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ILLIQUIDITY AND SPECULATION

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In the absence of perfect capital and futures markets and the presence of future investment opportunities, illiquidity is an undesirable feature of an asset, since it prevents the investor from moving his funds to new and better assets.

The type of liquidity preference which will be examined here depends on a special case of the Keynesian "speculation-motive" [Keynes 1969, Ch. 13, Sec. II). The basic idea is that the absence of perfect capital and futures markets forces one to hold liquid assets, if one wants to be able to take advantage of future opportunities (see also Baldwin and Meyer [1979]).

1. INTRODUCTION

Most investments, particularly those in real assets, have the property that a "too fast" sale will require a price discount. Such assets should thus pay a liquidity premium if future investment opportunities are partially unknown.

The present paper is in a sense complementary to Baldwin and Meyer's [1979], each representing a polar case of Tobin's [1958], concept of illiquidity as a relationship between the time needed to sell an asset and the transaction cost involved. Where Baldwin and Meyer look at illiquidity as the time needed to convert an asset directly into cash, I consider it as the transaction cost accruing from an instantaneous, noninfinitesimal sale (see also Chen, Kim, and Kon [1975]). Moreover, where the framework in Baldwin and Meyer naturally leads to a consideration of hurdle rates, the present model will look at portfolio fractions as indicative of liquidity preference.

The idea here is that an investor can currently invest in two assets, one liquid and the other such that transaction costs have to be paid for nonsmooth changes in the level at which it is held. It is shown that if a better opportunity is expected to come up in the future, the demand for the illiquid asset will be smaller the more illiquid it is and the sooner the new opportunity is expected to arrive.

The model has several shortcomings, most of which are imposed for purposes of analytical convenience. No transaction costs are imposed on smooth changes in asset holdings, and the future opportunity will be available on a now-or-never basis and will be such that a certain fraction of one's wealth has to be committed to it. While one would expect these assumptions to make liquidity preference more dramatic, they are, in my opinion, not critical to the results. In addition, the nature of the model, whereby the agent only looks ahead to the next attractive opportunity, is, in my judgment, unlikely to prove crucial to the results (see Baldwin, and Meyer [1979]). A more serious problem is that we are dealing with a partial framework returns and transaction costs are given.

2. THE MODEL

The model is kept as simple as possible. At the moment, the agent has all his wealth in one asset, paying r_0 and being fully liquid. He can, however, use part of his wealth to buy another asset, which pays r_1 but is illiquid, in the sense that it sells at a discount if it has to be sold too quickly. At some unknown future point in time, called T, a third asset, paying r_2, will be available on a now-or-never basis. For ease of calculation I shall assume that nature of this opportunity is such that, beginning at T, one has to commit a certain fraction of his wealth, s_2, to this asset at all future times. The model is myopic, in that no further opportunities are expected to appear after T.

By assuming $0 < r_0 < r_1 < r_2$, we get a highly nontrivial trade-off into the model. If I invest a lot at r_1 now, I gain more in the immediate future, but will have to take a larger loss later if I sell some of it to invest at r_2. Let me now try to analyze this trade-off by means of dynamic programming.

After T, no wealth is kept at r_0, since the liquidity argument, which might favor this to r_1, has disappeared. As the fractions of wealth kept at r_1 and r_2 ($1-s_2$ and s_2, respectively) have been frozen, all that remains after T is a trivial consumption-investment problem, which I will model as

$$\underset{\hat{y}}{\text{MAX}} \int_T^{\infty} e^{-\rho(t-T)} y^a(t)dt,$$

subject to

$$dS(t) = [s_2(r_2-r_1)S(t) + r_1S(t)-y(t)]dt \quad , \; S(T^+) > 0,$$

where

$\hat{y}$ [S(t),t] express nonnegative consumption y(t), as a function of time t and nonnegative wealth S(t);

T(ω) is the value of T for a particular realization ω of the process governing its arrival;

$S(T^+)$ is wealth just after T.

The constant scalars ρ and a satisfy

$$0 < r_0 < r_1 < r_2 < \frac{\rho}{a}$$

$$\frac{1}{2} < a < 1.$$

As is well known, this problem leads to a policy y^*, whereby $\frac{y^*}{S}$ is time-independent. The value function resulting from the optimal policy is

$$(1) \quad W[S(T^+)] = \left(\frac{\rho-ar_1-as_2(r_2-r_1)}{1-a}\right)^{a-1} S(T^+)^a .$$

If we denote by s_1 the fraction of wealth held at r_1 at time T^-, immediately before T, the amount of this sold at discount is $[s_1-(1-s_2)](S(T^-)$. Let the loss per unit resulting from the fast sale at T be called γ, $0 < \gamma < 1$, in this case,

$$(2) \quad S(T^+) = [1-\gamma(s_1-1+s_2)]S(T^-), \quad 1 - s_2 < s_1.$$

We can now maximize $W[S(T^+)]$ with respect to s_2 and thus find the optimal decision at T.

$$s_2^* = \frac{[1+\gamma(1-s_1)](r_2-r_1)(1-a)-\gamma(\rho-ar_1)}{(1-2a)(r_2-r_1)\gamma}$$

turns out to be the optimal value of s_2.

In order to make sure that $1 - s_2 < s_1$ and that $W(T^+) > 0$, we make the assumption

$$(3)\quad 1 + \frac{(1-a)}{a}\frac{1}{\gamma} - \frac{\rho/a - r_1}{r_2 - r_1} < s_1 < 1 + \frac{1}{\gamma} - \frac{\rho/a - r_1}{r_2 - r_1},$$

whereas the need for s_2^* to be between 0 and 1 demands

$$(4)\quad 0 < 1 + \frac{(1-a)}{a}\frac{1}{\gamma} - \frac{\rho/a - r_1}{r_2 - r_1} - s_1\frac{(1-a)}{a} < 2 - \frac{1}{a}.$$

After I have found s_1, I shall come back to these.[2]

We can now find $W^*[S(T^-)]$, the value function resulting from the optimal policy at T, s_2^*, and later, y^*, by inserting s_2^* into (1), (2):

$$(5)\quad W^*[S(T^-)] = \left(\frac{\rho-ar-a(1/\gamma+1-s_1)(r_2-r_1)}{1-2a}\right)^{2a-1} S(T^-)^a\,\gamma^a(r_2-r_1)^{-a}.$$

If we now add the assumption that T has an exponential distribution (subjectively) with parameter λ, we are ready to analyze the decision at time zero.

$$\underset{(\hat{y},\hat{s}_1)}{\text{MAX}}\ E\int_0^\infty e^{-\rho t}\, y(t)^a dt$$

subject to

$$dS(t) = [s_1(t)(r_1-r_0)S(t)+r_0S(t)-y(t)]dt + dN[\lambda,s_1(T^-),S(T^-)], S(0) > 0,$$

where no discount has been imposed on smooth changes in the level of holdings paying r_1.

$\hat{s}_1[S(t),t]$ express the fraction of wealth $s_1(t)$ put in the asset paying r_1, as a function of S and t. (I earlier defined $s_1 = s_1(T^-)$; it turns out that a constant s (t) is optimal.)

$dN[\lambda, s_1(T^-), S(T^-)]$ is the jump resulting from the appearance of the r_2 opportunity at $T(\omega)$ and the execution of the optimal policy at T.

So the value function jumps to the value given in (5) by dN at T. By dynamic programming, the value function for the entire problem should satisfy

$$0 = \underset{(\hat{y},\hat{s}_1)}{\text{MAX}} \{y^a - \rho W + W_S[s_1(r_1 - r_0)S + r_0 S - y] + \lambda(W^* - W)\},$$

where

$$E\{e^{-\rho t} W[\cdot]\} \to 0 \text{ for } t \to \infty.$$

The consumption policy is again time-independent, and W is of the form

$$W(S,t) = B\, S(t)^a \qquad , B = B_0 \text{ for } t < T,$$

where B_0 is a constant function of the parameters of the problem.

$s_1^*(t)$, the optimal investment decision before T, is

$$(6) \quad s_1^* = 1 + \frac{1}{\gamma} - \frac{\rho/a - r_1}{r_2 - r_1} - (2 - \frac{1}{a}) \left[\frac{\lambda\, \gamma^a}{B_0 (r_2 - r_1)^{1-a} (r_1 - r_0)}\right]^{\frac{1}{2(1-a)}} .$$

So the content of (3) and (4) is

$$(3') \quad \frac{1}{\gamma} > \left[\frac{\lambda\, \gamma^a}{B_0 (r_2 - r_1)^{1-a} (r_1 - r_0)}\right]^{\frac{1}{2(1-a)}}$$

$$(4') \quad \frac{(1-a)}{a} \frac{1}{\gamma} < \frac{\rho/a - r_1}{r_2 - r_1} < 1 - \frac{1}{\gamma} \left(\frac{a-1}{2a-1}\right) .$$

It is trivial, but tedious, to find B_0 as the positive of two roots to

$$0 = (1-a) B_0^{\frac{-a}{1-a}} - \rho B_0 + a B_0 [s_1^* (r_1 - r_0) + r_0] + \lambda [W^* (s_1^*) S^{-a} - B_0] .$$

It turns out that $\frac{\partial B_0}{\partial \lambda} < 0$ and $\frac{\partial B_0}{\partial \gamma} < 0$.

3. CONCLUSION

The motivation behind this exercise was to establish the liquidity preference effect of future investment opportunities. In order to do so, we look at s_1^*, the demand for the illiquid asset, in (6) and find that $\frac{\partial s^*}{\partial \gamma} < 0$ and $\frac{\partial s^*}{\partial \lambda} < 0$; thus, the greater the illiquidity of an asset and the larger the prospect of a better opportunity, the smaller the demand for the asset.

While a number of implications of this phenomenon have been pointed out in Baldwin and Meyer [1979], a possible macroeconomic consequence, not mentioned in Keynes [1969], might be worth conjecturing at this time. Suppose that first advantages and irreversibilities are significant in new product development and that firms look at such opportunities as random and exogenous. In this case, it is possible that periods with few new product opportunities will be characterized by low levels of productive investments, and may lead to lower levels of effective demand.

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FOOTNOTES

1. I have normalized so that y is bought at unit price.

2. These and other constraints are imposed for expositional ease only.

3. As suggested in Magill and Constantindes [1976], a massive amount of trading is imposed on the agent, especially if the yields are made stochastic. Progress towards greater realism is being made, however, both in the above paper and in Richard [1977].

4. See Stone [1973, thrm. 4.5] or Chen, Kim, and Kon [1975, thrm. 5.6].

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