Modeling Investment Uncertainty in the Costs of Global CO2 Emission Policy

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Modeling Investment Uncertainty in the Costs of Global C02 Emission Policy

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abstract: This paper investigates the effect that explicit modeling of stochastic returns to investment has on the CO_2 abatement policy returned by a large scale macroeconomic model of the United States economy. We find that a policy derived from the mean valued deterministic model in which the random variables of the stochastic model have been replaced by their expected value poorly approximates the optimal policy returned by solving the stochastic programming model. We measure this nonoptimality by determining the value of the stochastic solution and investigating the different evolutionary paths that various macroeconomic variables follow. Macroeconomic variables which stray far from their optimal paths when derived under the assumption of a certain mean valued future are as follows: The level of carbon taxation, investment in new energy production technologies, exploration for nonrenewable resources and investment in improved macroeconomic efficiency.

Key words: economics, environment, stochastic programming

1 Introduction

Atmospheric levels of CO_2 have dramatically increased during the past century due to the rapid rise in the use of fossil fuels. This increase is cause for worldwide concern. The concentration of CO_2 in the atmosphere plays an important role in regulating terrestrial temperature and weather patterns. Many people believe that the reduction in future aggregate consumption that this phenomenon may cause will not be averted by individual utility maximizing decisions made by consumers and producers. As a result, these concerned individuals believe that this externality can only be accounted for in the evolution of the world's economy through governmental policy.

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One suggestion is for the nations of the world to set scientifically established targets for the yearly emissions of CO_2 and other "greenhouse" gases. Each nation would have some "greenhouse" constraint, a percentage of the global emissions ceiling which its yearly emissions could not exceed. Given this constraint, individual national governments will have to determine what policies to implement to move their evolving economies towards a low carbon future with as little loss of output as possible.

The principal policy option our government has to alter the behavior of an economic system is to lower the activity of a particular sector of the economy by taxing its output and increasing the activity of another sector by subsidizing its production. As a result, a carbon tax is the most straightforward method for decreasing CO_2 production through utilization of carbon intensive fossil fuels in a nation's economy. At the same time, to minimize the reduction in consumer and producer surplus caused by the tax, the tax revenues collected need to be wisely redistributed throughout the economy. This redistribution may be in the form of funding for alternative low carbon energy development programs, investment for the improvement of the efficiency of a nation's productive capacity, exploration for low carbon exhaustible resources and investment for the development of atmospheric CO_2 extraction technologies. Policy options, such as fuel economy standards, can be considered a form of tax and subsidy as they force automakers to play the part of the government, raising their prices on gas guzzlers to subsidize the sale of fuel efficient cars.

Many questions surround the generic policy options introduced above. What should be the rate of taxation for different fossil fuels? Should this rate change over time? What investment options for maintaining economic growth should receive funding and what should be the timing of that funding? To help answer these more complicated questions of policy, decision makers rely on economic resource models that simulate how an economy allocates resources and evolves over time and how a government may most efficiently intervene to change that evolution.

Alan Manne and Richard Richels have used their model, Global 2100 [13], to determine how the tax rate on the carbon content of fossil fuels varies over time for each of the five regions that partition the world's economy in their model. They have also investigated, for each of the regions, what kinds of new technologies are developed to take the place of conventional fossil fuel driven energy production. They have considered these questions for two cases. In the first case, they allow carbon rights trading. That is, regions desiring to produce less than their quota of "greenhouse" gases are able to sell their surplus to those regions wishing to produce more than their quota. In the second case they allow no such market. For both cases they were able to determine a carbon tax path and trends in the development of replacement energy technologies.

An important consideration, when determining how tax and investment policy is to be formulated, is the uncertainty inherent in the future. In other words, policy that is put in place today must be derived so that it drives the economy in a way that optimally hedges against uncertainty. To do this, policy must maximize the expected sum of consumer and producer surplus over the distribution describing the behavior of the uncertain elements in the future.

One area of the economy that is especially uncertain is the relationship between investment initiatives and their payoff. Particular uncertainties include the relationship between present day investment and the future availability of noncarbon based energy technology, the availability of nonrenewable resources through exploration and the extent to which macroeconomic energy efficiency improvements are made. The extent to which these investment initiatives are successful will have a great bearing on the world economy's ability to continue to grow without producing ever greater quantities of "greenhouse" emissions.

Deterministic models, in which the random variables are replaced by their expected values, will provide policy suggestions which only approximate actual optimal hedging policies (those policies which will maximize the expected sum of consumer and producer surplus). To determine the actual optimal hedging policy, stochastic models that explicitly model uncertainty need to be developed. In this paper, we develop a stochastic programming model of the United States economy based on the Global 2100 macroeconomic submodel [13] of the United States. Our model differs from the Global 2100 model in that we make the future availability of alternative energy technologies, supplemental improvements to the nation's productive efficiency and increased stocks of nonrenewable resources a random function of investment.

Using this model, we see that policy options derived using the deterministic approximation of the stochastic program differ from the optimal hedging policy options derived directly from the stochastic program. We also see that the use of non-optimal deterministically derived policy decisions leads to an appreciable reduction in the expected sum of consumer and producer surplus over that resulting from the optimal hedging policy. This difference we refer to as the *value of the stochastic solution*. Finally, using the price path for carbon rights determined in Manne and Richels paper, we find that the existence of a market for the sale and purchase of carbon rights substantially lowers the *value of the stochastic solution*. That is, trade is a hedge against uncertainty.

2 The Stochastic Global 2100 Model - U.S.A.

2.1 Overview

The Stochastic Global 2100 Model - U.S.A. models the evolution of the United States' economy during the period lasting from 1990 to 2100. A schematic of the model is pictured in figure 1. The model's basic structure is a dynamic nonlinear multistage program entirely based on the Global 2100 U.S. submodel. The model divides the economy into two sectors: Energy and Nonenergy. Aggregate economic output in each period is a function (Cobb-Douglas type) of four aggregate variables: Labor, capital stocks, electrical and nonelectrical energy utilization. The by-product of energy production, CO_2, must remain below the sum of the U.S. quota plus any carbon rights that are purchased.

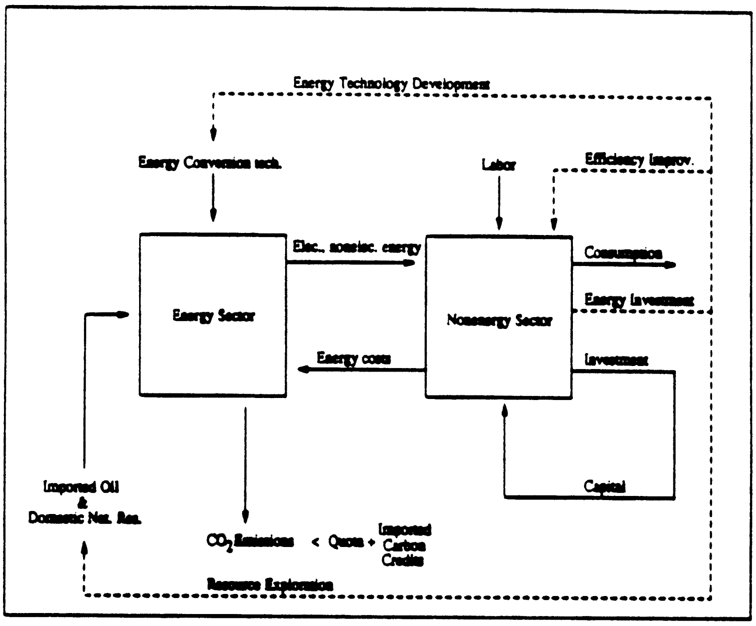

Figure 1: Stochastic global 2100 model

The aggregate output circulates back through the economy satisfying final consumption, investment, payments for the energy utilized during that period and carbon rights purchases. Capital investment builds up the depreciating capital stocks in the period subsequent to its expenditure. The model accounts for an economy's inertia by adopting Manne and Richels use of time lags. The model constructs time lags by dividing capital, electrical and nonelectrical energy utilization in two sets. Old capital, electrical and nonelectrical energy utilization depreciates each period at a constant rate and represents inflexible patterns in the economy. New capital, electrical and nonelectrical energy utilization is flexible and is decided upon each period in response to the prevailing price and availability of energy resources and technology. Manne and Richels refer to this type of formulation as a putty-clay model because existing energy use patterns, like hard baked clay, are fixed, unable to respond to price and availability changes, while new capital and energy use patterns are malleable, like putty, in that they can be changed by the model in response to changes in resource price and availability. The objective function of the nonlinear optimization problem is the discounted sum of logs of consumption in each period. The last term of the sum is the log of the post terminal period consumption. Post terminal consumption is included in the model to reduce end of study effects [12]. The model assumes that consumption will grow at a constant yearly rate indefinitely after the end of the horizon.

This model differs from the Global 2100 U.S. submodel in that stochastic multipliers connect energy investment with the resulting payoff from that investment. Energy investment can be used to increase the potential availability of alternative electric and nonelectric energy producing technologies. Thus, the rate at which capacity can be brought on line is a random multiple of the level of investment. In the Global 2100 U.S. submodel, uncertainty is not explicitly considered as a function of any decision variables. Thus, the creation of capacity in these technologies occurs no faster than a predetermined rate obtained from surveyed energy and technology experts.

Energy investment can also be used to increase the availability of nonrenewable resources through exploration. Manne and Richels chose to model resource availability deterministically, beginning the horizon with a known stock of each of the grades and types of resources. Because there is so much uncertainty regarding the reserves of oil, gas and coal that remain undiscovered, we have changed this to make resource discoveries a random multiple of investment dollars allocated to resource exploration.

Manne and Richels included a parameter in their model representing autonomous energy efficiency improvements. These are efficiency improvements made at no economic cost. That is, they are made not as a way to avoid the use of costly fossil fuels, but automatically over time as the economy matures. We have continued to include this, but have added efficiency investments that bring about uncertain improvements in the economies overall energy efficiency.

As will be seen later on, the inclusion of uncertainty in the investment portion of the model has a significant impact on the evolution of the economic system simulated by this model.

2.2 Values of Interest in Stochastic Programming

This paper uses an economic model to determine governmental policy for limiting the United States' economy's production of CO_2 emissions. This policy is derived by solving a stochastic programming optimization model that simulates the U.S. economy. The solution of the stochastic program is the sequence of period decisions that do not anticipate future uncertainty prior to its occurrence and maximize the expected consumption over the study horizon.

This solution is known as the "here and now" solution because it provides the modellers with the best first period decision they can make in light of the uncertainty about the future confronting them. That is, it is the solution which best hedges against the uncertainty. The problem is denoted as:

$$HN = \max_{x_t(\boldsymbol{\xi}_t)\in X} \int_{\Xi} \sum_{t=1}^{T} f_t(x_t(\boldsymbol{\xi}_t), x_{t+1}(\boldsymbol{\xi}_t), \boldsymbol{\xi}_t) P(d\boldsymbol{\xi}_t) \tag{1}$$

where $f_t(.)$ represents the discounted consumption in the t^{th} period along with all constraints linking the t^{th} and $(t+1)^{st}$ periods, $x_t(\boldsymbol{\xi}_t)$ represents the decision variables in the t^{th} period subject to previous random realizations, ξ_t, X represents the nonanticipative condition and $\boldsymbol{\xi}_t$ is a random vector that represents all uncertain parameters up to and including period t. The nonanticipative condition forces decision variables from different scenarios that are identical up until period t to be equal.

The "wait and see" solution is another quantity of interest. It is the expected solution where the decision makers are fully aware of the future at the time they make all of their decisions and optimize accordingly. It is as follows:

$$WS = \int_{\Xi} \max_{x_t(\boldsymbol{\xi}_t)\in X} \sum_{t=1}^{T} f_t(x_t(\boldsymbol{\xi}_t), x_{t+1}(\boldsymbol{\xi}_t), \boldsymbol{\xi}_t) P(d\boldsymbol{\xi}_t) \tag{2}$$

The "wait and see solution is always greater than the "here and now" solution for f_t concave because the future is known at the time when decisions must be made i.e. there is no uncertainty. The difference between these two values is called the *expected value of perfect information* (EVPI) Raiffa [16]:

$$EVPI = WS - HN \tag{3}$$

It tells the modellers how much better they could do if all uncertainty could be resolved. In essence it gives them an upper bound on how much they should be willing to spend to have perfect foresight about the future with respect to their problem.

Because both of these problems are difficult if not impossible to solve, modellers often solve simplified versions of the problem where all random variables are replaced by their expected values. This produces a solution called the "mean value solution":

$$MV = \max_{x_t(\overline{\xi_t}) \in X} \sum_{t=1}^{T} f_t(x_t(\overline{\xi_t}), x_{t+1}(\overline{\xi_t}), \overline{\xi_t}) \tag{4}$$

where $\overline{\xi_t} = \int_{\Xi} \boldsymbol{\xi}_t P(d\boldsymbol{\xi}_t)$. It can be shown by Jensen's inequality that this value exceeds the "wait and see" solution or is consistently optimistic. A good way to judge the sequence of decisions produced by the "mean value solution" is to determine the expected value of the problem if these decisions were actually to be used. This is called the "expectation of the mean value solution":

$$EMV = \int_{\Xi} \sum_{t=1}^{T} f_t(\overline{x_t}, \overline{x_{t+1}}, \boldsymbol{\xi}_t) P(d\boldsymbol{\xi}_t) \tag{5}$$

where $\overline{x_t}$ is the solution to equation 4. The "expectation of the mean value solution" is always less than the "here and now" solution because $\overline{x_t}$ is just one of many possible solutions from which HN picks the best. The difference between HN and EMV gives the *value of the stochastic solution* (VSS) Birge [1]:

$$VSS = HN - EMV \tag{6}$$

This quantity measures the benefit, in terms of increased expected return, from explicitly accounting for uncertainty in the model. This value has definite policy implications in that if governmental policy (e.g. rates of taxation, etc) is designed using the "mean value solution" , the VSS indicates to the modellers how much worse the economy will perform than if governmental policy had been designed using the "here and now" solution.

Technology name	Identification
Existing:	
HYDRO	Hydroelectric, geothermal, and other renewables
GAS-R	Remaining initial gas fired
OIL-R	Remaining initial oil fired
COAL-R	Remaining initial coal fired
NUC-R	Remaining initial nuclear
New:	
GAS-N	Advanced combined cycle, gas fired
COAL-N	New coal fired
ADV-HC	High-cost carbon free
ADV-LC	Low-cost carbon free

Table 1: Electrical Technologies

2.3 Energy Technologies

Our model includes the same types of technology present in the original Global 2100 U.S. submodel. One class of these energy producing technologies are the conventional technologies. That is, the technologies that have relatively low capital requirements and rely on the availability of easily obtainable natural resources that exist in finite quantities, such as oil, gas or coal. In most cases the pool of any one kind of resource is not homogeneous in its ease of discovery, extraction or refinement. Thus, each resource is divided into grades that are ordered according to the cost incurred in bringing them to market.

Another class of energy technologies tend to be more capital intensive and rely on an energy resource that is less easily captured. Some of these technologies may have limits to their expansion while others may be, for all practical purposes, unlimited. Examples of both types include hydroelectric technology, limited by the availability of rivers with the proper characteristics, and fusion power, with an essentially unlimited fuel source. The third energy technology does not involve the the transfer of a resource into an energy form, but rather the efficiency with which usable energy is transformed into useful output. A listing of both the electrical and nonelectrical energy producing technologies used in this model are shown in tables 1 and 2.

Technology name	Identification
OIL-MX	Oil imports minus exports
CLDU	Coal-direct uses
OIL-LC	Oil-low cost
GAS-LC	Gas-low cost
OIL-HC	Oil-high cost
GAS-HC	Gas-high cost
RNEW	Renewables
SYNF	Synthetic fuels
NE-BAK	Nonelectric backstop

Table 2: Nonelectrical Technologies

2.4 CO_2 Emissions Restrictions

Manne and Richels used the Global 2100 model to try to determine how the evolution of the economic system would change in the face of CO_2 restraints. In particular, they sought the costs of CO_2 restriction in terms of reduced GNP. They modeled several different restriction scenarios including a proposal for developed countries to reduce their yearly CO_2 emissions to a level 20% below 1990 levels. It is believed by many in the scientific community that yearly emissions of this level will have little environmental effect. Any associated temperature rise will be entirely absorbed by the oceans where it is believed little damage will be done. The proposal calls for the reduction in emissions to be phased in over a period of time so that by the year 2010 the U.S. will be restricted to produce no more than 1.144 billion tons of CO_2 yearly.

Manne and Richels did find a significant change in the economic system as a result of the imposition of CO_2 constraints. In particular, the mix of energy production technologies shifted towards those technologies that produce little carbon as a by-product of energy production. This meant, for example, that coal and the alternative synthetic fuels based on coal were de-emphasized while natural gas and alternative energy sources like renewables and large scale solar power were developed. This shift from easily extractible resources like coal to more capitally intensive and expensive technologies like solar power reduced economic output as compared to the unconstrained case by an amount that rose over the horizon to about 2.5% per year. This was for the case in which there was no international trade in carbon rights so that the CO_2 emissions limit was a hard constraint. When international trade in emissions rights was allowed, the penalty on economic output was somewhat reduced.

We use this same CO_2 restriction scenario and explore the impact on the economy from the uncertainty surrounding how much investment is required to bring alternative energy technologies to market at competitive prices. We also explore the impact on the economy resulting from uncertainty regarding returns from nonrenewable energy exploration and uncertainty regarding returns from investment allocated towards improving the efficiency of the economy's aggregate production function. Our goal is to determine how the simplifying assumption of a mean valued future changes the predicted evolution of the economy under CO_2 restrictions from that predicted by a stochastic programming model. This will indicate how far from optimality is government policy designed to curb CO_2 emissions under the assumption of a certain future. Our principal measure of this nonoptimality will be the size of the *value of the stochastic solution* [1].

The potential results of this investigation could be one of the following. The *value of the stochastic solution* is low, indicating that policy decisions derived by assuming a certain future are close to the optimal policy decisions. The *value of the stochastic solution* is high, indicating that it is important to explicitly consider this uncertainty when designing policy or policy decisions will be far from optimal. Finally, the *value of the stochastic solution* is infinite, indicating that policy decisions derived under an assumption of certainty will actually drive the economy to such a state that there exist scenarios under which the economy will be unable to meet CO_2 emissions targets.

We seek to determine the importance of this uncertainty by investigating for two cases with four scenarios each how much value the solution of the stochastic program has (i.e. how valuable is it to explicitly account for the uncertainty of investment payoffs).

The two cases to be explored are a restricted carbon production future in which carbon rights may be traded and one in which carbon rights may not be traded. This will provide information not only on the value of accounting for uncertainty, but the value of establishing an international market in carbon rights.

2.5 Uncertain Investment Returns

Obviously great uncertainty surrounds the payoffs, defined as increased potential capacity, that can be expected from a given level of investment dollars devoted to energy exploration or technology development. To reflect the wide range of outcomes that are possible, we have chosen scenarios that range from extremely optimistic to extremely pessimistic regarding the success of investment initiatives and have given them each equal weighting in the calculation of an optimal hedging strategy. It is assumed that success in these investment initiatives will be quite low until early in the next century (2020) when payoffs will either remain as they were or increase markedly. Again, in the next period of the study, returns on investment will either remain constant or increase again so that by the beginning of period 2040 all is known that will ever be known about the success rate of the individual investment initiatives. At this point, the stochastic tree has branched into the four scenarios that will be studied as in figure 2. A summary of the returns on investment used for the model is shown in table[2] 3.

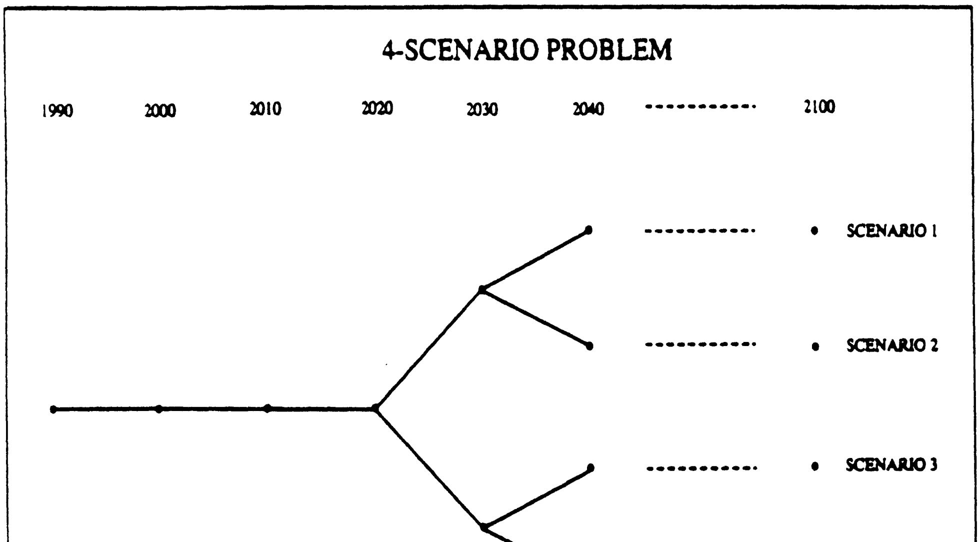

Figure 2: Scenarios

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[2] Nonelectrical technologies: EXAJ/10^{12} \$. Electrical technologies: TKWH/10^{12} \$. Efficiency: (Percentage reduction in energy intensity)/10^{12} \$.

Technology name	Scen 1	Scen 2	Scen 3	Scen 4
ADV-HC	0.094	0.33	3.3	10.328
ADV-LC	0.115	0.4	4.0	12.6492
OIL-LC	7	10	25	48
GAS-LC	7	10	25	48
OIL-HC	7	10	25	48
GAS-HC	7	10	25	48
RNEW	0.89	3.1	30.9	97.98
SYNF	2.36	8.17	81.7	258.2
NE-BAK	0.632	2.2	21.9	69.3
EFFICIENCY	0.06	0.12	0.15	0.17

Table 3: Returns on Investment

2.6 Hedging strategy

As already discussed, the stochastic program returns a strategy that optimally hedges against the future uncertainty. What is meant by hedging? Clearly if future uncertainty could be resolved immediately, this information could be taken advantage of to produce better present decisions. In fact, for each possible future scenario, a different set of optimal present decisions could be developed. The optimal hedging strategy is a strategy that lies somewhere between all these strategies based on perfect information until the point at which uncertainty is resolved. At this point, the optimal hedging strategy branches, in light of the new information, and attempts to rejoin optimally and feasibly the path corresponding to decisions made under perfect information. Clearly, the path of the optimal hedging strategy does not converge to the path of the perfect information strategy immediately, but only gradually over time. It is this deviation which is the price of uncertainty regarding the future (i.e. the expected value of perfect information). We see this clearly in figure 3 by looking at the time path development of nonelectric energy under perfect information and under uncertainty for scenarios one and four of the international trade in carbon rights case of the model. In this figure the hedging strategy path lies between the two perfect information paths until 2020 when the uncertainty regarding the return on investment is resolved. At this point, the hedging strategy paths attempt to converge towards their respective perfect information paths which they succeed in doing after a period of about 60-70 years.

We refer to the expected return under perfect information as the "wait

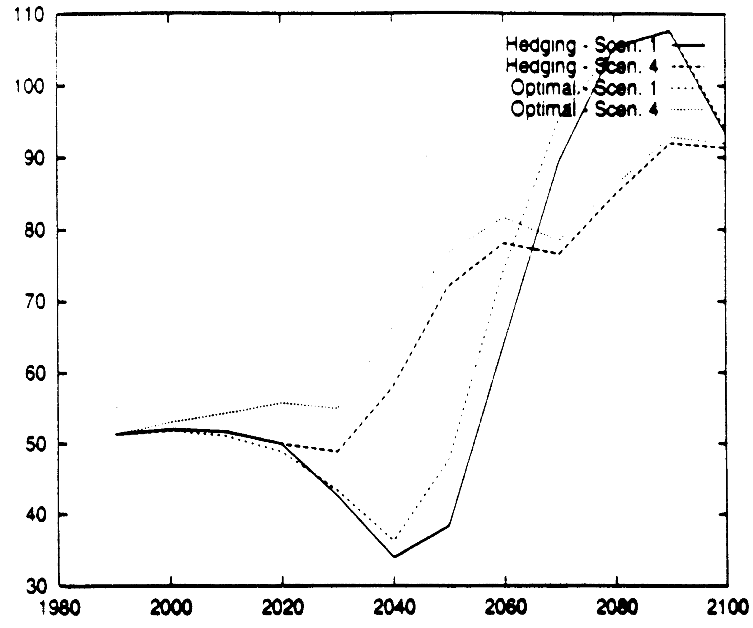

Figure 3: Nonelectric energy production

and see" solution (WS). The expected return from the hedging strategy solution is the "here and now" solution (HN) and the return that is expected when the deterministically derived mean value solution is used is called the "expected mean value" solution (EMV). The value of the stochastic solution (VSS) is simply the difference between the HN and EMV.

3 International Trade in Carbon Emissions

In the first case, an international market exists for the trade of CO_2 permits. The cost of a one billion ton credit for CO_2 production is assumed to be equal to that used by Manne and Richels in their model. They derived the time price of CO_2 permits using all five regions in their Global 2100 model and determining that string of prices which would approximately balance supply and demand between regions. This string of prices will no doubt change with the inclusion of uncertainty, but for the purposes of this study, we consider this to be an acceptable approximation.

How important is the uncertainty of investment in this model? Is the strategy returned by solving the EMV problem sufficiently close to the actual optimal hedging strategy that the extra effort required to explicitly model

Technology name	1990-2020	2030	2040-2100
ADV-HC	0.094	1.697	3.513
ADV-LC	0.115	2.0575	4.29105
OIL-LC	7	16	22.5
GAS-LC	7	16	22.5
OIL-HC	7	16	22.5
GAS-HC	7	16	22.5
RNEW	0.89	15.895	33.2175
SYNF	2.36	42.03	87.6075
NE-BAK	0.632	11.266	23.508
EFFICIENCY	0.06	0.105	0.125

Table 4: Expected returns on Investment

Strategy	Expected Return	% from WS	% from HN
WS	58848.349	-	-
HN	58745.605	0.175	-
EMV	57952.984	1.52	1.35

Table 5: The value of the stochastic solution - trade

the investment uncertainty is unwarranted? The following discussion will help to answer these questions.

The mean value problem develops an optimal strategy by assuming an expected valued future. In our formulation this means that the four scenarios are collapsed into one, each weighted equally, so that the following returns to investment, listed in table 4, are available in each period. The expected mean value solution is determined from the mean value solution strategy. The expected mean value solution problem is simply the original stochastic program used to determine the optimal hedging strategy, but with all decisions prior to the realization of uncertainty fixed at the values determined by the mean value problem. As a first, and perhaps most important, measure of the difference in quality between the two strategies, the value of the stochastic solution was calculated. The results in table 5, indicate that the optimal hedging strategy performs markedly better than the expected mean value strategy where the objective function is the expected utility of consumer and producer surplus. This is especially in light of how close the hedging strategy comes to the expected return under perfect information. In fact, the expected mean value strategy is a factor of nine farther from the WS solution than the optimal hedging strategy. This means that if the governmental policy to force compliance with CO_2 constraints is derived from a deterministic model, it will drive the economy to perform worse than governmental policy derived from a stochastic model by the value of the stochastic solution.

Consumption

The value of the stochastic solution provides a good measure of the improved quality of the strategy provided by solving the stochastic program. In addition to this though, it is important to look at the behavior of individual economic variables to understand how it differs from the mean value strategy and why the optimum hedging strategy is so much better. Consumption in the model is the most important variable to look at because, through the utility function, it drives the economy by establishing the preference for consumables over time. Figure 4 shows consumption in scenario

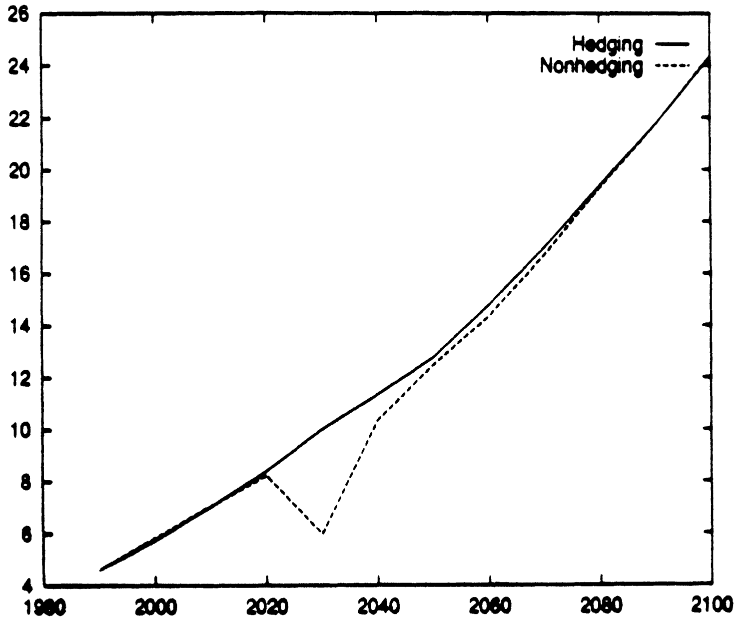

Figure 4: Consumption - Scenario 1

one over time for both strategies. Initially, mean value consumption increases at about the same rate as the consumption of the hedging strategy. When the results are revealed as to the success of investments initiated in

2020 though, mean value consumption drops severely. For the remainder of the horizon it converges from below towards the consumption path of the hedging strategy. A similar result follows for scenario two. Consumption for both strategies in scenarios three and four are almost identical.

This example demonstrates one of the principal weaknesses of using the mean value solution. An optimal strategy is found under the assumption that returns will be sizeable in 2030. When they are not, the unexpected shortfall must be made up by a reduction in consumption. The expected cumulative shortfall in consumption ends up totaling 0.3322 and 0.7912 trillion dollars when discounted, respectively, at 5% and 3% annually. In 2030 if scenario one occurs, the consumption returned by the mean value problem is actually 4.04 trillion dollars lower than that returned by the hedging strategy. And of perhaps most importance, consumption in the mean value strategy decreases from 2020 to 2030 by approximately 2 trillion dollars. This is a massive shift in the economy.

Investment

Since the shortfall is the result of lower than expected returns on investment, the timing of investment in the mean value strategy is a problem. Consider investment into the development of synthetic fuel technology. Figure 5 shows investment in scenario one for both strategies. The optimum hedging strategy invests moderately in both the periods, 2000 and 2010. On the other hand, the mean valued strategy, anticipating large returns from investment in 2020, over invests. The return on the nearly quarter of a trillion dollars is much less than anticipated and results in a shortfall. In contrast, figure 6 shows that in scenario 3, the hedging strategy invests moderately before it is known how successful investment will be and is, thus, able to invest intensely in period 2030 when returns are better than expected. The decision to invest large amounts early by the mean value strategy results in a loss because the bulk of investment occurs before it is clear whether it will be successful or not. This case is representative of the differences in the timing of all investment with uncertain returns between the mean value and optimum hedging strategies. One notable exception is investment into resource exploration which is zero for all resources and grades in both strategies.

Natural Resource Utilization

Another impact of the mean value assumption is to accelerate the utilization of the domestically produced resources: oil-lc, gas-lc, oil-hc and gas-hc. Figure 7 shows the depletion curve of low cost oil in scenario one. This depletion curve is almost identical for all scenarios and all resources and

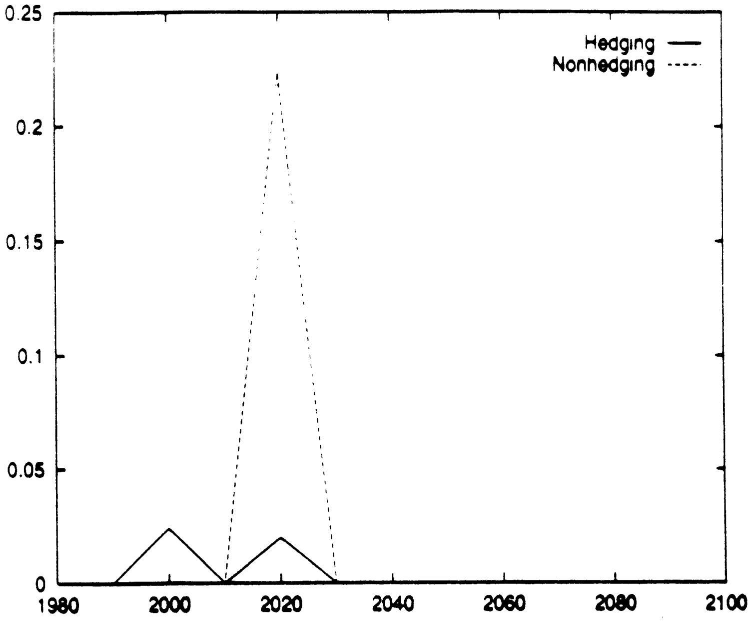

Figure 5: Synthetic fuel tech. investment - Scenario 1

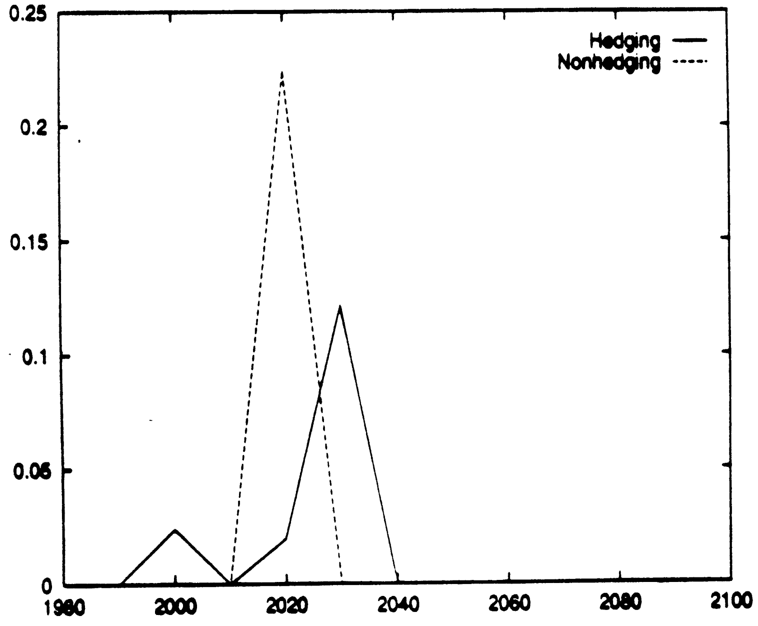

Figure 6: Synthetic fuel tech. investment - Scenario 3

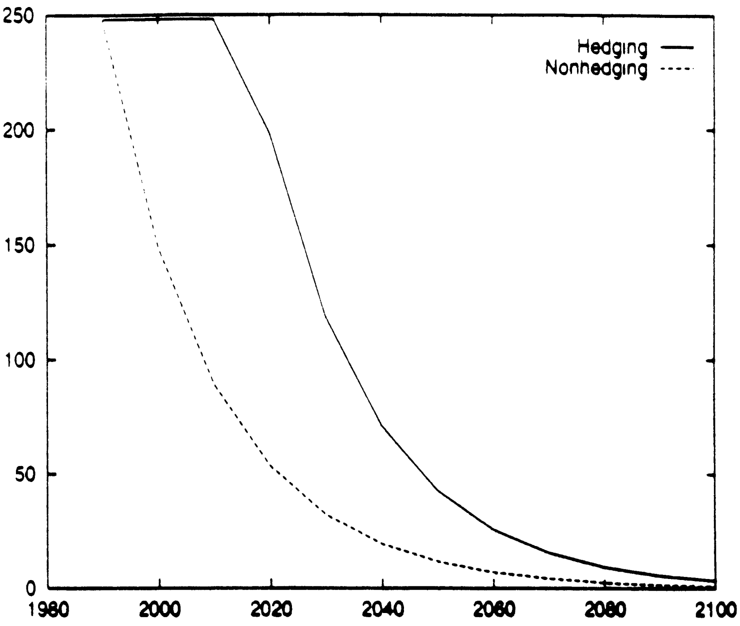

Figure 7: Depletion curve - low cost oil - scenario 1

grades.

One interesting side light to this is that the hedging strategy partially makes up for its slower utilization of domestic resources by increasing the percentage of its nonelectric needs that are met by imported oil. Figure 8 shows this for scenario one. The model indicates that it is optimal to pay a premium for imported oil in the present to conserve domestic reserves as a hedge against the possibility that investment initiatives will not succeed.

4 No International Trade in Carbon Emissions

The second case considered is that in which there is no international trade in CO_2 rights. The U.S. has a strict carbon limit that cannot be relaxed. This case is considered primarily to demonstrate the value that an international carbon rights market has for all parties involved.

Why does it have value? Because different regions have different costs associated with complying with their respective CO_2 constraints (assuming that an internationally agreed upon set of restrictions can established). Some regions do not produce enough CO_2 to even reach their constraint while others would be willing to pay a very high premium to relax theirs (i.e.

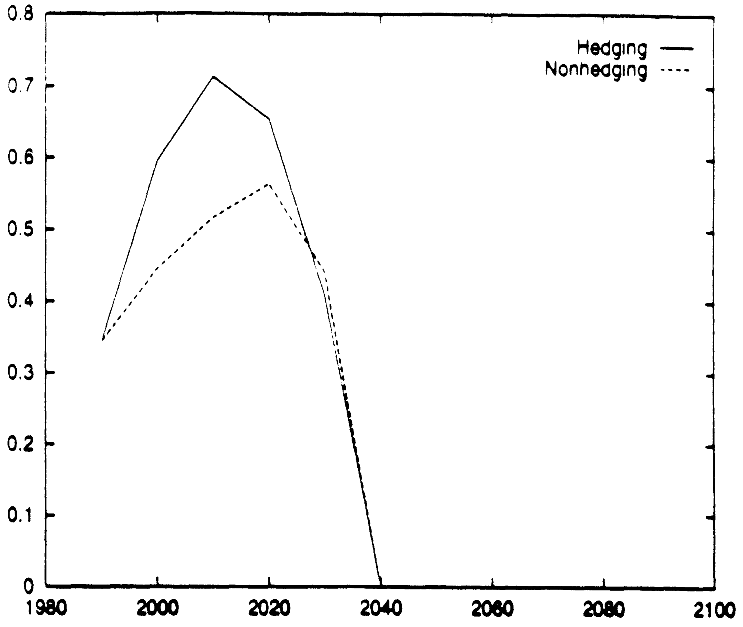

Figure 8: Percentage Nonelectric Energy - Imported Oil - scenario 1

the dual on their CO_2 constraint is high). An international market works by allowing seller regions (i.e. those regions with slack CO_2 constraints or duals on these constraints lower than the prevailing price of carbon rights) to sell until the price they receive equals the marginal cost of their constraint and buyer regions to buy until they pay a cost equal to their marginal benefit. This boosts the economies of all involved. A comparison of the U.S. in this second case with the previous trade case will help to quantify exactly how much this region would benefit.

In addition, a strict limitation on CO_2 production means that there is a much higher willingness to pay for the development of alternative low carbon technologies. This should accentuate the differences between the strategies returned by the mean value solution and the stochastic programming solution as it is these technologies which have uncertain returns to investment. All this should result in the value of the stochastic solution being different than in the previous case.

As with the first case, the values of the WS, HN and EMV solutions were calculated. These are displayed in table 6, as before. As with the trade case, the value of the stochastic solution is quite high (over 2% of the value of the HN solution). This improvement is especially dramatic when one notes

Strategy	Expected Return	% from WS	% from HN
WS	58589.584	-	-
HN	58470.243	0.204	-
EMV	57289.807	2.218	2.02

Table 6: The value of the stochastic solution - notrade

how much closer to the WS solution is the HN solution than the solution returned by the EMV problem. The factor of almost ten improvement is larger than that for the previous case. As expected, the VSS, as a percentage of the value of the HN solution, is significantly higher in this second case than in the trade case.

Consumption

The absence of any trade to lower the marginal cost of the CO_2 emissions constraint means that greater investment must be directed into the alternative technologies. This, of course, increases the impact that uncertainty has on the model. The principal failure of the mean value strategy once again shows up most dramatically in consumption over time. Figure 9 shows con-

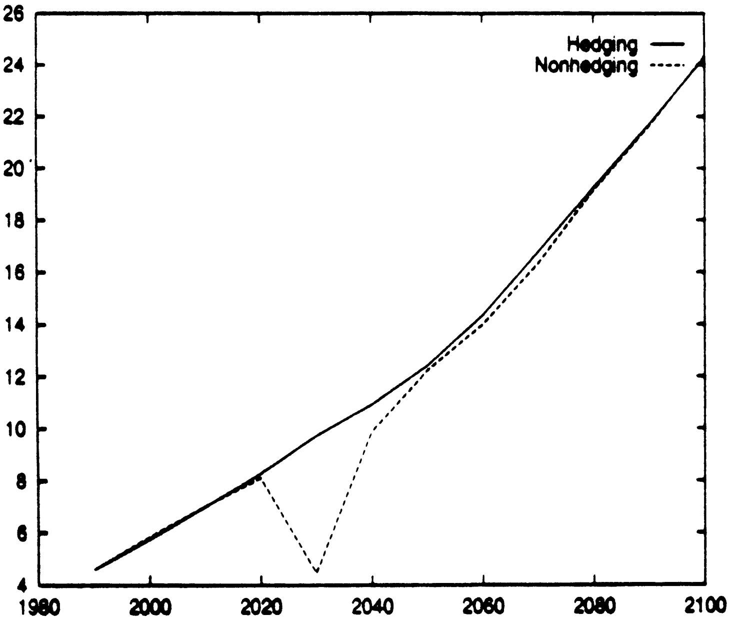

Figure 9: Consumption - Scenario 1 - notrade

sumption for both strategies in scenario 1. As before, consumption suffers as a result of lower than expected returns on investment. Discounted cumulative consumption is diminished by 0.9829 or 0.420 trillion dollars given a discount rate of either three or five percent respectively. In scenario 1 during the period 2030 at the point where consumption bottoms out for the mean value strategy, it has dropped 3.66 trillion dollars from the previous period and is 5.27 trillion dollars less than it might otherwise have been if the optimal hedging strategy had been followed. All these numbers are larger than the previous case and testify to the importance that trade in carbon rights plays as a hedge against the uncertainty of technology development.

In addition to no investment being devoted towards resource exploration in either strategy for the previous case, no investment was devoted to the development of the adv-hc technology. This occurs despite the fact that the adv-hc technology becomes available for research development ten years earlier than the adv-lc technology. The optimal hedging strategy under the no trade case also leaves this resource unexploited. In contrast to this result, the mean value strategy in the no trade case under all scenarios includes investment in and development of a small amount of this expensive resource, peaking at 0.0912337 TKWH in 2070 under scenarios one and two. The use of such a high cost technology provides further evidence that the marginal cost of the carbon constraint is high, especially in the first and second scenarios where development attempts have little success. Even though investment occurs in this high cost technology, no resource exploration of any kind occurs. This is also true for the hedging strategy. Apparently with the stocks of domestic resources that still remain and the opportunity for importing oil, little is gained by exploring for more of any grade of resource.

5 Comparisons between the Two Cases

Clearly the two cases produce differing economic paths as a result of the absence of trade in the second case. Given that the government derives its policy by explicitly accounting for uncertainty, the presence of trade increases the value of the optimal expected welfare by 275.362 units or 0.5%. In the case where government policy is suboptimal, the ability of regions to trade carbon rights at the prices suggested by Manne and Richels will increase the expected welfare function of the U.S. by 663.177 or 1.2%. This indicates that the presence of trade becomes even more crucial when uncertainty is present and unaccounted for by government policy. As suggested above, trade acts as a buffer that reduces the marginal cost of CO_2 compliance, thus, lessening the effect that unaccounted for uncertain returns to investment in low carbon technology will have on overall welfare.

Carbon Production

To give a clear idea of the value that trade has, consider figure 10, showing the production of carbon over time under both cases for scenarios one and four. This figure shows that in scenario one with the ability to

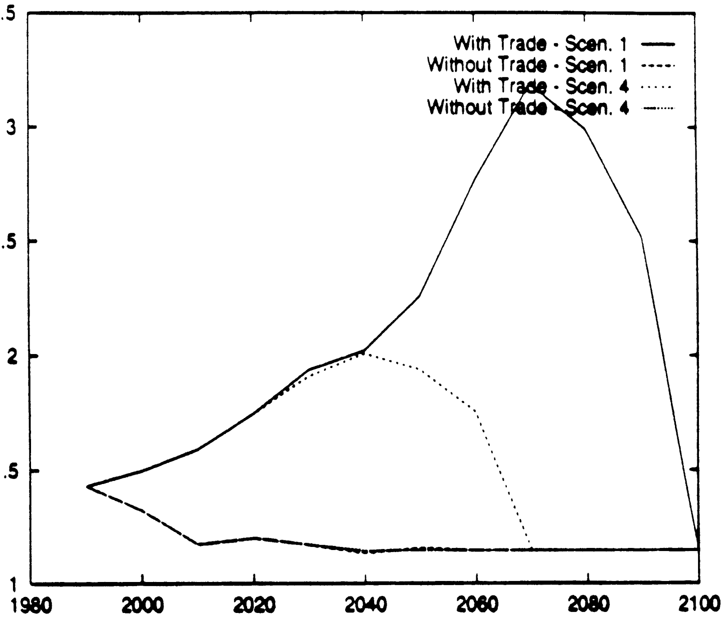

Figure 10: Carbon Production - Scenario 1 and 4

purchase carbon rights, carbon production grows in the U.S. until by 2070 it has topped three billion tons yearly. In this pessimistic scenario, it is better to produce carbon and pay for the emission rights than to invest in the carbon free technology. This is in contrast to scenario four where it is optimal to purchase a lesser amount of carbon rights for a much shorter time because the money is better spent developing the carbon free energy technology which has such a high return on investment. In this scenario, the U.S. discontinues the purchase of carbon rights at this stream of prices after 2080. This point is pushed off 20 years in the pessimistic scenario. In both cases of course, the U.S., under the nontrade restriction, can only produce as much carbon as its limit will allow. It is interesting to note that even with optimistic returns to investment, trade is still beneficial as the U.S.

prefers to produce amounts of carbon above its limit given the opportunity and the prices used in this study.

Consumption

Despite the fairly dramatic difference in carbon production between the two cases, figure 11 indicates that the lack of trade does not diminish consumption significantly if government follows policies derived using stochastic programming. The expected present value of the decrease in consumption under the assumption of no trade is 0.07317 or 0.19068 trillion dollars using a discount rate of 5% or 3%. This is between 1.5% and 4.1% of consumption in 1990. This loss is dramatically less than the loss associated with government policy derived using the mean value problem , which in the no trade case ranged between 9% and 21% and for the trade case ranged between 7% and 17% of 1990 consumption.

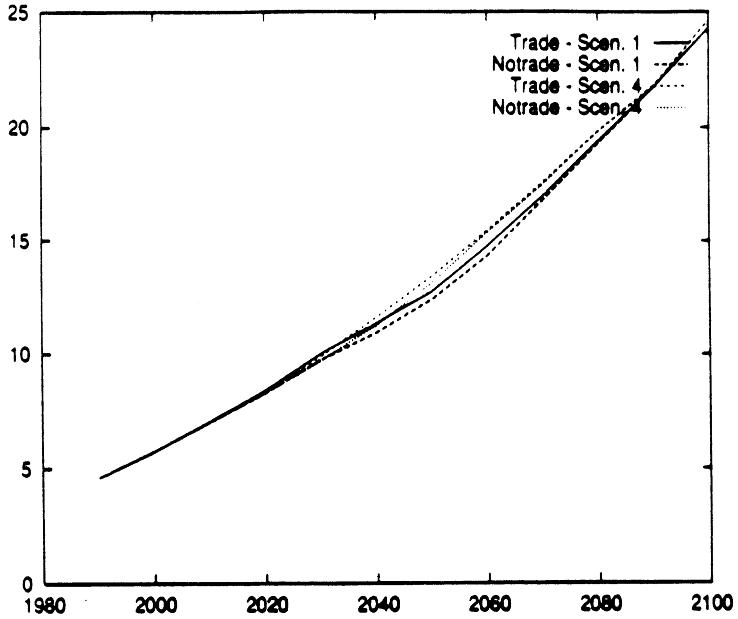

Figure 11: Consumption - Scenario 1 and 4

Energy Utilization

Consumption is maintained to this extent despite the relatively severe reduction in total primary energy utilization shown in figure 12 for both cases and scenarios 1 and 4.

Investment

Investment rates differ between the two cases because the marginal costs

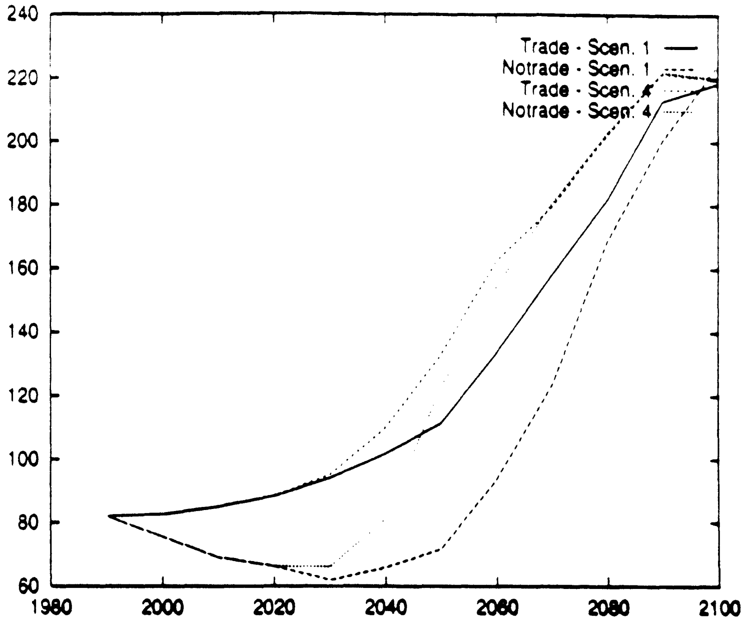

Figure 12: Total primary energy - Scenario 1 and 4

of carbon compliance are reduced so much by trade. Figure 13 illustrates the more rapid development of the ADV-LC technology in the no trade case for scenario one. Although development occurs earlier in scenarios three and four because of the higher returns, the no trade case still develops this technology more quickly.

Similarly, the NE-BAK technology is utilized more rapidly as is shown in figure 14 for both cases and scenarios one and four. For this rapid utilization to occur, higher levels of investment are expended in the no trade case. For ADV-LC technology development this means 0.075 versus 0.032 trillion dollars of investment in 2020 and for the NE-BAK investment the no trade case investment far overshadows the first case as shown in figure 15.

One interesting exception to this trend is for synthetic fuels. Synthetic fuels are derived from coal but, because of the process by which they are made, have a higher ratio of carbon content to energy potential: 0.04 versus 0.0241 billion metric tons per exaj. As a result, the economy in case two, which is severely constrained in the amount of carbon it can produce, develops a much smaller capacity of this technology than does the economy in the trade case. Figure 16 illustrates this for both cases and scenarios one and

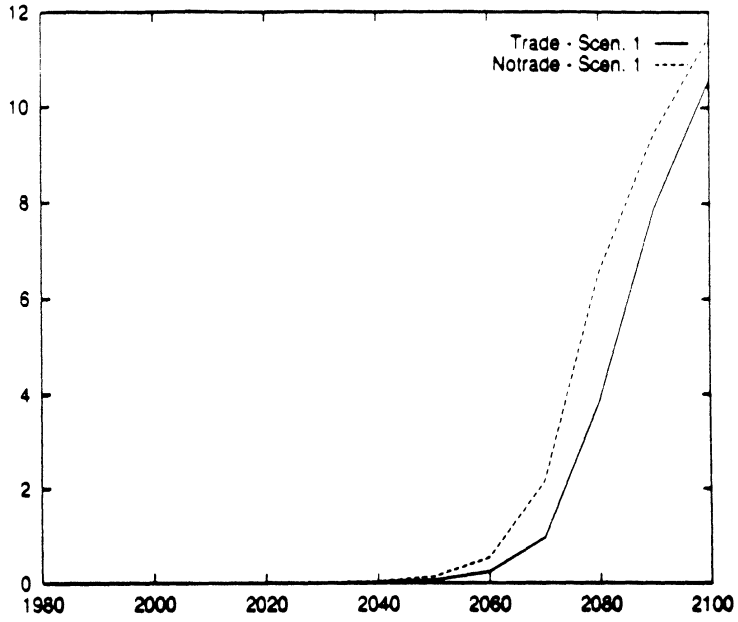

Figure 13: ADV-LC capacity (TKWH) - Scenario 1

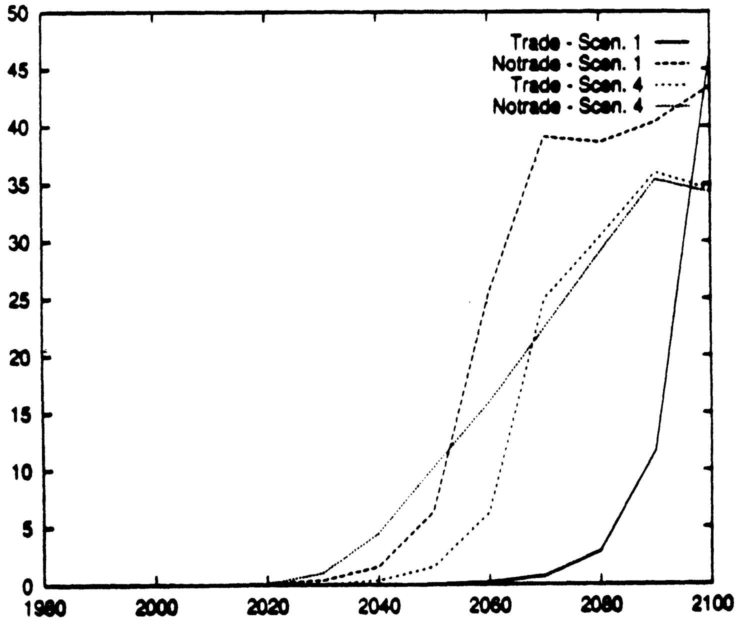

Figure 14: NE-BAK capacity (EXAJ) - Scenario 1 and 4

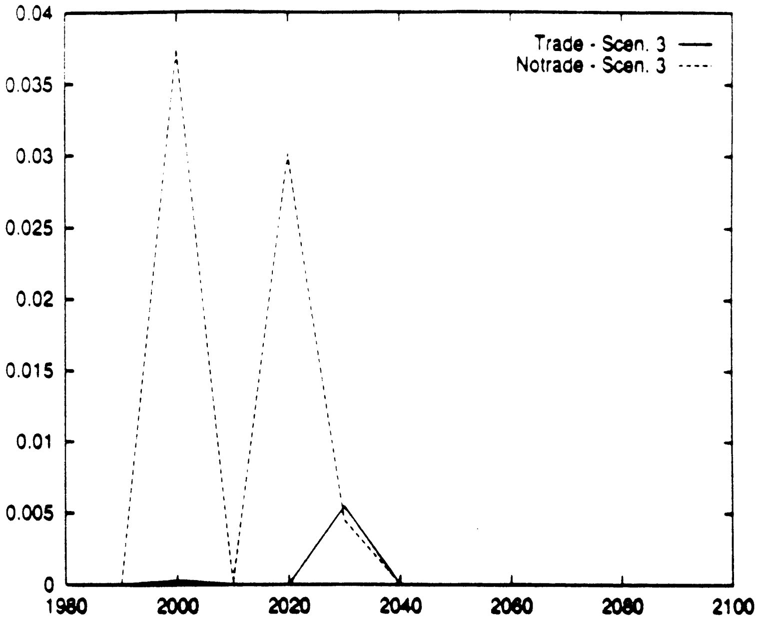

Figure 15: NE-BAK investment (10^{12} $) - Scenario 3

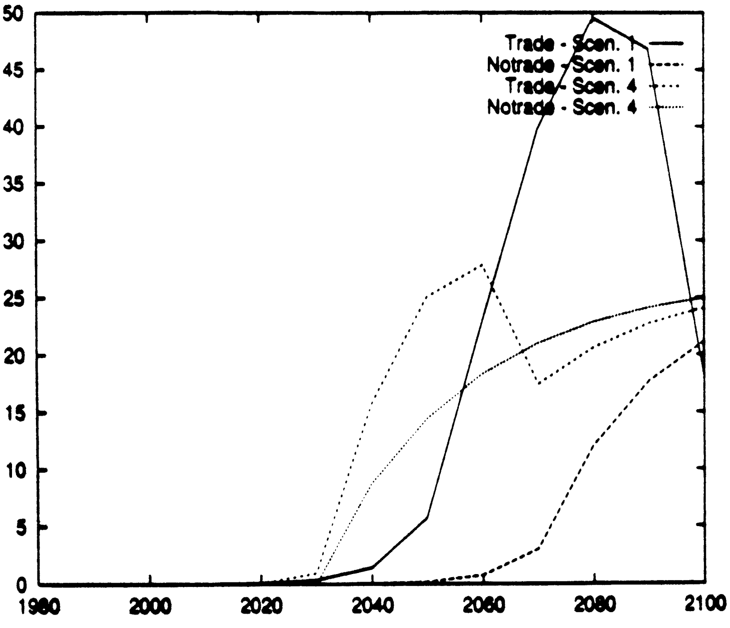

Figure 16: SYNF capacity (EXAJ) - Scenario 1 and 4

four. Note also that in the trade case, the maximum synthetic fuels capacity developed drops as returns on investment increase. This is because the emissions of synthetic fuels must be paid for through the purchase of extra emissions rights. As the returns on investment grow, it becomes cheaper to use low carbon technologies than to pay for the carbon rights.

6 Carbon Taxation

Explicitly accounting for the uncertainty of returns on investment results in a significantly higher level of expected consumption over the study horizon for both the case of carbon constraints with and without carbon rights trading. The question for policy makers is how to induce private decision makers in the U.S. economy to adapt to the low carbon future in a way that optimally hedges against the uncertainty surrounding investment decisions. The most straightforward method for doing this is to use a carbon tax. That is, a tax on those fuels containing carbon with the proceeds used to subsidize the use of low or no carbon alternatives. Inducing decision makers to follow the expected value maximizing path simply means charging the carbon tax that is derived from the HN problem rather than that derived from the EMV problem.

The value of the carbon tax varies from period to period and scenario to scenario and is calculated from the dual values of the constraints on both carbon production and energy costs. This is accomplished in the following way. The dual of the constraint governing the production of CO_2 indicates the penalty that would have to be charged on each unit of CO_2 produced for the economy to optimally produce no more than the amount that is called for by the constraint. To convert the penalty on carbon production into a tax on the price of each commodity based on the carbon content of the commodity, we divide the dual of the carbon constraint by the dual on energy costs. This gives us the uniform tax on each billion tons of carbon produced by each commodity that is to be added to the cost of the commodity. Clearly, those commodities with higher levels of carbon will be taxed more heavily, while those with little or no carbon content will be taxed little or not at all. The tax revenue recirculates within the economy, subsidizing those low carbon inputs to production that maximize expected consumption.

The carbon tax schedule is most dramatically different between the optimal hedging strategy and the EMV solution for the case of no trade. This makes sense as world trade in carbon emissions rights helps to smooth over not only regional differences in the value of carbon emissions, but scenario differences as well. Because of this, we will focus the discussion on the no trade case as this will most clearly show how the carbon tax schedule changes in response to the presence of uncertainty. Figure 17 shows the time path of a carbon tax for scenario one for the HN and EMV strategies. The most important area to look at in the plot is the level of taxation prior to the realization of uncertainty as this is the situation policy makers are in now. Prior to 2030, it is quite clear that the approximate hedging strategy undertaxes the production of carbon. The difference in the two tax rates is only \$41.6/billion tons of CO_2 (17%) in 2000 but rapidly grows to \$365.8 (100%) by the year 2020, ten years prior to the realization of uncertainty regarding the success of investment.

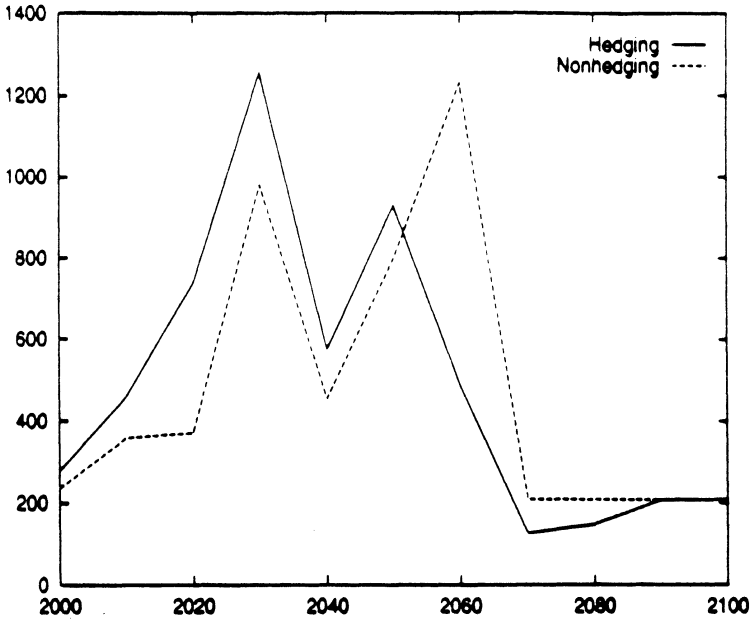

Figure 17: Carbon tax rate - Scenario 1

This higher early tax rate enables the economy to adapt more easily to the possibility of low returns on investment. As seen earlier, the U.S. economy following the EMV solution pays for its ill-preparedness by suffering a massive drop in consumption in 2030. What is most remarkable about this is that in spite of the much higher levels of taxation in the hedging strategy during the early periods, consumption and economic output during these periods differs only marginally from these same variables in the EMV solution.

After investment returns become known, the hedging strategy still maintains a higher level of taxation than the EMV strategy. This continues until 2050, at which time the taxation rate for the EMV strategy increases dramatically while the tax rate in the hedging strategy declines and converges to the steady state tax of \$208/billion tons of CO_2. This steady state rate arises because in the long run both synthetic fuels and the NE-BAK technology are used to provide energy. For them to both be equally attractive to purchasers of energy services, the tax rate must make the synthetic fuels as expensive as the NE-BAK technology. This is accomplished with the steady state tax rate, equal to their cost differential divided by their carbon content differential.

The time path of the carbon tax is different after 2030 for the other scenarios. In particular, for scenarios three and four, the tax rate in the hedging strategy is greater over the entire horizon than that for the EMV solution and the steady state tax rate is reached forty years earlier than in scenarios one and two.

Utilizing the carbon tax determined from the optimal hedging strategy is the simplest policy option available for inducing the U.S. economy to evolve optimally into a future of lower carbon emissions. Of course, imposing the tax is only part of the solution. The money collected from the tax must be redistributed within the economy in the same way as is done by the optimal hedging strategy. In particular, greater investments into the development of low carbon technologies. The tax revenues can be used directly by the government to fund research (e.g. national laboratories) or can be returned to the private sector in the form of tax incentives to private firms with an interest in developing alternative energy technologies.

7 Conclusion

As the discussion in this paper has indicated, uncertainty about the future directly impacts decisions made today in our economy. As a result, they must be accounted for when making predictions about what impact public policy will have on the direction of an economies development. When uncertainty is not explicitly considered in policy development we see that:

- Undertaxation of carbon production occurs early in the planning horizon.

- Consumption experiences severe shocks in the pessimistic scenario.
- Overinvestment in new technologies occurs prior to the realization of uncertainty followed by underinvestment.
- Domestic stocks of nonrenewable resources are depleted more quickly than is optimal.

In addition, it has been shown that the presence of an international market for the trade of carbon rights significantly decreases the negative impact that nonoptimal governmental policy (policy derived using the expected value model) has on the health of the economy and markedly improves economic welfare (expected consumer and producer surplus) regardless of the quality of governmental policy.

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