OPTION METHODS FOR INCORPORATING RISK INTO LINEAR PLANNING MODELS

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Technical Report 95-8

May 1995

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5/23/95

Abstract: Capacity and resource limits directly affect project risk. While risk consideration is well developed in finance through efficient market theory and the capital asset pricing model, management science has been slow to adapt these principles into planning models. One reason for this apparent inconsistency may be that analysis of a management science model does not reveal the level of risk until the model is solved. Using results from option pricing theory, we show that this inconsistency can be avoided in a wide range of planning models. By assuming the availability of market hedges, we show that risk can be incorporated into planning models by simply adjusting capacity and resource levels. The result resolves some possible inconsistencies between finance and management science and provides sound financial basis for many planning problems.

[1]This work is supported by the National Science Foundation under Award Number DDM-9215921.

1 Introduction

Large management science models often involve a variety of limited resources and uncertain parameters that may all affect decisions based on these models. The combination of limited resources and uncertainty makes attitudes toward risk an important consideration. Utility functions can capture these attitudes (see, for example, Keeney and Raiffa [1976]), but the functions may be difficult to evaluate in the most general cases.

For profit-making corporations, investor considerations can often be used for general attitudes toward risk. In this case, the capital asset pricing model (see, for example, Sharpe [1964]) provides a foundation for a utility tradeoff between risk and return that can be observed in the market. With this relationship defined, adjustments for risk can be incorporated into discount rates on future returns. The result is a project evaluation based on discounted expected cash flows.

Problems may occur if cash flows are skewed in a way that might make investors prefer risks of one nature ("upside" over "downside") over another. This problem is often avoided by assuming sufficient diversification that total returns appear symmetric. The question for an individual project is only the fraction of risk that it might contribute to the overall portfolio of investments.

Measuring this risk contribution can often be done using β values for common stocks and other market instruments. The difficulty in these assessments with capacitated problems is that the risk contribution varies with the capacity level and often cannot be determined at the outset. A major contribution of option theory has been to avoid this difficulty through the risk-neutral valuation method (see, for example, Cox and Ross [1976]), in which, option values can be found by treating all investors as if they were risk neutral.

This foundation of option pricing theory has broad implications for decisions on projects with limited capacity (see, for example, Andreou [1990] and Myers [1984]). In this paper, we apply the basic principle of risk-neutral valuation to general forms of constrained resource problems, such as capacity planning. The application of these results for linear problems leads to a modification of the constraints while other characteristics of the problem remain the same. In this way, linear models can still incorporate risk without changing the linear functional form in the objective.

In the next section, we give the basic option approach. Section 2 presents the application of option pricing methods for simple capacity evaluation. In Section 3, we extend this approach to constrained linear models. Section 4 applies this result to a capacity planning model for a manufacturing firm. Section 5 presents a summary and conclusions.

2 Basic Model for Call Option Valuation

The basic observation begins with option pricing theory. For simplicity, we consider a European call option on a non-dividend paying common stock. The option is to buy one share of the stock at a fixed *strike price*, K, at a given time, T. We assume the current time is t when the share price is S_t. The question is how to value the call option given that we know the stock's volatility (or annual standard deviation on return, assumed constant), σ, and a constant riskfree rate, r_f. The other assumptions are a frictionless market and that the share prices follow an Ito process (see Black and Scholes [1973]).

With these assumptions, it is always possible to hold some amount of shares with written calls to form a perfect hedge, a riskless portfolio that earns the riskfree rate. Given this observation, the calculation of the value of the call option does not depend on the investors' attitudes toward risk. We then obtain the risk-neutral valuation method, which states that the call option can be valued as if investors were neutral toward risk. The price in this case is the same as if investors require a premium for risk.

A result of the risk-neutral approach is the Black-Scholes formula (see Black and Scholes [1973]) and various extensions. In the simple European call option we have described, this formula amounts to assuming that stock price returns are lognormally distributed with logarithmic annual mean return, $\mu = E[log(S_1/S_0)]$, and annual standard deviation on the logarithm of returns, $\sigma = \sqrt{Var[log(S_1/S_0)]}$. This implies $E[S_1] = S_0 e^{\mu+\frac{\sigma^2}{2}}$, where we assume $r_f = \mu + \frac{\sigma^2}{2}$ under the risk-neutral condition. Denoting the resulting distribution function on prices, S_T, at time T by F_f, the call evaluation at time t is to find:

$$C_t = e^{-r_f(T-t)} \int_K^\infty (S_T - K) dF_f(S_T). \tag{1}$$

Note that the distribution function in (1) assumes investors are indifferent toward risk. In reality, expected annual returns should show a premium for risk, but we can still use (1) to evaluate the call option because of the risk neutral equivalence.

To illustrate how these ideas are used, we consider a binomial approximation of the lognormal distribution on stock prices. This approximation is well known to approach the continuous time model as the number of intervals used in the approximation increases (see, e.g., Cox, Ross and Rubinstein [1979] and Jarrow and Rudd [1983]). The idea is that prices follow a random walk, increasing from $S(= S_t)$ to Se^{u_f} with probability p_f or decreasing to Se^{d_f} with probability $1 - p_f$. We can choose these parameters in a variety of ways. Following Jarrow and Rudd [1983], we suppose that the interval to expiration, $\tau = T - t$, is divided into I equal subintervals. Then, by choosing

$$u_f = (r_f - \sigma^2/2)\tau/I + \sigma\sqrt{\tau/I}, d_f = (r_f - \sigma^2/2)\tau/I - \sigma\sqrt{\tau/I}, p_f = \frac{1}{2}, \tag{2}$$

we achieve a distribution with the same first two moments as the lognormal for every I and convergence in distribution of this binomial model to the lognormal as $I \to \infty$.

We can also use this model to show how the risk-neutral assumption arises. Suppose just a single interval ($I = 1$) and a call exercise price between Se^{u_f} and Se^{d_f} ($Se^{d_f} \leq K \leq Se^{u_f}$). We next show how to construct a riskfree hedge with call writing plus some amount of the share.

The written call requires payment of 0 if Se^{d_f} occurs and $Se^{u_f} - K$ is Se^{u_f} occurs. The return on δ shares is $\delta S(e^{d_f} - 1)$ if Se^{d_f} occurs and $\delta S(e^{u_f} - 1)$ if Se^{u_f} occurs. The future payoff is then:

$$FV = \begin{cases} -Se^{u_f} + K + \delta S(e^{u_f} - 1) & \text{if } Se^{u_f} \text{ occurs,} \\ \delta S(e^{d_f} - 1) & \text{if } Se^{d_f} \text{ occurs.} \end{cases} \tag{3}$$

A hedge represents an equating of these payoffs. For $\delta = \frac{Se^{u_f}-K}{Se^{u_f}-Se^{d_f}}$, the FV in (3) is the same in each outcome. Discounting by the riskfree rate, we achieve an equivalent present value of

$$PV = e^{-r_f\tau}(\frac{(Se^{u_f} - K)(Se^{d_f} - S)}{Se^{u_f} - Se^{d_f}}). \tag{4}$$

Note that we were able to construct a hedge in (3) by choosing δ independently from the probability of rise or fall in the price, S. We can continue to find these hedges (recursively) with increasing I. This result allows us then to choose a probability consistent with any risk attitude, justifying the risk neutral assumption. In a single period, we then have the call value given by $C_t = e^{-r_f\tau}p_f(Se^{u_f} - K)$.

3 Simple Capacity Valuation

We now apply the option valuation model to a simple capacity problem. Suppose that the present value of demand (sales) forecasts for time T also follows an Ito process as in the stock example above. In this case, we assume that all demand can be met and that revenues are linear in demand with some margin, c_T. Assuming that investors require a return, r, based on the contribution to market risk (measured by firm β for example) from these revenues, the present value at t for revenues at time T should be c_T multiplied by:

$$\bar{S}_T^t \equiv (e^{-r(T-t)}) \int_{S_T} S_T dF(S_T), \tag{5}$$

where F is now the distribution function on demand S_T at T.

Actual revenues are, however, limited by capacity. If we assume that capacity limits actual sales to at most K units, then the actual present value of revenues is $c_T(\bar{S}_T^t - C_t)$ where C_t is the value of sales in excess of the capacity K. We can use the formula in (1) for C_t but must first convert from the original distribution in (5) to the risk-neutral distribution in (1). We do this by transforming S_T (with distribution function F) to S_{Tf} (with distribution function F_f) where $E[S_{Tf}]$ discounted with rate r_f is equivalent to $E[S_T]$ discounted with rate r. Defining this equivalent risk-neutral distribution is not, however, necessary as shown in the following theorem.

Theorem 1 *If F_f is the equivalent risk-neutral distribution function on S_T and r is the rate of return expected on all sales in the case with risk premium, then C_t in (1) is equivalent to*

$$C_t = e^{-r\tau} \int_{Ke^{(r-r_f)\tau}}^{\infty} (S_T - Ke^{(r-r_f)\tau}))dF(S_T). \tag{6}$$

Proof. Since we often will deal with discrete distributions, we use the binomial approximation to establish this result. From the binomial derivation (Jarrow and Rudd [1983]), we have that

$$C_t = \lim_{I\to\infty} e^{-r_f\tau}\sum_{l=0}^{I} q_I(l)(S_t(e^{u_f})^l(e^{d_f})^{I-l} - K)^+, \tag{7}$$

where $q_I(l)$ is the binomial probability that $S_T = S_t(e^{u_f})^l(e^{d_f})^{I-l}$.

Now, if we suppose an environment with an annual expected rate risk premium of $\alpha = r - r_f$, then the binomial model in the risk averse case would replace u_f and d_f in (2) with

$$u = (r - \sigma^2/2)\tau/I + \sigma\sqrt{\tau/I}, d = (r - \sigma^2/2)\tau/I - \sigma\sqrt{\tau/I}, p = \frac{1}{2}. \tag{8}$$

Thus, we would have a binomial distribution with $Se^u = Se^{((r-r_f)\tau/I)u}$ in place of Se^{u_f} and $Se^d = Se^{((r-r_f)\tau/I)d}$ in place of Se^{d_f}. The result is then that $q_I(l)$ is the binomial probability in the risk averse case for $S_T = S_t(e^u)^l(e^d)^{I-l} = e^{\alpha\tau}S_t(e^{u_f})^l(e^{d_f})^{I-l}$. Substituting in (7), we obtain:

$$\begin{aligned} C_t &= \lim_{I\to\infty} e^{-r_f\tau}e^{-\alpha\tau}\textstyle\sum_{l=0}^{I} q_I(l)(S_te^{\alpha\tau}(e^{u_f})^l(e^{d_f})^{I-l} - Ke^{\alpha\tau})^+, \\ &= \lim_{I\to\infty} e^{-r\tau}\textstyle\sum_{l=0}^{I} q_I(l)(S_t(e^u)^l(e^d)^{I-l} - Ke^{\alpha\tau})^+, \\ &= e^{-r\tau}\textstyle\int_{Ke^{(r-r_f)\tau}}^{\infty}(S_T - Ke^{(r-r_f)\tau})dF(S_T), \end{aligned} \tag{9}$$

where we use the binomial approximation result again in the last step to obtain the desired result. □

From the result in Theorem 1, we have that the present value of a finite capacity plant is given by:

$$PV = e^{-r\tau}c_T(\int_0^{Ke^{(r-r_f)\tau}}(S_T)dF(S_T) + Ke^{(r-r_f)\tau}(1 - F(Ke^{(r-r_f)\tau})), \tag{10}$$

which involves the discount rate r that applies to the uncapacitated case, the original distribution function on S_T, and an adjusted capacity level from K to $Ke^{(r-r_f)\tau}$. Note that when capacity is tight so that $F(Ke^{(r-r_f)\tau}) = 0$ is low, then the first term in (10) disappears yielding:

$$PV_{tight} = e^{-r_f\tau}c_TK, \tag{11}$$

which is indeed the present value for a fixed, riskless future value K at $\tau = T - t$ periods into the future. If the capacity is loose $(F(Ke^{(r-r_f)\tau}) = 1)$, then we obtain

$$PV_{loose} = e^{-r\tau}c_T(\int_0^{\infty} S_TdF(S_T)), \tag{12}$$

which is again the uncapacitated present value.

The advantage of (10) over manipulation of the distribution function into (1) is that the only necessary additions to reflect risk attitude are the uncapacitated discount factor, $e^{-r\tau}$, and an adjustment of the overall capacity. In optimization, this is especially useful because the actual distribution on S_T is often determined within the problem. The transformation in Theorem 1 allows the modeler to proceed without determining a risk-neutral equivalent at the start. The next section describes our generalization.

4 Generalized Problem and Multistage Stochastic Linear Program

To generalize the result in the previous section, suppose instead of a single dimensional S_T that S_T is a random vector with associated probability space, $(\Sigma_T, \mathcal{B}_T, P_T)$, where the support, $\Sigma_T \subset \Re^n$, the corresponding Borel field is $\mathcal{B}_T$, and the probability measure is P_T. Suppose a linear revenue vector c_T so that the uncapacitated present value of the period T revenue is:

$$PV = e^{-r\tau}(\int_{\Sigma_T} c_T'(S_T)P_T(dS_T)). \tag{13}$$

The result in (13) may also be expressed in an optimization problem. The main requirement is that the returns have a symmetric form that enables determination and use of the appropriate discount factor with rate r.

Now, suppose that actually revenues, x_T, are restricted so that x_T must satisfy both $x_T \leq S_T$, and $Ax_T \leq h_T$. The expected future revenues become:

$$FV = c_T'(\int_{\Sigma_T}(S_T)P_T(dS_T) - \int_{\Sigma_T} \max_{Ax_T \leq h_T}(S_T - x_T)^+ P_T(dS_T)), \tag{14}$$

which has the same form as the simple capacity evaluation with the second term being analogous to (1).

To evaluate the present value of the contingent claim term, $\int_{\Sigma_T} \max_{Ax_T \leq h_T}(S_T - x_T)^+ P_T(dS_T)$, in (14), we suppose again a frictionless market with no riskless arbitrage, short selling, continuous trading, and that the expectation process on each component of S_T follows an Ito process. We also need to assume n instruments other than the contingent claim term that also depend on the components of S_T. These instruments may for example represent other products that can depend on the same demand factors as S_T and that would be available in the market in some way. With a wide range of products, this assumption becomes quite realistic. With these assumptions, we can again (see, for example, Hull [1993]) assume that a riskless hedge is possible with these securities and that, therefore, we can again employ the risk neutral valuation method.

We continue to make the assumptions consistent with risk neutral valuation. We can then find a present value for FV in (14) by using a probability measure, P_{Tf}, that includes no risk premium on the rate of increase in the present value of the expectation of S_T. The present value of the contingent claim portion of (14) becomes:

$$C_t = e^{-r_f\tau} c_T' \int_{\Sigma_T} \max_{Ax_T \le h_T} (S_T - x_T)^+ P_{Tf}(dS_T). \tag{15}$$

By applying an annual risk premium, e^{r-r_f} to each component of the present value of the expectation of S_T as in Theorem 1, we obtain a probability measure, P_T, such that $P_T(A) = P_{Tf}(e^{(r_f-r)\tau}A)$ for any $A \subset \mathcal{B}$ and thus obtain:

$$\begin{aligned} C_t &= e^{-r_f\tau} e^{-\alpha\tau} c_T' \textstyle\int_{\Sigma_T} \max_{Ax_T \le h_T} e^{\alpha\tau}(S_T - x_T)^+ P_{Tf}(dS_T) \\ &= e^{-r\tau} c_T' \textstyle\int_{\Sigma_T} \max_{Ax_T \le h_T} (S_T - e^{\alpha\tau} x_T)^+ P_T(dS_T) \\ &= e^{-r\tau} c_T' \textstyle\int_{\Sigma_T} \max_{Ax_T' \le e^{\alpha\tau} h_T} (S_T - x_T')^+ P_T(dS_T), \end{aligned}$$

where in the last step we have substituted x_T' for $e^{\alpha\tau}x_T$.

The result is the following corollary to Theorem 1.

Corollary 1 *Assuming the conditions for a riskless hedge given above, the present value of the result in (14) is equivalent to*

$$e^{-r\tau} c_T' \left(\int_{\Sigma_T} (S_T) P_T(dS_T) - \int_{\Sigma_T} \max_{Ax_T \le e^{\alpha\tau} h_T} (S_T - x_T)^+ P_T(dS_T) \right). \tag{16}$$

From Corollary 1, we can derive present value equivalents for a broad range of stochastic optimization problems with linear constraints. We consider the general form of a multistage stochastic program with fixed linear recourse to maximize expected utility. We express this model as

$$\min_{x_0, \boldsymbol{x}^1, \ldots, \boldsymbol{x}^H} \quad cx_0 + E_{\boldsymbol{\xi}}[U^1(\boldsymbol{c}^1\boldsymbol{x}^1) + \ldots U^H(\boldsymbol{c}^H\boldsymbol{x}^H)] \tag{17}$$

$$\begin{array}{lll} \text{s.t.} & Ax^0 & = b, \\ & \boldsymbol{T}^1 x^0 + \boldsymbol{W}^1 \boldsymbol{x}^1 & = \boldsymbol{h}^1, \quad a.s., \\ & \vdots & \vdots \\ & \boldsymbol{T}^H \boldsymbol{x}^{H-1} + \boldsymbol{W}^H \boldsymbol{x}^H & = \boldsymbol{h}^H, \quad a.s., \\ & 0 \le x^0, 0 \le \boldsymbol{x}^t, & t = 1, \ldots, H, \quad a.s., \\ & \boldsymbol{x}^1, \ldots, \boldsymbol{x}^H \text{nonancitipative}, & \end{array}$$

where bold face indicates random quantities, the decisions, $x^t \in \Re^{n_t}$, and the right-hand side parameters, $b \in \Re^{m_0}$ and $h^t \in \Re^{m_t}$, with other matrices defined accordingly. The functions, U^t, represent the present utility of objective outcomes in time t. The term *nonanticipative* indicates that $\boldsymbol{x}^t$ can only depend on outcomes up to time t (see Wets [1980] or Dempster [1988] for details). We assume the stochastic elements are defined over a canonical probability space $(\Xi, \sigma(\Xi), P)$, where $\Xi = \Xi_1 \otimes \cdots \otimes \Xi_H$, and the elements of Ξ_t are $\{\xi^t = (T^t, W^t, h^t, c^t)\}$.

If a set of decisions are defined for each element or *scenario* in Ξ, the expected value can be written as a finite sum of the objective values of these decisions. The requirement that each stochastic constraint must hold almost surely may be enforced by defining the same set of constraints for each realization. We can use the result in Corollary 1 now to construct an equivalent deterministic linear program to (17). First, we assume a discrete distribution with N_t possible outcomes for ξ_t in stage t yielding $\bar{N}_t = N_1 \times \cdots \times N_t$ total scenarios at time t. We also assume a financial objective so that, if we knew the contribution of the current project to overall portfolio risk, we could find a discount factor to represent the utility and reduce (17) to a linear program. Using Corollary 1. we do not need to determine this contribution completely. We assume only that we know an appropriate discount factor for the revenues without explicit capacity constraints. With these assumptions, we find an equivalent program to (17) as:

$$\max \quad cx^0 + \sum_{k=1}^{\bar{N}_1} p_k^1 e^{-r} c_k^1 x_k^1 + \cdots + \sum_{k=1}^{\bar{N}_H} p_k^H e^{-rH} c_k^H x_k^H$$

$$\begin{array}{lll}
\text{subject to} & Ax^0 & = b, \\
& T_j^t x_{\gamma(j,t)}^{t-1} + W^t x_j^t & = h_j^t, \quad j = 1, \ldots, \bar{N}_t, \quad t = 1, \ldots, H, \\
& T_j'^t x_{\gamma(j,t)}^{t-1} + W'^t x_j^t & \leq e^{\alpha t} h_j'^t, \quad j = 1, \ldots, \bar{N}_t, \quad t = 1, \ldots, H, \\
& 0 \leq x_j^t, & j = 1, \ldots, \bar{N}_t, \quad t = 1, \ldots, H,
\end{array} \tag{18}$$

where

$$\begin{array}{rcl}
x_i^t & = & \text{Decision vector to take in stage } t \text{ given outcome } i; \\
p_i^t & = & \text{Probability that scenario } i \text{ in stage } t \text{ occurs, } i = 1, \ldots, \bar{N}_t; \\
(c_i^t, h_i^t, T_i^t) & = & \text{Cost and RHS vectors and Technology matrix for scenario } i \text{ in stage } t; \\
W^t & = & \text{Recourse matrix for stage } t, \ W^t \in \Re^{m_t \times n_t}; \\
\gamma(j,t) & = & \lfloor (j-1)/N_t \rfloor + 1, \text{ the } \textit{ancestor} \text{ (scenario sharing history up to time } t-1) \\
& & \text{of node } j \text{ in stage } t-1.
\end{array}$$

In (18), we have split the period t constraints into two components, T, W, h and T', W', h'. The first set without prime superscripts are constraints that depend on overall market conditions (such as maximum possible demand) and not on specific capacity decisions. The second set with coefficients, T', W', and h', distinguishes specific resource restrictions that prevent realizations of full market potential. The application of this to capacity planning is given in the next section.

5 A Capacity Planning Example

We consider a manufacturer with certain plants with installed capacity to produce specific products. The capacity planning question is to determine whether additional capacity should be installed at a plant where no capacity for a product

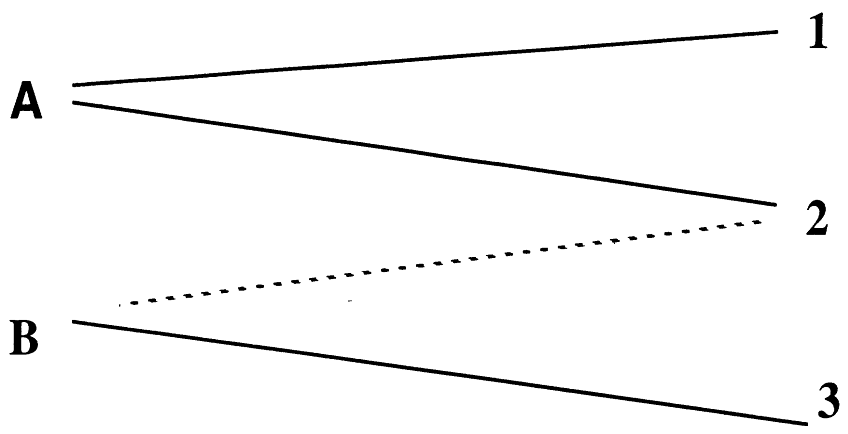

Figure 1: Adding flexible capacity at Plant 2.

currently exists. This additional capacity would allow the plant to continue production if demand for the new product is higher than existing capacity at other plants and if the demand for other products at the new plant is lower than the existing plant capacity.

As an example, consider Figure 1. Here, there are two products, A and B, and three plants, 1, 2, and 3. The solid lines in the diagram indicate that each plant currently only produces a single product. We could assume that each of the plants is built to meet the mean demand exactly. In that case, if demand for a product ever exceeds the mean, then potential sales are lost.

By building additional capacity at 2 for product B (the dotted line in Figure 1), if demand for product A is lower than the mean and demand for product B is higher than its mean, then the excess product B demand can be produced at 2 and fewer sales would be lost. That is the motivation for our previous development and the basic goal of flexible capacity. The decision problem is to trade off the costs of adding additional capacity against the potential revenue from additional sales due to the extra capacity. This basic problem has been considered by Eppen et al. [1989] where they use a one-sided loss utility function and apply a mixed integer, stochastic linear programming model. With our results above, we can avoid this somewhat ad hoc procedure and use an objective consistent with the overall performance of the firm.

For two-stages, this model becomes:

$$\max \quad \sum_{i=1}^{L}\sum_{j=1}^{R}[c_{ij}x_{ij}^0 + e^{-r}\sum_{k=1}^{\bar{N}_1} p_k^1 c_{k,ij}^1 x_{k,ij}^1]$$

subject to $\sum_{i=1}^{L}\sum_{j=1}^{R} c_{ij}x_{ij}^0 \quad \leq b,$ (19)

$$\begin{array}{ll} \sum_{i=1}^{L} x_{k,ij}^{1} & \leq h_{jk}^{1}, \quad j=1,\ldots,R; k=1,\ldots,\bar{N}_1, \\ T_{ij}^{\prime 1} x_{ij}^{0} + x_{k,ij}^{1} & \leq e^{\alpha} h_{1,ij}^{\prime 1}, \quad i=1,\ldots,L; j=1,\ldots,\ldots,R; k=1,\ldots,\bar{N}_1, \\ \sum_{j=1}^{R} x_{k,ij}^{1} & \leq e^{\alpha} h_{2,i}^{\prime 1}, i=1,\ldots,L; k=1,\ldots,\bar{N}_1, \\ x_{ij}^{0} \in X_{ij}^{0}, & x_{k,ij}^{1} \geq 0, i=1,\ldots,L; j=1,\ldots,\ldots,R; k=1,\ldots,\bar{N}_1, \end{array}$$

where

$$\begin{array}{rcl} L & = & \text{Number of plants}, \\ R & = & \text{Number of products}, \\ x_{ij}^{0} & = & \text{New capacity installed for product } j \text{ at plant } i, \\ c_{ij} & = & \text{Cost (negative revenue) for installing each capacity unit at start}, \\ x_{k,ij}^{1} & = & \text{Actual production of product } j \text{ at plant } i \text{ minus revenue from } j\ , \\ c_{k,ij}^{1} & = & \text{Unit production margin for } j \text{ at } i \text{ under scenario } k, \\ b & = & \text{Initial budget constraint}, \\ h_{jk}^{1} & = & \text{Maximum possible sales (demand) of product } j \text{ under scenario } k, \\ T_{ij}^{\prime 1} & = & \text{Negative of factor for increase in production capacity for } j \text{ at } i, \\ h_{1,ij}^{\prime 1} & = & \text{Original production capacity for } j \text{ at } i, \\ h_{2,i}^{\prime 1} & = & \text{Total production capacity for all products at } i, \\ X_{ij}^{0} & = & \text{Set of possible capacity levels (may be discrete).} \end{array}$$

Notice that (19) has the same form as (18). This model was used in Sims [1992] to determine optimal additional flexible capacity levels for a class of products with eight plants and sixteen products (as in Jordan and Graves [1991]). The assumption was that capacity decisions were made once and that production could not be held in inventory from one period to the next (sales were lost above the capacity limit).

The scenarios were chosen to reflect known characteristics of the sales forecasts by using approximation techniques that bound the expected values with limited information (see, for example, Birge and Wets [1986]). The model was then solved to observe the effects of increasing lifetimes (production periods) on the installed capacity. The expected number of lost sales and expected utilizations were also considered but not added directly into the objective. In general, flexible capacity increased as lifetimes increased, with corresponding increased utilizations (to 98%) and decreased numbers (by 80%) of lost sales. The linear model allowed for efficient solutions with binary capacity decisions using IBM's OSL [1991] package on an IBM RS/6000 workstation.

6 Conclusions

Measuring risk attitudes in management science problems often causes difficulties for modelers as well as decision makers. We have shown that application of option pricing can readily incorporate financial risk attitudes into linear models. Our capacity planning example illustrates an especially important case. In general, however, a modeler must distinguish constraints as related to market or project-specific restrictions. This distinction may not always be obvious and requires careful consideration within a project context.

Other issues of importance to this development include the assumptions of the completeness of the market and the absence of transaction costs. Specific evidence may be necessary to justify these assumptions completely in practice, but the model matches general portfolio conditions in the extreme cases of tight and loose constraints. Further studies should explore the decision-making significance of the additional assumptions.

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