ENGINEERING RESEARCH INSTITUTE
UNIVERSITY OF MICHIGAN
ANN ARBOR

THEORETICAL STUDY, DESIGN AND CONSTRUCTION OF C-W MAGNETRONS FOR FREQUENCY MODULATION

QUARTERLY PROGRESS REPORT NO. 1

Period Covering December 1, 1949, to March 1, 1950
Electron Tube Laboratory
Department of Electrical Engineering

BY

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Project M762

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TABLE OF CONTENTS

PERSONNEL

Scientific and Engineering Personnel		* Time worked in man-hours.
W. G. Dow	Professor of Electrical Engineering	Supervisor
H. W. Welch	Research Physicist	1.49
J. R. Black	Research Engineer	2.52
G. Hok	Research Engineer	.44
H. A. Martens	Research Engineer	.65
G. R. Brewer	Research Associate	1.54
H. W. Batten	Student Assistant	.23
W. W. Peterson	Student Assistant	.20

Service Personnel		
V. R. Burris	Machine Shop Foreman	1.44
R. F. Steiner	Assembly Technician	2.83
J. W. VanNatter	Assembly Technician	1.58
Mrs. G. R. Merithew	Draftsman	1.93
N. I. Navarre	Draftsman	.75
J. Mossar	Technician	1.26
M. J. Walker	Stenographer	.86
S. Spiegelman	Stenographer	.47
D. L. McCormick T. G. Keith E. A. Kayser	Laboratory Machinists	4.00

* Time worked is figured on the basis of 172 hours per month.

THEORETICAL STUDY, DESIGN AND CONSTRUCTION OF

C-W MAGNETRONS FOR FREQUENCY MODULATION

QUARTERLY PROGRESS REPORT NO. 1

MARCH, 1950

I. OBJECTIVES FOR THE PERIOD (H. W. Welch, Jr.)

The purpose of this report is to summarize the progress in the University of Michigan Electron Tube Laboratory during the period from December 1, 1949, to March 1, 1950, on Contract No. W-36-039 sc-35561.

The general objectives of the program under this contract are to increase the knowledge of space charge effects and frequency characteristics of c-w magnetrons and apply this knowledge to the development of methods for the frequency modulation of magnetrons in the 2000-2400 mc range. The general technique which has been adopted for development is the use of the magnetron-type space charge as a modulating element.

In summary the status quo at the beginning of December was as follows:

(a) The understanding of the magnetron type of space charge insofar as effects on frequency were concerned was fairly complete, based on experimental and theoretical observations presented in Technical Report No. 1, issued in November, 1948, on Contract No. W-36-039 sc-32245.

(b) Three designs for frequency modulation magnetrons had been developed. Six tubes of one design (Model 6) had been started in construction.

Four of these operated in the desired mode. Sketchy modulation data were obtained on one. A second type of tube (Model 5) was under construction. Cold tests were being made on the third type (Model 8) to obtain data necessary to complete design.

(c) A program to supplement previously obtained experimental data on the characteristics of the space charge had been initiated with the purpose of checking unexplored regions of magnetic field, thus providing further check on theoretical analysis.

The primary objectives for the period covered by this report were the following:[1]

(a) Construction and testing of more Model 6 f-m magnetrons

(b) Construction of Model 5 f-m magnetrons

(c) Completion of design of the Model 8 f-m magnetrons

(d) To obtain experimental data on the magnetron space charge using Model 4 interdigital magnetron and other specially constructed tubes

(e) To study further the theory of the space charge with emphasis on statistical methods and any other methods, theoretical or experimental, not heretofore considered in detail.

This report is intended only to report on progress in construction of tubes and development of theory and design during the period. The following previously issued reports should be consulted if further information is desired.

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[1]Model numbers referred to here are explained in the text of the report. Drawings of all tubes are also included.

On Contract No. W-36-039 sc-32245

Space-Charge Effects and Frequency Characteristics of C-W Magnetrons Relative to the Problem of Frequency Modulation, Technical Report No. 1, issued November, 1948.

Operation of Interdigital Magnetrons in the Zero Order Mode, Technical Report No. 2, issued May, 1949.

Theoretical Study, Design and Construction of C-W Magnetrons for Frequency Modulation, Final Report, Technical Report No. 3, issued May, 1949.

On Contract No. W-36-039 sc-35561

Theoretical Study, Design and Construction of C-W Magnetrons for Frequency Modulation, Interim Report, issued December, 1949.

The basic principles of operation of all of the f-m tubes (Models 5, 6 and 8) are the same. Each tube has two cathodes and two anode sets in a single cavity. The cathodes are essentially separated for d-c or low frequency. The anodes are at ground potential. One cathode is located in the oscillator set of anodes and is supplied with the d-c voltage necessary to produce oscillation. The other cathode is located in the modulator set of anodes and is to be supplied with the modulating voltage. The modulating voltage varies the diameter or the density of the magnetron-type space charge swarm in the modulator set of anodes. This variation affects the capacitance of the resonant system. The resonant frequency is, therefore, changed at a rate corresponding to the rate of change of the modulating voltage.

The modulator section takes different forms in the different tubes and is subject to any changes which may be indicated necessary by test results. The interaction space of the oscillator section is, at present, essentially the same in Model 5 and Model 6. The design has been changed in the Model 8 to improve oscillator performance. Models 5 and 6 will probably be changed when tests on the new structure are complete. The resonant system in each of the f-m magnetrons is unique except for the use of interdigital anode sets. Each of these tubes will be discussed in detail as modulation data become available, thus making possible a more integrated picture of their performance.

II. CONSTRUCTION AND TESTING OF F-M MAGNETRONS

Although only one Model 5 tube was built (Serial No. 27), the tube has been tested and reassembled with minor changes three times. Only one Model 6 tube was successfully completed during the period (Serial No. 31). Another tube (Serial No. 29) was lost in brazing.

Design of a third type of f-m magnetron called Model 8 is nearly complete. This tube will use an anode design different from the Model 5 and Model 6 in an attempt to improve oscillator characteristics.

An experimental tube for space charge studies is near completion. This tube is discussed briefly in Section III.

A. Model 5 F-M Magnetron (H. W. Welch, Jr., J. R. Black)

The Model 5 is a nontunable interdigital tube utilizing coupling to the cathode line to introduce the effect of the modulating space charge supplied by a second cathode. (An assembly drawing is shown in Figure 1.) The first magnetron of this type was completed in January. The tube would not oscillate in the zero order mode and sparking between the oscillator and modulator cathode was observed when pulsed voltages were applied to the oscillator cathode. X-ray showed the modulator cathode to be distorted and making near contact with the oscillator cathode in the by-pass sleeve between them. Upon taking the tube apart it was discovered that a lavite spacing insulator between the two cathodes (not shown in the drawing) had been displaced and wedged between the cathodes in such a way as to cause

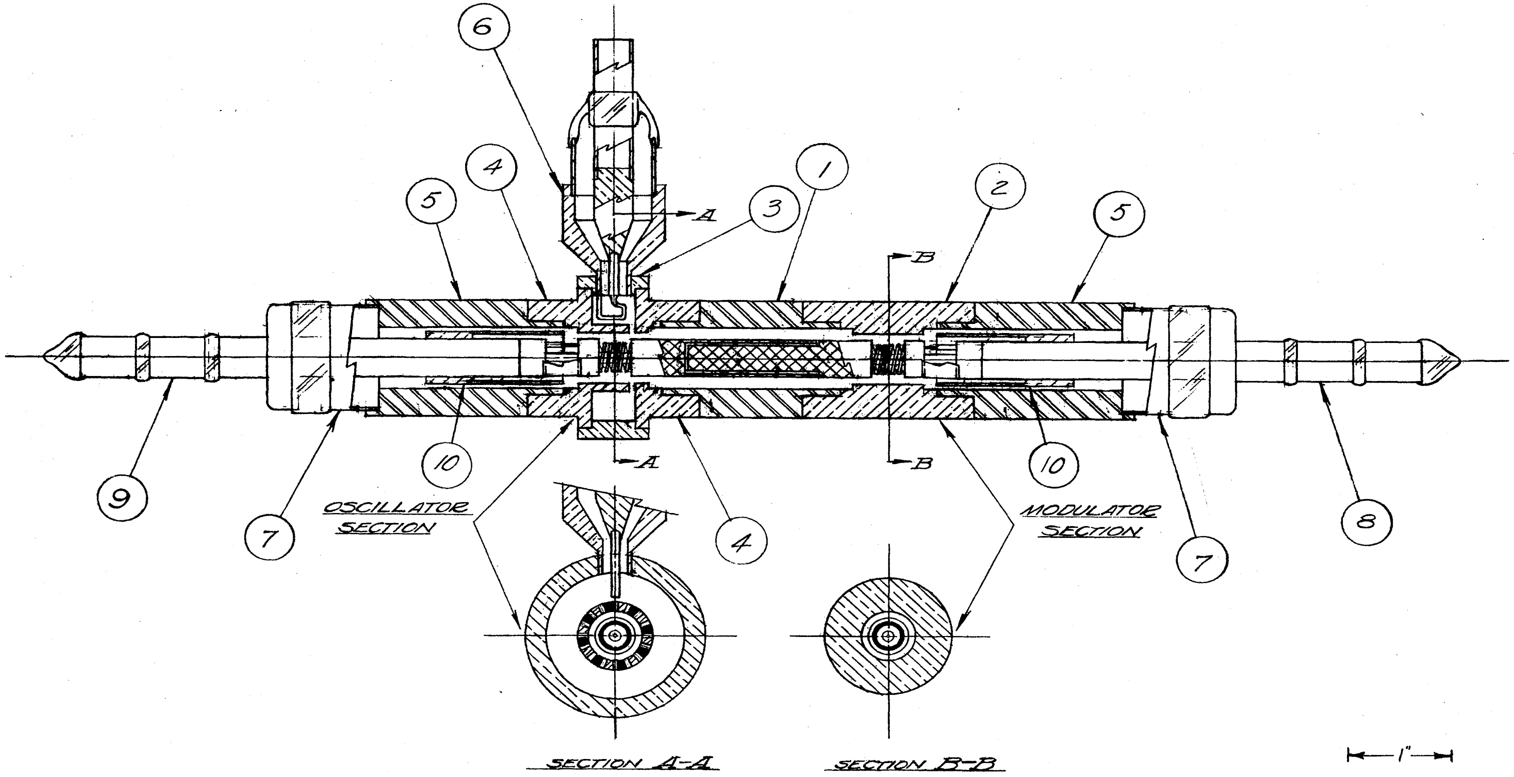

FIG. 1

ALL DIMENSIONS UNLESS OTHERWISE SPECIFIED MUST BE HELD TO A TOLERANCE - FRACTIONAL ± 1/64" DECIMAL ± .005" ANGULAR ± 1/2°

ISSUE	DATE			
3	12/20/44	DEPARTMENT OF ENGINEERING RESEARCH, UNIVERSITY OF MICHIGAN, ANN ARBOR MICHIGAN	DESIGNED BY H.W.W / DRAWN BY 9HW / CHECKED BY H.W.W.	APPROVED BY H.W.W / SCALE FULL SIZE / DATE 4/14/48
2	7/6/50	PROJECT M-694	TITLE FM MAGNETRON	
1	4/14/48	CLASSIFICATION	DWG. NO. B-10,005	

distortion on expansion of the cathodes when they were heated.

The tube was reassembled without the lavite spacer. The same results were observed on testing as before, and another X-ray showed the cathode to be out of line again. This time it had occurred in assembly. Needless to say, after reassembly the tube was X-rayed immediately. This time the cathode was all right.

The results of hot tests on this tube (finally satisfactorily assembled about March 1) may be summarized as follows:

(a) Oscillations were observed in the desired mode at 15.34 cm only by unloading the tube to the point where power output was not measurable.

(b) Rather strong coupling to both cathodes is observed at 15.34 cm. This is to be expected since the cathode chokes are designed for 14 cm. No coupling of the 10-cm mode to the cathode was observed. A check of data on the Model 4 magnetron which has the same oscillator section (A-A in Figure 1) indicates that the anode spacing should be .080 inch instead of .050 inch as it is in the Model 5 for oscillation at 14 cm. (See Interim Report issued December 15, 1949, for details on the Model 4.) In order to shift the resonant wavelength of this particular tube, the cathodes have been removed and the chokes (part no. 10 in Figure 1) shortened by the insertion of a copper sleeve .75 cm in length between the cathode stem and the by-pass sleeve at the base of the choke. Effectively the cathode line, which is part of the resonant circuit, is shortened 1.5 cm. This should change the resonant wavelength by about the same amount. The tube is being reassembled with this change.

(c) No modulation data were obtained because of the erratic behavior of the oscillator.

B. Model 6 F-M Magnetron (H. W. Welch, Jr., J. R. Black)

The Model 6 f-m magnetron has a coaxial resonant cavity, two anode sets, and two cathodes. There are sixteen anodes in the oscillator section and four in the modulator section. Resonant wavelength in the desired mode is approximately 13 cm. For this mode, the coaxial cavity is one wavelength long with a voltage maximum at each anode set. Eight tubes have been started in construction; five have been tested with the oscillators in the desired mode. The other three were lost during assembly. The tube is not tunable, but because of the simplicity of the coaxial cavity it has good possibilities from this point of view. An assembly drawing is given in Figure 2. This drawing shows the changes made in Model 6, Serial No. 31. In order to shift the 10.96-cm mode, which is a vane resonance, to a shorter wavelength and thus increase mode separation, the ring (part no. 14) was placed in the cavity to shorten the vanes. The ring has an insignificant effect on the desired mode for which the vanes represent a lumped capacitance. Serial No. 31 also has a slightly larger diameter modulator anode than previous Model 6 tubes. However, this change produced no appreciable effect.

Performance data on Model 6, No. 26 with modulator cathode removed are given in Figure 3. For this tube the higher order mode was 10.96 cm at a voltage about 10 percent higher than the desired 13-cm mode. A pulsed characteristic of tube No. 31 is shown in Figure 4. Here the higher order mode is shifted to 9.2 cm and voltage separation is correspondingly greater. The modulator cathode, which has always interfered with operation in the wanted mode, seemed to have an especially strong effect in this tube. By selective shielding of the glass seals it was discovered that the power

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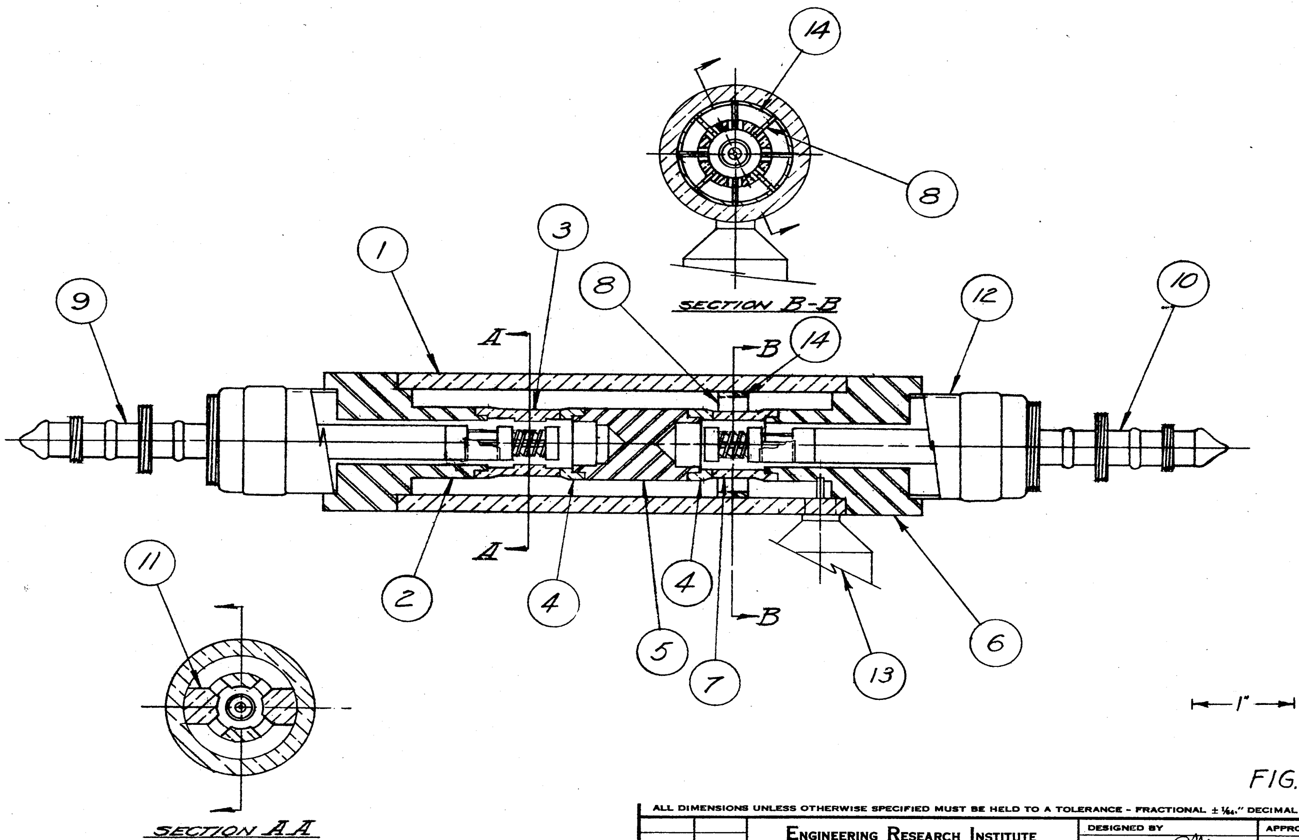


FIG. 2

ALL DIMENSIONS UNLESS OTHERWISE SPECIFIED MUST BE HELD TO A TOLERANCE - FRACTIONAL ± 1/64," DECIMAL ± .005," ANGULAR ± 1/2°

ENGINEERING RESEARCH INSTITUTE UNIVERSITY OF MICHIGAN ANN ARBOR MICHIGAN	DESIGNED BY DRAWN BY JAM CHECKED BY VRB	APPROVED BY SCALE FULL SIZE DATE 2-20-50	
PROJECT M-762	TITLE FM MAGNETRON MODEL #6A		
ISSUE 1 · DATE 2-20-50 · CLASSIFICATION	DWG. NO. B-10,006A		

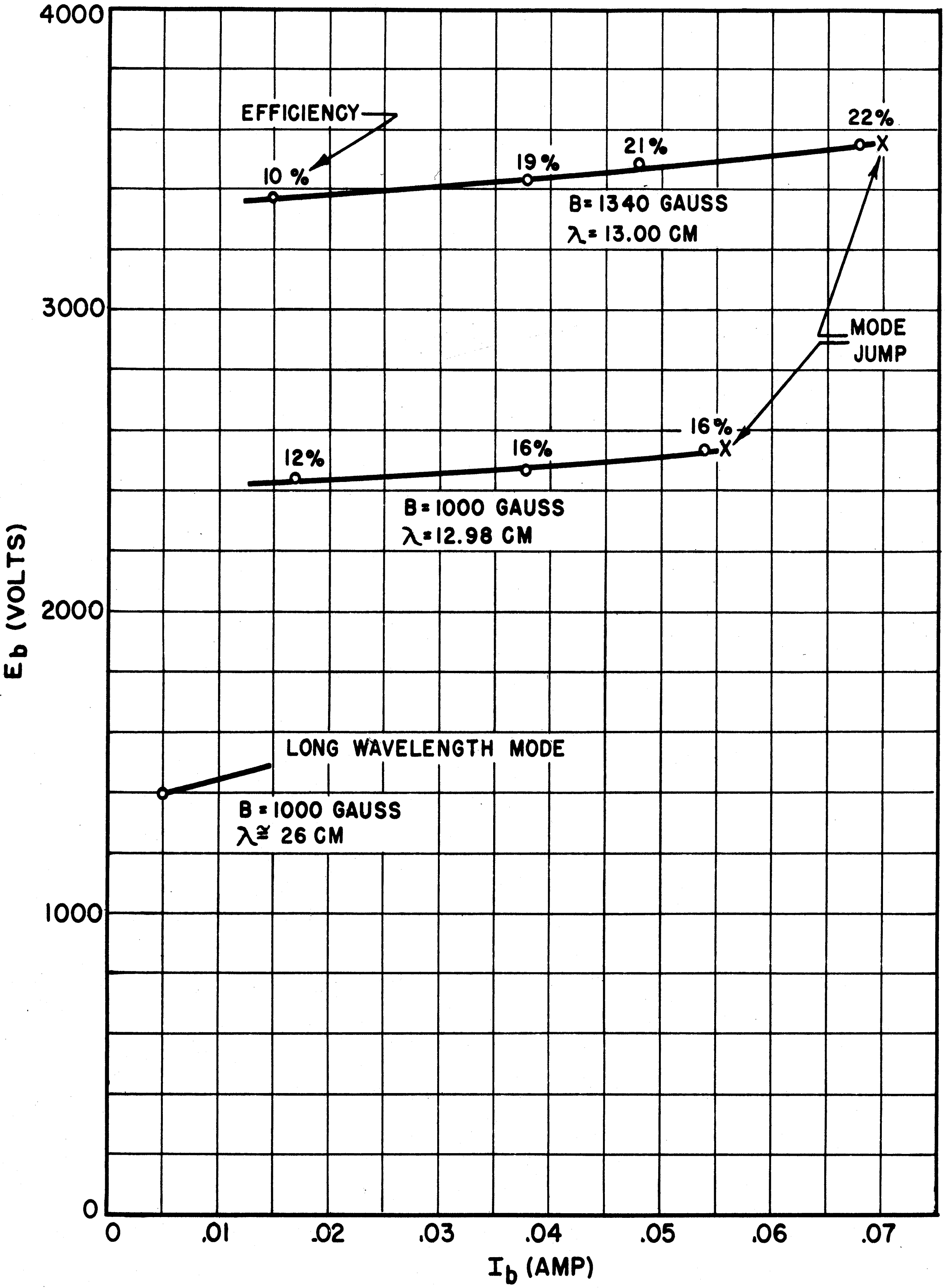


FIG. 3 PERFORMANCE CHARACTERISTICS MODEL 6 #26

MODULATOR CATHODE IS REMOVED

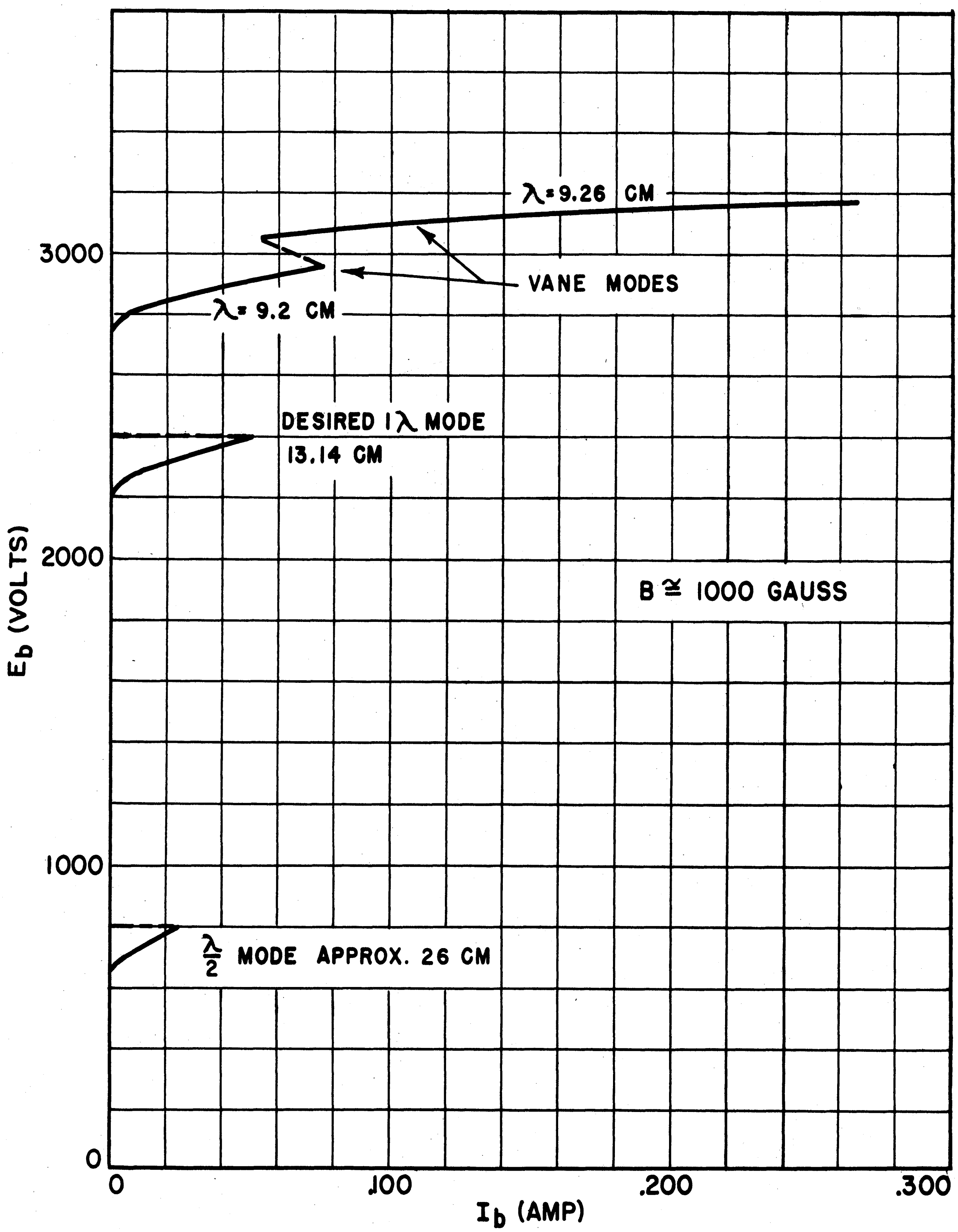


FIG. 4 VOLT-AMPERE CHARACTISTIC PULSED MODEL 6 #31

MODULATOR FILAMENT LEAD SEALS ARE SHIELDED. MAIN CATHODE SEAL IS NOT SHIELDED. THE 13.14 CM MODE DOES NOT APPEAR WITHOUT SHIELDS.

leakage was out the filament leads, not the outside of the stem. This coupling could only take place at the helix or through holes in the stem provided for evacuation. The exact nature of the coupling has not been determined yet, but the cathode has been removed and will be replaced by an oxide cathode where this type of power leakage will be impossible.

Modulation data were obtained on the Model 6, No. 26. With the modulation cathode in place it was necessary to unload the tube to make it oscillate in the desired mode, so no power measurements were made. However, some indication of modulation performance is given and the results are encouraging when one considers that as an oscillator the tube is still behaving very poorly for reasons completely independent, but possibly detrimental to the electronic modulation. Theoretically predicted values of wavelength shift and r_H/r_c (space charge swarm radius/cathode radius) are given with the experimental results in Figure 5. These predictions are based on the assumption that the capacitance between the cathode surface and the modulator anodes is 5.15 percent of the total modulator anode capacitance. This figure is obtained from a flux plot of the modulator structure with the assumption that the cathode potential was halfway between the potentials of the two sets of modulator anodes. This percentage is probably the least accurate determination in the calculation because of the complexity of the modulator construction, as is fairly obvious from examination of Figure 2. If the cathode is assumed to be at the potential of the inner conductor of the coaxial cavity, a figure as high as 24 percent is obtained.

If it is assumed that the modulating capacitance, i.e., anode to emitting surface capacitance, is 5.15 percent of the total anode capacitance, then the following steps lead to the calculation of wavelength shift.

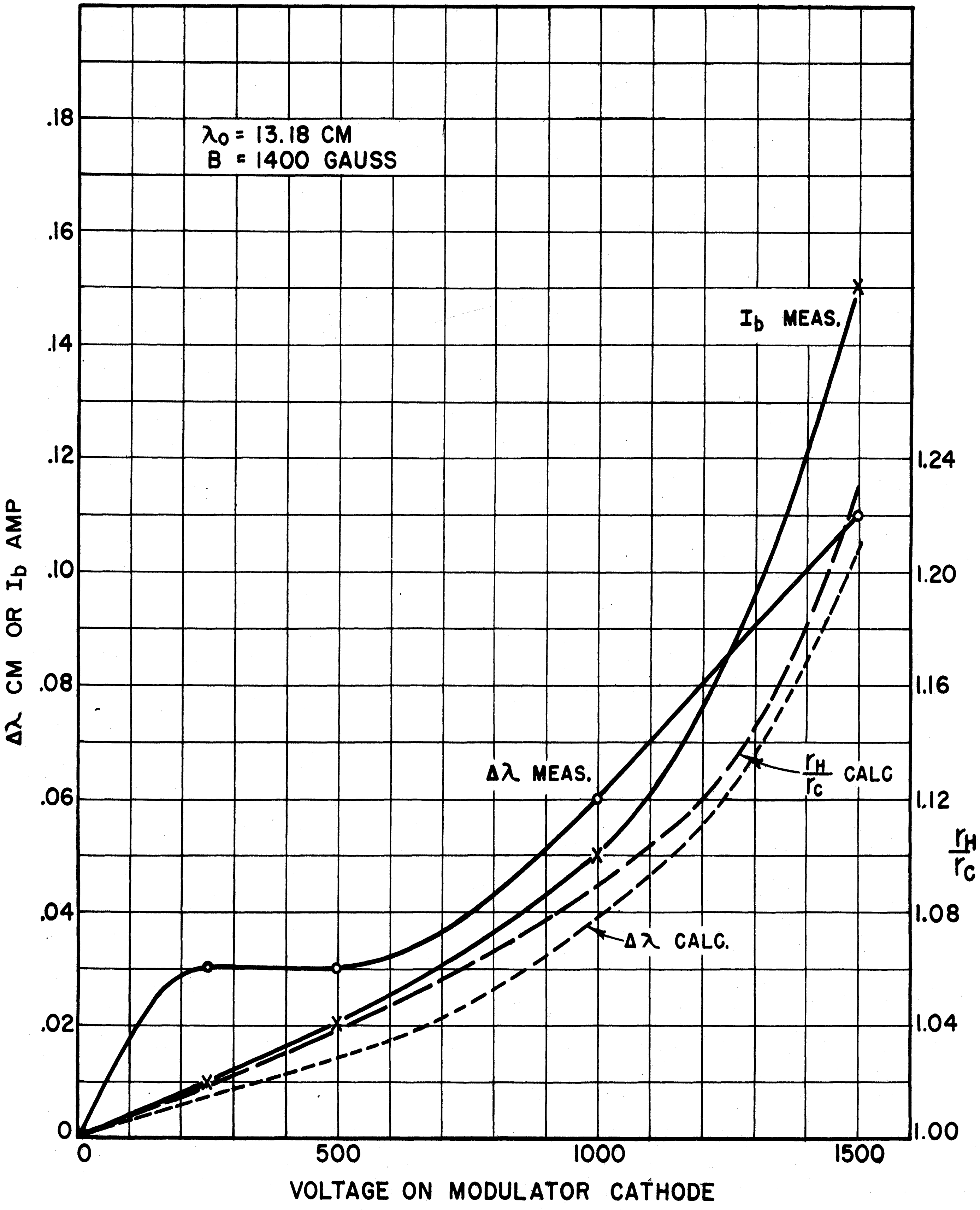


FIG. 5 MODULATION DATA ON MODEL 6 #26

The formula for resonance in a half-wavelength cavity corresponding to the modulator half of the cavity is

$$\frac{1}{\frac{2\pi c}{\lambda} C_A} = Z_o \tan \frac{2\pi \ell}{\lambda} \qquad (1)$$

If C_A is changed by dC_A to produce a wavelength shift $d\lambda$, then differentiating the above equation

$$\frac{d\lambda}{\lambda} = \frac{dC_A}{C_A} (1 + 2\theta \csc 2\theta)^{-1} \qquad (2)$$

where

$$\frac{dC_A}{C_A} = \frac{\Delta C_c}{C_c} \frac{C_c}{C_A}$$

and $\theta = 2\pi\ell/\lambda$

C_c = capacitance to cathode surface

C_A = total modulator anode capacitance

$\frac{\Delta C_c}{C_c}$ = percentage change in cathode capacitance caused by space charge (see Technical Report No. 1, Equation 7.5).

Approximately half of the total energy storage in the cavity is in the oscillator section, so the result, Equation (2), is multiplied by 0.5 to get the over-all estimate of wavelength shift. The function of θ is plotted in Figure 6. For this particular case:

C_c/C_A = 5.51 percent

θ = 65°

$\Delta C_c/C_c$ = 123 percent for maximum r_H/r_c .

r_H/r_c was determined from Figure 13 of Technical Report No. 1 (r_a/r_c = 1.33).

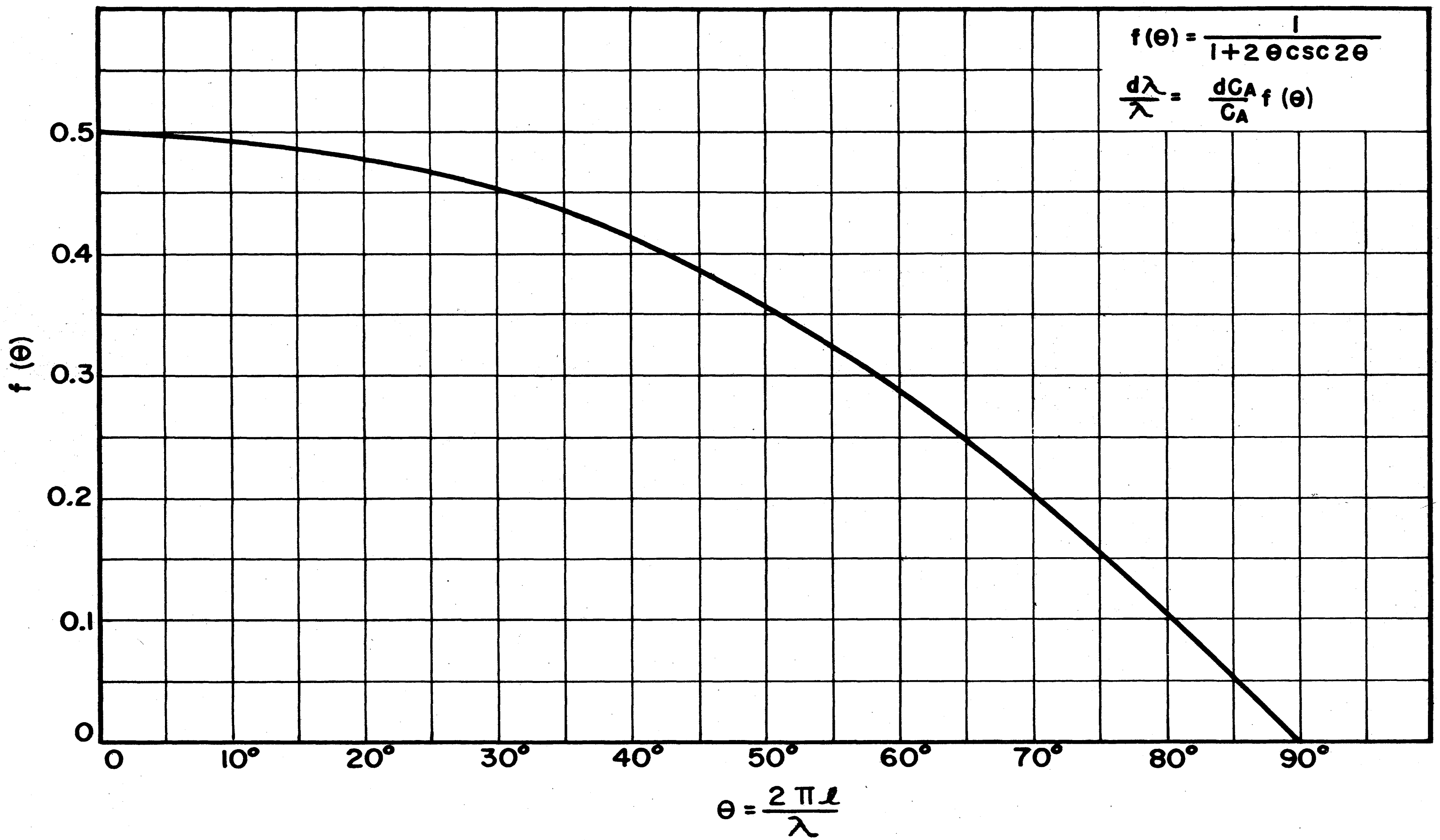


FIG. 6 FACTOR USED IN CALCULATING WAVE-LENGTH SHIFT IN MODEL 6

As soon as the problem of coupling to the cathode is eliminated and normal oscillation is obtained, the design of the Model 6 will be changed to promote more efficient behavior of the oscillator section and to make the modulating capacitance a larger percentage of the total. However, it does not seem desirable to make more than one or two changes at a time in the tube structure until the design reaches a more clear-cut stage. Otherwise it becomes difficult to compare performance of successive tubes.

C. Model 8 Rectangular Cavity F-M Tube (J. R. Black)

Due to the difficulties encountered in the Models 5 and 6 f-m magnetrons, a stronger emphasis has been placed on the study of the Model 8 structure. As seen from Figures 7 and 8, this structure is essentially a capacity loaded full-wavelength cavity, having two sets of anodes, each set placed at a voltage maximum. If one anode set is designed as an oscillating magnetron and the other as a variable reactance, the structure would form an f-m magnetron. Also, it is immediately obvious that if both anodes are designed as oscillators, an increase in output power would be realized in a push-pull type of oscillation. Very high power tubes might be constructed by lengthening the cavity and employing several cathodes and sets of anodes.

A sketch of a brass Model 8 cavity for cold testing is shown in Figure 7. The ratio of length to width of the cavity has been made exactly 2 to 1 in an attempt to eliminate complex resonance observed in earlier cavity. (See Section 13c of Final Report, Technical Report No. 3.) The brass model was built with the same type of anodes as in Model 5 magnetron. (See Figure 1.) The seven holes evenly spaced along one side enable one to determine the field pattern by means of a small probe. Figure 8 shows the field

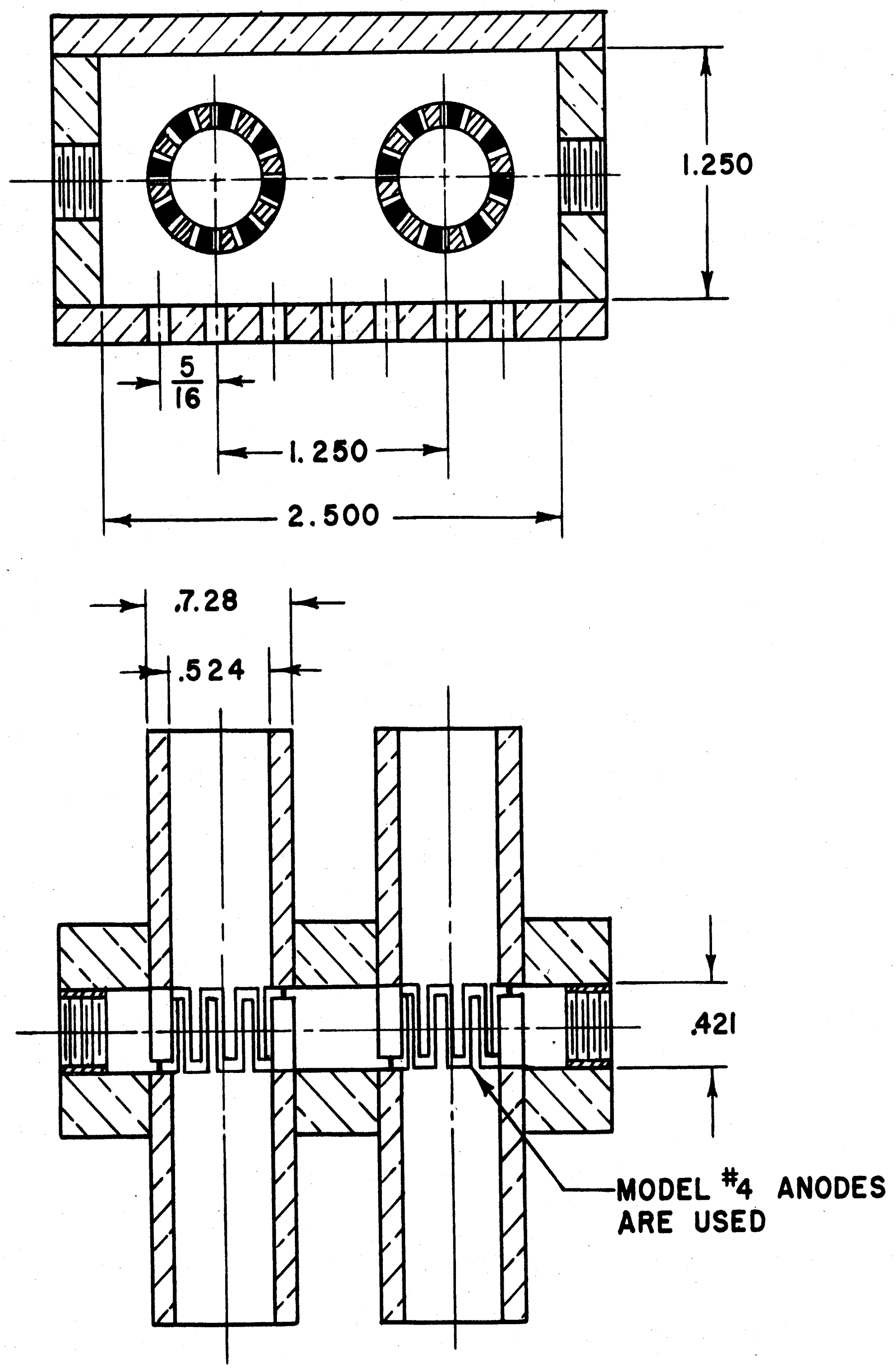


FIG. 7 SKETCH OF MODEL #8 MAGNETRON USED FOR COLD TEST

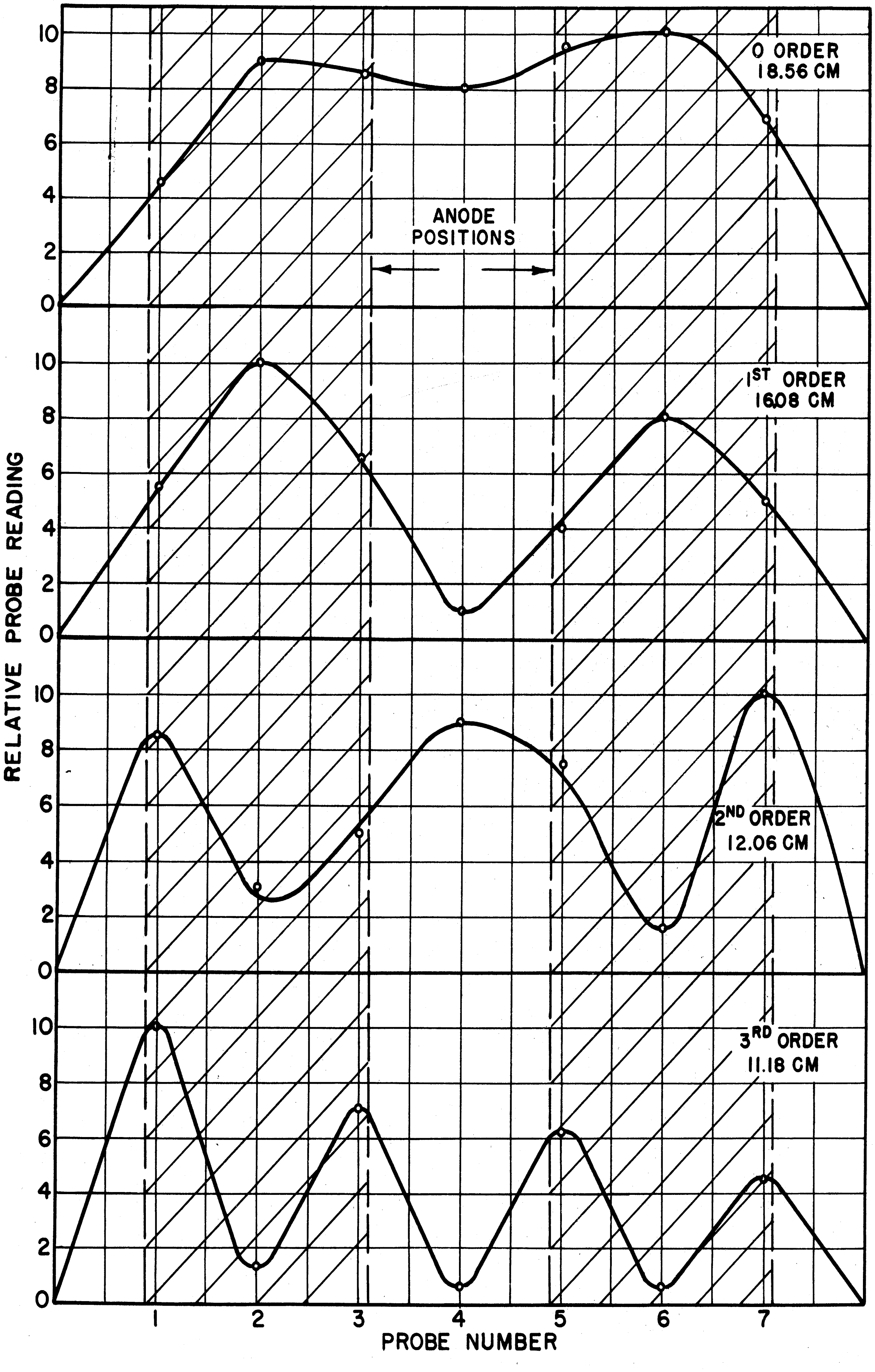


FIG. 8 RESONANCES IN RECTANGULAR CAVITY MAGNETRON

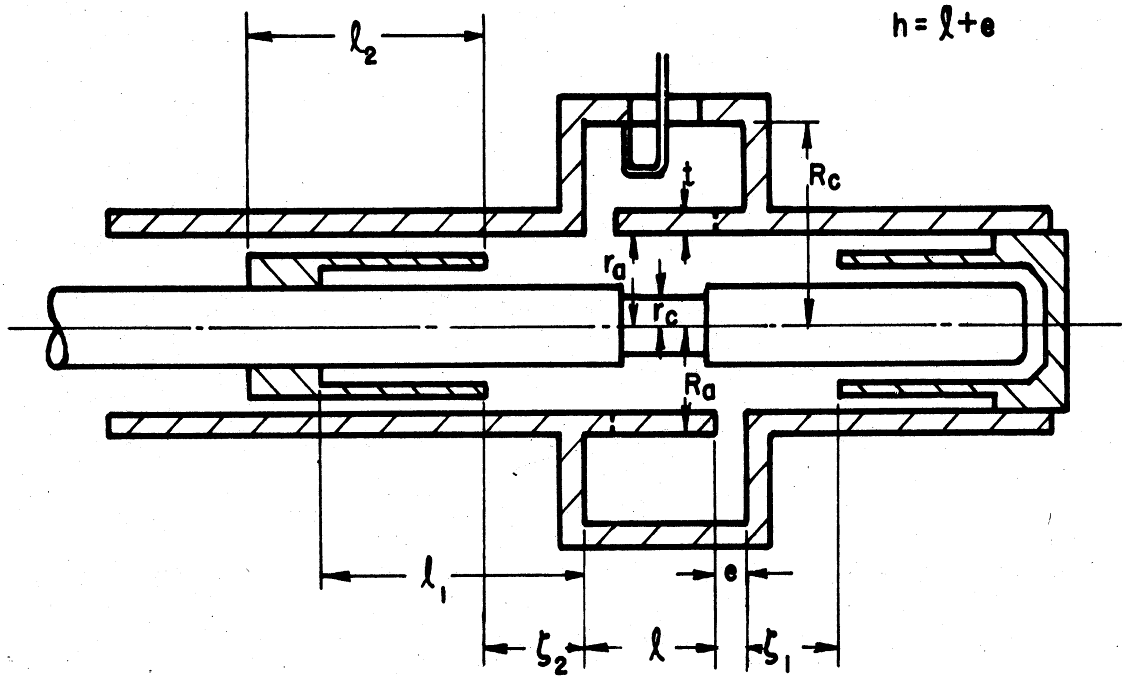
patterns for the zero, first, second, and third order modes. The two shaded areas indicate the positions of the two anodes. The desired mode of operation is the first order at 16.08 cm for this model; however, it would probably be capable of operating all four such modes. The zero and second order modes could easily be suppressed by short circuiting them in the center of the cavity where the desired mode has a node.

A working model of this structure was designed from the data obtained from the cold test model, and the parts are now being constructed in the shop. The frequency was scaled to be 13 cm. A new interaction space design will be used which should work more efficiently than the design which has been used in the Models 4, 5 and 6.

The following are the design parameters used for the Model 8 tube which will immediately be started in construction. For symbols used, see Figure 9. Chokes will not be designed until the resonant wavelength of the cavity is determined exactly from cold tests.

r_a	.45 cm	cavity	3 x 6 cm
r_c	.30 cm	h	1.02 cm
r_a/r_c	1.5	ℓ	.90 cm
L	.724 cm	d	.089 cm
R_a	.762 cm	N	16
		C_A	4 $\mu\mu$f

In this tube both sets of anodes were designed as oscillatory structures. This will check the possibility of push-pull operation as well as f-m features of the structure.



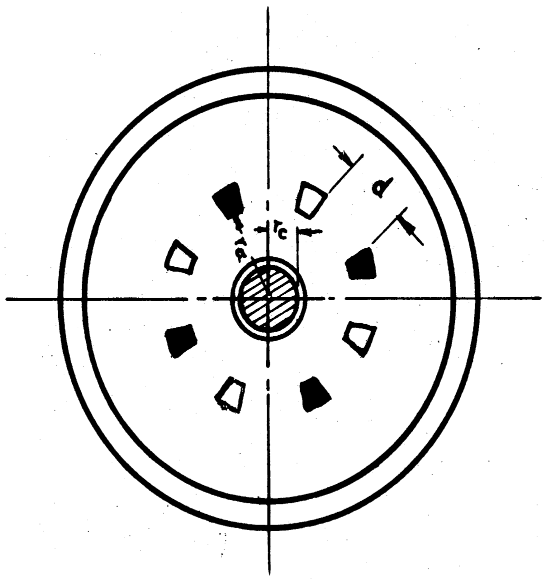


FIG. 9 SCHEMATIC AND EQUIVALENT CIRCUIT OF INTERDIGITAL MAGNETRON

SK-195

III. STUDY OF MAGNETRON SPACE CHARGE

The study of magnetron space charge was more or less deemphasized during 1949, under the pressure of development of the interdigital tubes and the f-m magnetrons. However, for several reasons, it has been resumed during the period covered by this report. These reasons are the following:

(a) Experimental data supporting the general theory presented in Technical Report No. 1 were inadequate, although important reactive properties of the space charge which are usable in the production of frequency modulation were conclusively verified. In order to complete the picture, a detailed survey of space charge swarm properties insofar as they effect wave propagation would be necessary for wide ranges of magnetic field and for various orientations of direction of propagation and polarization of the wave with respect to the magnetic field.

(b) No analysis or extensive experimental data exist which give an understanding of the nature of losses in the magnetron space charge swarm. This is a rather serious problem in the use of such a swarm for modulation and may be the limiting feature. Knowledge of the magnitude of this loss, the relative importance of back bombardment of the cathode and collection of current by the anode, and a theoretical understanding of basic phenomena could be very useful in devising methods for minimizing or eliminating the loss.

(c) During the development of f-m magnetrons of more or less unconventional design, problems involving the space charge in the operating magnetron have been continually recurrent, i.e., mode jump current, optimum loading conditions, symmetry of fields in the interaction space, etc. In trying to understand behavior of the oscillator section of these magnetrons, the answer is continually that knowledge of the space charge under these conditions is inadequate. An experimental approach to augment this knowledge seems most desirable as a first step. Two general methods are available: the actual exploration of the potential and current distribution in the space charge by probe or electron beam; and the detailed study with the aid of Rieke diagrams of the effects of loading and interaction space design on frequency pushing, mode jump current and slope of the volt-ampere characteristic, all of which are related to space charge behavior.

An attractive feature of the program to increase understanding of the space charge is that positive results obtained in any part of the program will probably be helpful to the understanding of all three of the phenomena mentioned above and the over-all results contribute to the knowledge of both frequency modulation and magnetron oscillation.

A. R-F Properties of Magnetron Space Charge Swarm (H. W. Welch, Jr., G. R. Brewer)

The general purpose of this program is, by hot impedance tests, to supplement the experimental data presented in Technical Report No. 1 and to get some quantitative idea of the loss inherent in the use of the magnetron space charge as a modulating element. It was intended to make initial measurements on a Model 4 magnetron. This magnetron is particularly suited to the purpose because the cyclotron resonance, a critical point in any space charge phenomena, occurs at a point where the magnetron is normally expected to operate. (See Interim Report of December, 1949.) However, this tube operates at 14 cm and the behavior of signal generators available in this laboratory is very poor at this wavelength, making accumulation of a large number of impedance measurements a very discouraging undertaking. After considerable effort in this direction, it was decided to try the measurements on a 10-cm magnetron where klystron signal generators are satisfactory. An absorption technique was worked out which was usable for measurement of resonant wavelength although Q measurements could not be made.

A QK59 magnetron was used. This is not an ideal tube for the purpose, since the capacitance to the cathode is a very small fraction of the total anode capacitance, thus making effects of an expanding space charge swarm small. However, with care, reproducible readings of wavelength shift could be obtained. The data are presented in Figures 10 through 26. Resonant wavelength of the magnetron is plotted against E_a/B^2, where E_a is the anode potential in volts and B is the magnetic field in gauss. In a given magnetron, the radius of the static magnetron space charge swarm is determined by E_a/B^2 through the following relationship.

$$\frac{E_a}{B^2} = \frac{e}{8m}\left(\frac{r_H}{r_c}\right)^2 \left[2\left(1 - \frac{r_c^2}{r_H^2}\right)\log\frac{r_a}{r_c}\frac{r_c}{r_H} + \left(1 - \frac{r_c^2}{r_H^2}\right)\right] \qquad (3)$$

where

r_H = radius of space charge swarm

r_c = radius of cathode

r_a = radius of anode

e/m = electronic charge/electronic mass

Figure 27 is a plot of this equation for $r_a/r_c = 1.65$, as is the case in the QK59. Figure 28 is a plot of points taken from the data in Figures 10 to 26 for various values of swarm radius, i.e., values of E/B^2. These curves illustrate effects due to the space charge density or motion independent of extent of the space charge.

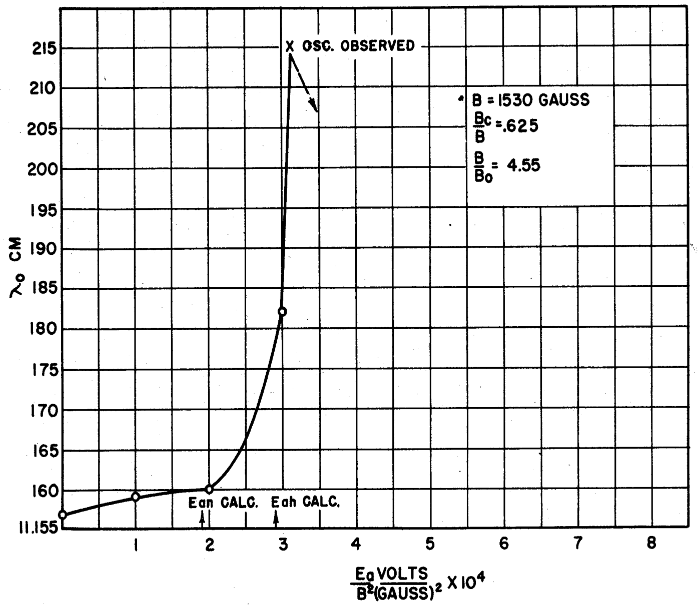


FIG. 10 HOT IMPEDANCE TEST ON QK59 NO. 2483

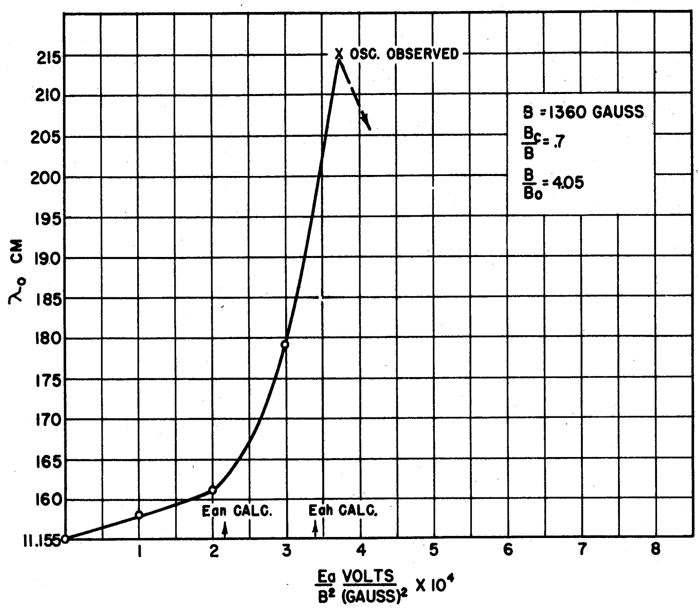


FIG. 11 HOT IMPEDANCE TEST ON QK59 NO. 2483

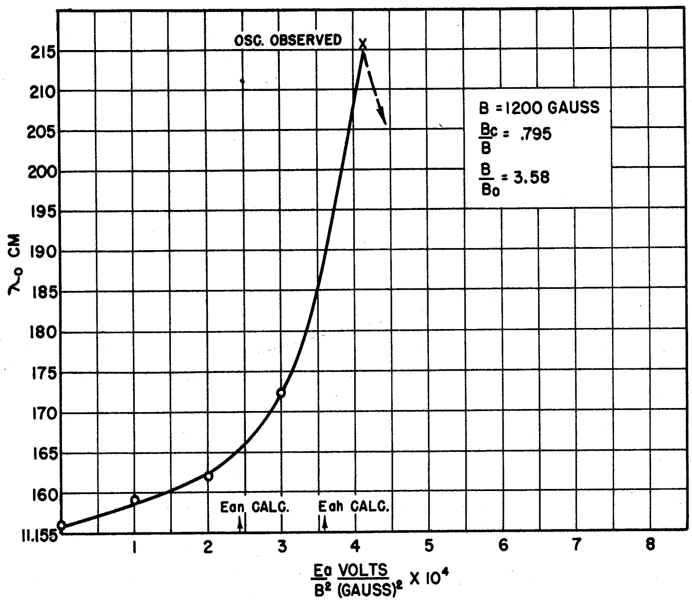


FIG. 12 HOT IMPEDANCE TEST ON QK59 NO. 2483

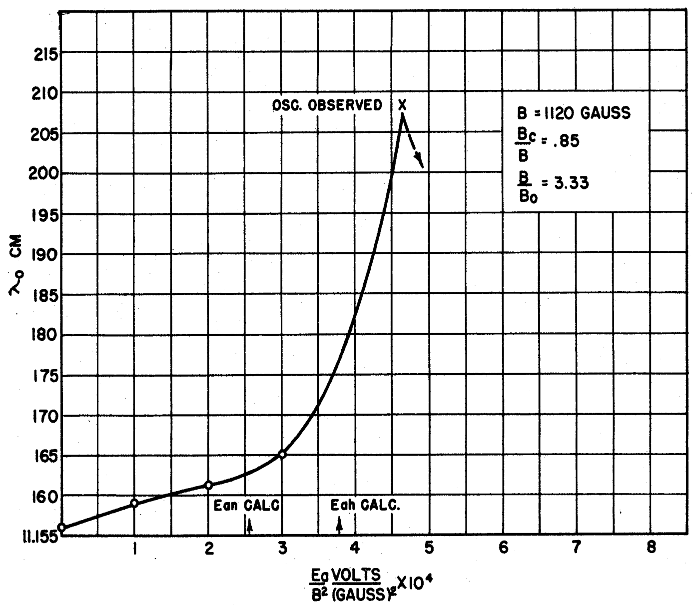


FIG. 13 HOT IMPEDANCE TEST ON QK59 NO. 2483

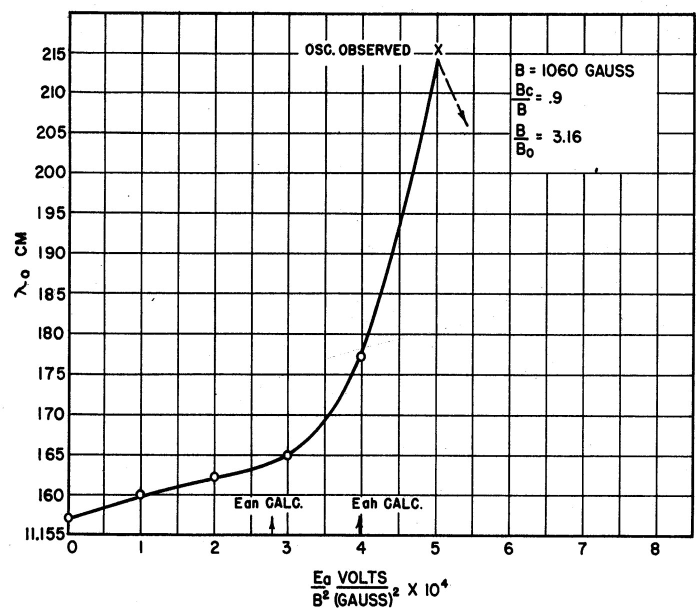


FIG. 14 HOT IMPEDANCE TEST ON QK59 NO. 2483

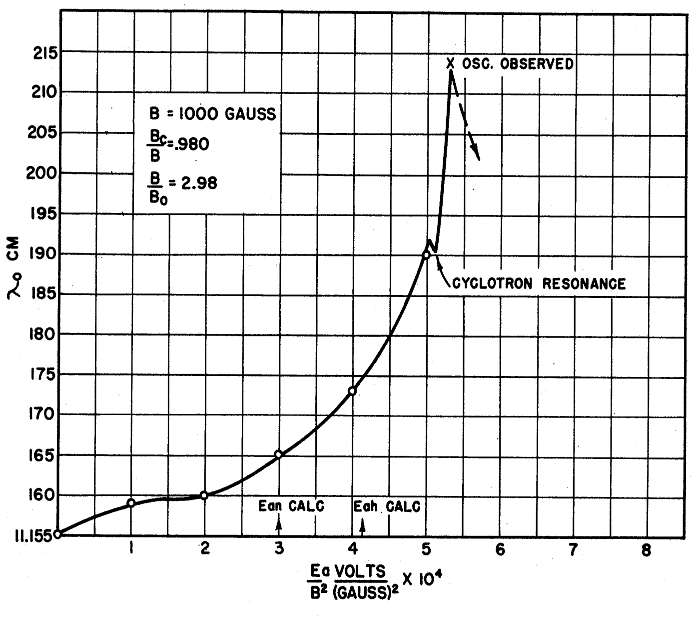


FIG. 15 HOT IMPEDANCE TEST ON QK59 NO. 2483

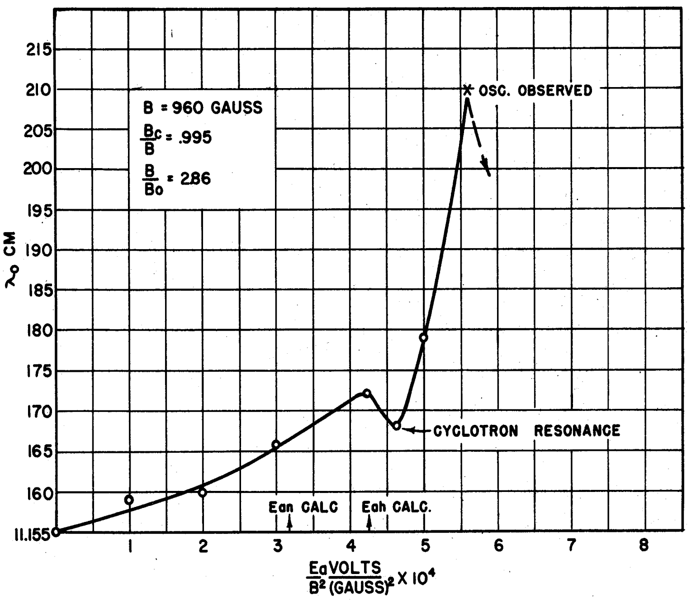


FIG. 16 HOT IMPEDANCE TEST ON QK59 NO. 2483

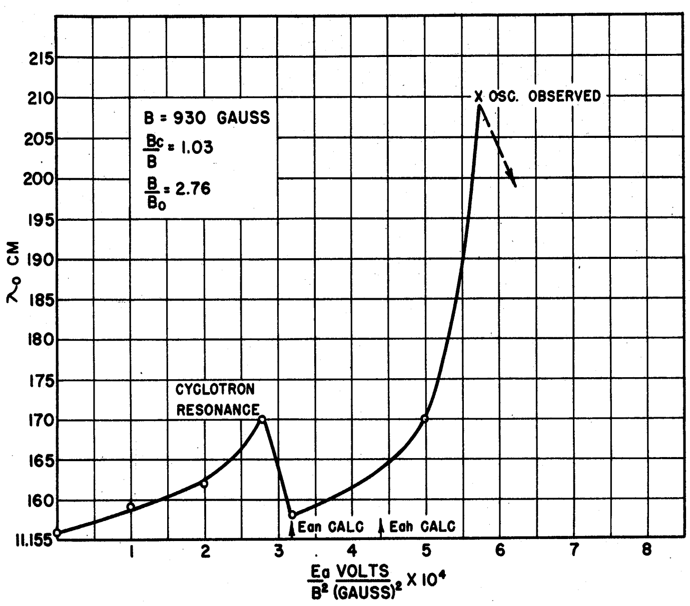


FIG. 17 HOT IMPEDANCE TEST ON QK59 NO. 2483

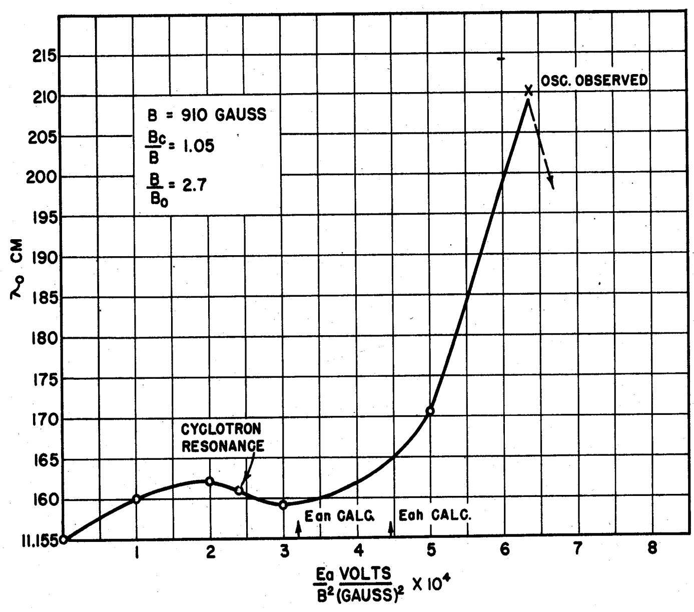


FIG. 18 HOT IMPEDANCE TEST ON QK59 NO. 2483

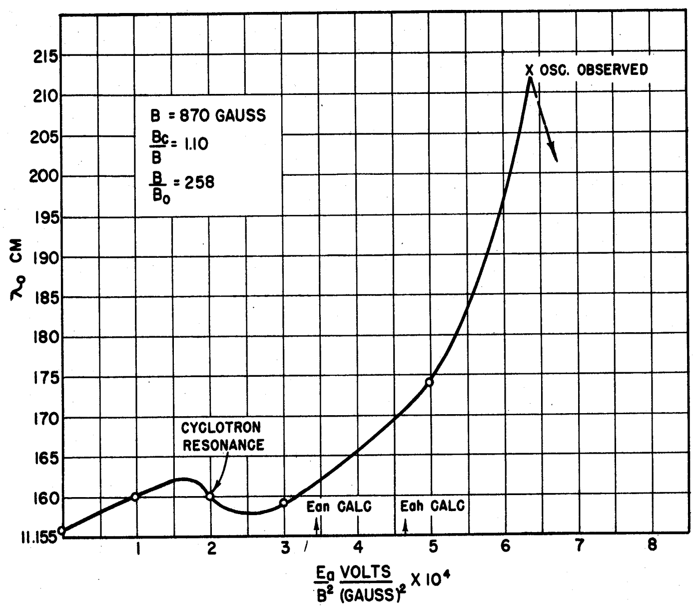


FIG. 19 HOT IMPEDANCE TEST ON QK59 NO. 2483

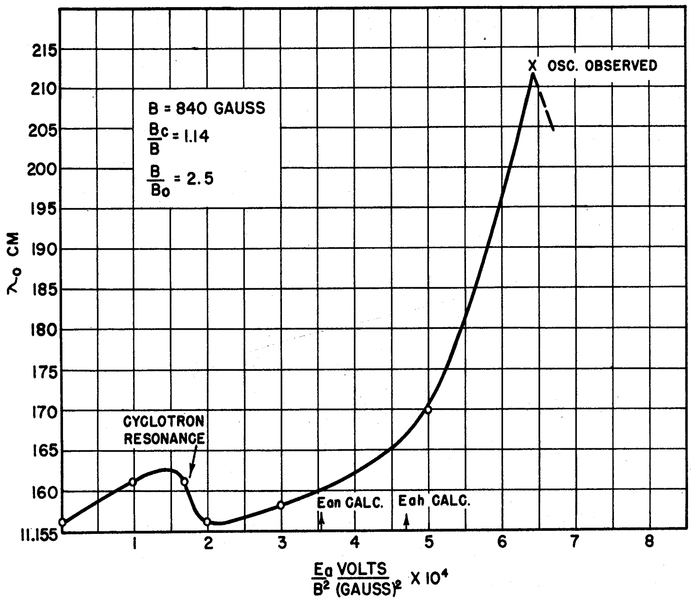


FIG. 20 HOT IMPEDANCE TEST ON QK59 NO. 2483

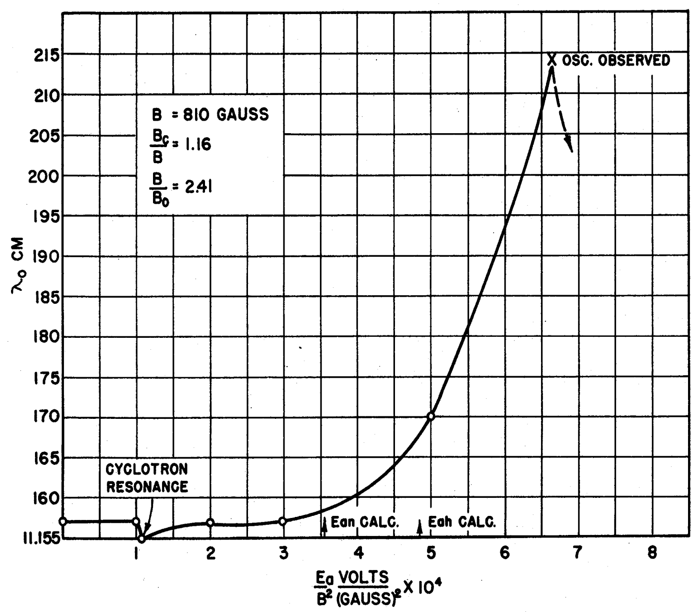


FIG. 21 HOT IMPEDANCE TEST ON QK59 NO. 2483

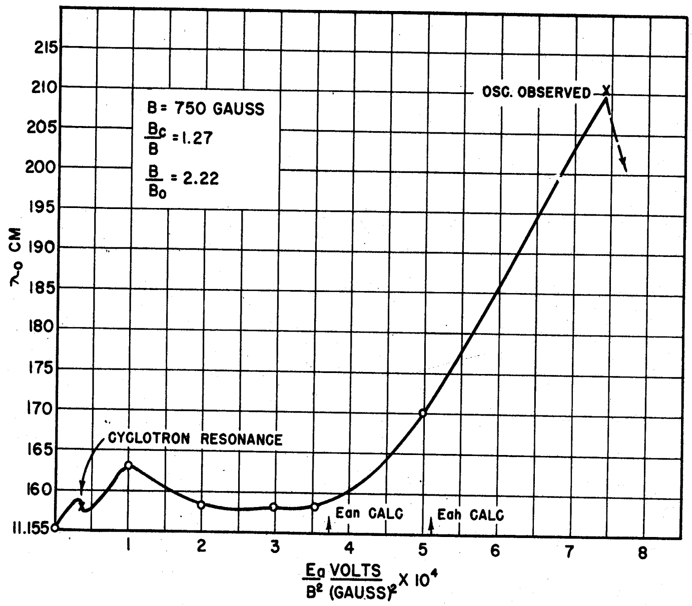


FIG. 22 HOT IMPEDANCE TEST ON QK59 NO. 2483

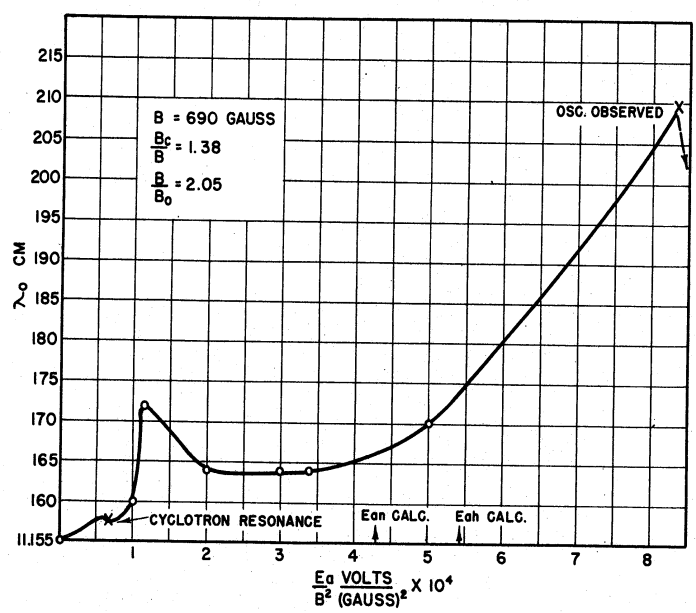


FIG. 23 HOT IMPEDANCE TEST ON QK59 NO. 2483

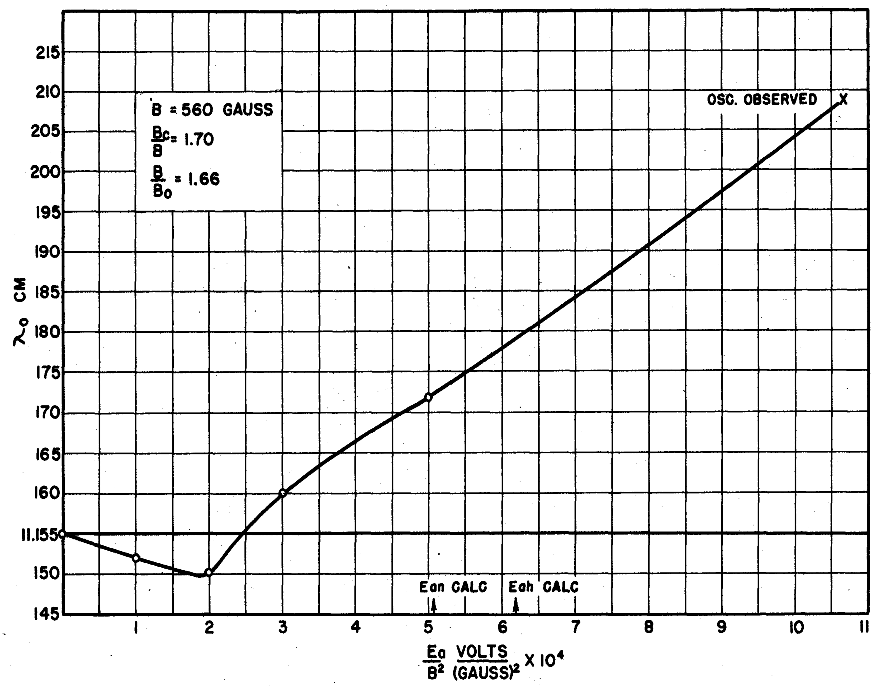


FIG. 24 HOT IMPEDANCE TEST ON QK59 NO. 2483

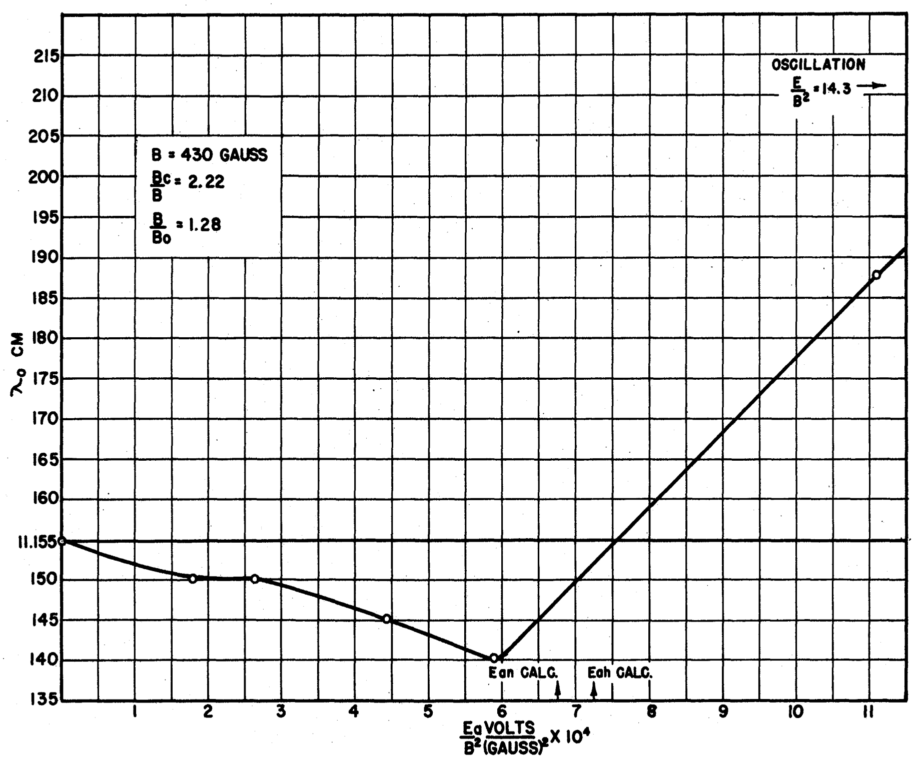


FIG. 25 HOT IMPEDANCE TEST ON QK59 NO. 2483

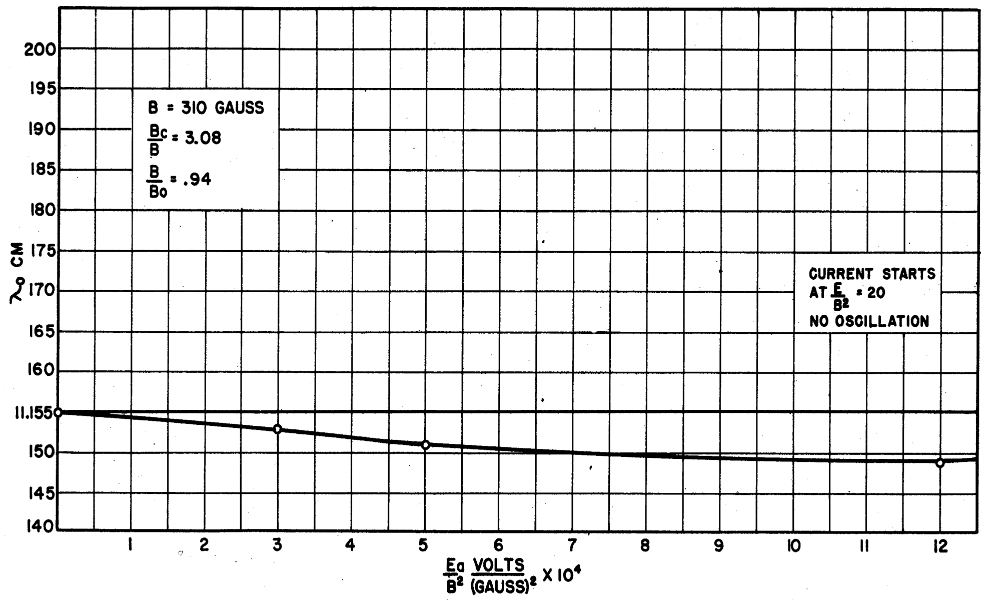


FIG. 26 HOT IMPEDANCE TEST ON QK59 NO. 2483

In the magnetron with r-f imposed on the anodes there are three critical conditions involving magnetic field and voltage. These are expressed in the following three equations:

$$\omega_c \cong \frac{B_c e}{m}$$

where

$$\omega_c = \frac{2\pi c}{\lambda_c} \qquad (4)$$

$$c = \text{velocity of light.}$$

This equation defines the so-called cyclotron resonance. λ_c is the critical wavelength for the particular magnetic field B_c at which the period of natural rotation of an individual electron in the magnetic field is equal to the period of the r-f cycle. This effect shows up as a perturbation in the resonant wavelength when the critical field is reached.

$$\frac{E_{an}}{E_o} = \frac{B_n}{B_o}\,\frac{r_c^2/r_a^2}{\frac{B_n}{B} - \left(1-\frac{r_c^2}{r_a^2}\right)}\left[2\left(\frac{B_n}{B_o}\right)\left(\frac{2}{1-\frac{r_c^2}{r_a^2}} - 1\right)\log\frac{\sqrt{\frac{B_n}{B_o} - \left(1-\frac{r_c^2}{r_a^2}\right)}}{\frac{r_c}{r_a}\sqrt{\frac{B_n}{B_o}}} + 1\right] \qquad (5)$$

$$E_o = \frac{m}{2e}\,\omega_n^2\, r_a^2$$

$\omega_n = 2\pi c/n\lambda$ = angular velocity synchronous with travelling r-f wave in interaction space

n = mode number = 1/2 number of anodes in π mode

$$B_o = \frac{2m}{c}\,\omega_n\,\frac{1}{(1 - r_c^2/r_a^2)}$$

Equation (5) relates the anode voltage (E_{an}) and magnetic field (B_n) for which outer electrons in the space charge swarm reach a velocity synchronous with the angular velocity of r-f wave travelling around the

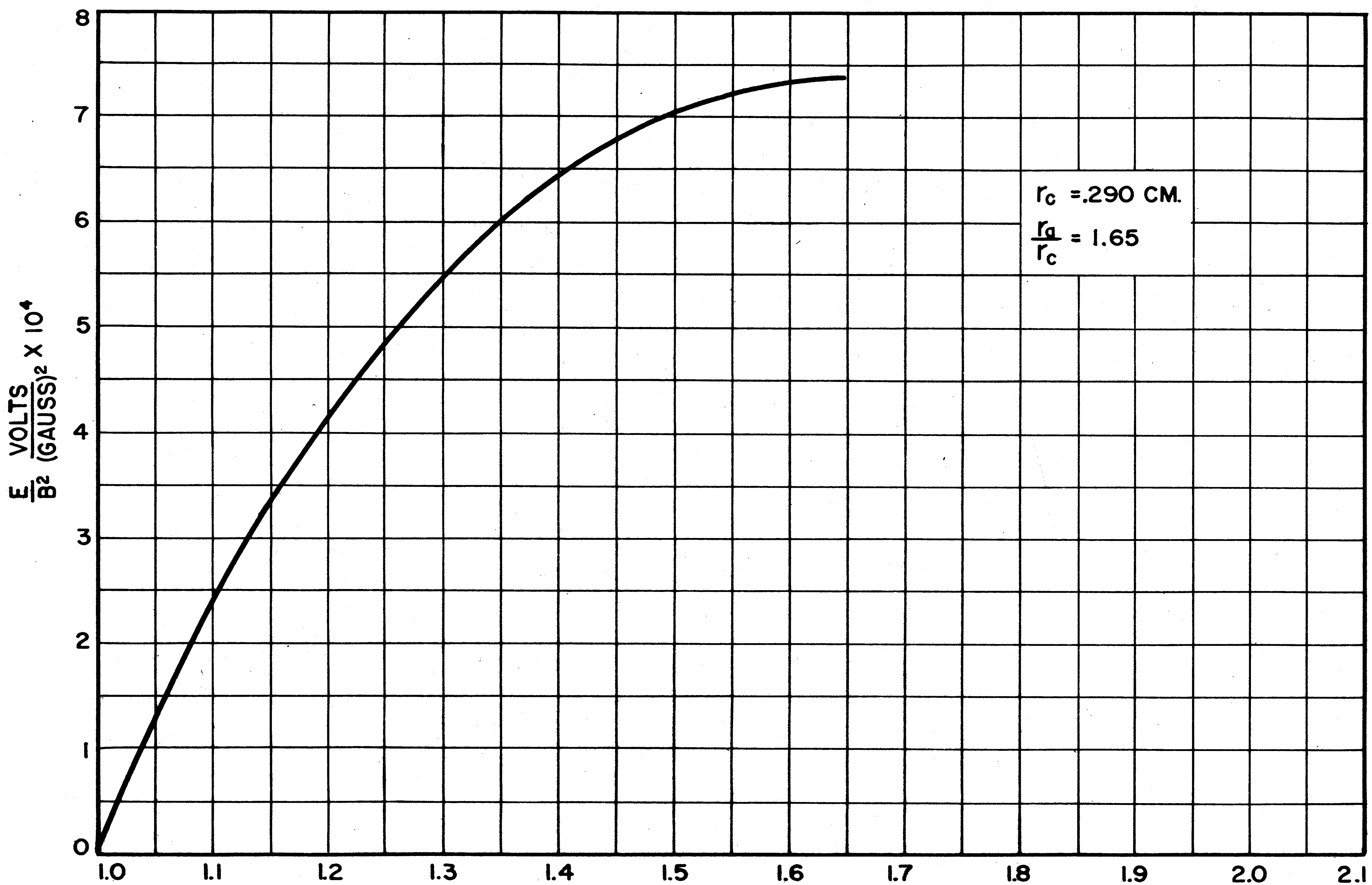


FIG. 27 $\frac{r_H}{r_c}$ $\frac{\text{RADIUS OF STATIC MAGNETRON SPACE CHARGE}}{\text{RADIUS OF CATHODE}}$

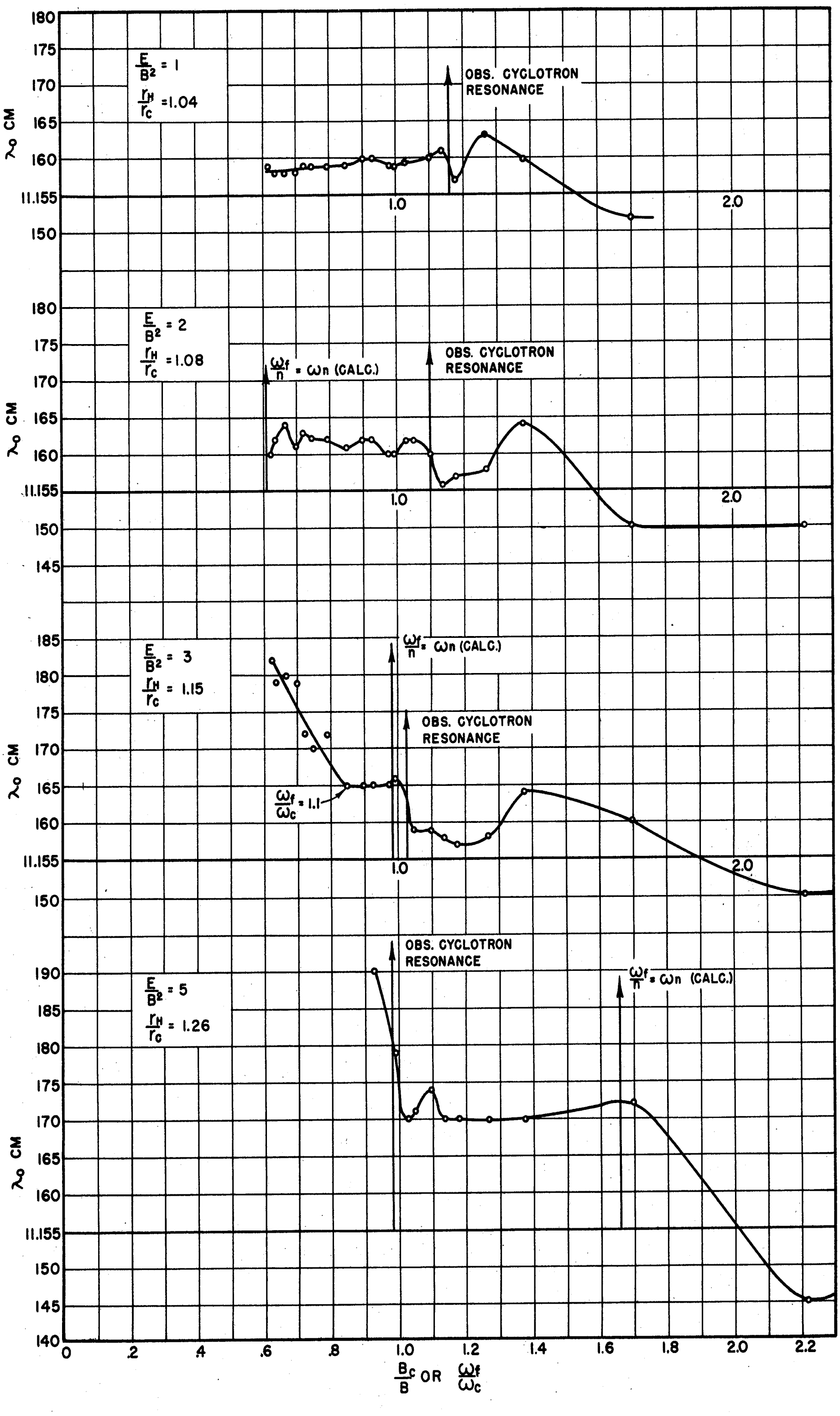


FIG. 28 HOT IMPEDANCE TEST ON QK59 NO. 2483

interaction space. For higher voltages than E_{an} the electrons are trying to move faster than synchronism and can give energy to the r-f field. This condition shows up as a contribution of negative conductance and positive susceptance to the system, thus causing a rather sharp increase in Q and resonant wavelength.

$$\frac{E_{ah}}{E_o} = 2\frac{B}{B_o} - 1 \qquad (6)$$

This equation defines the anode voltage at which synchronous electrons can reach the anode with no radially directed velocities (Hartree voltage). This voltage is approximately the voltage at which oscillations begin.

Figure 29 presents summary data of these three conditions extracted from Figures 10 to 26 with the calculated values for comparison. A considerable spread is represented for points on the curve for E_{an}/E_o, since the point at which a sudden increase in resonant wavelength occurs on the experimental curves is not definite.

In addition to the critical conditions just discussed, the space charge has what may be called "bulk" properties which affect the velocity of propagation of an electromagnetic wave in the space charge. These properties depend on the density of the space charge and the relative orientation of the polarization of the wave, the direction of propagation of the wave, and the direction of the magnetic field. In the multianode magnetron the direction of propagation and polarization are both perpendicular to the magnetic field. In this case the effective dielectric constant (as derived in Technical Report No. 1) is given by the following relationship.

$$\varepsilon_r = 1 - \frac{M}{2} \frac{\omega_c^2}{\omega_f^2} \frac{\left(\omega_f^2/\omega_c^2\right) - \frac{M}{2}}{\left(\omega_f^2/\omega_c^2\right) - \left(1 + \frac{M}{2}\right)} \tag{7}$$

$$M = \left[1 + \frac{r_c^2}{r_H^2}\right]$$

A plot of this equation is shown in Figure 30 for $r_H/r_c = 1.08$. The direction of wavelength shift to be expected due to the "bulk" space charge properties may be predicted as follows:

$$\varepsilon_r < 0 \qquad \Delta\lambda \text{ positive}$$

$$\varepsilon_r > 1 \qquad \Delta\lambda \text{ positive}$$

$$0 < \varepsilon_r < 1 \qquad \Delta\lambda \text{ positive}$$

The wavelength shift from Figure 28 for the case of $r_H/r_c = 1.08$ is replotted **for** comparison on Figure 30. The approximate wavelength shift to be expected is shown by the dotted line.[1] Except for the region between $B_c/B = .575$ and the observed cyclotron resonance, the data seem to be fairly consistent with the theory. We can offer no explanation at the moment for the inconsistency.

The over-all picture presented by these data is fairly clear while attention to particular points may be misleading because of experimental inaccuracies or insufficient theoretical understanding. Some of the important points are summarized below.

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[1]Note: There is no check on amplitude of shift in this data; the amplitudes of the two curves were made the same.

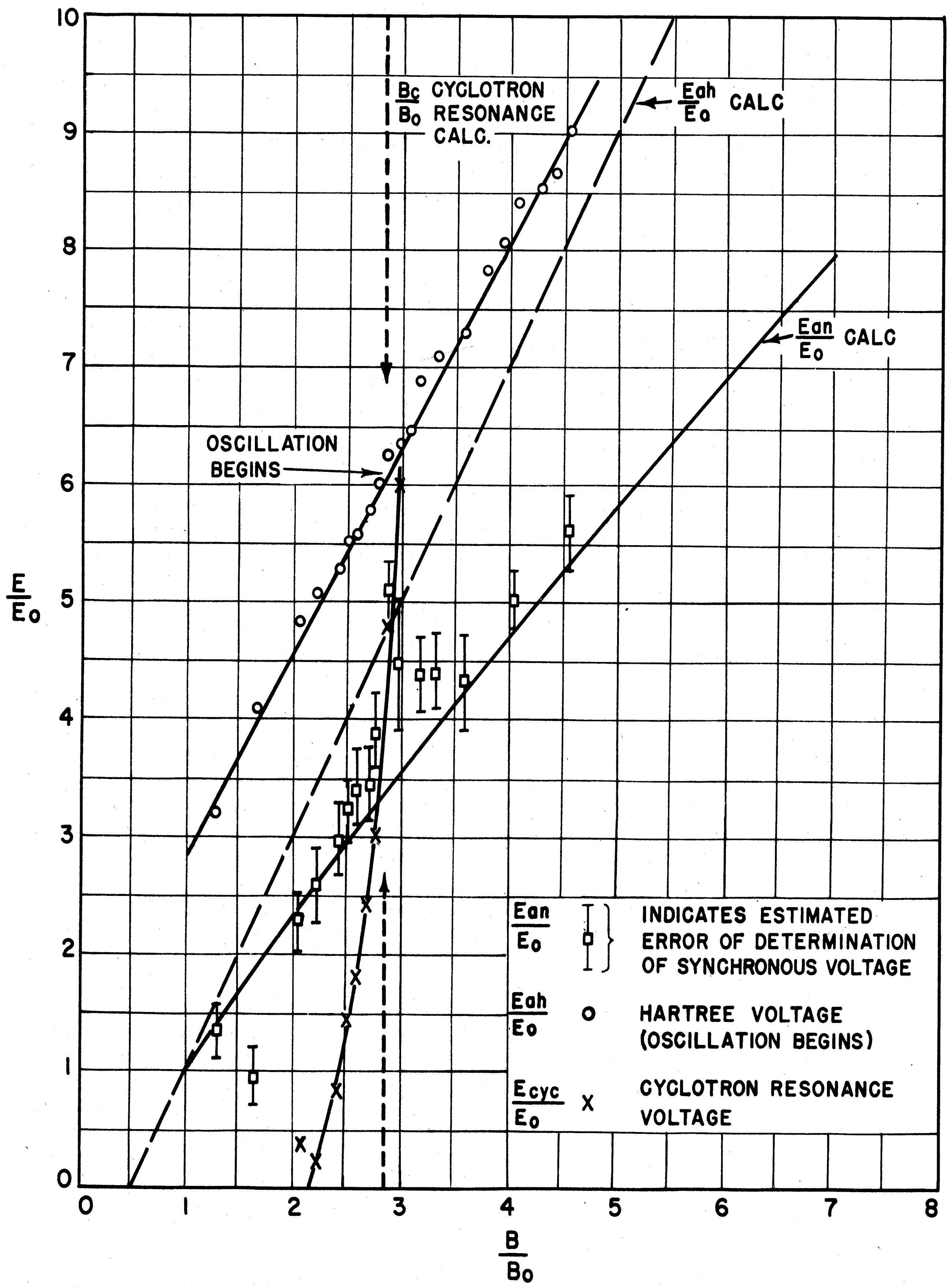

FIG. 29 SUMMARY DATA ON QK59 NO. 2483 ILLUSTRATING EQUATIONS 4,5 AND 6

(a) The three types of effects on frequency are clearly represented by the data.

"Bulk" effects are illustrated as wavelength shift at sub-synchronous voltages and to the right of the line $\omega_f/n = \omega_n$ in Figure 28. The curves in Figure 30 which pertain to these effects have been discussed in the last paragraph.

Synchronous reactance is demonstrated by the relatively sharp increase in wavelength above synchronous voltage and to the left of the line $\omega_f/n = \omega_n$ in Figure 28. The case for $E/B^2 = 5$ is particularly interesting in this connection. The decrease in wavelength showing up in the other curves between $B_c/B = 1.05$ and $B_c/B = 1.35$ is cancelled by the effects of synchronism. This shows up more clearly upon careful examination of Figures 18 to 23.

The cyclotron resonance shows up quite obviously in Figures 15 to 23 and is, as would be expected, dependent upon anode voltage. It is not clear from the data whether this point is intimately related to the other two effects. It does appear in Figure 28 that a sharp increase in wavelength with increasing magnetic field should be associated with the cyclotron resonance.

(b) Critical voltages and magnetic field given by Equations (4), (5), and (6) are represented by the data as presented in Figure 29. The voltage at which oscillation starts is affected by the loading on the magnetron and does not necessarily coincide with the Hartree voltage but should be near the Hartree voltage. A possible theory of the effects of loading is being studied which will be presented in a later report if it proves worth while. The critical magnetic field for the cyclotron resonance is quite

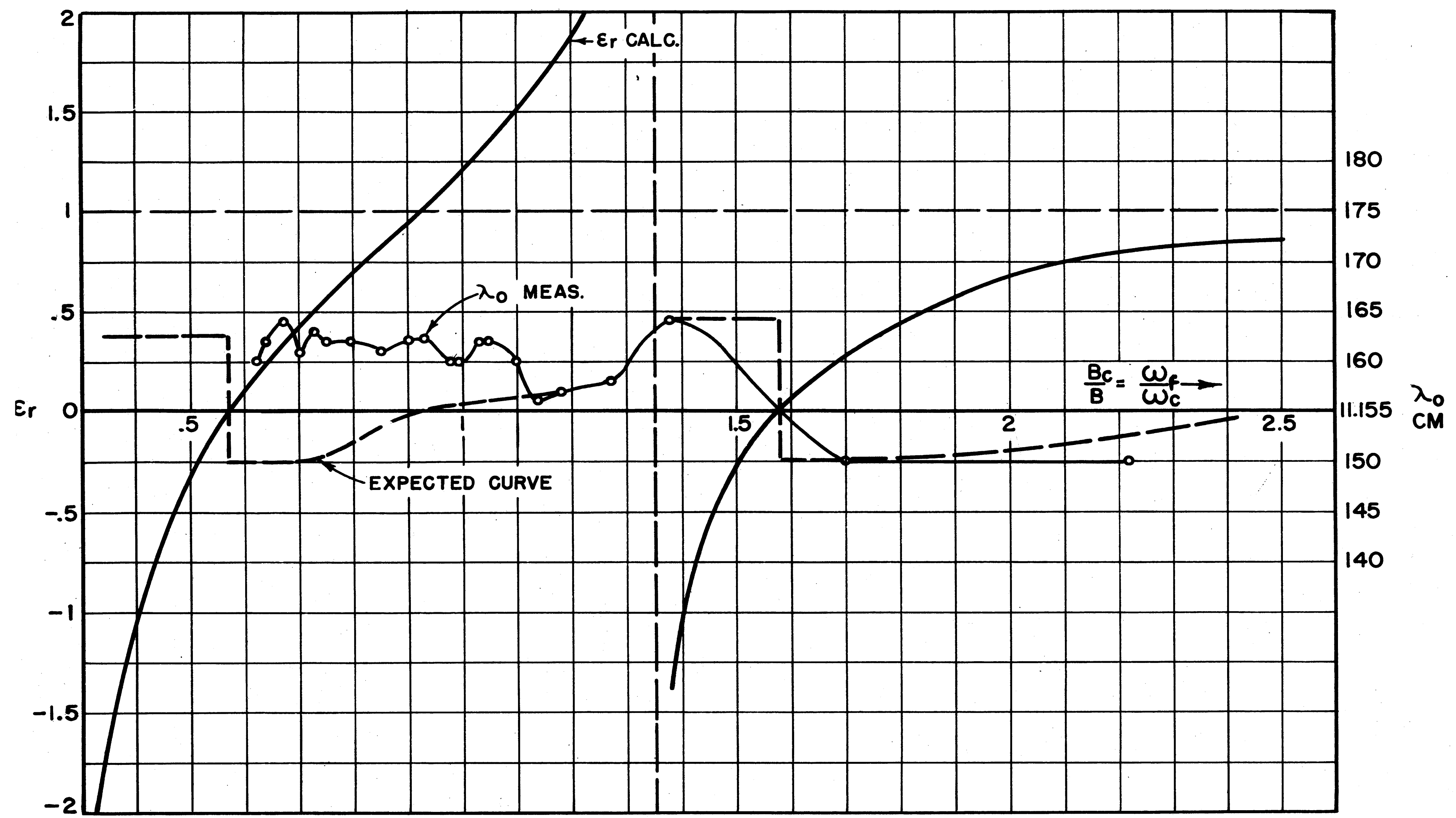

FIG. 30 EFFECTIVE DIELECTRIC CONSTANT OF MAGNETRON SPACE CHARGE COMPARED WITH WAVE-LENGTH SHIFT FOR $\frac{r_H}{r_c} = 1.08$

obviously affected by voltage. No satisfactory theory has been developed to give quantitative explanation of this effect. The synchronous voltage checks very well, considering the region of doubt which exists in its determination. It seems to be strongly perturbed in the region of the cyclotron resonance. It is interesting to note, however, that the cyclotron resonance seems to have no particular effect on the starting voltage for oscillation.

(c) Other unexplainable effects are observed.

The sharp rise at near $E/B^2 = 1$ in Figures 22 and 23 was quite real and noticeable in the process of taking data but is yet unexplained.

The Hull voltage (where anode current should begin in the static magnetron) is calculated to be $E/B^2 = 7.4$. It is quite obvious in Figures 23 to 26 that this is not checked for low magnetic field.

(d) Most probable sources of error are the following:

Magnetic field for the entire series of measurements may be off by 75 to 100 gauss. It is fairly certain that the calibration was not changed during the series of measurements because it was possible to recheck critical points at the cyclotron resonance and starting voltage.

Dimensions of the magnetron, particularly r_a/r_c, may be in error by 10 to 20 percent. Another tube which was taken apart had a cathode larger in this proportion. The dimensions used were taken from data in the Panel on Electron Tube reports.

Wavelength measurements were made with a Mico wave meter. The estimated error is $\pm$ .002 cm. The direction of wavelength shift could always be observed by watching the screen. Determination of the exact resonant wavelength near the cyclotron resonance was difficult because of the distortion of the scope pattern.

In order to obtain data for other orientations of polarization and direction of propagation than can be obtained in a magnetron, a simple magnetron diode has been constructed. (See Figure 14 of Interim Report, December, 1949.) Cold impedance tests have been made on this tube in order to ascertain loop size for the desired coupling. Completion and hot impedance tests have been delayed by pressure of the f-m tubes.

B. Theoretical Analysis of Space Charge (G. Hok)

In the small-signal analysis of the behavior of the space charge in a magnetron, it is usual to consider the r-f phenomena as small perturbations on space charge cloud determined by the Hull-Brillouin solution of the d-c magnetron, or an approximation of single-swarm double-stream solution, although the latter is not known in explicit form. However, since experiments with d-c magnetrons indicate that neither of these solutions agrees closely with the actually observed space charge, it has become evident that an investigation of the modifications of the space charge, because of statistical interaction between the electrons, is desirable..

The first step was to find a space charge in statistical equilibrium. This was found to be so radically different from the magnetron space charge that no relevant positive conclusion could be drawn. The reason is that the boundary conditions at the anode and cathode prevent any approximation of statistical equilibrium to establish itself.

The next attempt will be to start out from the single-swarm double-stream solution and study the probability of a certain angle of deflection and change of energy of an electron from the undisturbed orbit. Two different statistical processes act to modify this state: close interaction between electrons in interacting orbits; and the combined effect of all other electrons on the motion of each electron. The first objective is to find at least approximately the equilibrium state of diffusion of the electrons in phase space due to these two processes.

C. Experimental Study of Static Magnetron Space Charge (W. Peterson, H. W. Welch, Jr.)

The problems of attaining complete understanding of the static magnetron space charge from a theoretical approach are rather imposing. Serious efforts to improve this understanding with an experimental method have not been numerous. However, there are certain facts which indicate further experimental study would be desirable. Particularly significant known facts are the following:

(a) Pre-cut-off anode current in the nonoscillating magnetron is quite appreciable and may occur at a small fraction of the cut-off voltage (i.e., less than 10 percent).

(b) Current at cut-off is between .7 and .8 of ordinary diode space-charge limited current.

(c) In the oscillating magnetron mode jump current (or drop-out current) is strongly affected by the available current, whether it is temperature limited or space-charge limited.

There are other facts of interest which might be mentioned, such as data on the nature of back-heating and noise in magnetrons. However, the main interest in this laboratory is in factors affecting mode-jump current. Knowledge of the space charge distribution and current paths in the static space charge could be very useful in this connection. The present program is to build a tube in which an electron beam can be sent through the space charge axially. The beam will start out grazing the cathode.

Its exit point can be determined by a fluorescent screen. By regulating beam voltage it should be possible to determine the path of an individual electron for a particular anode potential and magnetic field. It is hoped that the method can be extended to preoscillating and oscillating magnetrons, thus to determine what changes, if any, occur in the core of the magnetron space charge as oscillation starts or mode jump occurs.

Simpler tubes for cut-off measurement and for probe measurement of distributions within the space charge are also contemplated.

IV. CONCLUSIONS (H. W. Welch, Jr.)

Since all work in progress is in a state of incompletion, conclusions must be tentative. However, the progress during the period covered by this report may be summed up as follows:

(a) The Model 6 f-m magnetron is beginning to show promise. Constructional problems have been solved.

(b) A Model 5 f-m tube has been completed. The erratic behavior of the first model indicates no very interesting conclusions.

(c) Design of the Model 8 f-m magnetron is complete. A tube should be completed within the next quarterly period.

(d) The experimental data on magnetron space charge clearly present the complexity of the over-all picture with enough quantitative correlation to verify parts of the theory hitherto untested.

(e) Very little time has been devoted to theoretical and experimental study of the static magnetron space charge. The work has been started and should produce preliminary results during the next quarterly period.

V. WORK IN PROSPECT (H. W. Welch, Jr.)

Work will continue on all phases of the program discussed in this report. In addition an increased effort will be expended toward the understanding of factors affecting current drop-out and frequency pushing with emphasis on obtaining very wide frequency pushing above 3000 megacycles. This type of operation has been developed at G. E. for lower frequency, lower power magnetrons in recent months. A detailed study of effects of loading and interaction space design on magnetron operation will be required. Additional funds have been made available by the Signal Corps for this program and preliminary results should be expected within the next period.

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