TACIT COLLUSION AND
MULTIPLE POINT COMPETITION

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by

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Using theories of self-enforcing agreements, the paper demonstrates how a proliferation of competitive parameters can facilitate oligopolistic coordination. For example, we analyze the effect of introducing advertising or shifting the cost function in a quantity game. We state explicit conditions under which a new variable can improve payoff in games of the prisoners' dilemma type.
I. Introduction

In the vast majority of real oligopolies, the firms compete on several parameters and/or in several markets. Yet almost all our theory is developed for price competition in single markets. This paper is an attempt to offer a first cut at analyzing the competitive implications of multiple point competition.

The general heading of multiple point competition covers three different types of situations. First, there is competition on several parameters, e.g., price and advertising, in a single market. In such situations the conventional wisdom since Chamberlin (1933), is that competition on advertising somehow facilitates collusion on price. Unfortunately, no theory is available to rationalize this phenomenon. In the present paper we will provide such a theory and show how advertising can facilitate collusion among a given number of oligopolists. A related but different line of work focuses on the use of advertising as an entry barrier and thus its ability to limit the number of participants in a price game (Salop, 1979). Secondly, there are situations where the same firms compete simultaneously in several distinct but related markets. This can, for example, be product with close demand interrelatedness, e.g., trains and buses or razors and blades. Alternatively, the product can be linked in terms of costs by use of the same inputs or processes or they can be vertically linked, one being an input in the production of another. Some of these latter situations have been the subject of research, e.g., on "vertical squeeze" strategies (Williamson, 1970), or on "counter competition" (Watson, 1982). None of this is, however, very general. The present paper will give a set of general conditions under which some parameters, interacting with others in the profit functions, can be used to facilitate collusion on the latter parameters and perhaps vice versa. Finally, there is the case of
conglomerate competition in which firms compete in several unrelated markets. This type of situation has been described by the "spheres of influence" hypothesis according to which the danger of multimarket retaliation hampers the tendency to start local price wars (Edwards, 1955; Stocking and Watkins, 1946; Scherer, 1980, pp. 340-2). Although this conjecture has been the subject of extensive empirical research (Scott, 1982; Heggestad and Rhoades, 1978; Feinberg, 1982), it has not been given a solid theoretical underpinning. In the present paper we will demonstrate that multiple point collusion with additively separable profit functions is substantially different from other cases. (See also Karnani and Wernerfelt, 1983).

We will nest our work in the theory of self enforcing agreements in repeated games of the prisoners' dilemma type. In Section II, we will summarize most of the many different models leading to cooperation in such games. Although the results are derived under very different assumptions, the ability to collude in all cases depend on the profit from doing so, the profit from competition, the profit from cheating on cooperating partners and the profit (loss) from being penalized. The central idea in this paper is, that multiple point competition offers possibilities to shift these profit functions, such that their relative size change to allow more cooperation. Section III will contain four simple examples of this. First we will look at a duopoly where full quantity collusion is not feasible without advertising but becomes feasible if the firms start to advertise competitively. Secondly, we will look at a converse example where advertising at the competitive level, as opposed to zero advertising, ruins full volume collusion. Thirdly we will show how firms sometimes can pick an "unnatural" level of advertising (that is neither competitive nor fully collusive) and thereby make the package deal consisting of that level of advertising and volume collusion a self-enforcing
agreement. Finally we will see how a change in technology, though not rational if we stay in Nash equilibrium, can be rational because it may allow a cooperative equilibrium. In Section IV we will derive the general conditions under which changing a non volume parameter can pay off in the sense that it allows more volume collusion and yet is relatively cheap. Section V will contain a discussion of the results.

II. The Repeated Prisoners' Dilemma

By a self enforcing agreement, we mean an agreement which is such that it is rational for all parties involved to keep it. In general, oligopolistic games have a prisoners' dilemma type structure and securing cooperation therefore becomes a nontrivial task. In particular, if we model an oligopoly as an n-stage repetition of an identical (finite) prisoners' dilemma, with actions (high price, low price), we have a unique equilibrium in which both firms charge the low price. To see this, recognize that both firms will charge the low price in the last period, that they therefore have nothing to gain from not doing so in the next-to-last period and so on. The literature contains several variations of the above model, each of which change it in a special way to obtain an equilibrium where both firms cooperate (charge the high price). We will now go through these models.

First, if there is no observation of retaliation lags, such that each player's action at time t can take the actions of all the other players, also at time t, into account, then any price between the monopolistic and the competitive can be a perfect equilibrium (Laitner, 1982). By a perfect equilibrium we mean one in which the conjectural variations are consistent in the sense that each player's conjectures about the actions of other players, should he himself deviate from the equilibrium, are consistent with payoff
maximization on their part given their conjectural variations. More intuitively, the threats which sustain the equilibrium are credible, not of the type "if you do such and such, I will kill us both." In the following we shall only deal with perfect equilibria unless otherwise stated.

Within the class of games with a reaction lag (defining the length of one period), Friedman (1971) presents the first and simplest analysis. He assumes an infinite game to have an infinite horizon and furthermore requires firms to believe that once an agreement is violated, a new one will never be established. So a Cournot-Nash equilibrium will result. In this case, we can find the maximum degree of sustainable cooperation as a simple function of $\pi^+$, the per period profit from the agreement, $\pi^0$, the profit in Cournot-Nash equilibrium, and $\pi^*$, the profit to a cheater in the period before the reaction is implemented by the other players. Accordingly, the gain from cheating is $\pi^* - \pi^+$ and the loss is $\frac{\alpha}{1-\alpha} (\pi^+ - \pi^0)$, where $\alpha$ is the discount factor. So we can find the maximum $\pi^+$ which satisfy $\pi^* - \pi^+ \leq (\pi^+ - \pi^0) \frac{\alpha}{1-\alpha}$. (Note that $\pi^*$ and $\pi^+$ vary together in this inequality). From a similar set of arguments, Salop (1982) gets $\pi^+ \geq b\pi^* + (1-b)\pi^0$, where $b$ is some constant.

The assumption that an agreement, once broken, can never be reestablished is clearly unreasonable. And yet the Cournot-Nash behavior after a breach depends critically on it. If we allow an agreement to be reestablished, it might pay off to penalize a cheater beyond the competitive level. Still using an infinite game in an infinite horizon setting, Abreu (1982), has done an admirable job of analyzing this. He shows that under certain conditions the maximum credible threats are of the stick and carrot nature. If someone breaks the agreement, there is collective punishment in the next period followed by new collusion if everyone participated. If someone cheated on the
penalty, the next period is again a penalty period. Accordingly, Abreu finds
the following condition on \( \pi^+, \pi^* - \pi^+ \leq \alpha(\pi^+ - \pi^-) \), where \( \pi^- \) is the pen-
alty profit (loss). Some similar, but less general, results are obtained in
a stochastic setting by Porter (1983) and Green and Porter (1983).

In finite time horizons, one has to resort to uncertainties to make
collusion self-enforcing. A simple model of infinite games is due to Telser
(1982). He ruins the backwards recursive deadlock described above by making
the time horizon stochastic. (An idea originally conceived by Luce and Raiffa,
1957). Since the players never know which period is the last, the logic never
gets started. As expected, Telser finds a condition of the form \( \pi^* - \pi^+ \leq
\beta(\pi^+ - \pi^-) \), where \( \beta \) depends on the discount rate and the expectations about
the length of the game.

Another, more realistic analysis of the finite horizon case for finite
games is given by Kreps, et al., (1982), Kreps and Wilson (1982), and Milgrom
and Roberts (1982). The basic idea is to postulate uncertainty, on the part
of other players, about a particular "mystical" player. This uncertainty
could concern his payoff structure. Is there a small chance that it would be
rational for him to penalize very dramatically? If so, you want to think
twice before you challenge him. Following this, the gain from cheating should
be weighted against \( (\pi^+ - (1-p) \pi^0 - p \pi^-) \), where \( p \) is the subjective probability
of a violent reaction. With Bayesian players, \( p \) can furthermore be massaged
during the game. In particular, the mystical firm can behave as if it is
rational for it to penalize violently, thereby increasing \( p \). Consequently,
there is incentive to behave tougher than you are, to get a reputation. So
in the early stages of the game, the mystical firm will be tough even if
others put a very small probability \( p \) on this being rational in the last
period where reputation building is wasteful. In a given period therefore,
the maximum degree of self-enforcing collusion is given by \( \pi^* - \pi^+ < \frac{\alpha}{1-\alpha} \pi^+ \pi(p,t) \), where \( Q(p,t) \) is the net present value of the optimal penalty, given \( p \) and \( t \). If \( p \) is small or \( t \) is big \( \pi(p,t) = \frac{\alpha}{1-\alpha} \pi^- \) such that no excess penalty is imposed. So a bigger \( p \) and a smaller \( t \) helps increase \( \pi^+ \). Note finally that the uncertainty could pertain to almost anything, the strategy of a player, his rationality, etc.

In all the inequalities above, the maximal degree of collusion depends on \( \pi^*, \pi^+, \pi^0, \pi^- \) and \( p \), as well as the discount factors, etc. Of these \( \pi^+, \pi^0 \) and \( \pi^- \) are endogenous or depend mainly on the cost and demand conditions, whereas \( p \) depends on the reputations of the players and \( \pi^* \) mainly on the timelag between cheating and reaction. (Stigler, 1964, and Spence, 1978; analyze the consequences of uncertainty in detection, but we will not complicate the analysis further.) In this paper, we will concentrate on the effects of cost or demand interactions on \( \pi^*, \pi^+, \pi^0, \) and \( \pi^- \).

III. Some Examples

In order to motivate the general analysis in Section IV we will look at three very simple examples of multiple point competition and self enforcing agreements. To keep things as simple as possible we use Telser's (1982) framework as a vehicle and assume that full cooperation, competition or cheating are the only three feasible actions. So we look at a finite game.

We will assume that the expected number of future games, called \( \mu \), is independent of the number of games already played. If the discount rate is zero, an agreement yielding per period payoffs of \( \pi^+ \) is self-enforcing if \( \pi^* - \pi^+ \leq \mu(\pi^+ - \pi^-) \).

Let us look at a symmetric duopoly where two firms set outputs \( x_1, x_2 \) in the face of an inverse demand curve of the form \( P = a - b(x_1 + x_2) \),
\((a,b) \in \mathbb{R}^2_+\). Assuming costs to be zero, it is well known that Nash outputs and profits will be \(x^0 = \frac{a}{3b}, \pi^0 = \frac{a^2}{9b}\) respectively. Cooperative outputs and profits will be \(x^+ = \frac{a}{3b}, \pi^+ = \frac{a^2}{8b}\); while a firm which cheats a cooperative partner will produce \(x^* = \frac{3a}{8b}\) and receive profits of \(\pi^* = \frac{9a^2}{64b}\). Based on this, full cooperation is not a self-enforcing agreement if:

\[
\frac{9a^2}{64b} - \frac{a^2}{8b} - \mu\left(\frac{a^2}{8b} - \frac{a^2}{9b}\right) > 0 \iff \mu < \frac{9}{8}
\]

Let us introduce the possibility of advertising and model the price consequences of the firms spending \(y_1^2\) and \(y_2^2\) dollars of advertising as \(P = a - b(x_1 + x_2) + c(y_1 + y_2)\). If firms choose their advertising competitively to maximize \(P \times x_i - y_i^2\), \(i = (1,2)\), the levels of advertising will be \(y_i = \frac{c}{2}x_i\), \(i = (1,2)\).

In this case it is trivial but tedious to find outputs and profits from competing, cooperating and cheating (in the quantity game) as:

\[
x^0 = \frac{a}{3b - c^2}, \quad x^+ = \frac{a^2}{(3b - c^2)^2}
\]

\[
x^* = \frac{a(3b - c^2)}{4b - c^2}, \quad \pi^* = \frac{a^2(3b - c^2)^2}{2(b - 1/4c^2)(4b - 3/2c^2)^2}
\]

If, for example, \(a = 1, b = 2/3, c = 1, \mu = \frac{17}{16}\), we find \(\pi^0 = \frac{5}{12}, \pi^+ = \frac{3}{7}, \pi^* = \frac{108}{245}\) and \(\pi^* - \pi^+ - \mu(\pi^+ - \pi^0) < 0\).

Given the above parameter values, cooperation in the advertising game is not a self-enforcing agreement. Assuming full cooperation in the quantity game, the profits from competing, cooperating and cheating in the advertising game are \(\frac{5}{12}, \frac{3}{4}\) and \(\frac{21}{16}\) respectively. One may check that this requires a \(\mu\) much above \(\frac{17}{16}\) to yield self-enforcing cooperation. So the mere addition of competitive advertising, while not cooperative in itself, changed the odds in the quantity game and made cooperation rational.
To see that advertising can work both ways, consider the situation where \( a = 1, \ b = \frac{2}{5}, \ c = 1 \) and \( \mu = \frac{5}{4} \). In this case one may verify that the introduction of advertising shift the equilibrium away from full cooperation in the quantity game.

The above examples deal with "semi-fuzzy" games, in which a separate decision to cooperate or compete is made for each decision variable (see the Appendix for a brief description of the properties of such games). If we allow "package deals," even more striking results appear. For example, in the case \( a = 1, \ b = \frac{2}{5}, \ c = 1, \ \mu = \frac{5}{4} \), the agreement to collude on quantity and not advertise is self enforcing relative to competing on both parameters. In these particular situations the package deal—collusion is less profitable than package competition, but as we shall see in the next example, this is not always the case.

Instead of advertising we can look at shifts in the technology. Suppose that our duopolists, with \( a = 1, \ b = \frac{2}{3} \) and \( \mu = \frac{17}{16} \), have a production cost of \( \frac{3}{20} \) per unit. In this case cooperation is not sustainable and Nash profits are \( (1 - \frac{3}{20})^2 \frac{6}{6} = .1204\ldots \). Assume that they can agree to change the linear technology to one with a quadratic cost function such that profits equal \( P_x - dx^2 \). In this case we find:

\[
\begin{align*}
X^0 &= \frac{a}{3b + 2d}, \quad Z^0 = \frac{a^2(b + d)}{(3b + 2d)^2}, \\
X^+ &= \frac{a}{4b + 2d}, \quad Z^+ = \frac{a^2}{4(2b + d)} \\
X^* &= \frac{(3b + 2d)}{4(b + d)(2b + d)}, \quad Z^* = \frac{a^2(3b + 2d)^2}{16(b + d)(2b + d)^2}
\end{align*}
\]

One may verify that \( a = 1, \ b = \frac{2}{3}, \ d = \frac{2}{3} \) and \( \mu = \frac{17}{16} \) makes cooperation unsustainable, such that profits of .125 result. With competition, profits are only
.12, so the shift in technology only pays off because it allows a shift in
equilibrium to self-enforced cooperation.

IV. General Analysis

In this section, we will look at the maximum degree of self-sustaining
cooperation, rather than the full cooperation case used in the examples above.

If $x^+$ denotes the level of $x$ in the strongest feasible self-enforcing
agreement, the theories from Section II all tell us that $x^+$ is constrained by
an inequality of the form.

$$F(x^+) \equiv \pi^*(x^+) - \sigma^{+}(x^+) + \nu_0 + \gamma^- \leq 0$$

If, as in the quantity game interpretation, $\frac{\delta F}{\delta x^+} \leq 0$, we have a non-trivial
coordination problem if $\frac{\delta F}{\delta x^+} < 0$ and $F(x_1) > 0$, where $x_1$ is the fully coop-
erative quantity. In this case $x^+$ is implicitly defined by $F(x^+) = 0$.

Let us now introduce another parameter $y$, such as product design, dis-
tribution channels, overseas prices, manufacturing technology or anything else
which influence the components of $F(\cdot)$. Assume that $y$ is a continuous vari-
able and that $F(\cdot)$ is a $C^1$ function of $y$. So we can write the implicit
definition of $x^+$ as

$$\pi^*(x,y) - \sigma^{+}(x,y) + \nu_0(x,y) + \gamma^-(x,y) = 0$$

Using the implicit function theorem, we find the effect of $y$ on $x^+$ as

$$\frac{dx^+}{dy} = - \frac{\delta F}{\delta y} \left( \frac{\delta F}{\delta x^+} \right)^{-1}$$

So, depending on the sign of $\frac{\delta F}{\delta y}$, we can decrease $x^+$ (increase $\pi^+(x^+)$) by
increasing or decreasing $y$, as long as $\frac{\delta F}{\delta y} \neq 0$. So any parameter which
influence $F(\cdot)$ can be used to decrease $x^+$. If the parameter is at its
minimum feasible value, it can be used if $\frac{\delta F}{\delta y} < 0$, if it is at its maximum, it can be used if $\frac{\delta F}{\delta y} > 0$. A change in $y$ will be profitable as long as

$$-\frac{\delta F}{\delta y} + \frac{\delta F}{\delta x} \left(\frac{\delta F}{\delta x}\right)^{-1} + \frac{\delta F}{\delta y} \frac{\delta F}{\delta x}$$

has the same sign as $-\frac{\delta F}{\delta y}$. So the optimal level of $y$

can be found at $\frac{\delta F}{\delta y} = \frac{\delta F}{\delta x} \left(\frac{\delta F}{\delta x}\right)^{-1}$, assuming the involved functions are sufficiently well behaved.

The functional form of (1) has interesting implications, in some sense contrary to the spheres of influence hypothesis, for conglomerate competition. If all the markets have similar structure, adding more will merely increase all terms on the l.h.s. of (1) in the same proportion and thus not influence $x^+$. Furthermore, most of the theories in Section 2 lead to conditions of the form:

$$\pi^* - \pi^+ + \sigma(\pi^* - 0) - \nu(\pi^+ - 0) - \gamma(0 - \pi^-) \leq 0.$$

In this case, if a new market merely adds the same constant to all the profit terms above, nothing is gained as far as collusion goes. The most dramatic collusive advantages of multiple point competition must therefore be expected for single markets with many decision parameters or many related markets. In conglomerate competition, one must often resort to other means, such as "limited wars" (Schelling, 1960, 1966) to influence reputations or "mutual footholds" to decrease reaction time ($\sigma, \nu, \gamma$). For details on this case, see Karnani and Wernerfelt (1983).

The analysis above has been for "package deal" collusion, where firms agree to shift from $y_1$ to $y_2$ because $x^+$, $y_2$ is self-enforcing relative to $x^-, y_1$. Situations where a new competitive tool is activated such that independent collusion on old tools become feasible only exist under rather strong assumptions, but may be easier to take advantage of than the package deals.
V. Discussion

As we have seen above, by going from single to multiple point competition oligopolists may facilitate collusion. While it presumably will be necessary to stop the proliferation of competitive parameters before monitoring costs grow too big (Stigler, 1964, Spence, 1978), it seems fair to conjecture that the optimal number of competitive parameters in several cases will exceed one. If the market is such that suitable differentiation (advertising, promotion, quality, delivery, etc.), cannot be achieved; it is feasible but probable less effective to segment the market along geographical lines and play the game that way.

In addition to these more conventional examples of multiple point competition, firms can "post a bond" to guarantee cooperation in many other ways. For example, they can pick a technology with steeply increasing marginal costs or they can concentrate on product design which are not suitable for mechanization. In fact we could show that almost any parameter could influence the maximum degree of sustainable collusion.

While we derived our general results in the most favorable setting, that of "package deals" and continuous parameters, it seems intuitively clear that opportunities for multiple point collusion occur frequently also under other circumstances. Unfortunately, it seems difficult to derive any general results for the case of discrete parameters. A pursuit of the case where each parameter is set individually would, however, be a useful extension of this work. Another pertinent question is how firms can go about establishing "package deals." If no reasonable prescriptions can be offered, it may well be that the semifuzzy case, where each parameter is set individually, is the more realistic.
A direct empirical test of the theory seems difficult since we have no general way of identifying the parameters whose shifts allow increased collusion. Instead one could perhaps draw on more specific or indirect tests, based on the relationship of profits to advertising to sales ratios (Shepherd, 1972), capital intensity (Schoeffler, et al., 1974) or multimarket contact (Scott, 1982). It would finally be useful, but outside the scope of this work, to examine the public policy consequences of multiple point collusion.

The present paper has attempted to present a richer framework for tacit coordination. While it is hard to judge the actual importance of multiple point competition, the increase in several factors, such as the fraction of the economy controlled by diversified firms, the international character of markets and the degree of product differentiation, seems to indicate that these concerns will grow in importance in the future.

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APPENDIX: SEMIFUZZY GAMES

Formally, we start with an ordinary n-person game with superadditive characteristic function \( W: \{0,1\}^n \rightarrow A \), where \( A \) is an \( n \)-dimensional space of multiutilities in the no side payment case and the nonnegative real line in the side payment case. Assume that we can decompose the actions of each player into \( m \geq 1 \) identical, mutually exclusive and collectively exhaustive subsets, such that the set of feasible actions in one subset is independent of the choice made in another subset. Call these subsets "scenes." Given enough information about the game, we can now construct the \( m \)-scene decomposition as an "\( n \)-person, \( m \)-scene semifuzzy game," defined by its superadditive characteristic function \( V_1: (\{0,1\}_i)^m \rightarrow B_i, i \in \mathbb{N} \). Here \( \{0,1\}_i \) is the set of \( n \)-vectors with 1 in the \( i \)'th argument and 0 or 1 elsewhere. \( B_i \) is the \( i \)'th argument of \( A \) in the no sidepayment case and the nonnegative real line in the sidepayment case.

Because of the superadditivity of \( V_1 \), the tightest core constraints will be those of the (full) coalitions also permitted in the original game. So the core of \( W \) and \( V_1, i \in \mathbb{N} \) coincides. Furthermore, it can be shown (under appropriate conditions) that a semifuzzy game approaches a fuzzy game (Aubin, 1981) as the number of scenes grow. For details see Wernerfelt, 1983.
REFERENCES


