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DYNAMIC DUOPOLY, SEARCHING CONSUMERS, AND SCALE ADVANTAGES

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We look at Nash equilibrium in feedback strategies of a differential game. Under reasonable assumptions, it is demonstrated that a firm's growth maximizes early, never late, in the product life cycle. It is also shown that prices eventually may rise and that the market share of the large firm may go down.

It has recently been suggested, that "practical art" is a fertile basis for scientific generalization [Christenson 1976]. "Practical art" means skills developed by practitioners to deal with the topics under investigation. This paper will try to develop this suggestion.

I. INTRODUCTION

The concrete "practical art," which I shall use as a point of departure, is part of the approach to corporate strategy taken by the Boston Consulting Group (BCG). That part of the BCG theory which I shall use provides a rough framework for assessing investment opportunities under capital rationing. It is known as the product portfolio theory and is, in my judgment, the major factor behind the spectacular success of the firm during the past two decades. Recent research [Haspeslagh 1981] indicates that most Fortune 500 firms use the framework (see also Fortune, [Sept. 24, 1979]).

The reasoning behind the product portfolio theory is simple and intuitively appealing. Assume the following four premises to be true:

1. All costs follow "experience curve" reductions, where experience is a measure of accumulated, firm-specific sales.
2. In order to grow, a firm must, even in a friendly capital market, be able to retain some earnings.
3. Market shares exhibit declining price sensitivity over time.
4. Growth rates for individual products decline over time.[1]

BCG has developed an extensive body of data purporting to back up premise 1, while premises 2, 3, and 4 are easier for most practitioners to evaluate.

Let us look at BCG's practical conclusions from the above four points:

1. If you have a big market share and have moved far down the experience curve, you will have lower unit costs than your competition. (Some empirical evidence is provided by Schoeffler, Buzzell, and Heany [1974]).

2. This condition can be reached by an all out effort in the early stages of the life cycle.

3. In the late stages, market shares stabilize and total market growth slows down; investments are thus nonrewarding, and you should channel funds to younger industries.

The purpose of this paper is to provide a rational reconstruction of the recommendation that firms should maximize growth early in the product life cycle. Another reconstruction can be found in Spence [1979], whereas Shepherd [1962] contains a more traditional argument for equivalence of growth and profit maximization.

2. ANALYSIS

2.1 The Model

I will look at a duopoly where both firms operate under the financial constraint

(1) $\dot{K}_{it} \leq b_0 \Pi_{0it} - b_1 K_{it}$; $\quad i = 1, 2,$

where b_0, b_1 are two positive scalars, and K_{it} and Π_{0it} are capital and operating profits, respectively, of the i'th firm at time t.

Suppose further that there are TM_t customers in the market ($\dot{TM} \geq 0$, $\ddot{TM} \leq 0$) and that the i'th firm has a share MS_{it} of these, each of which is consuming at a positive level y_{it}.

I will assume the capital costs of producing $TM_t MS_{it} y_{it}$ units to be given by:

(2) $K_{it} = a_1 (TM_t MS_{it} y_{it})^{\phi}$; $\quad i = 1, 2,$

where a_1, ϕ are positive scalars and $0 < \phi \leq 1$, such that returns to scale may be present.

As defined,

$$\Pi_{0it} \equiv TM_t MS_{it} y_{it} (p_{it} - c_{it}); \qquad i = 1, 2,$$

where p_{it}, c_{it} are price and average unit costs, respectively, of the i'th firm. We get from (1) and (2):

$$(3)\ \frac{\dot{MS}_{it}}{MS_{it}} + \frac{\dot{y}_{it}}{y_{it}} + \frac{\dot{TM}_t}{TM_t} + b \leq a(TM_t MS_{it} y_t)^{1-\phi}(p_{it} - c_{it}); \qquad i = 1, 2,$$

where a, b are positive scalars.

The cost function exhibiting static and dynamic returns to scale,

$$(4)\ c_{it} = c_{0t}(TM_t MS_{it} y_{it})^{\alpha} \varepsilon_{it}{}^{\beta}; \qquad i = 1, 2,$$

where c_{0t}, is a declining industry-shared factor, α, β are nonpositive scalars; and ε_{it} is the experience of the i'th firm at t, such that

$$(5)\ \dot{\varepsilon}_{it} = \chi TM_t MS_{it} y_{it}; \qquad i = 1, 2,$$

where $\varepsilon_{i0} = 1$ and χ is positive.

Let me combine this with a simple demand-flow curve resulting from brand shifting by searching consumers:

$$(6)\ \dot{MS}_{it} = f(t)\ MS_{it} \sum_{j=1}^{n} MS_{jt}(p_{jt} - p_{it}),\ 0 \leq MS_{i0} = 1 - MS_{j0} \leq 1; \qquad i = 1, 2;\ i \neq j,$$

where the declining speed coefficient f(t) converges ($\dot{f}(\dot{t}) \geq 0$) to a nonnegative level.

I will further assume a certain adjustment period in the consumption level of consumers; thus,

$$(7)\ \dot{y}_{it} = g(p_{it}, y_{it}),\ 0 = g(p_{i0}, y_{i0}); \qquad i = 1, 2,$$

where

$$\frac{\delta g}{\delta p}, \frac{\delta g}{\delta y}, \frac{\delta^2 g}{\delta p^2} < 0 \text{ and } \frac{\delta^2 g}{\delta p \delta y},\ g\ \frac{\delta^2 g}{\delta y^2} > 0.$$

As illustrated in Figure 1, y converges to the stable points $\bar{y}(p)$, given by $0 = g(p,y)$, and forms an ordinary demand curve. For a reasonably small and stable value of $\dot{p}$, y will run almost parallel to y and $\frac{\delta y}{\delta p} \simeq \frac{\delta \bar{y}}{\delta p}$. In the following, I shall use the above approximation and, in addition, will assume y to be "close to" $\bar{y}$.

FIGURE ONE

Assume that each firm tries to find the "best" piecewise continuous feedback strategy $p_{it} \equiv \hat{p}_i\ (t,x)$, $i = 1, 2$, where x is the 6 vector formed by the two 3 vectors of type $(MS_{it}, y_{it}, \varepsilon_{it})$. The aim is to maximize discounted operating profit:

$$(8)\ \underset{\hat{p}_i}{MAX}\ \int_0^T e^{-\rho t} TM_t MS_{it} y_{it}(p_{it}-c_{it})dt + B_1 MS_{iT} + B_2 y_{iT} + B_3 \varepsilon_{iT}; \qquad i = 1, 2,$$

where ρ, B_1, B_2, B_3 are nonnegative scalars.

The two-person, general sum differential game defined by (3), (5), (6), (7), and (8) leads to the Hamiltonian:

$$(9)\ H_i(t, x_t, \mu_{it}, \nu_{it}, \eta_{it}, \hat{p}_1, \hat{p}_2) = e^{-\rho t} TM_t MS_{it} y_{it}(p_{it}-c_{it})$$

$$\sum_{j=1}^{2} (\mu^j_{it} f(t) MS_{jt} [\sum_{k=1}^{2} MS_{kt}(p_{kt}-p_{jt})] + \nu^j_{it} g(p_{jt}, y_{jt}) + \eta^j_{it} \chi TM_t MS_{jt} y_{jt}); \ i=1,\ 2,$$

where μ_{it}, ν_{it}, η_{it} are the three 2 vectors with typical arguments μ^j_{it}, ν^j_{it}, η^j_{it}, $j = 1, 2$. These, of course, form the dual system. Note now how $H[\cdot]$ is concave in p_{it} and how we can find the optimal policy at a given point, $\hat{p}_{it}$, independent of competitor action.[2]

In order to simplify the analysis as much as possible, I will demand that, away from (3), the optimal $\hat{p}_i$ depend on $(t, MS_{1t}, MS_{2t}, y_{it}, \varepsilon_{it})$ only.

This demand allows me to operate with much simpler costate dynamics, since I avoid the unknown derivatives of $\hat{p}_i$ with respect to y_{jt} and ε_{jt} $(i \neq j)$. As $MS_{jt} = 1-MS_{it}$, the influence of MS_{jt} poses no special problem. Note that a possible interpretation of this is, that each firm is ignorant of the experience and the unit volume of the other, while market shares are publicly known. From now on, subscript 1 will refer to the initially larger firm, whereas the smaller firm is indicated by subscript 2.

2.2 Growth Maximization

Denoting the growth-maximizing price strategies by $p_1^0(\hat{p}_2), p_2^0(\hat{p}_1)$, (3) gives for i = 1, 2,; j ≠ i:

$$(10) \quad f(t)(1-MS_{it})(p_{jt}-p_{it}^0) + \frac{1}{y_{it}} q(p_{it}^0, y_{it}) + \\ + \frac{\dot{TM}_t}{TM_t} + b - a(TM_t MS_{it} y_{it})^{1-\phi}(p_{it}^0 - c_{it}) = 0.$$

From (10) we see that unless there are no economies of scale present ($\alpha=\beta=1$ $1-\phi=0$), $p_{it}^0, (p_2^0) < p_{2t}^0(p_1^0)$; thus, the big firm can keep on gaining market share in the growth-maximizing game, a very intuitive result.

Let me now use the implicit function theorem to get an idea about price dynamics. Considering everything except p_{it}^0 and y_{it} to be functions of time, I get the following time derivative of the l.h.s. of (10):

$$(10t) \quad \dot{f}(t)(1-MS_{it})(p_{jt}-p_{it}^0) - f(t)\dot{MS}_{it}(p_{jt}-p_{it}^0) + f(t)(1-MS_{it})\dot{p}_{jt} + \left(\frac{\ddot{TM}_t}{TM_t}\right)$$

$$-a(TM_t MS_{it} y_{it})^{1-\phi}\left[(1-\phi)\left(\frac{\dot{TM}_t}{Tm_t} + \frac{\dot{MS}_{it}}{MS_{it}}\right)p_{it}^0 - (1-\phi+\alpha)\left(\frac{\dot{TM}_t}{TM_t} + \frac{\dot{MS}_{it}}{MS_{it}}\right)c_{it} - \dot{c}_{0t}\right],$$

where $(\frac{\ddot{TM}_t}{TM_t}) \equiv \frac{d(\dot{TM}_t/TM_t)}{dt}$.

Assume now that the prices are not too different, such that for

$$\frac{\dot{MS}_{it}}{MS_{it}} + \frac{\dot{TM}_t}{TM_t} \geq 0; \qquad i = 1, 2.$$

If, furthermore, $\dot{p}_{jt}$ is small and

$$(\frac{\ddot{TM}_t}{TM_t}) \leq \frac{\dot{TM}_t \dot{f}(t)}{TM_t f(t)},$$

(10t) is negative.[3]

The price derivative of the l.h.s. of (10) is:

$$(10p) \quad -f(t)(1-MS_{it}) + \frac{1}{y_{it}}(\frac{\delta q \delta y}{\delta y \delta p} + \frac{\delta q}{\delta p} - \frac{\dot{y}_{it}\delta y}{y_{it}\delta p})$$

$$-a(TM_t MS_{it} y_{it})^{1-\phi}[(1-\phi)\frac{p^0_{it}\delta y}{y_{it}\delta p} - (1-\phi+\alpha)\frac{c_{it}\delta y}{y_{it}\delta p} + 1].$$

If we now assume that the demand elasticity and the scale advantages are limited, such that

$$-\frac{\delta y p^0_{it}}{\delta p y_{it}} < (1-\phi)^{-1} \text{ and } 1-\phi+\alpha \geq 0;$$

and if, in addition, $\dot{p}^0_{it}$ is relatively small, such that we can use $\frac{\delta \bar{y}}{\delta p}$ as a proxy for $\frac{\delta y}{\delta p}$ and use that $\dot{y}_{it}$ and $\frac{\delta g}{\delta y}$ are small, then (10p) is negative.

The result is <u>declining growth-maximizing prices, for both firms, as $\dot{p}^0_{it}(\hat{p}_j) \leq 0$</u>.

2.3 Profit Maximization

Denoting the price strategies which maximize profit in the unconstrained game by $p_1^*(\hat{p}_2)$, $p_2^*(\hat{p}_1)$, we find from (9) an implicit definition of p_1^* as:

$$(21) \quad e^{-\rho t}TM_t MS_{it} y_{it} - \mu_{it} f(t) MS_{it}(1-MS_{it}) + \nu_{it}^i \frac{\delta g}{\delta p} = 0.$$

The dual dynamics are for μ_{it}, ν_{it}^i:[4]

$$(12) \quad \dot{\mu}_{it} = -e^{-\rho t}TM_t y_{it}[p_{it}^* + \eta_{it}^i \chi e^{\rho t} - c_{it}(1+\alpha)] - \nu_{it}^j \frac{\delta g}{\delta p_j} \frac{\delta \hat{p}_j}{\delta MS_i} + \eta_{it}^j \chi TM_t y_{jt}$$

$$f(t)[(p_{jt}-p_{it}^*)(1-2MS_{it}) + MS_{it}(1-MS_{it})\frac{\delta \hat{p}_j}{\delta MS_i}]\mu_{it} \qquad , \mu_{iT} = B_1.$$

$$(13) \quad \dot{\nu}_{it}^i = -e^{-\rho t}TM_t MS_{it}[p_{it}^* + \eta_{it}^i \chi e^{\rho t} - c_{it}(1+\alpha)] - \nu_{it}^i \frac{\delta g}{\delta y} \qquad , \nu_{iT}^i = B_2.$$

If we insert the solutions to (12) and (13) into (11), we get:

$$(11') \quad e^{-\rho t}TM_t MS_{it} y_{it} - e^{G^i(t)} B_1 f(t) MS_{it}(1-MS_{it}) + e^{F^i(t)} B_2 \frac{\delta g}{\delta p}$$

$$-e^{G^i(t)} f(t) MS_{it}(1-MS_{it}) \int_t^T (e^{-G^i(s)-\rho s} TM_s y_{is}[p_{is}^* + \eta_{is}^i \chi e^{\rho s} - c_{is}(1+\alpha)] +$$

$$+ \nu_{is}^j \frac{\delta g}{\delta p_j} \frac{\delta \hat{p}_j}{\delta MS_i} - \eta_{is}^j \chi TM_s y_{is}) ds$$

$$+ e^{F^i(t)} \frac{\delta g}{\delta p} \int_t^T e^{-F^i(t)-\rho s}(TM_s y_{is}[p_{is}^* + \eta_{is}^i \chi e^{\rho s} - c_{is}(1+\alpha)]) ds = 0,$$

where $G^i(t) = \int_t^T f(s)[(p_{js}-p_{is}^*)(1-2MS_{is}) + MS_{is}(1-MS_{is}) \frac{\delta \hat{p}_j}{\delta MS_i}] ds$

$$F^i(t) = \int_t^T \frac{\delta g}{\delta y} ds.$$

In order to get a hold on this complex equation I will exploit the fact that the costate variables go arbitrarily close to their terminal values, as time comes close to T. Therefore, since the last two terms on the l.h.s. of (11') go to 0 as $t \to T$, we can, for large t, reason on the first three terms, in which $F^i(t)$ and $G^i(t)$ also go to 0. Considering everything as functions of either MS or p, we find that for large t, $p^*_{2t}(p^*_1) < p^*_{1t}(p^*_2)$; thus, late in the life cycle of the unconstrained game, the market share of the big firm goes down (see Scherer [1970], pp. 217-218).

The mechanism at work here is that the marginal costs of a price cut are larger for the larger firm, whereas the marginal utility is the same for both firms late in the life cycle, when the net present value of investments in market share, volume, and experience are the same for both firms.

The fact that a firm which can monopolize an industry quite often chooses not to do so is well known to practitioners--and, indeed, to BCG ([Henderson 1979, pp. 90-94]). Theoretical explanations of the phenomenon, however, have been rather poor, concentrating either on chance, segmentation, or fear of regulation. In the above model, the explanation emerges directly from the economics of the problem. (It would be desirable to extend the result beyond the duopolistic setting so that a new theory of the size distribution of firms could be found.)

Coming now to the time dependence of p^*_{it}, I consider again y and p as functions of p and take the total time derivative of all other variables in (11):

$$\text{(11t)}\quad e^{-\rho t}TM_t MS_{it} y_{it}\Big[-\rho + \frac{\dot{TM}_t}{TM_t} + f(t)(1-MS_{it})(p_{jt} + \eta^i_{it}\chi e^{\rho t} - c_{it}[1+\alpha])$$

$$- \frac{1}{y_{it}}\frac{\delta g}{\delta p}(p^*_{it} + \eta^i_{it}\chi e^{\rho t} - c_{it}[1+\alpha])\Big] - \nu^i_{it}\,\frac{\delta g \delta g}{\delta y \delta p}$$

$$-f(t)MS_{it}(1-MS_{it})[(\eta^j_{it}\chi TM_t y_{jt} - \nu^j_{it}\,\frac{\delta g\ \delta\hat{p}_j}{\delta p_j\,\delta MS_i}) + \mu_{it}(\frac{\dot{f}(t)}{f(t)} - f(t)[1-MS_{it}MS_{it}\,\frac{\delta\hat{p}_j}{\delta MS_i})].$$

Since the price derivative of the l.h.s. of (11) is negative, $\dot{p}^*_{it}(\hat{p}_j) > 0$ if (11t) is positive.[5] Assume now that all firms operate at a profit; that

$$\rho < \frac{\dot{TM}_t}{TM_t} + f(t)(1-MS_{it})(p_{jt}-c_{it}[1+\alpha]) - \frac{1}{y_{it}}\frac{\delta g}{\delta p}(p^*_{it}-c_{it}[1+\alpha]);$$

and that $\dot{p}^*_{it}$ is rather small and stable, such that $\frac{\delta g}{\delta y}$ is small. If the opponent plays p^0_j, $\frac{\delta p^0_j}{\delta Ms_i}$ is positive; and under the stated assumptions, $\dot{p}^*_{it}(p^0_j) > 0$. As we just saw, if the opponent plays p^*_j, $\frac{wp^*_j}{wMS_i}$ is negative late in the life cycle. In simultaneous development, however, f(t) might grow very small as loyalty from consumers increases towards segmentation of the market. In this case (11t) might still be positive, such that <u>we get inflation late in the life cycle of the profit maximizing game</u>.

The underlying mechanism is that both the elasticity of demand and the competitive efforts go down, while the cost decline is slowing down (see also Dolan and Jenland [1981]).

2.4 Constrained Profit Maximization

Let me now use the results from the two simple games to analyze the constrained profit maximizing game. Note, first, that the price charged is the larger of p^0_{it} and p^*_{it}. Since under the stated assumptions, $\dot{p}^0_{it}(\hat{p}_j) < 0$ and $\dot{p}^*_{it}(p^0_j) > 0$, we see that the only possible development is from the pair (p^0_1,p^0_2) to (p^*_1,p^0_2) or (p^0_1,p^*_2) and then to (p^*_1,p^*_2). If we now consider a hypothetical shift late in the life cycle, from p^0_j to the higher valued p^*_j, this implies that $\frac{\delta \hat{p}_j}{\delta MS_i}$ changes from positive to negative. From (12) we see that this corresponds to a lower location of the μ_i trajectory, and from (11), that this causes p^*_{it} to increase. So even though $p^*_{it}(p^*_j)$ might not increase all the time (as if $f(t) \approx 0$), it lies on a higher level than the increasing $p^*_{it}(p^0_j)$, which stays above the decreasing $p^0_{it}(\hat{p}_j)$. Thus, <u>late in the life cycle we never leave the (p^*_1,p^*_2) pair</u>. We thus have the BCG result, that <u>firms should make an all-out effort to maximize growth early in the life cycle and then take the profits home later</u>. From this viewpoint, <u>growth maximization can be seen as a means to profit maximization</u>.

Further analysis of the results would confirm that the BCG premises constitute an overkill, in the sense that the growth constraint and any one of the three trends (1, 3, and 4) alone would be sufficient.

3. CONCLUSION

The above model reconstructs the BCG argument for early growth maximization. In the presence of experience curves, declining price sensitivity, and declining growth rates, growth maximization early in the product life cycle can be a means to profit maximization. It is further demonstrated that the larger firm will gain market share early in the life cycle and lose some of it later. Finally, prices will decline early and may increase late.

Even though we reproduce the BCG result, the complex nature and strong assumptions of the model leave several open questions, the answer to which may or may not require further assumptions. One such question is under which conditions a firm which has left the growth constraint will ever go back. Another interesting issue is which firm is the first to leave the growth constraint for good.

One should also remember that the analysis presented here is for one market only, and that the outlined path is the Nash equilibrium path. If the firm participates actually or potentially in more the one market, we may get a different behavior. It might be, for example, that the bigger firms would want to withdraw cash early in order to support divisions in younger or more promising markets. It might also be, that firms competing simultaneously in more markets would want to disturb each other's cash transfer plans.

Even more important are the limitations of the Nash equilibrium concept. In practice one quite often sees the establishment of some sort of price leadership such that the largest firm, which could become a monopolist if it wanted to, sets the price and gets the highest unit profit.

Note that under the stated assumptions, regardless of when and how we depart from the Nash path and enter price leadership, <u>the profits per unit will be greater for the biggest firms in all phases: growth maximization, unconstrained profit maximization</u>, and <u>price leadership</u>.

Three developments of the model--the use of advertising as a control variable, the extension to n firms, and the introduction of firm-specific scaling factors on the cost function--will presumably be relatively easy. The ease with which uncertainty can be dealt with will clearly depend on the way it is introduced. On the surface, one would expect it to enhance the advantage of the larger firm, but to leave the qualitative results unchanged.

Finally, there are two issues which are too important to be neglected and yet seem difficult to treat formally: those of entry and heterogeneous products.

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FOOTNOTES

1. Points 1, 2, and 4 can be found in a slightly less precise form on p. 164 in Henderson [1979], whereas point 3 appears on p. 163.

2. By Uchida's [1978] theorem 2, a Nash equilibrium exists for the stochastic game which results from adding Wiener processes to (5), (6), and (7). The theorem does not allow state-dependent constraints on the policy variables, such as (3), whereas fixed constraints of, e.g., nonnegativity type are allowed. To apply the theorem one therefore has to reformulate the problem, such that "foregone growth" instead of price becomes a policy variable. In this formulation H_i is still concave in the own policy variable, while it is convex in the competitor policy variable.

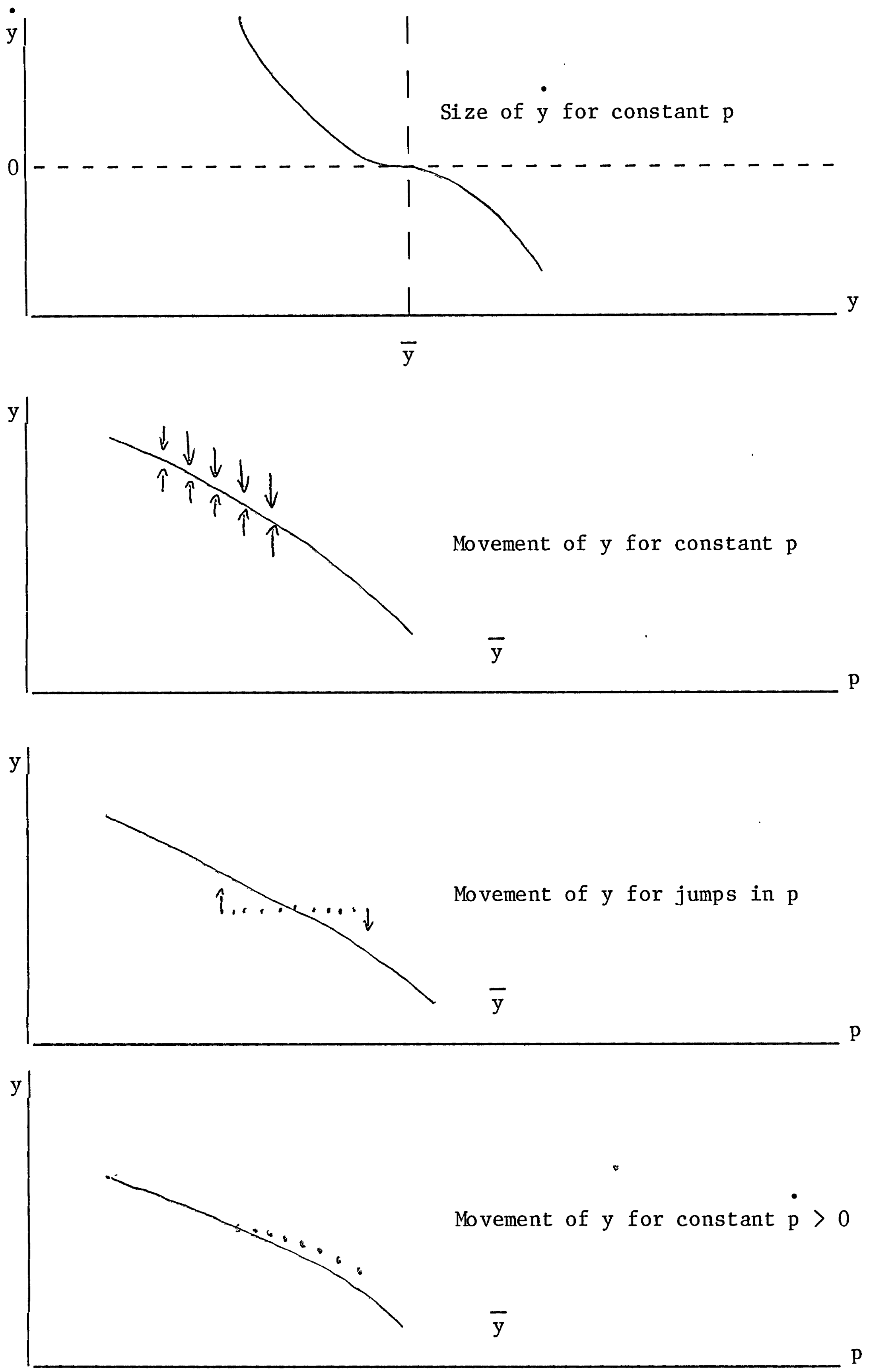
   Unfortunately, the theorem does not cover the deterministic game presented here.

3. Both here and later, I prefer early interpretable to weaker assumptions. I furthermore make my arguments on both prices, where statements about average price or leader price would require fewer assumptions.

4. I here use that, by (11), $\frac{\partial H_i}{\partial p_i} = 0$, and that $\frac{\partial \hat{p}_j}{\partial y_i} = \frac{\partial \hat{p}_j}{\partial \varepsilon_i} = 0$, as has been assumed for $\hat{p}_j = p_j^*$, and, by (10), is almost true for $\hat{p}_j = p_j^0$.

5. I here and later use that, under the stated assumptions, $\mu_i, \nu_i^i, \eta_i^j, -\nu_i^j, -\eta_i^j$ are nonnegative if the opponent plays p_j^* or p_j^0. This can be seen from the dual dynamics, by finding contradictions between the opposite signs and the ability to meet the terminal conditions.

Figure 1


$\dot{y}$
0
Size of $\dot{y}$ for constant p
y
$\bar{y}$
y
Movement of y for constant p
$\bar{y}$
p
y
Movement of y for jumps in p
$\bar{y}$
p
y
Movement of y for constant $\dot{p} > 0$
$\bar{y}$
p