Exhaustible Resource Models with Uncertain Returns from Exploration Investment

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Abstract

Exhaustible resource models that do not consider exploration investment typically have low values of perfect information and sometimes even optimal myopic policies. In this paper, we add exploration and capacity investment and allow the returns from exploration to be stochastic. We show that, in this model, the stochastic program solution may be quite valuable and that myopic policies are far from optimal.

1. Introduction

Exhaustible resource models have been studied by a number of authors. Hotelling [3] initially formulated a model that demonstrated that the market price of an exhaustible resource grows exponentially as it is depleted. Nordhaus [7] introduced the idea of a "backstop" technology to this model. The result is the Hotelling-Nordhaus model in which a finite resource is used until its production cost exceeds that of the inexhaustible backstop technology. The backstop technology is then introduced and the two technologies are never used simultaneously.

Manne [5] and Manne and Richels [6] use the Hotelling-Nordhaus model in their analysis of the effect of the uncertainty of the introduction date of the fast breeder reactor. They formulate a stochastic linear program and solve it to find the expected value of perfect information (EVPI). Their results indicate that the expected value of perfect information in this model is low and that, therefore, deterministic problem solutions provide close approximations to the solution of the stochastic problem.

Chao [2] presents an analytical justification for the observations of Manne and Richels. He formulates a mathematical program for the Hotelling-Nordhaus model. Under certain assumptions that include a demand that is independent of price, Chao shows that a myopic policy of using the most inexpensive available technology first is optimal. He also introduces a price responsive demand function to his model and again shows that the EVPI is low.

In this paper, we expand upon Chao's model by allowing exploration investment that could yield additional resource supplies. The amount of increase in the supply per unit of investment is however uncertain. We show that the EVPI and the value of the stochastic solution (VSS) (Birge [1]) can be large when this type of uncertainty is included. We give examples illustrating these observations.

2. The Basic Model

Our results concern two measures of the effect of uncertainty in stochastic programs, the expected value of perfect information and the value of the stochastic solution. We present these measures in the context of two-stage stochastic programs with recourse. We first formulate the deterministic program

$$\text{Minimize} \quad \phi(x,\xi) = cx + \text{Min}[qy \mid Wy = \xi + Tx,\ y \geq 0] \tag{1}$$
$$\text{subject to} \quad Ax = b,\ x \geq 0$$

where the vectors $c \in R^n$, $q \in R^n$, and $b \in R^m$ are known, the m_2-vector ξ is a random vector defined on the probability space (Ξ, F, F), and A, W, and T are correspondingly dimensioned known real-valued matrices. A decision vector $\bar{x}(\hat{\xi})$ obtained in Program 1 represents an optimal first period decision given a realization $\hat{\xi}$ of the random vector.

If an optimal first period decision is taken for all possible realizations of the random vector, then we obtain in expected value the "wait-and-see" (WS) solution value (Madansky [4]), where

$$WS = E_\xi\,[\underset{x}{\text{Min}}\ \phi\,(x,\xi)].$$

The stochastic program with recourse (Wets [8]) involves optimizing after taking the expected value. We write the value of this program as

$$RP = \underset{x}{\text{Min}}\ \ E_\xi\,[\phi(x,\xi)].$$

For $E(\xi) = \bar{\xi}$, we obtain a third value that is the expectation of the expected value (EEV) solution $\bar{x}(\bar{\xi})$ that is optimal in (1) for $\xi = \bar{\xi}$. This quantity is

$$EEV = E_\xi\,[\phi(\bar{x}(\bar{\xi}),\xi)].$$

The effects of uncertainty are measured by differences among WS, RP, and EEV. The expected value of perfect information represents the amount one is willing to spend in gaining information about the stochastic variables. It is calculated as

$$EVPI = WS - RP.$$

The value of the stochastic solution, on the other hand, measures the additional value of solving the stochastic program over solving the deterministic expected value problem. We define

$$VSS = EEV - RP.$$

In the discussion below, we describe VSS and EVPI in the context of an exhaustible resource model originally due to Chao.

Chao's basic model is a linear program to determine an optimal dynamic production schedule to minimize the present value of the cost of satisfying an increasing sequence of demand requirements over time. The demand may be satisfied by any of m-1 substitutable technologies, each using one distinct finite resource, and by one backstop technology with no resource limit. The resulting linear program is

$$\min \sum_{i=1}^{m} \sum_{t=1}^{\infty} \beta^t c_i y_{it} + \sum_{i=1}^{m} \sum_{t=0}^{T} \beta^t k_i x_{it} \tag{2}$$

$$\text{s.t.} \quad \sum_{t=0}^{\infty} y_{it} \leq R_i, \; i=1, \ldots, m;$$

$$\sum_{t=1}^{m} y_{it} = D_t, \; t=1, \ldots, T;$$

$$y_{i,t+1} = y_{it} + \sum_{s=0}^{\infty} (\delta_s - \delta_{s-1}) x_{i,t-s}, \; t=0,1,\ldots,$$

$$y_{it} \geq 0, \; x_{it} \geq 0; \; t=0,1,\ldots; \; i=1,\ldots,m;$$

where y_{it} is the amount of period t demand, D_t, satisfied by resource i at time t, x_{it} is the amount of resource i committed at t to be extracted later, c_i is the current cost of technology i, k_i is the capital cost of i, β is the discount factor, δ_t is the extraction rate, and R_i is the initial availability of the resource used by technology i. It is assumed that y_{io} and x_{it} are known for i=1, ..., n and for t=0, -1,..., and that $y_{io} = \sum_{t=0}^{\infty} \delta_{-t} x_{it}$. It is also assumed that $D_1 \leq D_2 \leq \ldots \leq D_{T-1} \leq D_T$.

Chao defines γ as the capital recovery factor for the standard time profile where $\gamma = 1/(\sum_{s=0}^{\infty} \beta^s \delta_s)$ and lets d_t be the demand for new resource commitments where $D_t = \sum_{s=0}^{\infty} \delta_s d_{t-s}$. The result is that (1) can be rewritten as

$$\min \sum_{i=1}^{m} \sum_{t=0}^{T} (k_i + c_i/\gamma) \beta^t x_{it} \tag{3}$$

$$\text{s.t.} \quad \sum_{t=0}^{T} x_{it} \le R_i - \sum_{t=-1}^{-\infty} (\sum_{s=-t}^{-\infty} \delta_s) x_{it}, \; i=1,\ldots,m;$$

$$\sum_{i=1}^{m} x_{it} = d_t, \; t=0,\ldots,T;$$

$$x_{it} \ge 0, \; i=1,\ldots,m; \text{ and all } t.$$

Chao uses Program 3 to derive his results on myopic solutions. He shows that the corresponding transportation problem can be solved optimally by the Northwest Corner Rule if the resource costs $k_i + c_i/\gamma$ are arranged in increasing cost order within each period.

The result leads to an expected value of perfect information of 0 because the WS solution is the same as the RP solution. It also yields a VSS of 0 because the EEV value is the same as RP when myopic solutions are optimal.

Chao introduces price-responsive demands to the basic model in (3) and obtains a nonlinear programming model that does not have myopic optimal decisions. He computes an upper bound on the EVPI and shows that distant future uncertainties and low price elasticities lead to a small EVPI. In the next section, we introduce investment uncertainty into the basic model and show that this may lead to a significant EVPI and VSS.

3. A Model with Uncertain Exploration Returns

We assume that R_i in Program 3 represents the amount of resource i that is known to be available at time 0. This amount can be increased by exploration investment, but the amount of the increase is uncertain. We also assume that there is a capacity limit L_i on the the amount of a resource which may be committed at time 0. This amount may also be increased by investment in new capacity and that return is assumed known with certainty. The stochastic linear program derived from (3) is then

$$\min \sum_{i=1}^{m} (k_1+c_i/\gamma)x_{io} + \sum_{i=1}^{m} d_i u_{io} + \sum_{i=1}^{m} g_i v_{io} + \sum_{t=1}^{T} \sum_{i=1}^{m} \sum_{j=1}^{K_t} p_t^j \beta^t \{(k_i+c_i/\gamma)x_{it}^j + d_i u_{it}^j + g_i v_{it}^j\} \quad (4.0)$$

subject to

$$x_{it}^j \leq R_i + \sum_{s=0}^{t-1} \alpha_{is}^{a(j)} u_{is}^{a(j)} - \sum_{s=0}^{t-1} x_{is}^{a(j)} \quad (4.1)$$

$$i=1,\ldots,m;\ t=0,\ldots,T;\ j=1,\ldots,K_t;$$

$$x_{it}^j \leq L_i + \sum_{s=0}^{t-1} v_{is}^{a(j)}, \quad (4.2)$$

$$i=1,\ldots,m;\ t=0,\ldots,T;\ j=1,\ldots,K_t;$$

$$\sum_{i=1}^{m} x_{it}^j = d_t;\ t=0,\ldots,T;\ j=1,\ldots,K_t; \quad (4.3)$$

$$x_{it}^j \geq 0,\ i=1,\ldots,m;\ t=0,\ldots,T;\ j=1,\ldots,K_t; \quad (4.4)$$

where d_i is the cost of one unit of exploration for resource i, u_{it}^j is the amount of exploration, g_i is the cost of capital investment in resource i, v_{it}^j is the amount of that investment, p_t^j is the probability of scenario j at time t, K_t is the number of scenarios at time t, and α_{it}^j is the return per unit of exploration for resource i under scenario j. Each scenario j is preceeded by ancestor scenarios in previous periods which are designated by a(j).

The stochastic nature of Program 4 is contained only in the return on exploration investment, α_{it}^j. In general, these values may vary continuously, but the discrete formulation in (4) is used for simplicity. This program involves a stochastic technology matrix, but it may be formulated with stochastic right-hand sides by defining new variables w_{it}^ℓ, $\ell \geq 0$, such that

$$u^{a(j)}_{i,t-1} = \sum_{\ell=1}^{L^t_i} w^{\ell}_{it}, \qquad (5)$$

and

$$x^{j}_{it} \leq R^{a(j)}_{i,t-1} + \sum_{\ell=1}^{L^t_i} \alpha^{\ell}_{it} \; w^{\ell}_{it} - x^{a(j)}_{i,t-1},$$

where $R^{a(j)}_{i,t-1}$ is the availability of resource i in period t-1, there are L^t_i different values of $\alpha_{i,t-1}$, and $w^{\ell}_{it} \leq 0$ for all ℓ except for $\ell = \ell^j$ such that $\alpha^{\ell j}_{it} = \alpha^{a(j)}_{i,t+1}$. The upper bound on $w^{\ell j}_{it}$ is sufficiently large to allow any investment level. The stochastic right-hand side problem is then formed by substituting (5), (6), and a constraint where R^{j}_{it} is set equal to the right-hand side of (6), for Constraint 4.1 in Program 4.

In the deterministic version of (4), the investment decisions may skip from investment in one resource to another according to the values of α^{j}_{it}. This is due to the basic property of the linear program in which extreme point values correspond to investments in single resources. The solution of (4) allows for many more combinations of alternatives investment decisions and, hence, provides for hedging against other possibilities. This hedging characteristic yields a positive VSS for many cases and the value of knowing the investment return yields a positive EVPI. An example of these occurrences appear in the next section.

4. Example

We consider a two period problem to demonstrate the potential effect of investment uncertainty. In this example, we consider three technologies. The first technology uses a resource in which investment return is highly variable. The second technology corresponds to a resource in which investment in additional capacity results in certain returns. The third technology is an infinitely available backstop. The data for the model is contained in Table 1.

Resources	Current Cost	Initial Availability
Res 1	5.0	25.0
Res 2	10.0	10.0
Backstop	16.7	$+\infty$
Investment	**Cost**	**Return**
Res 1 - Good Luck	1.0	1.0
Bad Luck	1.0	0.1
Res 2	1.0	1.0
Periods	**Demand**	
First	15.0	
Second	25.0	
Scenarios	**Probability**	
Good Luck	0.5	
Bad Luck	0.5	
Discount Factor	$\beta = 0.6$	

Table 1. Model Input Data

The only uncertainty in this model is in the return for Resource 1 exploration investment. Resource 2 investment can be interpreted as building additional capacity. This model can be formulated as a stochastic linear program with recourse and with uncertainty in the right-hand side by using constraints as in (5) and (6). In this case, we obtain the following two-stage stochastic linear program in which x represents first period decisions and y represents second period decisions.

$$
\begin{aligned}
\min z = {} & 5x_1 + 10x_2 + 16.7x_3 + x_4 + x_5 + E_{\xi}[3y_5 + 6y_6 + 10y_7] && (7)\\
\text{s.t.} \quad & x_1 \le 25\\
& x_2 \le 10\\
& x_1 + x_2 + x_3 \ge 15\\
& -x_1 + y_1 + .1y_3 + y_4 = 0\\
& x_4 - y_3 - y_4 = 0\\
& -x_2 + x_5 + y_2 = 0\\
& y_4 \le \xi\\
& y_1 + y_5 \le 25\\
& y_2 + y_6 \le 10\\
& y_5 + y_6 + y_7 \ge 25,\\
& x_1, \ldots, x_5 \ge 0, \quad y_1, \ldots, y_7 \ge 0,
\end{aligned}
$$

where $P\{\xi = 0\} = 0.5$ and $P\{\xi = 10\} = 0.5$. In this program, x_1, x_2, and x_3 represent commitments of the resources, x_4 and x_5 are investment variables, y_1 and y_2 represent the net changes in resource availabilities, y_3 and y_4 represent the amount of new Resource 1 availability obtained through investment, and y_5, y_6, and y_7 represent commitments in the second period.

The alternatives to Program 7 are to solve deterministic models that assume good luck, bad luck, a mean value with $\xi = \bar{\xi} = 5$, or a single myopic solution. For each of these solutions, we obtain the expectation of the two period costs after using the first period solution obtained by these deterministic problems (as in finding the EEV). These values are

Scenario	Deterministic Value	Expectation Value
Good Luck	175.0	196.5
Bad Luck	200.0	200.0
Mean	185.0	200.75
Myopic	215.0	215.0

These values can be compared to the value of the stochastic program (7), which is 192.5.

We can then obtain the information values, EVPI and VSS. The expected value of perfect information is

$$EVPI = RP - WS = 192.5 - 187.5 = 5.0.$$

The value of the stochastic solution is

$$VSS = EEV - RP = 200.75 - 192.5 = 8.5.$$

The value of the stochastic solution relative to the myopic, or no investment, solution is also of interest. It is 215.0 - 192.5 = 22.5.

The difference between the EVPI and VSS values demonstrates how these quantities reflect different values of uncertainty. The EVPI is lower than the VSS because the RP solution can fairly adequately hedge against either of the future outcomes. In the RP solution, there is investment in both Resource 1 and Resource 2 capacity ($x_4 = 10$ and $x_5 = 4$) so that no backstop usage is necessary in either scenario. The mean value solution, however, only involves investment in Resource 1 so that the backstop must be used in the bad luck scenario. This leads to a higher VSS than EVPI and shows the merit of using the stochastic program solution.

Investment in two resource is unique to the stochastic program solution. Any deterministic scenario only involves investment in one resource. This again shows the utility of the stochastic program. It is able to blend the deterministic solutions so that the decision maker does not have to decide among two completely different solutions.

We also note that the addition of investment has a significant effect on the value relative to the myopic solution. If no investment is allowed then the myopic solution would be optimal, and the backstop would necessarily be used to satisfy five units of demand in the second period. An exhaustive resource model with investment therefore clearly must consider future scenarios, and the solution of an equivalent stochastic program can have significant advantages over the solution of a deterministic expected value problem.

References

[1] J.R. Birge, "The value of the stochastic solution in stochastic linear programs with fixed recourse," Mathematical Programming 24(1982) 314-325.

[2] H.P. Chao, "Exhaustible resource models: the value of information," Operations Research 29(1981) 903-923.

[3] H. Hotelling, "The economics of exhaustible resources," J. Political Econ. 39(1931) 173-175.

[4] A. Madansky, "Inequalities for stochastic linear programming problems," Management Science 6(1960) 197-204.

[5] A.S. Manne, "Waiting for the breeder," in: Review of Economic Studies Symposium, (1974), pp. 47-65.

[6] A.S. Manne and R.G. Richels, "A decision analysis of the U.S. breeder reactor program," Energy 3(1978) 747-767.

[7] W.D. Nordhaus, "The allocation of energy resources," Brookings Papers on Economic Activity 3(1973) 529-576.

[8] R.J-B Wets, "Stochastic programs with fixed recourse: the equivalent deterministic program," SIAM Review 16(1974) 309-339.