_i.[3] Again, this is likely if marginal costs decline steeply around average market share. We thus have

THEOREM 2: <u>Under (E), the symmetric steady state is locally unstable</u>.

3. ASYMMETRIC STEADY STATES

Let us first make some assumptions to guarantee that the dynamical system is well behaved.

By (B) and (6), the steady-state price for $s_{it} \to 1$ goes to the monopoly price, given by

$$p_{i1}^{0} = C(y(p_{i1}^{0})) + y(p_{i1}^{0})\, C'(y(p_{i1}^{0})) - \frac{y(p_{i1}^{0})}{y'(p_{i1}^{0})} \qquad (i \in N),$$

whereas (C) and (6) give the competitive price for $s_{it} \to 0$:

$$p_{i0}^{0} = C(0) \qquad (i \in N).$$

Let me now assume the symmetric steady-state price to be between the competitive price and the monopoly price, such that

$$\text{(F)} \quad p_{i0}^{0} < p^{0} < p_{i1}^{0} \qquad (i \in N).$$

For ease of exposition, I will furthermore assume that

$$\text{(G)} \quad \frac{\partial p_i^0}{\partial s_i}(s_i) = 0 \quad \text{has at most two solutions in } [0,1] \qquad (i \in N).$$

Under these assumptions, we can sketch the following graph (Figure 1) of $p_i(s_i)$.

Figure 1 here

All that is needed now to prove the existence of an asymmetric steady state with n_1 big and $n_2 = (n - n_1)$ small firms is to assure the existence of a price, such that these market shares will add up to 1. If $p_i^0(s_i)$ has its interior maximum and minimum at s_a and s_b, respectively, we can define

$\bar{s}^1 = 1$ if $p_{i1}^0 < p_i^0(s_a)$ or else $\bar{s}^1 = \text{MAX}\{s_i | p_i^0(s_i) = p_i^0(s_a)\}$,

$\bar{s}_1 = s_b$ if $p_i^0(s_b) > p_{io}^0$ or else $\bar{s}_1 = \text{MAX}\{s_i | p_i^0(s_i) = p_{i0}^0\}$,

$\bar{s}^2 = s_a$ if $p_i^0(s_a) < p_{i1}^0$ or else $\bar{s}^2 = \text{MIN}\{s_i | p_i^0(s_i) = p_{i1}^0\}$,

$\bar{s}_2 = 0$ if $p_{i0}^0 > p_i^0(s_b)$ or else $\bar{s}_2 = \text{MIN}\{s_i | p_i^0(s_i) = p_i^0(s_b)\}$,[4]

such that $\bar{s}^1$, $\bar{s}_1$, $\bar{s}^2$, $\bar{s}_2$ are the maximal and minimal possible market shares for big and small firms, respectively. So to assure feasibility we need

(H) $n_1 \bar{s}_1 + n_2 s_2 < 1$,

(I) $n_1 \bar{s}^1 + n_2 \bar{s}^2 > 1$.

3.1 Existence of Asymmetric Steady States

We can here prove the existence of two asymmetric steady states by the Poincaré-Hopf theorem. The theorem uses the Poincaré index, which is based on the sign of the determinant of the negative of the linearized system at an equilibrium of a dynamical system. According to the theorem, in a well-behaved dynamical system, the number of positive such determinants exceeds the number of negative ones by 1 (see, e.g., Varian [1981, Sect. 2.3]). Since M had all positive eigenvalues at the symmetric steady state, the relevant sign at that equilibrium is negative and we thus have two more steady states. So:

THEOREM 3: Under (B) and (I), there exists an asymmetric steady-state industry structure with n_1 big and n_2 small firms.[5]

3.2 Stability of Asymmetric Steady States

To analyze this case, we shall need to think of the two types of derivatives below:

$$\frac{\partial p_i^{\,0}}{\partial s_{it}} = 2\, y_i C_i' + s_i y_i^2 C_i'' + \frac{\frac{y_i^2}{\rho}\frac{1}{s_{it}}\frac{\partial^2 g_i}{\partial p_i \partial s_{it}} - \frac{\partial g_i}{\partial p_i}\frac{1}{s_{it}}}{y_i' + \frac{y_i}{\rho}\frac{1}{s_{it}}\frac{\partial g_i}{\partial p_i}^{2}} \qquad (i \in N),$$

$$\frac{\partial p_j^{\,0}}{\partial s_{it}} = \frac{\frac{y_j^{\,2}}{\rho}\frac{1}{s_{jt}}\frac{\partial^2 g_j}{\partial p_j \partial s_{it}}}{y_j' + \frac{y_j}{\rho}\frac{1}{s_{jt}}\frac{\partial g_j}{\partial p_j}^{2}} \qquad i \neq j,\ (i,j \in N).$$

It seems reasonable to assume $\partial^2 g_j/\partial p_j \partial s_{it}$, $i \neq j$, $(i,j \in N)$ to be rather small and negative. So we might assume

$$\text{(J)} \quad \frac{\partial^2 g_j}{\partial p_j\, \partial s_{it}} < 0 \qquad i \neq j,\ (i,j \in N).$$

In this case all diagonal elements of M,

$$\frac{\partial g_i}{\partial s_{it}} = \sum_{j \neq i}^{n} \frac{\partial g_i}{\partial p_j}\frac{\partial p_j}{\partial s_{it}} + \frac{\partial g_i}{\partial p_i}\frac{\partial p_i}{\partial s_{it}} \qquad (i \in N),$$

are negative at asymmetric steady states, since these also have the property that $\partial p_i/\partial s_{it} > 0$, $(i \in N)$.

It furthermore seems reasonable to assume that $\partial g_i/\partial p_k$, $i \neq k$, $(i,k \in N)$ is small relative to $-\,\partial g_i/\partial p_i$, $(i \in N)$. So we could assume that, at a given asymmetric steady state,

$$\text{(K)} \quad -\sum_{k \neq i,j}^{n} \frac{\partial g_j}{\partial p_k}\frac{\partial p_k^{\,0}}{\partial s_{it}} < \frac{\partial g_j}{\partial p_j}\frac{\partial p_j^{\,0}}{\partial s_{it}} + \frac{\partial g_j}{\partial p_i}\frac{\partial p_i^{\,0}}{\partial s_{it}} \qquad i \neq j,\ (i,j \in N).$$

In this case all off-diagonal elements of M are positive.[6]

Thus, under (A), (J), and (K), all column sums of M are zero, all diagonal elements are negative, and all off-diagonal elements are positive. Therefore, any (n - 1) x (n - 1) matrix, formed by removing one row and corresponding column from M, will be a Hadamard matrix with negative diagonal. All eigenvalues of these matrices will thus have negative real parts (see, e.g., Murata [1977, Ch. 1, Thrm 20]). We then have

THEOREM 4: Under (A) and (J), asymmetric steady states which satisfy (K) are locally stable.

4. CONCLUSION

A theoretical drawback of infinite-horizon differential games is that neither existence (see, however, note 1) nor uniqueness results are available for such models. An advantage, however, is that economies of scale and demand rigidities can be treated more readily in the dynamic framework.

On the empirical side, a problem with the particular model analyzed here is that it seems unrealistic to argue for more than two firm sizes in the deterministic open-loop format used. As mentioned in note 3, however, it is possible that a closed-loop format could overcome this problem. An attractive feature of the model is that the unit markup grows with market share with the result that the well-known relation between market share and profitability (see, e.g., Buzzell, Gale, and Sultan [1975]) is refound.

If the mechanisms of the present model work to turn a small difference in market share into a bigger one, compounded by a markup difference, there are interesting strategic implications for the involved firms. Assuming that the small firm decides against trying to fight the odds, the natural response is to try to segment the market in such a way that it can sustain a larger equilibrium market share. Conversely, the bigger firms will try to prevent or preempt this segmentation. This, together with the equilibrating mechanisms described here, will tend to shift competitive emphasis from price to advertising. The major public policy implications of the model would seem to follow from the result that economies of scale do not necessarily lead to monopoly.

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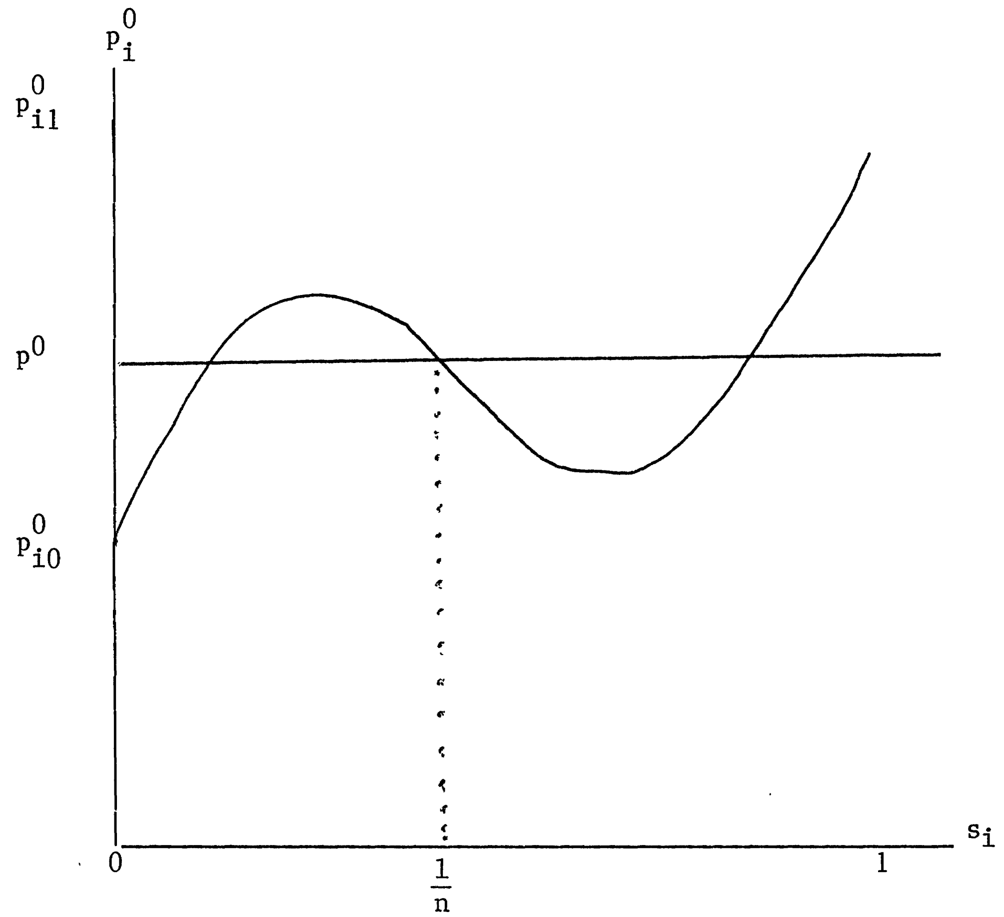
FIGURE 1

Steady-State Prices as Functions of Market Shares



1. While no proof of the existence of such a Nash path is known at the moment, a discrete time version of the game can be shown to have a Nash path by Theorem 9.6 in Friedman [1977].

2. We thus check for the stability of small unilateral shocks to market shares, pretending to forget the constraint $\sum_{i=1}^{n} s_{it} = 1$. A check for the stability of small shifts in shares will ease the proof of the analogy to Theorem 3 and complicate that of the analogy to Theorem 4.

3. Note that (I) is only slightly different from (D).

4. One could operate with a threshold market share, if one changed (C) and (E) accordingly and allowed three solutions in (F).

5. A closed-loop model might make it defensible to assume more than the two solutions in (F) and thus allow explanation of a more complex industry structure.

6. Unfortunately, the "easy" assumption $\partial^2 g_j / \partial p_j \partial s_{it} = 0$, $i \neq j$, $(i,j \in N)$ is unrealistic.