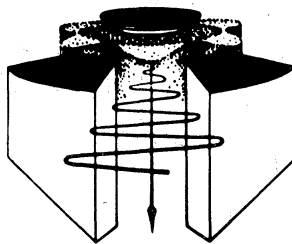


THE UNIVERSITY OF MICHIGAN

DIGITAL COMPUTER ANALYSIS OF CROSSED-FIELD ELECTRON GUNS

TECHNICAL REPORT NO. 71

ELECTRON PHYSICS LABORATORY
Department of Electrical Engineering



By: J. E. Boers

Approved by: J. E. Rowe
April, 1964

CONTRACT WITH:

ELECTRONIC TECHNOLOGY DIVISION OF THE AIR FORCE AVIONICS LABORATORY,
RESEARCH AND TECHNOLOGY DIVISION, AIR FORCE SYSTEMS COMMAND
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ABSTRACT

A digital computer program written in the IBM 7090 FORTRAN programming system is described and then employed in the analysis of several crossed-field guns.

The digital program employs relaxation techniques while alternately computing electric fields and trajectories. One cycle through the program consists of three basic parts.

1. Calculation of the voltages within the gun by solving Poisson's equation, in difference form, on a 7200 point matrix on which electrode potentials are laid out.

2. Calculations of new current densities along the cathode and the setting up of trajectory starting points along the cathode to simulate these densities.

3. Calculation of trajectories and comparison of the new beam with the previous one.

Thermal effects and direct particle-particle interaction effects are ignored but space-charge effects, introduced through the field calculations, tend to prevent excessive or unusual trajectory crossings. Execution time on a 7090 varies from 5 to 10 minutes depending on gun design, with better designed guns generally running faster.

Results produced by the program are presented and compared with experimental and analytical results. Agreement with experimental results within 5 percent should be obtainable.

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DIGITAL COMPUTER ANALYSIS OF CROSSED-FIELD

ELECTRON GUNS

I. INTRODUCTION

A digital computer program for the solution of nonre-entrant crossed-field electron guns has been written. The program to be described employs relaxation techniques to determine the current and voltage distribution within the gun. The program alternately computes voltages within the gun and trajectories through the gun until two consecutive sets of trajectory calculations essentially duplicate each other.

The program is written in the IBM 7090 FORTRAN programming system and requires at least a 32,000 word core for execution. Execution time on an IBM 7090 digital computer is usually between 5 and 10 minutes, with better designed guns generally running faster. The program employs a 7200-point matrix on which the gun electrode potentials are laid out, with intermediate potentials computed by relaxation techniques using Poisson's equation. A single core load is used with one tape unit needed for temporary storage of data.

Agreement with experimental results within 5 percent should be attainable.

Output from the program includes complete voltage and space-charge density distributions in the gun, variation in current density across the cathode, and the trajectories through the gun. Thermal effects and crossing of trajectories are ignored (the beam does not

have to be laminar), but moderate amounts of crossing do not seem to reduce the validity of the solutions.

II. PROGRAM FLOW

Before discussing the program in detail let us first examine its flow. Figure 1 shows the general flow diagram for the program to be described.

The first section reads in all the data needed to describe the gun and control the flow of the program. This is followed directly by the calculation of constants and the setting up of tables to be used later in the program.

The next step is the setting up of initial voltage and space-charge distributions on the appropriate matrices. If a previous run has been made for a particular gun it is possible to read the previous results (the voltage and space-charge matrices) into the computer and resume calculations from that point.

The calculations of the electric fields are then carried out using Poisson's and Laplace's equations as developed below. After the field calculations have terminated, the current densities along the cathode are calculated and a new set of trajectory starting points is set up to simulate this distribution.

The new set of trajectories is then traced through the gun using the equations developed below and at the same time the new space-charge matrix is calculated from the trajectories.

The program then compares the new beam with the previous one. If substantial changes have occurred, a new cycle is started by resuming the field calculations with the new space-charge matrix. If the beam

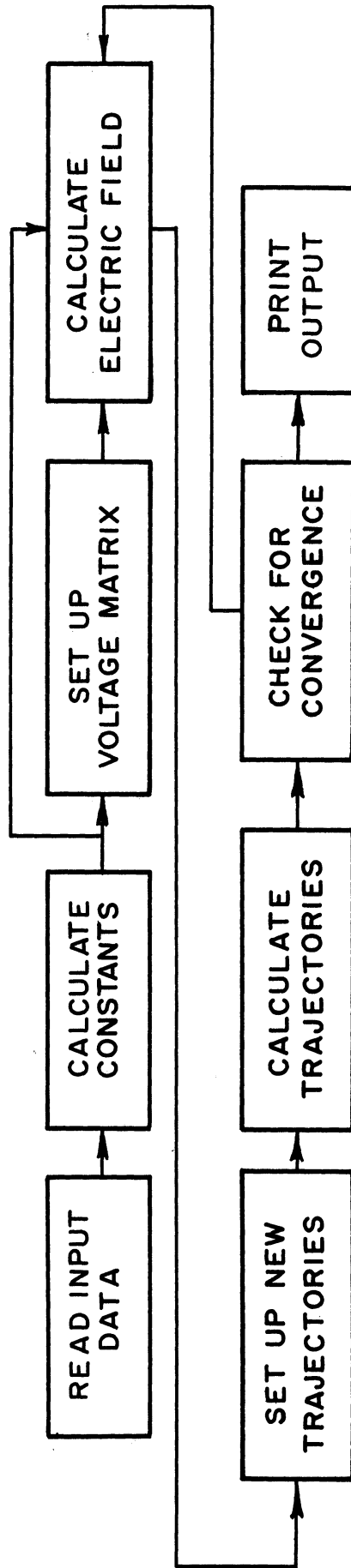


FIG. 1 FLOW CHART FOR CROSSED-FIELD PROGRAM.

essentially repeats itself, the program terminates by printing out all the useful data including the voltage and space-charge matrices and the final set of trajectories. Also the final voltage and space-charge matrices can be saved so that it would be possible to continue the calculations from this point at a later time if it is desired.

III. DEVELOPMENT OF THE COMPUTER EQUATIONS

Since the electric fields are calculated by relaxation techniques on a discrete matrix, it is necessary to develop Poisson's equation in a finite difference form which the computer can easily handle. The electron trajectory equations are then derived from the Lorentz force law in a form which would use the data available from the field calculations and again which the computer can handle. A brief discussion of the starting conditions for the trajectories is also presented.

3.1 The Electric Field Equations

Since the digital computer works only with finite differences and not with continuously varying functions it is necessary to develop any differential equation (such as Laplace's or Poisson's) in difference form. Thus, to make it acceptable to the computer, Poisson's equation,

$$\nabla^2 V = -\frac{\rho}{\epsilon_0} \quad , \quad (1)$$

must be put into (in this case) two-dimensional difference form:

$$\frac{V_{i+1,j} - 2V_{i,j} + V_{i-1,j}}{(\Delta x)^2} + \frac{V_{i,j+1} - 2V_{i,j} + V_{i,j-1}}{(\Delta y)^2} = -\frac{\rho}{\epsilon_0} \quad , \quad (2)$$

where the $V_{i,j}$'s are defined in Fig. 2. Solving for $V_{i,j}$

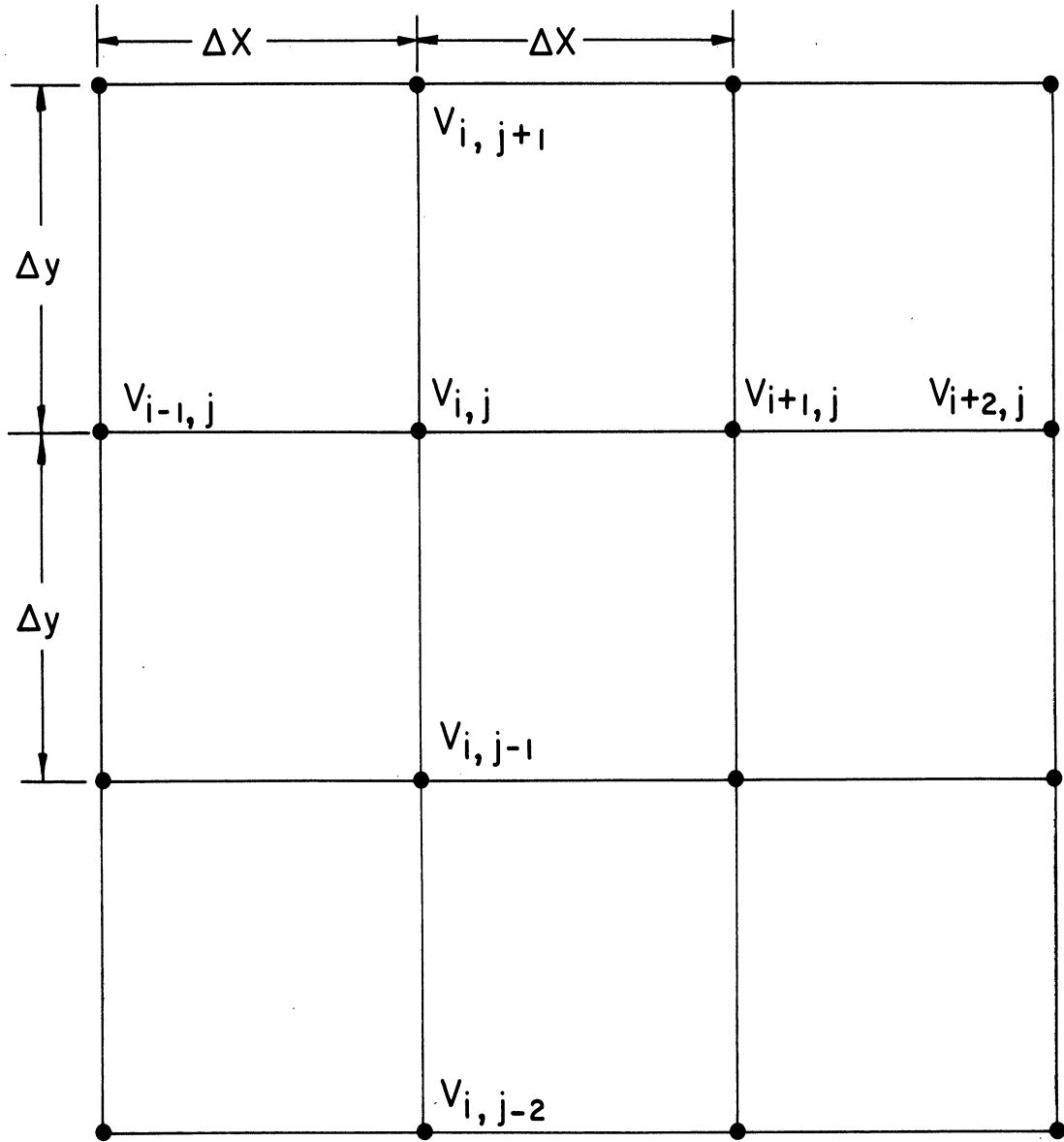


FIG. 2 DEFINITIONS OF ELECTRIC FIELD QUANTITIES.

$$V_{i,j} = \frac{V_{i+1,j} + V_{i-1,j} + V_{i,j+1} + V_{i,j-1}}{4} + \frac{(\Delta x)^2 \rho}{4\epsilon_0}, \quad (3)$$

where $\Delta x = \Delta y$, an expression is obtained for the voltage at a point in terms of the voltages at adjacent points and the space-charge density.

If the last term in Eq. 3 is defined to be

$$\rho_{i,j} = \left| \frac{(\Delta x)^2 \rho}{4\epsilon_0} \right|, \quad (4)$$

Eq. 3 becomes simply

$$V_{i,j} = \frac{V_{i+1,j} + V_{i-1,j} + V_{i,j+1} + V_{i,j-1}}{4} - \rho_{i,j}, \quad (5)$$

where $\rho_{i,j}$ is now proportional to the space-charge density at the matrix point i,j . If $\rho_{i,j}$ can be determined the field calculations become quite simple and very fast using Eq. 5.

The space-charge density (ρ) can be expressed in terms of the current density and velocity at any point by the expression

$$\rho = \frac{J}{v}, \quad (6)$$

where J is the current density and v the velocity. Substituting Eq. 6 into the definition of $\rho_{i,j}$ (Eq. 4), the following expression is obtained for $\rho_{i,j}$,

$$\rho_{i,j} = \left| \frac{(\Delta x)^2 J}{4\epsilon_0 v} \right|. \quad (7)$$

If the current density and velocity can be determined at each matrix point from the trajectory calculations, $\rho_{i,j}$ can be computed before the fields are computed and hence produce very rapid field calculations.

If the trajectories are calculated in increments of time Δt (see below) at discrete points (i.e., $x_{l,k}, x_{l,k+1}, \dots; y_{l,k}, y_{l,k+1}, \dots$; where l is the number of the electron and k is the number of the trajectory points) the velocity v becomes

$$v = \frac{\sqrt{(x_{l,k+1} - x_{l,k})^2 + (y_{l,k+1} - y_{l,k})^2}}{\Delta t} \quad (8)$$

at each point k . If each trajectory is assigned the same initial current density (J_0), then

$$\rho_{i,j} = \sum_{l=1}^{\text{NOEL}'} \sum_k \frac{J_0 \Delta t (\Delta x)^2}{4\epsilon_0 \sqrt{(x_{l,k+1} - x_{l,k})^2 + (y_{l,k+1} - y_{l,k})^2}}, \quad (9)$$

where the prime on the summation over l means that only those electrons passing within one matrix square length (Δx) of the point i,j are used, and the prime on the summation over k means that only those points where trajectories cross the horizontal or vertical lines defined through the points i,j are used.

The use of Eq. 9 means that a new matrix (ρ , or in the computer RHO) is defined giving the space-charge density at any point in the gun region. Since this density is a summation over all the "electrons" injected from the cathode it is directly proportional to the total current density at any point, therefore allowing for nonuniformities in space-charge density in the beam and in emission densities from the cathode.

Since the electron trajectories are not likely to pass directly through matrix points the following approximations are made for Eq. 9.

Whenever a trajectory crosses from one square to the next on the matrix the following additions are made to the ρ matrix. If the path crossed a horizontal line (i.e., j increased or decreased by one),

$$\rho_{i,j} = \rho_{i,j} + \frac{J_0 \Delta t (1 - R_x) \sin^2 \theta (\Delta x)^2}{4\epsilon_0 \Delta s} \quad (10a)$$

and

$$\rho_{i+1,j} = \rho_{i+1,j} + \frac{J_0 \Delta t R_x \sin^2 \theta (\Delta x)^2}{4\epsilon_0 \Delta s} , \quad (10b)$$

where θ is the angle between the electron path and a horizontal line, R_x is the distance between the i th vertical line and $x_{l,k+1}$ divided by Δx (the matrix square length) and $\Delta s = v \Delta t$. Similarly for the crossing of a vertical line

$$\rho_{i,j} = \rho_{i,j} + \frac{J_0 \Delta t (1 - R_y) \cos^2 \theta (\Delta x)^2}{4\epsilon_0 \Delta s} , \quad (11a)$$

$$\rho_{i,j+1} = \rho_{i,j+1} + \frac{J_0 \Delta t R_y \cos^2 \theta (\Delta x)^2}{4\epsilon_0 \Delta s} , \quad (11b)$$

where R_y is now defined as the distance between the j th horizontal line and $y_{l,k+1}$ divided by Δx .

3.2 The Trajectory Equations

The solution of the trajectory equations is somewhat simpler. Starting with the Lorentz force equation

$$\vec{F} = -e (\vec{E} + \vec{v} \times \vec{B}) , \quad (12)$$

two scalar equations can be written assuming no z -directed motion and

$$\vec{B} = \vec{B}_z = B\vec{k} ,$$

$$\ddot{x} = \eta B \dot{y} - \eta E_x, \quad (13a)$$

and

$$\ddot{y} = -\eta B \dot{x} - \eta E_y. \quad (13b)$$

Again forming the difference equations

$$\frac{x_{k+1} - 2x_k + x_{k-1}}{(\Delta t)^2} = \eta B \frac{y_{k+1} - y_{k-1}}{2 \Delta t} - \eta E_x \quad (14a)$$

and

$$\frac{y_{k+1} - 2y_k + x_{k-1}}{(\Delta t)^2} = -\eta B \frac{x_{k+1} - x_{k-1}}{2 \Delta t} - \eta E_y. \quad (14b)$$

To compute the trajectories it is necessary to solve these equations for

succeeding points (i.e., y_{k+1} , x_{k+1}) rather than the center points

(i.e., y_k , x_k). Solving for y_{k+1} and x_{k+1} with $z = \eta B \Delta t$, gives

$$x_{k+1} = \left[2x_k - \left(1 - \frac{z^2}{4}\right) x_{k-1} + z(y_k - y_{k-1}) - (E_x + \frac{z}{2} E_y) \right] / \left(1 + \frac{z^2}{4}\right) \quad (15a)$$

and

$$y_{k+1} = \left[2y_k - \left(1 - \frac{z^2}{4}\right) y_{k-1} - z(x_k - x_{k-1}) - (E_y - \frac{z}{2} E_x) \right] / \left(1 + \frac{z^2}{4}\right) \quad (15b)$$

If the first derivatives (\dot{x} , \dot{y}) in Eqs. 14 are defined in a slightly different manner, i.e.,

$$\dot{x} = \frac{x_k - x_{k-1}}{\Delta x}, \quad (16)$$

the expressions for the trajectories become somewhat simpler than those given in Eq. 15. These equations,

$$x_{k+1} = 2x_k - x_{k-1} + z(y_k - y_{k-1}) - \eta(\Delta t)^2 E_x \quad (17a)$$

and

$$y_{k+1} = 2y_k - y_{k-1} - z(x_k - x_{k-1}) - \eta(\Delta t)^2 E_y, \quad (17b)$$

are good approximations to Eqs. 15.

The electric field forces (E_x, E_y) have been represented by several combinations of voltages from the voltage matrix. The most accurate trajectories computed so far have been obtained using the following second-order forms:

$$E_x = \left[(2V_{i,j} - V_{i+1,j} - V_{i-1,j})d_x - \frac{(V_{i+1,j} - V_{i-1,j})}{2} \right] \frac{1}{\Delta x} \quad (18a)$$

and

$$E_y = \left[(2V_{i,j} - V_{i,j+1} - V_{i,j-1})d_y - \frac{(V_{i,j+1} - V_{i,j-1})}{2} \right] \frac{1}{\Delta x}, \quad (18b)$$

where d_x is the normalized (to Δx) distance between x_k and the i th matrix point, and d_y is similarly defined with respect to y_k and j . The second-order terms are needed when the electron trajectories tend to travel normal to the electric field. When an electron passes from one matrix square to the next, abrupt changes in the electric field can cause deflections of the trajectories; the second-order terms tend to smooth these reactions and improve their accuracy.

For most purposes simple first-order expressions are adequate for the electric field, i.e.,

$$E_x = - \frac{V_{i+1,j} - V_{i,j} + V_{i+1,j+1} - V_{i,j+1}}{2 \Delta x} \quad (19a)$$

and

$$E_y = - \frac{V_{i,j+1} - V_{i,j} + V_{i+1,j+1} - V_{i+1,j}}{2 \Delta x} . \quad (19b)$$

In this case i, j are the coordinates of the lower left-hand corner of the matrix square in which the electron lies.

Interactions between individual "electrons" are ignored since the computing time would become excessive. Even for a simple diode the time needed to compute trajectories with interactions requires several hours on the computer.

Moderate amounts of trajectory crossing do not seem to reduce the validity of the solutions, and excessive amounts usually indicate a poor design.

3.3 Trajectory Starting Conditions

Since it is very difficult to calculate electron trajectories in the first matrix square in front of the cathode it has been found desirable to start the electron trajectories at the upper edge of these squares. Due to the magnetic field the trajectories are deflected in this first square and it is necessary to determine both the distance deflected and the slope of the trajectories at the edge of this first square in terms of the current density and the magnetic field.

In order to obtain values for the deflection and slope near the cathode an approximate voltage distribution was assumed, and trajectories traced using the above difference equations. The voltage distribution used consisted of a $4/3$ power curve with a potential minimum 0.015 mm in front of the cathode. The depth of the minimum was a function of the current density and the distance to the minimum. All x- and z- directed electric fields were assumed to be zero.

The slope and position of the trajectory 0.005 and 0.010 inch in front of the cathode were determined for magnetic fields from 100 to 700 gauss and current densities from 0.1 to 1.3 amps/cm². From these results empirical relationships for the slope and displacements were chosen.

The slope of the trajectories relative to a normal to the cathode obtained by the above method is approximately given by

$$\theta = \frac{B}{0.002} \left(\frac{1}{0.46 + \frac{J}{2.4}} \right) \left(\frac{3y_0 + y}{4y_0} \right), \quad (20)$$

where θ is the angle in degrees, B is the magnetic flux density in webers/m², J is the current density in amperes/cm², y is the position above the cathode in meters and y_0 is 1.27×10^{-4} meters (0.005 inch). The displacement (in meters) from the normal to the cathode is given by

$$\delta x = (2.9J^2 - 6.6J + 11.2)B \left[1 + (0.18J^2 - 0.48J + 0.55) \frac{y - y_0}{y_0} \right]. \quad (21)$$

It should be noted that these expressions are good only in the region near y_0 and are not accurate down to $y = 0$.

It is the limited range of usefulness of these equations that forces the requirement that the cathode surface lie along a row of the voltage matrix (see below).

IV. THE COMPUTER PROGRAM

The electric fields are represented on a 7200-point matrix laid out in a 120 x 60 array. Space charge is laid out on an identical array

so that there is an easy, one-to-one correspondence between the two matrices.

Numbering of these two matrices starts in the lower left-hand corner with the coordinates $I = 1, J = 1$. The I coordinates run along the long dimension (120) of the voltage matrix, and the J coordinates run perpendicular to the bottom edge. For simplicity and speed in making calculations the matrix is made up of squares and not rectangles.

The 120 x 60, 7200-point matrix has been found satisfactory for the representation of most guns and will be assumed in all the following discussions. The size and shape of this matrix can be modified by changing 11 cards (or less) in the FORTRAN program, subject only to the capacity of the machine.

The electric fields are calculated using Eq. 5 at all points within the gun geometry, with a linear field being set up along the left edge of the matrix while an approximate expression is used at the right edge. The regions where these calculations are carried out are specified by four integer matrices which describe the gun geometry on the voltage matrix. These four matrices (JL, JU, JC and JD) are the "J" coordinates for the electrodes, each matrix being 120 points long (same length as the V (voltage) matrix).

The calculations start in the lower left-hand corner and proceed up the columns in sequence across the matrix. JL (J lower electrode) is made up of the coordinates of the lowest electrodes including the cathode and sole. JU (J Upper electrode) is made up of the coordinates of the first electrode encountered as one travels up along the columns from JL. If there is a second region (beyond JL and JU) in which it is desired to make calculations, this can be carried out by specifying the

lower coordinate by JC and the upper coordinate by JD. The voltage calculations are carried out first from JL to JU and then from JC to JD unless JC(I) is zero in which case the calculation from JC to JD is deleted.

Figure 3 shows a typical layout for a gun showing the points specified by these matrices.

If these calculations are initially carried out on the V-matrix the convergence is extremely slow, and many hundreds of passes will be called for. It was found that if the calculations are first performed on a "coarse" matrix, i.e., using only the odd numbered points of the fine matrix, the computing time could be cut to a small fraction of that required for the fine matrix alone.

Convergence of the V matrix calculations is found by determining the number of points where the voltages change by more than some specifiable amount (usually 0.01 percent), when the number of such points falls below a specifiable number (as low as zero) the matrix will be assumed converged.

Approximate borders for the beam must be specified but it is not important to the program that these be particularly accurate. The borders are used by the program to set up an initial distribution on the space-charge (RHO) matrix. This distribution is reasonably accurate only in the immediate vicinity of the cathode. It is calculated by extending Eq. 7 one more step, i.e., using the fact that $v = \sqrt{2\eta V}$ to obtain

$$\rho_{i,j} = \left| \frac{J (\Delta x)^2}{4\epsilon_0 \sqrt{2\eta} \sqrt{V_{i,j}}} \right| , \quad (22)$$

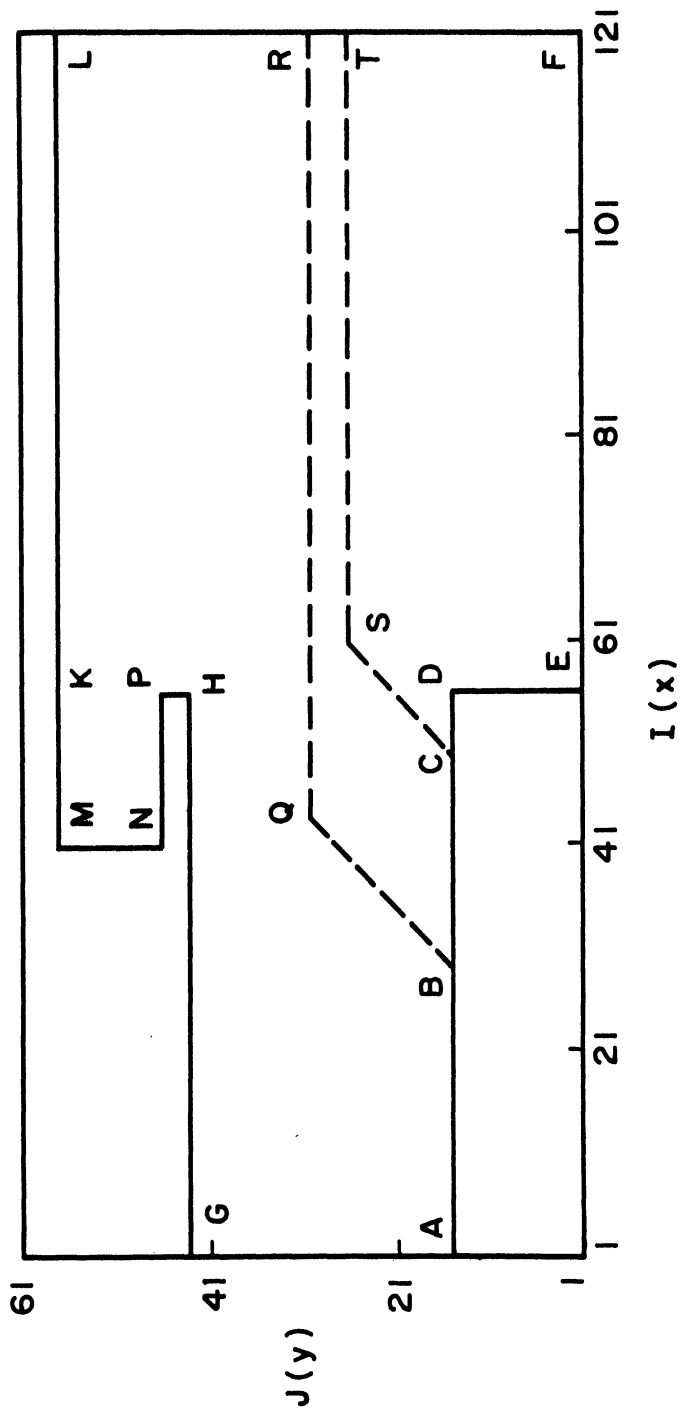


FIG. 3 SAMPLE GEOMETRY FOR CROSSED-FIELD GUN.

where $V_{i,j}$ is from the initial voltage distribution and J is the initial current density read in from external data. If the beam is convergent this calculation rapidly becomes inaccurate as one moves away from the cathode.

The cathode must be flat and parallel with the long dimension of the V matrix; it also must lie along a row of the voltage matrix. It is specified by giving the approximate right- and left-hand I coordinates and the exact location, in scaled units (to be explained below), of the right- and left-hand edges.

The "electron" trajectory starting points are spaced out along the cathode in inverse proportion to the current densities along the cathode. The spacing is widest where the current density is lowest, the maximum spacing being one matrix square. A maximum of 100 trajectories can be traced.

Cathode current densities are calculated using Child's law,

$$J = \frac{2.33 \times 10^{-6} V^{3/2}}{(2\Delta x)^2}, \quad (23)$$

where V is the voltage at the second matrix point in front of the cathode and $2\Delta x$ is the length of two matrix squares. This is calculated for each matrix point along the cathode and the minimum of all these values is the initial current density assigned to all the trajectories to be traced.

It was found that the current density along the cathode would fluctuate quite widely during the first few cycles of execution so it was necessary to use a relaxation procedure on both the total current density and on the variations.

This relaxation procedure on the current densities along the cathode consists of comparing the normalized inverse current densities

with a number which allows larger and larger variations along the cathode from cycle to cycle until some maximum allowed variation is reached. On an initial cycle the maximum permitted variation might be between 0.9 and one, on the second cycle between 0.8 and one down to some variation which could be as much as from 0.1 to one, a ten to one current density variation. Both the initial and final current variation limits are specified in the input data by VARYM and VARY. VARYM will be reduced by 0.1 on each cycle until VARY is reached.

It was also found necessary to average the new and the previous current densities. If the new current density were used directly the program would oscillate strongly above and below the proper solution.

The trajectory starting points are initially laid out along the cathode starting at the left edge of the cathode and working toward the right edge, stopping with the electron which falls closest to the right edge. All the spacings are then expanded (or contracted) so that the last trajectory lies exactly at the edge of the cathode. The current density is changed by the same amount (usually less than one or two percent) to take account of this expansion or contraction.

It was found that the trajectories are not accurate if it is attempted to compute them in the first matrix square off the cathode since an initially inaccurate field would have to be used to represent what should be a four-thirds power curve. To overcome this the trajectory starting points are moved forward to the first matrix point in front of the cathode and there assigned the proper initial velocity and direction (see above). This decreases the error (at least in velocity) from as high as 2.0 percent to less than 0.1 percent.

As mentioned above the second matrix point in front of the cathode is used to determine the cathode current densities. This is done because the voltages at the first matrix points are sensitive to fluctuations in the space charge at these points caused by small shifts in trajectory starting points. These first matrix point voltages are set using Child's law, solved for the voltage, and the current densities calculated from the second matrix point.

The new current densities along the cathode and the trajectory starting points are printed out for each cycle so that the calculations can be followed from cycle to cycle.

The trajectories are then traced through the gun. Up to 450 points can be calculated per trajectory. The trajectories are computed in groups of five and stored on tape (ITEMP) since there is not enough core space to store all of them in core at once. Δt (or DT in the program) should be selected so that at least one trajectory point is calculated in every matrix square through which the trajectories may pass.

At the same time that the trajectories are computed the space charge is computed using Eqs. 10 and 11. Also checks are made for interception along the electrode described by JU , and new borders (upper and lower) for the beam are computed.

The new beam borders, which are made up of the outermost coordinates of the trajectories in the beam, are compared with the previous borders to determine convergence of the program. It has been found that these borders are very sensitive to changes within the gun and furnish a very quick check for convergence. These borders are also printed out after every cycle and furnish a further check on the program progress from cycle to cycle.

When the beam borders essentially repeat themselves the results, the voltages, space-charge densities, and trajectories are all printed out. If it is desired the voltage and/or the space-charge density matrices can be saved on tape or cards for other uses or a possible restart in the calculations.

V. DATA REQUIRED

The following is a detailed description of the data required as input to the program. This data falls into three basic groups. The first group contains the constants which control the flow, convergence and scaling of the program. The second group contains the integer matrices which describe the gun geometry. The third group is concerned with the setting of initial voltages on V matrix electrodes.

The first data group consists of three cards. The information on these cards control the scaling of the gun on the matrix, the convergence requirements, some of the program flow and a detailed description of the cathode.

The first data card consists of 14 integers of which only the first 11 are used by the program. These integers are:

1. MM - the maximum number of cycles the program is to be allowed; this number can vary from 5 to 20, but in general 10 is an adequate number.

2. NM - the maximum number of passes through the coarse voltage matrix during each field calculation; 100 to 200 is a good range of values for this number, with 160 the most commonly used value. One fourth (25-50) of this number of passes are allowed through the fine matrix.

3. NVERRS - the maximum number of "errors" to be allowed in the coarse matrix at convergence; 0 to 200 is the usual range of values for

this number, with the number getting smaller as the accuracy desired increases. One fourth of this number (0-50) will be the number of errors allowed on the fine matrix.

4. NX - the length (in matrix points) of the voltage matrix to be used for a particular run; the length can be any value up to 120 but should be a multiple of 20 for convenience in making up and punching data.

5. NY - the width of the voltage matrix to be used for this run; can be any value up to 60.

6. NBMERS - the number of changes allowed in the beam border at convergence; 0 to 20 is the usual range of values, 5 is a typical value, smaller values demand a great deal more accuracy from the program.

7. IOTA - controls the saving and restoring of binary data. If it is 1 (one) no binary data will be read in or printed out; if it is 2 no binary data will be read in but the final voltage and space-charge matrices will be saved on tape or cards; if it is 3 the two matrices will be read in and then saved at the end of the run.

8, 9, 10, 11. IPQ, IPS, JPQ, JPS - the boundaries of a region of the V matrix to be multiplied by one half before calculations start; IPQ is the lower left-hand corner I coordinate of the region; IPS the lower right-hand corner I coordinate; JPQ the lower left-hand J coordinate and JPS the upper left-hand J coordinate of the region. This is used in front of the cathode to speed convergence of the field calculations.

The remaining numbers on this card are not used and can be left blank.

The second data card consists of 7 floating point numbers which control initial conditions, scaling, and convergence requirements for the gun being analyzed.

1. SZERO - the expected current density from the cathode in amperes per square meter, the closer this is to the actual value the faster the convergence; however, an error of 2 or 3 times the final result will be quickly corrected by the program. Excessive error could give rise to impossible conditions and result in early termination of the program.

2. VSCALE - the scaling voltage (in volts) for the gun, normally the highest actual voltage in the gun. All other voltages read in or printed out will be normalized to this value.

3. BMAG - the magnetic field strength in webers per square meter.

4. DX - the length of one matrix square in scaled units (see below).

5. XSCALE - the length (in meters) in the gun which will correspond to 1.0 in scaled (all data read in or printed out) units. This can usually be chosen such that material printed out will be in "nice" units, e.g., XSCALE = 0.00254, 1.0 units = 0.1 inch.

6. DT - the time increment for the trajectory calculation, chosen so that at least one trajectory point will be computed in each matrix square that the trajectory passes through. This can be found using

$$DT \approx \frac{DX \cdot XSCALE}{5.93 \times 10^5 \sqrt{VSCALE}} \quad (24)$$

7. EPSV - the convergence requirement for the voltage matrix calculation; a good value for this is 10^{-4} . EPSV is the largest change that can take place between succeeding values of V at convergence (in unscaled units this change is EPSV.VSCALE).

The third card describes the cathode geometry, it contains 2 integers and 4 floating point numbers.

1. IL - the first matrix point number (I coordinate) to the cathode's left, not necessarily the nearest matrix point.

2. IR - the I coordinate nearest the right-hand side of the cathode.

3. XLFT - the exact position (in scaled units) of the left-hand edge of the cathode; note that this does not necessarily have to correspond to the position of a matrix point. If this does happen to fall on a matrix point (i.e., IL = 21, DX = 0.1, XLFT = 2.0) it may be necessary to make XLFT a little larger to take care of the round-off error in the computer (i.e., XLFT = 2.00001).

4. XRT - the exact right-hand edge (in scaled units) of the cathode.

5. VARYM - the largest variation to be allowed in the current density across the cathode. This is used principally during the first few execution cycles to prevent unusual variations in current density along the cathode. Typical values for this are between 0.3 and 0.7, with smaller numbers permitting larger variations in current density.

6. VARY - the initial variation to be allowed in the current densities across the cathode. This number will be reduced by 0.1 on each cycle until the value for VARYM is reached. It should start at 1.0 or 0.9 for most guns.

The second data group consists of 6 integer matrices used to describe the gun geometry on the V matrix. All of these are read by a 2013 format and this is the prime reason it is desirable to make NX a multiple of 20.

In order to select these points properly it is first necessary to lay out the gun to be analyzed on graph paper. For this purpose the gun

must be accurately drawn on paper at least 120 squares long by 60 squares wide, if the full matrix is to be used. The points selected are those closest to the line to be represented, only the cathode must lie exactly on a matrix row. In general it is not necessary to modify voltages at matrix points which do not fall exactly on the gun geometry, although in some cases it may be desirable to modify the voltages in the immediate vicinity of the cathode to compensate for the approximation.

The first matrix to be read in is JL (J lower), the lower anode J coordinates; this includes the cathode, sole and cathode focusing anodes. These values are read serially across the V matrix from I = 1 to NX (up to a maximum of 120), remembering that the matrix begins at 1,1 and not 0,0 in the lower left-hand corner.

Figure 3 shows a sample geometry for a possible crossed-field gun. The matrix JL consists of the J coordinates along A-B-C-D and E-F.

The second matrix, JU (J upper), is made up of the J coordinate directly above JL. There must also be NX of these points as there was for JL. In Fig. 3 JU is made up of the points from G to H and K to L.

Since crossed-field gun geometries are usually quite simple, these two matrices will frequently be sufficient to describe the gun layout on the matrix. However, for some geometries such as in Fig. 3 there will be small region(s) such as NPKM where it will be desirable to compute the fields. The following two matrices permit these calculations.

The matrix JC describes the lower limit to any such region. In this case (Fig. 3) JC would be zero at all points except those along N-P. The upper limit of this region is described by JD which would also be zero except between M-K.

Both of these matrices should be zero at all points (1 to NX) where no calculations are desired. Calculations are made between these two matrices only where they are not zero.

The last two matrices describe approximate borders for the expected beam. The top beam border (JBU) is described by the dashed line B-Q-R in Fig. 3. The matrix will be zero from A to B. The bottom beam border (JBL) is described by the lines B-C-S-T in Fig. 3. This matrix is also zero from A to B.

These borders do not have to be accurate. They are used for two purposes. First, they are used in setting up an approximate space-charge distribution on the RHO matrix by the method described earlier. The second use is in determining the convergence of the program. After each cycle the new border is compared with the previous one, and then the new border replaces the previous one so that these matrices change from cycle to cycle.

The above 6 matrices are usually sufficient to describe the gun geometry on the voltage matrix.

At this point it is possible to read in any previous results (the V and RHO matrices) from binary cards or a binary save tape. This ability to restore data is especially useful if it is desired to make several runs with only slight variations in the geometry or other parameters. If multiple runs are not expected the save data routines can be deleted.

The third major data group sets the normalized voltages of anodes on the V matrix. This is done after the binary matrix (if any) is read in so that it is possible to make changes in the geometry from run to run. Three different methods of setting these voltages will be described.

Since most of the geometry can be described by the matrices JL, JU, JC and JD, it is possible to set most of the anode voltages using information from them. To do this it is noted that surfaces such as G-H in Fig. 3 are unipotential, so that it is sufficient to specify the voltage to be set on it and beginning and end I coordinates for this anode. The voltage can then be stored at the matrix points $V(I, JU[I])$ (i.e., V_{i, ju_i}). Up to three different voltages can be set along the surfaces described by JL. Since the regions described by JC and JD are small only one voltage can be set along these surfaces by this method.

A total of eight voltages can be set by this method; 3 along JU, 3 along JL, one along JC and one along JD. Any other points along the anodes must be set by one of the following two methods.

Since most of the voltages along horizontal and sloping surfaces can be set by the above method most of the remaining voltages to be set lie along vertical lines. Each card in this group has the lower and upper limits of J, the column I and the 6 normalized voltages to be stored, all in a 3I4,6E10.3 format. Note that lower to upper limits on J must describe exactly 6 points and at least one card must be read by this method. The number of cards to be read by this method is set by the first card in this group.

The last data set common to all three programs permits the storage of voltages at individual points on the voltage matrix. The number of points to be set is punched in the first card in an I4 format. All the following cards have the I,J coordinates and the (normalized) value to be stored at that point on the voltage matrix, arranged in a 2I4,F12.4 format. Note that at least one point must be read.

Since it is necessary to carry out the initial field calculations on the coarse (odd-numbered terms) matrix, sometimes points just off the

useful part of the voltage matrix will be used. An example of such a point is seen in Fig. 3 at $I = 55$, $J = 45$. When the voltage calculations are made in column 57 odd-numbered points in column 55 will be used. In some cases ($J = 3, 5, 7 \dots 13$; $I = 55$) these numbers will already be properly set (i.e., zero), but at $J = 45$ the voltage on this anode should be 0.25. Thus it is necessary to watch quite closely for such points as failure to assign the proper value can considerably slow the program convergence.

A more detailed development of the data required for the gun of Fig. 3 is shown in Appendix II.

VI. RESULTS

Results obtained with the program have been in general agreement with experimental and Poisson Cell¹ results. Due to the lack of detailed theoretical and/or experimental information about crossed-field guns it is difficult to say just how good the agreement may be.

Individual sections of the program have been found capable of producing results within 0.1 percent of theoretical results and in some cases the agreement is within 0.01 percent. It is difficult to estimate the overall accuracy of the program but agreement with experiment within 5 percent should be obtainable.

The following figures show the results for a typical computer run. The final results and also the cycle by cycle changes are presented for the run.

Figure 4 shows the final set of trajectories for this gun. The gun itself is a foreshorted Kino gun designed from the theoretical curves of Kino². The severe crossing of trajectories seen in this figure has

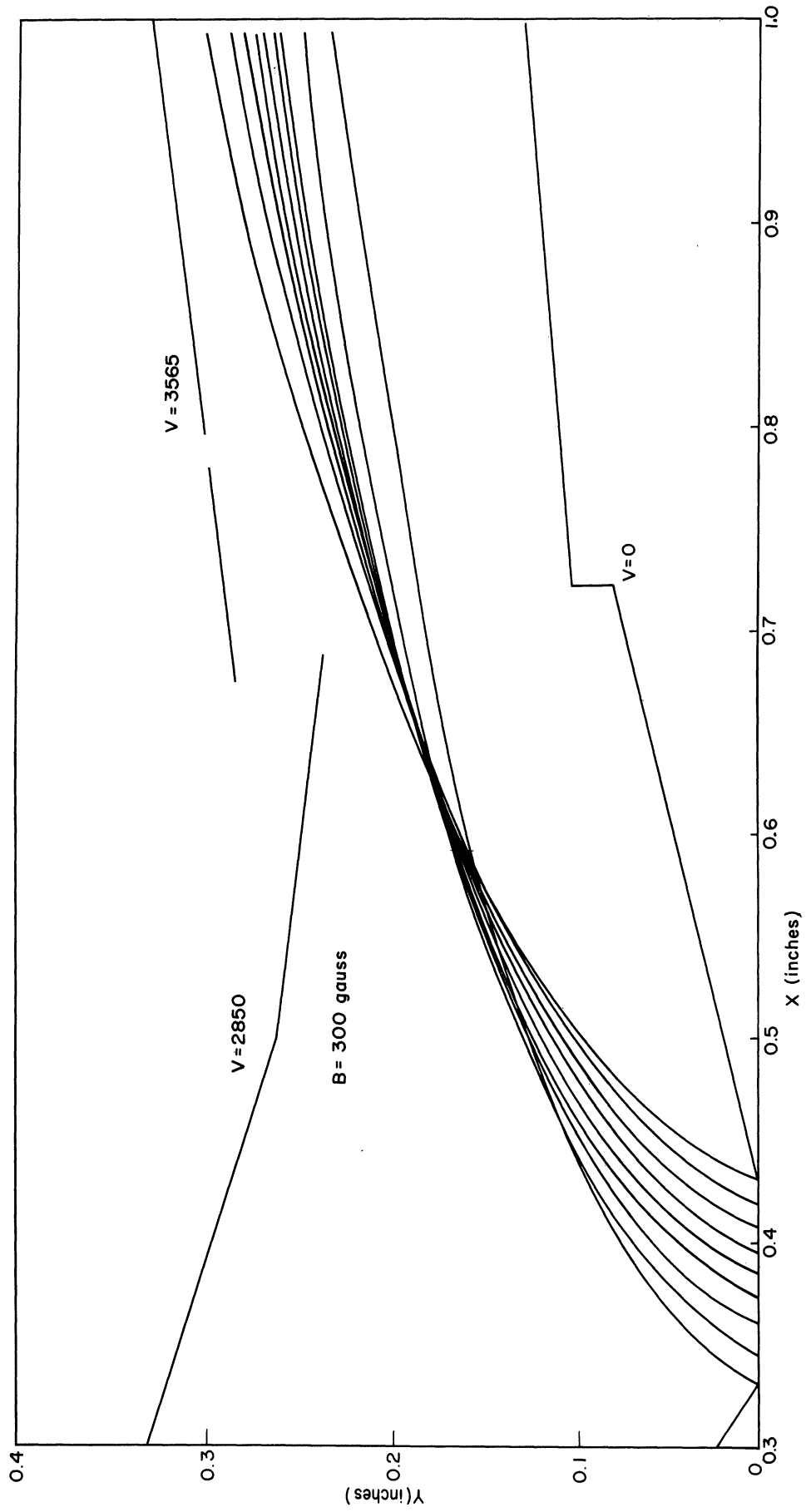


FIG. 4 TRAJECTORIES AND GEOMETRY FOR A FORESHORTENED KINO GUN.

been typical for the guns designed from these curves and analyzed by the program. Methods of improving the design will be discussed below.

The step by step variations of the current densities along the cathode are presented in Fig. 5. The general shape of this distribution is evident from the first cycle. By the fourth cycle the distribution has essentially reached its final form and changes by less than 4 percent at any point over the last two cycles. Cycles 5 and 6 have been combined with the x's representing the results from the sixth cycle. The current density over most of the cathode is seen to be higher than Kino's theoretical current density.

Figure 6 presents a summary of all the passes through the voltage matrix. A total of 230 passes through the coarse (odd-numbered terms) matrix and 152 passes through the fine matrix were required during execution. The number of passes on each cycle falls quite rapidly during the first few cycles and then levels off to between 22 and 25 passes through both the fine and coarse matrices over the last 3 cycles. This tendency has been typical for most of the designs that have been run on the computer, although the exact number of passes required for the later cycles will vary from as low as five to as high as 30. Figures 7a and 7b summarize this general result.

Figures 7c and 7d present the relative changes in the beam borders on each cycle. The dots represent the total change in these borders on each cycle and the crosses represent the number of points at which the borders changed. Both of the quantities being equal, as they are on the last four cycles, indicates that the program is very near convergence by the third cycle and it probably would have been desirable to be a little more liberal on the convergence requirements.

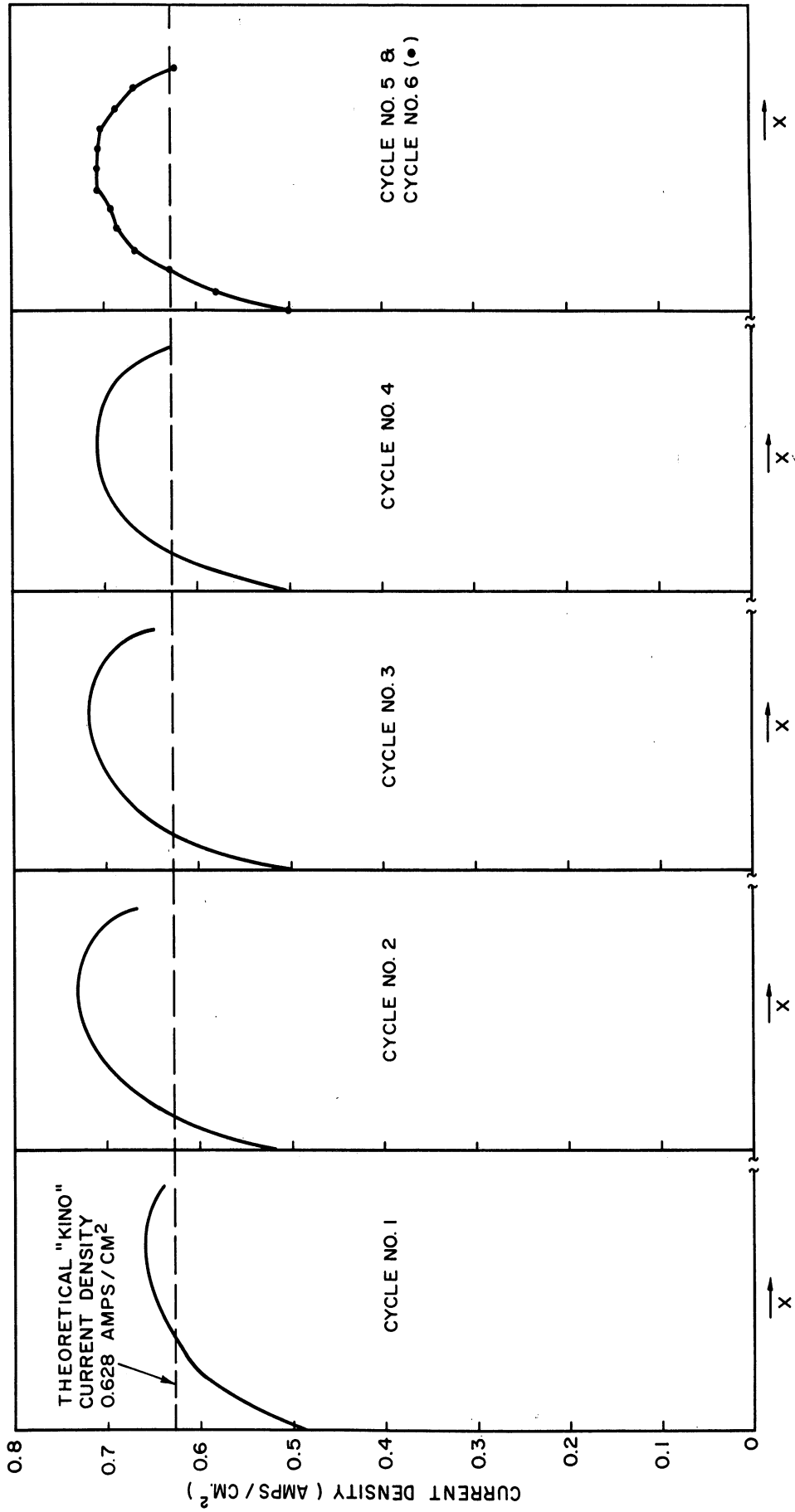


FIG. 5 CATHODE CURRENT DENSITY DISTRIBUTION ACROSS THE CATHODE FOR EACH CYCLE.

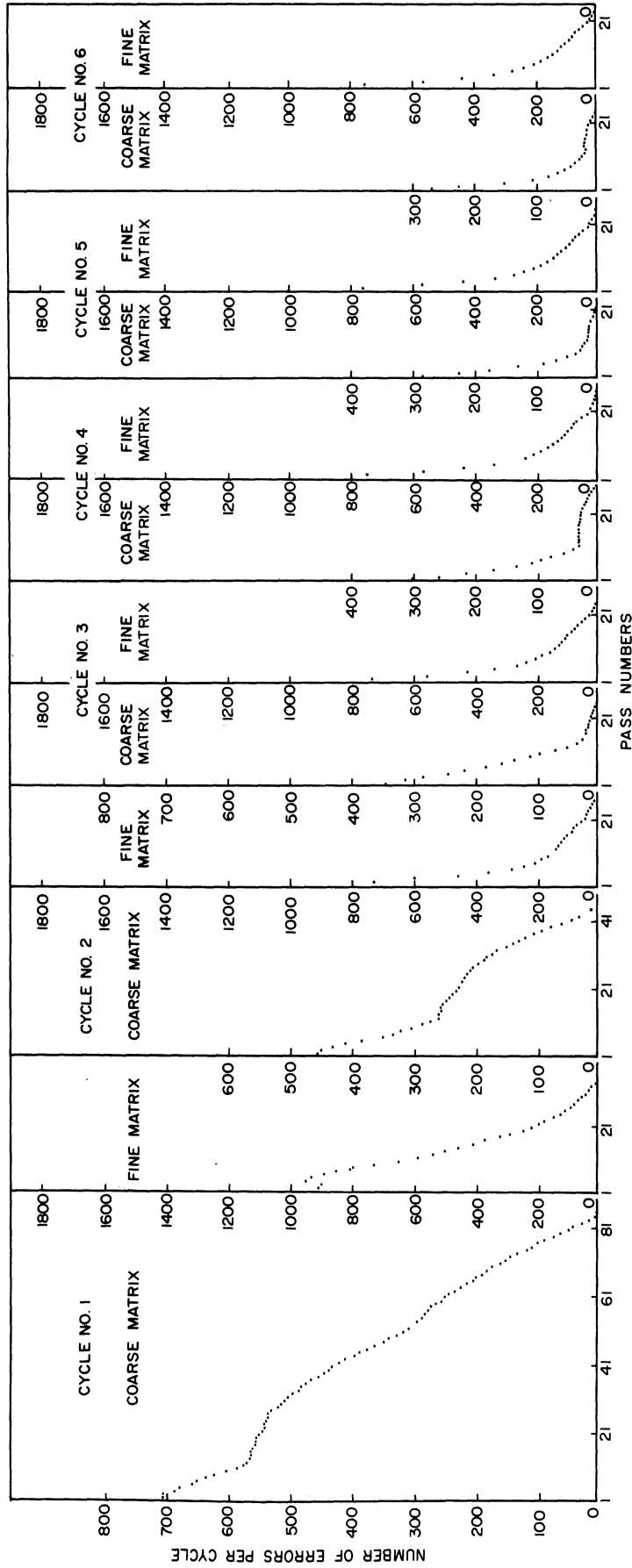


FIG. 6 VOLTAGE MATRIX ERRORS ON EACH PASS DURING FIELD CALCULATIONS.

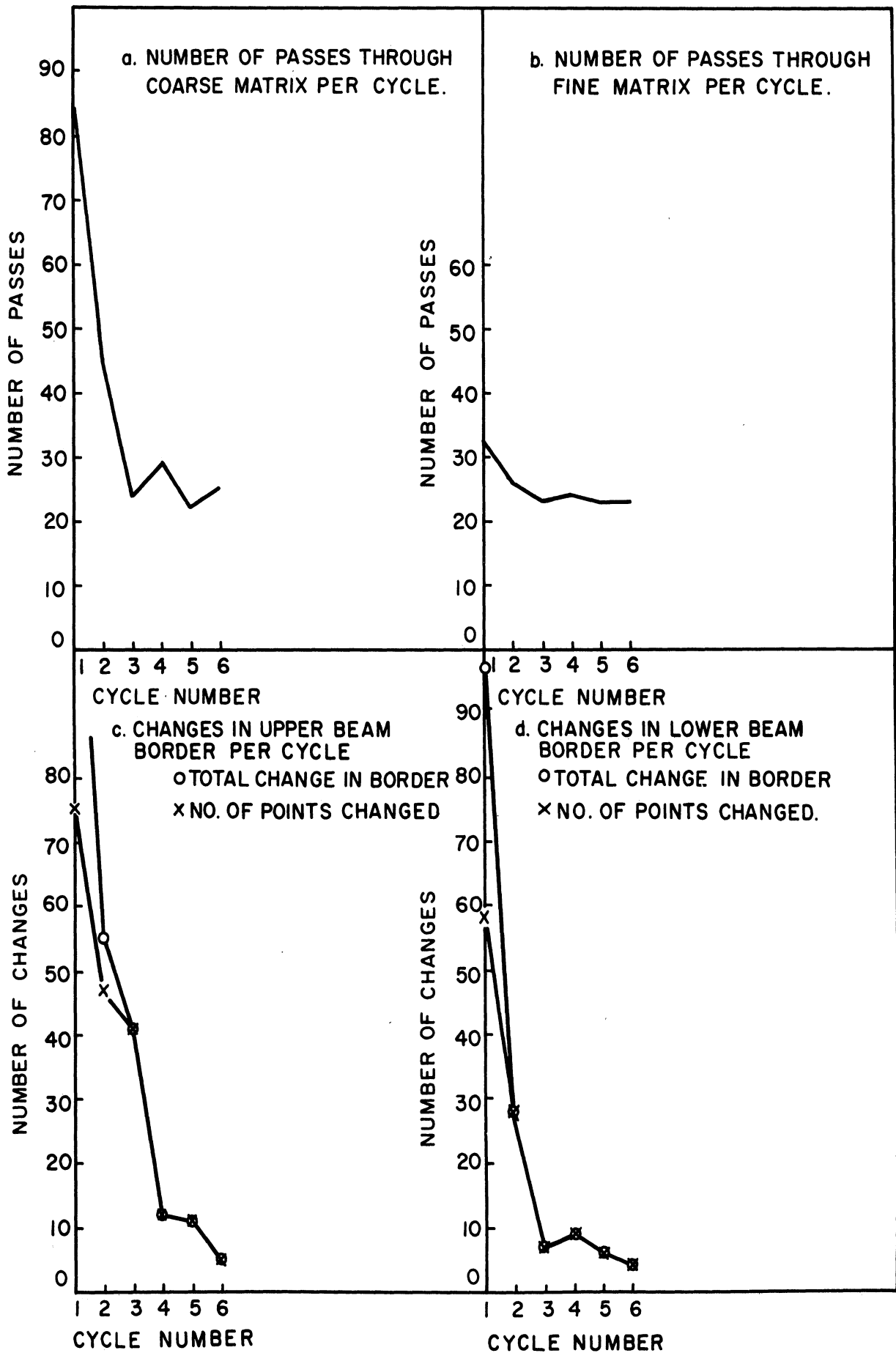


FIG. 7 CYCLE BY CYCLE VARIATION OF PARAMETERS.

The run described above is fairly typical for this program. However, some comments as to what may happen under different circumstances should be made. In general it has been found that the more poorly designed guns (those with excessive trajectory crossing) take longer to converge. Also, guns with larger cathodes (with respect to the matrix) tend to take more cycles to converge. On the other hand, a gun design producing a laminar beam will tend to converge faster.

A Kino gun for which some experimental data is available is presented in Fig. 8. The current distribution in the gun region seems to follow that implied by the photograph presented by Midford and Kino³. Their photograph shows that the emission from the back of the cathode is appreciably less than for the center and front regions. The computed current densities along the cathode are shown directly above the cathode in Fig. 8. Midford and Kino were able to improve this beam by tilting the gun forward 4 degrees and manipulating the drift space electrode voltages. They noted that the top of the beam appeared to be fairly stable while the bottom edge would ripple under some conditions. The computer results show strong cycloiding within the beam which results in the somewhat ragged lower edge.

The program results indicate a current of about 123 ma from the gun which is slightly higher than that of the experimental device.

An attempt was made to modify this design in order to obtain a better beam. The result of these modifications is shown in Fig. 9. It was necessary to change the shape of the first anode and in effect increase the emission from the rear of the cathode. The focusing anode in front of the cathode has also been lowered slightly. It has been found that the current density variations across the cathode can be

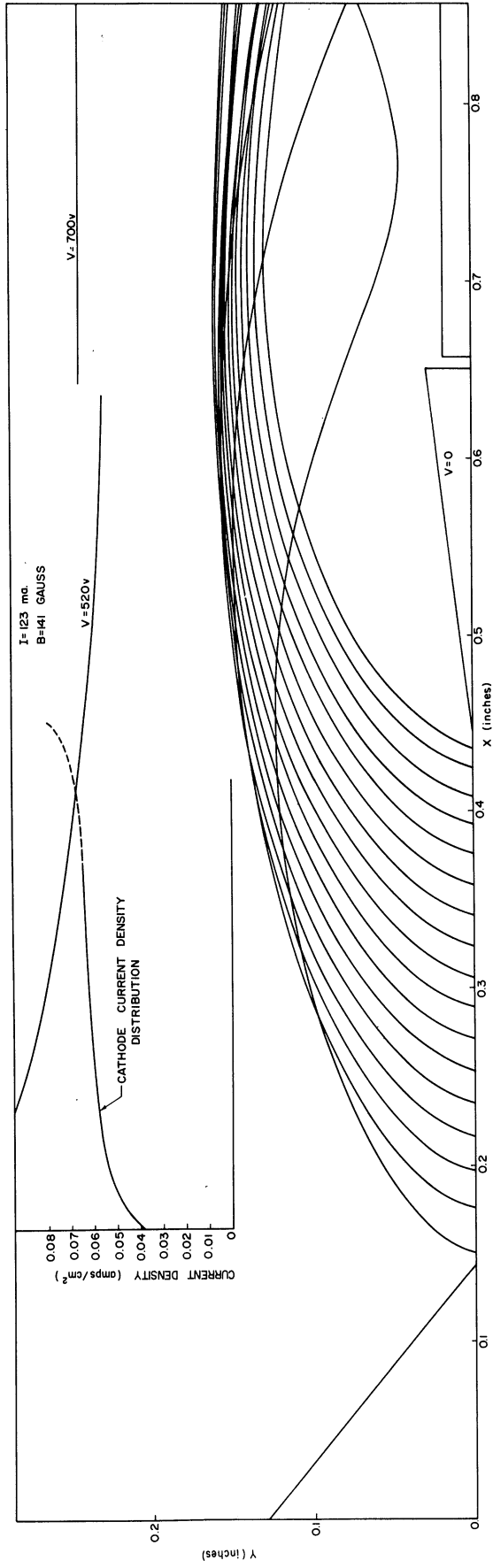


FIG. 8 TRAJECTORIES AND CATHODE CURRENT DENSITY VARIATIONS FOR KINO GUN.

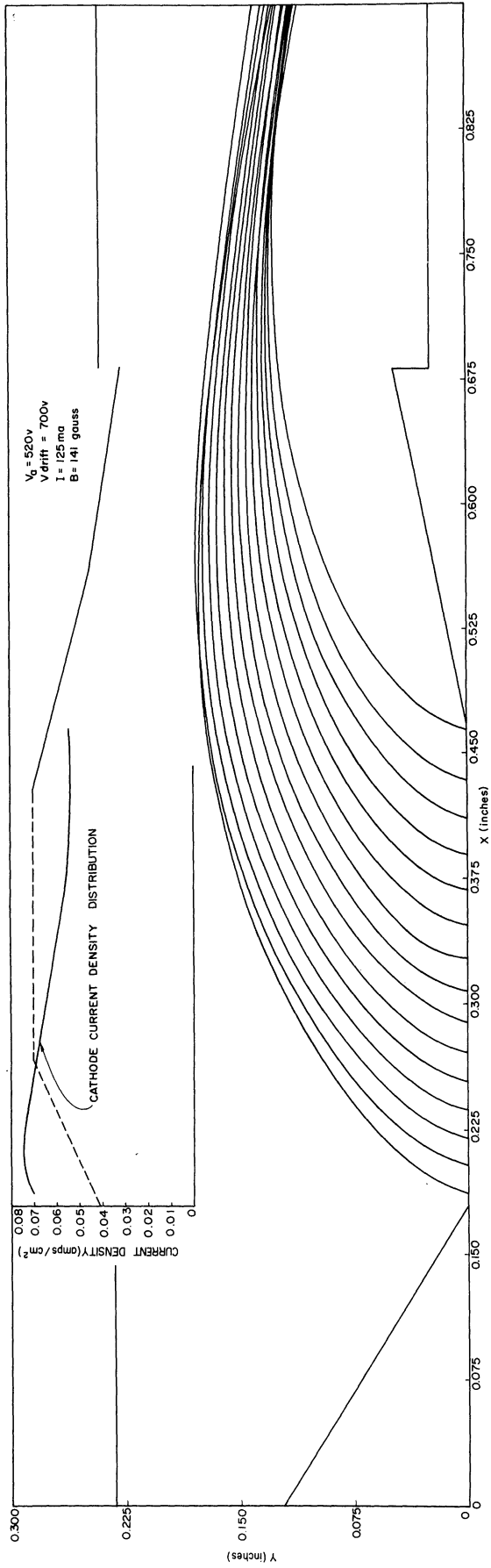


FIG. 9 TRAJECTORIES AND CATHODE CURRENT DENSITY VARIATIONS FOR MODIFIED KINO GUN.

controlled to a large extent by modifications of these cathode focusing anodes. Preliminary results suggest that the rear cathode focusing electrode should be closer to the Pierce angle (22.5 degrees from the horizontal).

At right the beam is beginning to ripple and further study of the drift space potentials is needed. Apparently a stronger focusing field is needed.

Figure 10 illustrates the effect of moving the cathode focusing electrodes back too far. A simple planar geometry for which the data is developed in Appendix II is shown. The trajectories from the back of the cathode rise well above the front trajectories although the front trajectories are pushed so far forward that they soon start cycloiding.

The space charge has also caused the zero equipotential to move around the corner of the lower electrode toward the cathode. If the cathode is made wider or moved closer to the corner the equipotential will rapidly move closer to the cathode.

The current density distribution across the cathode is shown directly below the cathode.

In Fig. 11 ramp electrodes have been added in front and in back of the cathode. The trajectories now show a substantially laminar beam through the gun and transition regions, but crossing still exists in the drift space. It should be noted that the magnetic field strength was reduced to 140 gauss for this gun.

The cathode current density distribution is also seen to be substantially improved. The current density variations have been reduced from nearly 90 percent in Fig. 10 to less than 20 percent in Fig. 11. This variation probably could be reduced still further by sloping the upper anode down to the right.

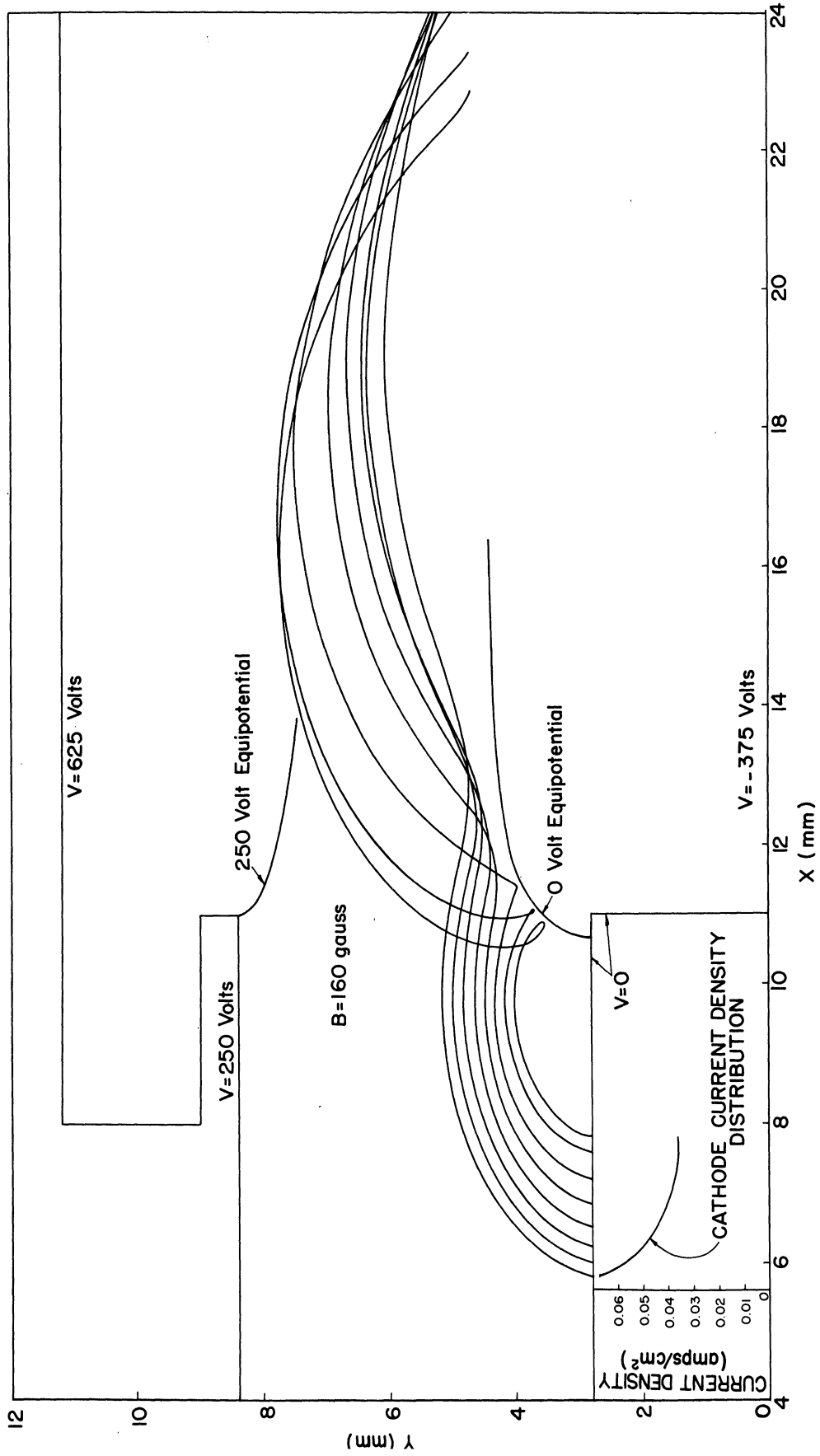


FIG. 10 TRAJECTORIES AND CATHODE CURRENT DENSITY VARIATIONS FOR SIMPLE PLANAR GEOMETRY.

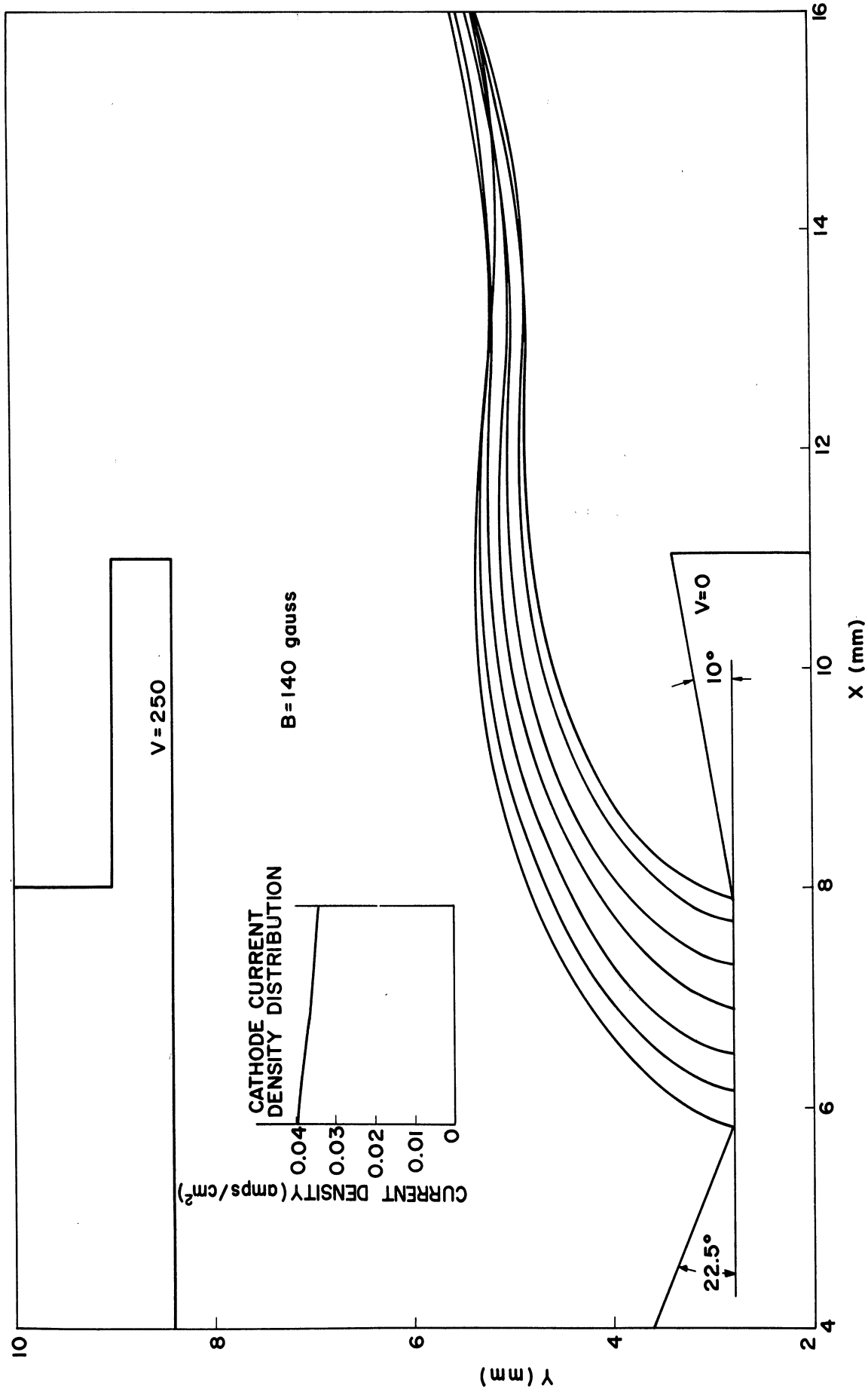


FIG. 11 TRAJECTORIES AND CATHODE CURRENT DENSITY VARIATIONS FOR IMPROVED VERSION OF ORIGINAL PLANAR GEOMETRY.

VII. CONCLUSIONS

A digital program for the analysis of crossed-field electron guns has been developed for use on a large-scale digital computer. It is written in the IBM 7090 FORTRAN programming system and requires less than 10 minutes on an IBM 7090 for execution. The program determines voltage and space-charge distributions, cathode current densities, and then beam shape.

The results have been in general agreement with experimental results. Agreement within 5 percent of experiment should be easily attainable.

The program has shown itself to be a very quick and accurate method of determining the feasibility of any gun design.

APPENDIX I. LISTING OF CEMFEG

The following pages contain a listing of the digital computer program CEMFEG (for Crossed Electric and Magnetic Field Electron Gun). The program as shown is set up to run under The University of Michigan Executive System and some minor modifications would have to be made to run this under the FORTRAN monitor or other systems. Some subroutines (e.g., FTRAP, ERROR) would not be needed and the calls for some of the others (e.g., SQRT, ELOG, etc.) would have to have a final F added to them.

The subroutines RDCHK and WRCHK, which read and punch binary cards, are presented in Appendix III. It may be desirable to replace the calls for these by tape reading and writing routines for data saving purposes.

Tape numbers are set on the first page of the listing, IIN is the input tape, IOUT is the output tape, ITEMP is the temporary storage tape for the trajectories, and ISAVE can be used as the binary "save" tape.

To modify the program for different size or shape matrices only cards on the first page of the program listing need be changed. If the matrix shape is changed the dimension statements must be changed.

The matrix constants are NVR, the number of points in the voltage and space-charge matrices; NTRPT, the number of points in each group of 5 trajectories; NTRA, the maximum number of trajectories; and NXY, one less than the maximum length of any trajectory.

\$ EXECUTE, FULL DUMP, I/O DUMP
\$COMPILE FORTRAN,PUNCH OBJECT,PRINT OBJECT

CMFGC001

C
C CROSSED ELECTRIC AND MAGNETIC FIELD ELECTRON GUN
C - -
C
C (CEMFEG2D) NONUNIFORM ELECTRON BEAM PROGRAM
C
C PROGRAMED BY J. E. BOERS , THE UNIVERSITY OF MICHIGAN
C
C *****

MATRIX DIMENSIONS

DIMENSION V(120,60), RHO(120,60), X(5,450), Y(5,450)

DIMENSION JL(120), JU(120), JC(120), JD(120), JBU(120), JBL(120),
1 NBU(120), NBL(120)

DIMENSION CM(120), XLOC(100), YLOC(100), XDOT(100), YDOT(100)

DIMENSION NOERRS(200), ERRR(100)

*****M*****

MATRIX MANIPULATION CONSTANTS

NVR=7200
NTRPT=2250
NTRA=100
NXY=449

CALL FTRAP(0)

TAPE NUMBERS

ISAVE=2
IIN=7
IOUT=6
ITEMP=3

```
C
C
C           INPUT DATA
C
C 5 READ INPUT TAPE I2N,1005,MM,NM,NVERRS,NX,NY,NBMERS,IOTA,IPQ,IPS,
C 1 JPQ,JPS,IOTAB,IOTAC,IOTAD,SZERO,VSCALE,BMAG,DX,XSCALE,DT,EPSV,BE
C
C READ INPUT TAPE IIN,1025,IL,IR,XLFT,XRT,VARYM,VARY
C
C REWIND ITEMP
C SENSE LIGHT 0
C
C DO 10 I=1,NVR
C V(I)=0.0
C 10 RHO(I)=0.0
C
C IF(IOTA-2)25,25,20
C
C 20 CALL RDCHK(V,NVR)
C CALL RDCHK(RHO,NVR)
C
C 25 READ INPUT TAPE IIN,1020,(JL(I),I=1,NX),(JU(I),I=1,NX),(JC(I),I=1,
C 1 NX),(JD(I),I=1,NX),(JBU(I),I=1,NX),(JBL(I),I=1,NX)
C
C DO 30 K=1,3
C
C READ INPUT TAPE IIN ,1025,ILOW,IHIGH,VSET
C
C DO 30 I=ILOW,IHIGH
C J = JU(I)
C 30 V(I,J) = VSET
C DO 35 K=1,3
C
C READ INPUT TAPE IIN,1025,ILOW,IHIGH,VSET
C
C DO 35 I=ILOW,IHIGH
C J = JL(I)
C 35 V(I,J) = VSET
C
C READ INPUT TAPE IIN,1025,ILOW,IHIGH,VSET
C DO 37 I=ILOW,IHIGH
C J=JC(I)
C 37 V(I,J)=VSET
C
C READ INPUT TAPE IIN,1025,ILOW,IHIGH,VSET
C DO 38 I=ILOW,IHIGH
C J=JD(I)
C 38 V(I,J)=VSET
C
C WRITE OUTPUT TAPE IOUT,1030,XSCALE,DX,VSCALE,SZERO,BMAG,DT,(JU(I),
C 1 I=1,NX)
C
C WRITE OUTPUT TAPE IOUT,1035,(JL(I),I=1,NX)
C
C READ INPUT TAPE I9N,1045,JJJ,((LL,LU,II,(V(II,JJ) ,JJ=LL,LU)),I=1
C 1 ,JJJ)
C
C READ INPUT TAPE IIN,1050,JJJ,((II,JJ,(V(II,JJ),),I=1,JJJ))
C
```

C
C
C

CONSTANTS

```
M=0
N2VERS=NVERRS/4
NMM=NM/4
IR1=IR-1
NX1=NX-1
NX2=NX1-1
DXI=1.0/DX
XLFT=XLFT/DX
XRT=XRT/DX
G=VSCALE/(XSCALE*XSCALE)
CONST=G*1.759E11*DT*DT/DX
Z=BMAG*DT*1.759E11
ZZ = Z*Z
ZZ1 = 1.0-ZZ/4.0
ZZ1I = 1.0/(1.0+ZZ/4.0)
ZO2 = Z/2.0
ECCO = 2.33E-6*VSCALE*SQRT(VSCALE)/(DX*DX*XSCALE*XSCALE)
ECCO=ECCO*0.25
SRHA=DX*DX*XSCALE*XSCALE*SZERO/(4.*8.854E-12*SQRT(2.0*1.759E11*VSC
1ALE)*VSCALE)
SCALE=5.93E5*EXP(ELOG(XSCALE*XSCALE/2.33E-6)/3.0)/XSCALE
THTCON=(3.14159*BMAG/0.36)*(3.31E-4+DX*XSCALE)/5.08E-4
YZR=(DX*XSCALE-1.27E-4)/1.27E-4
```

C
C


```
C
C      SET INITIAL FIELD VALUES
C
      IF(IOTA-2)15,15,60
15 DO 39 I=IL,IR
      JL(I)=JL(I)+1
      J=JL(I)
39 V(I,J)=EXP(2.0*(ELOG(SZERO/(ECCO*4.0)))/3.0)
      DO 45 I=1,NX
      JLI=JL(I)
      AL=JLI
      JUI=JU(I)
      AUAL=JUI-JLI
      VDIF=(V(I,JUI)-V(I,JLI))/AUAL
      DO 45 J=JLI,JUI
      V(I,J)=V(I,JLI)+VDIF*(FLOATF(J)-AL)
45 CONTINUE
      DO 49 I=1,NX
      IF(JC(I))49,49,46
46 JUI=JD(I)-1
      JLI=JC(I)+1
      AL=JLI+1
      AUAL=JUI-JLI+2
      VDIF=(V(I,JUI+1)-V(I,JLI-1))/AUAL
      DO 48 J=JLI,JUI
48 V(I,J)=V(I,JLI-1)+VDIF*(FLOATF(J)-AL)
49 CONTINUE
50 DO 55 I=IPQ,IPS
      DO 55 J=JPQ,JPS
55 V(I,J) = 0.5*V(I,J)
C
C      SET SPACE CHARGE (IOTA=1)
C
      DO 58 I=1,NX
      IF(JBU(I))58,58,56
56 JBUI=JBU(I)
      JBLI=JBL(I)+1
      DO 57 J=JBLI,JBUI
      RHO(I,J)=SRHA/SQRT(V(I,J))
57 CONTINUE
58 CONTINUE
C
```

C
C
C
C

COARSE FIELD CALCULATION

```
60 N = 0
65 NERRS = 0
   DO 105 I=1,NX,2
     JLI=2*(JL(I)/2)+3
     JUI=JU(I)-2
     DO 105 J=JLI,JUI,2
       IF (I-1)75,70,75
70  VNU = (V(I,J+2)+V(I,J-2))*0.5
     GO TO 90
75  IF(NX1-I)80,85,80
80  VNU=(V(I,J+2)+V(I-2,J)+V(I-2,J)+V(I,J-2))*0.25-4.0*RHO(I,J)
     GO TO 90
85  VNU = (V(I,J)+V(I-2,J)+V(I,J-2)+V(I,J+2))*0.25-RHO(I,J)*4.0
```

C

```
90 IF (ABSF(VNU-V(I,J))-EPSV)100,95,95
95 NERRS = NERRS +1
100 V(I,J) = VNU
105 CONTINUE
```

C

```
   N = N+1
   NOERRS(N) = NERRS
   IF (N-NM) 110,110,115
110 IF (NERRS-N2VERS) 115,65,65
```

C

```
C
C   FILL IN INTERMEDIATE POINTS FOR THE FINE FIELD CALCULATION
C
115 DO 120 I=1,NX,2
    JLI = 2*((JL(I)+2)/2)
    JUI=JU(I)-1
    DO 120 J=JLI,JUI,2
120 V(I,J) = (V(I,J-1)+V(I,J+1))*0.5
    DO 125 I=2,NX2,2
    JLI=JL(I)+1
    JUI=JU(I)-1
    DO 125 J=JLI,JUI
125 V(I,J) = (V(I-1,J)+V(I+1,J))*0.5
    JLI=JL(NX )+1
    JUI=JU(NX )-1
    DO 130 J=JLI,JUI
130 V(NX,J) = V(NX-1,J)
C
    WRITE OUTPUT TAPE IOUT,1055,(NOERRS(I),I=1,N)
C
C
```

```
C
C     FINE MATRIX CALCULATION
C
135 N = 0
140 NERRS = 0
    DO 180 I=1,NX
      JLI=JL(I)+1
      JUI=JU(I)-1
      DO 180 J=JLI,JUI
        IF(I-1)150,145,150
145 VNU=(V(I,J+1)+V(I,J-1))*0.5
    GO TO 165
150 IF (NX-I) 155,160,155
155 VNU=(V(I+1,J)+V(I,J+1)+V(I-1,J)+V(I,J-1))*0.25-RHO(I,J)
    GO TO 165
160 VNU = (V(I,J+1)+V(I-1,J)+V(I,J)+V(I,J-1))*0.25-RHO(I,J)
165 IF (ABSF(V(I,J)-VNU)-EPSV)175,170,170
170 NERRS = NERRS+1
175 V(I,J) = VNU
180 CONTINUE
C
    DO 1180 I=1,NX
      IF (JC(I)) 1180,1180,1144
1144 JLI=JC(I)+1
      JUI=JD(I)-1
      DO1175 J=JLI,JUI
        IF(I-1)1150,1145,1150
1145 VNU=(V(I,J+1)+V(I,J-1))*0.5
    GO TO1165
1150 IF(NX-I)1155,1160,1155
1155 VNU=(V(I+1,J)+V(I,J+1)+V(I-1,J)+V(I,J-1))*0.25-RHO(I,J)
    GO TO 1165
1160 VNU=(V(I,J+1)+V(I,J-1))*0.5-RHO(I,J)
1165 IF (ABSF(V(I,J)-VNU)-EPSV)1175,1170,1170
1170 NERRS=NERRS+1
1175 V(I,J)=VNU
1180 CONTINUE
C
    N=N+1
    NOERRS(N )=NERRS
    IF(N-NMM)185,185,190
185 IF (NERRS-NVERRS) 190,140,140
C
190 WRITE OUTPUT TAPE IOUT,1060,(NOERRS(I),I=1,N)
C
C
```

```
C
C
C           NEW CURRENT DENSITIES
C
C           DO 400 I=1,NX
400 CM(I)=0.0
C           DO 405 I=IL,IR
C           J=JL(I)+1
405 CM(I)=ECCO*V(I,J)*SQRT(V(I,J))
C
C           WRITE OUTPUT TAPE IOUT,1010,(CM(I),I=1,NX)
C
C           CMIN=CM(IR)
C           DO 415 I=IL,IR
C           IF(CMIN-CM(I))415,415,410
410 CMIN=CM(I)
415 CONTINUE
C           DO 420 I=IL,IR
420 CM(I)=CMIN/CM(I)
C
C
C           CHECK FOR LARGE CURRENT VARIATIONS
C
C           IF(SENSE LIGHT 1)425,427
425 SENSE LIGHT 2
427 IF(VARY-VARYM)430,430,428
428 VARY=VARY-0.1
430 DO 440 J=IL,IR
C           IF(CM(J)-VARY )435,440,440
435 CM(J)=VARY
C           SENSE LIGHT 1
440 CONTINUE
C           IF(SENSE LIGHT 2)445,455
445 IF (SENSE LIGHT 1)450,455
450 SENSE LIGHT 2
C
C
C           NEW X STARTING POINTS
C
C           455 XLOC=XLFT
C           I=2
C           DO 475 J=IL,IR1
460 XLOC(I)=XLOC(I-1)+CM(J)
C           I=I+1
C           IF(XFIX(XLOC(I-1))-J+1)460,460,465
465 IF(XLOC(I-1)-XRT-CM(IR1)*0.5)475,470,470
470 I=I-1
C           GO TO 465
475 CONTINUE
C           NOEL=I-1
C
```

C
C
C
CHECK FOR LARGE CURRENT CHANGE

IF(SENSE LIGHT 3)480,485
480 SENSE LIGHT 4
485 IF(CMIN-SZERO/VARYM)495,495,490
490 CMIN=SZERO*(1.0+V1RYM)/(0.5+VARYM)
GO TO 505
495 IF(CMIN-SZERO*VARYM)500,510,510
500 CMIN=SZERO*(0.5+VARYM)/(1.0+VARYM)
505 SENSE LIGHT 3
510 IF(SENSE LIGHT 4)515,525
515 IF(SENSE LIGHT 3)520,525
520 SENSE LIGHT 4

C
C
C
SET NEW VELOCITIES AND STARTING POINTS

525 DXN=(XRT-XLOC)*DX/(XLOC(NOEL)-XLOC)
CMIN=(XRT-XLOC)*CMIN/(XLOC(NOEL)-XLOC)
SZERO=(SZERO+CMIN)*0.5
RHOA=DT *DX*DX*XSCALE*SZERO / (4.0*8.854E-12*VSCALE)
XLOC=XLOC*DX
DO 530 I=2,NOEL
530 XLOC(I)=XLOC+(XLOC(I)-XLFT)*DXN

C
DO 535 I=IL,IR
J=JL(I)
535 V(I,J)=EXP(2.0*(ELOG(SZERO / (ECCO*CM(I)*4.0)))/3.0)

C
WRITE OUTPUT TAPE IOUT,1100,SZERO,(XLOC(I),I=1,NOEL)

C
DO 540 K=1,NOEL
I=XFIXF(XLOC(K)*DXI)+1
THT=THTCON/(0.46+0.4E-4*SZERO/CM(I))
VEL=SCALE*EXP(ELOG(DX*DX*SZERO/CM(I))/3.0)
S1=SZERO/CM(I)
S2=S1**2
XLOC(K)=XLOC(K)+DX*BMAG*(2.9E-8*S2-6.6E-4*S1+11.2)*(1.0+(0.18E-8*S
12-0.48E-4*S1+0.55)*YZR)
YDOT(K)=VEL*COS(THT)
YLOC(K)=DX*FLOATF(JL(I)-1)-1.01*YDOT(K)*DT
540 XDOT(K)=VEL*SIN(THT)

COMPUTE NEW TRAJECTORIES

```
DO 195 I=1,NVR
195 RHO(I)=0.0
DO 197 I=1,NX
NBU(I) = 0
197 NBL(I)=NY
DO 199 I=1,IL
199 NBL(I) = 0

NPTT=0
LL=0
200 DO 205 I=1,NTRPT
X(I) = 0.0
205 Y(I) = 0.0
210 DO 270 L=1,5
LL=LL+1
X(L,1)=XLOC(LL)
Y(L,1)=YLOC(LL)
X(L,2)=X(L,1)+XDOT(LL)*DT
Y(L,2)=Y(L,1)+YDOT(LL)*DT
I = 1+XFIXF(X(L,2)*DXI)
J = 1+XFIXF(Y(L,2)*DXI)
I1=I
J1=J
DO 250 K=2,NXY
I=I1
J=J1
IF (J-JU(I)) 215,255,255
215 EX = (V(I,J)+V(I,J+1)-V(I+1,J)-V(I+1,J+1))*0.5*CONST
EY = (V(I,J)+V(I+1,J)-V(I,J+1)-V(I+1,J+1))*0.5*CONST
X(L,K+1) = (X(L,K)+X(L,K)-ZZ1*X(L,K-1)+Z*(Y(L,K)-Y(L,K-1))-(EX+Z02
1 *EY))*ZZ1I
Y(L,K+1) = (Y(L,K)+Y(L,K)-ZZ1*Y(L,K-1)-Z*(X(L,K)-X(L,K-1))-(EY-Z02
1 *EX))*ZZ1I
C
I1 = 1+XFIXF(X(L,K+1)*DXI)
J1 = 1+XFIXF(Y(L,K+1)*DXI)
C
```

C
C
C

COMPUTE NEW SPACE CHARGE

```
IF (I1-I) 220,225,220
220 DELX = X(L,K+1)-X(L,K)
    DELXSQ = DELX*DELX
    DELY=Y(L,K+1)-Y(L,K)
    DELYSQ=DELY*DELY
    VSQ=DELXSQ+DELYSQ
    SINESQ=DELXSQ/VSQ
    RHA=RHOA*SINESQ/SQRT(VSQ)
    A1=J1-1
    RAT=Y(L,K+1)MDXI-A1
    RHO(I1,J1+1)=RAT*RHA+RHO(I1,J1+1)
    RHO(I1,J1)=R+A*(1.0-RAT)+RHO(I1,J1)

C
225 IF (J1-J) 230,245,230
230 IF (I1-I) 240,235,240
235 DELX = X(L,K+1)-X(L,K)
    DELXSQ=DELX*DELX
    DELY=Y(L,K+1)-Y(L,K)
    DELYSQ=DELY*DELY
    VSQ=DELXSQ+DELYSQ
240 COSSQ = DELYSQ/VSQ
    RHA=RHOA*COS SQ/SQRT(VSQ)
    A1=I1-1
    RAT=X(L,K+1)MDXI-A1
    IF (J1-J) 242,242,243
242 RHO(I1,J1+1) = RHA*(1.0-RAT)+RHO(I1,J1+1)
    RHO(I1+1,J1+1) = RHA*RAT+RHO(I1+1,J1+1)
    GO TO 245
243 RHO(I1+1,J1)=RHA*RAT+RHO(I1+1,J1)
    RHO(I1,J1)=R+A*(1.0-RAT)+RHO(I1,J1)
245 IF(NBU(I1)-J1)246,247,247
246 NBU(I1)=J1
247 IF(NBL(I1)-J1)249,249,248
248 NBL(I1)=J1+1
249 CONTINUE
    IF (I1-NX) 250,255,255
250 CONTINUE
    K = NXY
255 IF (NPTT-K) 260,265,265
260 NPTT = K
265 IF (LL-NOEL) 270,275,275
270 CONTINUE
275 WRITE TAPE ITEMP,(X(I),I=1,NTRPT),(Y(I),I=1,NTRPT)
    IF (LL-NOEL) 200,280,280
280 REWIND ITEMP
```



```
C
C          PRINT RESULTS
C
C          SAVE V AND RHO MATRICES
C
C 375 IF(IOTA-2)377,376,376
C
C 376 CALL WRCHK(V,NVR)
      CALL WRCHK(RHO,NVR)
C
C 377 WRITE OUTPUT TAPE IOUT,1065,(V(I),I=1,NVR)
C
      WRITE OUTPUT TAPE IOUT,1070,(RHO(I),I=1,NVR)
C
      NPTT=5 *NPTT+5
      K=1
C
C 380 READ TAPE ITEMP,(X(I),I=1,NTRPT),(Y(I),I=1,NTRPT)
C
      WRITE OUTPUT TAPE IOUT,1075,K,(X(I),Y(I),I=1,NPTT)
C
      K=K+5
      IF(K-NOEL)380,380,385
C
C 385 WRITE OUTPUT TAPE IOUT,1080
C
      CALL ERROR
C
      GO TO 5
C
```

```
C
C          FORMATS
C
1005 FORMAT(14I4/8E9.2)
C
1010 FORMAT(/1H1/////45X,31HCURRENT DENSITIES ALONG CATHODE///(10F12.3/
1))
C
1020 FORMAT ((20I3))
C
1025 FORMAT(2I4,4F12.5)
C
1030 FORMAT(/1H1/1H-/41X,37HCROSSED FIELD ELECTRON GUN SIMULATION/////
155X,10HPARAMETERS/////20X,8HXSCALE =,E12.5,44X,4HDX =,E12.5///20X,
28HVSCALE =,F10.1,43X,7HJZERO =,1PE12.4///20X3HB =OPF10.8,51X,4HDT =,
3=,1PE12.4/////46X,27HANODE COORDINATES - JU(I)///(10X10I3,5X,10I
43,5X,10I3/))
C
1035 FORMAT(/////50X,20HSOLE ANODE - JL(I)///(10X10I3,5X,10I3,5X,10I3
1/))
C
1045 FORMAT(I4/(3I4,6E10.5))
C
1050 FORMAT (I4/(2I4,F12.4))
C
1055 FORMAT(/1H1/1H-//44X,31HCONVERGENCE RATES (COARSE GRID)///(20I6/))
C
1060 FORMAT(/1H6,45X,29HCONVERGENCE RATES (FINE GRID)///(20I6/))
C
1065 FORMAT (1H1///53X,14HVOLTAGE MATRIX/////((10E12.5))
1070 FORMAT (/1H1///55X,10HRHO MATRIX/////((10E12.5))
C
1075 FORMAT (/1H1/////56X,7HX AND Y///43X,16HFOR ELECTRONS N=,I2,14H T
1THROUGH N=N+4/////14X,1HX,19X,1HY,19X,1HX,19X,1HY,19X,1HX,19X,1HY///
2(6E20.8/2X,4E20.8))
C
1080 FORMAT(/1H1/1X,119(1H*)/61X,7HTHE END)
C
1085 FORMAT (1H1/1H-/45X,5HAFTER,I2,14H OF A POSSIBLE,I2,7H CYCLES///2
13X,20HTHERE ARE A TOTAL OF ,I4,11H CHANGES IN ,I4,35H LOCATIONS IN
2 THE UPPER BEAM BORDER///10X,3HJBU///(10X,10I3,5X,10I3,5X,10I3/))
C
1090 FORMAT(/1H6 /23X,20HTHERE ARE A TOTAL OF,I4,11H CHANGES IN,I4,35H
1 LOCATIONS IN THE LOWER BEAM BORDER///10X,3HJBL///(10X,10I3,5X,10I
23,5X10I3/))
C
1095 FORMAT (1H1/1H-/53X,5HAFTER,I2,7H CYCLES/////23X,20HTHERE ARE A TOTA
1AL OF,I4,11H CHANGES IN I4,35H LOCATIONS IN THE UPPER BEAM BORDER/
2/10X,3HJBU///(10X,10I3,5X,10I3,5X,10I3/))
C
1100 FORMAT(/////43X,34HTHE NEW MINIMUM CURRENT DENSITY IS//50X,7HJZERO
1=,F12.5//47X,26HTRAJECTORY STARTING POINTS//(10F12.5/))
C
C          GLORIOUS END
C
C          END
```

APPENDIX II. SAMPLE DATA SET

Each of the cards and groups of cards for a particular data set are discussed below in some detail. First a copy of the individual data card(s) will be presented followed by a detailed description of the material on the card(s). At the end of this appendix a complete listing of the data is presented. The data describes the gun in Fig. II.1.

Data formats are discussed in most FORTRAN manuals and will not be discussed here.

Card 1, Format 1414

```
10 160 20 120 60 5 1 30 60 17 30
```

The integers on this card control the flow of the program to a large extent. The first number on this card (10) gives the maximum number of cycles the program is to be allowed. Ten seems to be a good number to use, many designs will converge in 5 or 6 cycles while others may require up to 15 or 16 cycles. Ten cycles usually produce good results even if not fully converged. Execution time should be less than 10 minutes for 10 cycles.

The second number (160) is the maximum number of cycles to be allowed during each field calculation on the coarse grid. One fourth of this (40) will be allowed on the fine grid. After the second or third cycle through the program as few as 10 or 20 cycles through the matrix may be sufficient, but for the first one or two cycles this full limit may be needed. If the field calculations are not converging by the third or fourth cycles the input data should be checked for possible mistakes.

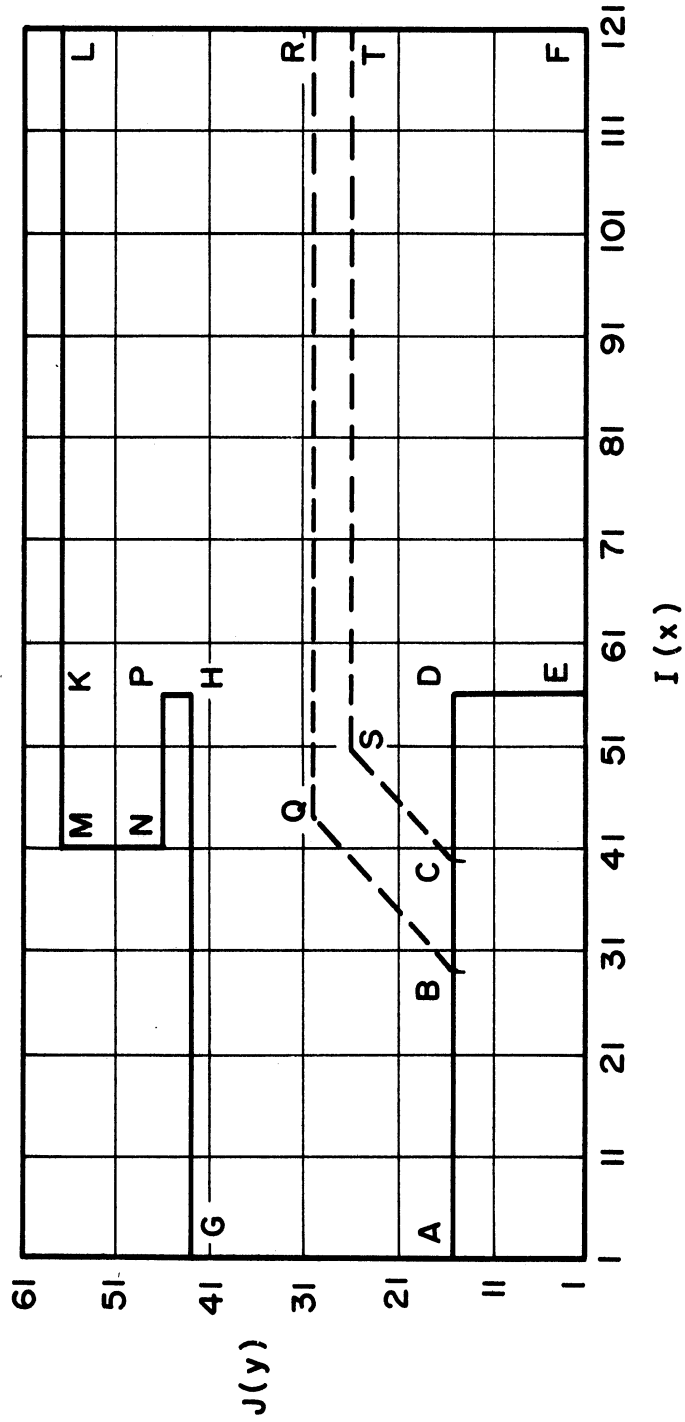


FIG. II.1 GEOMETRY LAYOUT FOR DATA DESCRIBED IN APPENDIX II.

The third number (20) is the number of errors that may be left on the fine voltage matrix at convergence. One fourth of this (5) will be allowed on the coarse matrix. The smaller this number the greater the accuracy required from the program, 20 is a fairly fine requirement, 400 would be a coarse requirement.

The length and width of the voltage matrix to be used are given respectively by the fourth (120) and fifth (60) numbers on this card. The gun geometry may be such that the whole matrix may not be needed in which case one (or both) of these numbers can be made smaller.

The next number (5) is the largest number of changes to be allowed in either beam edge at convergence. This variation is usually a fairly sensitive measure of the convergence of the program. If this were zero a very fine convergence would be required and if it were 20 or larger a coarser convergence would be called for, 5 is an intermediate to fine convergence requirement.

The seventh number (1) controls the restoring and saving of the voltage and space-charge matrices. Being set equal to one means there is no binary data to be read in and the results are not to be saved (in binary form). If equal to two no binary data will be read but the matrices will be saved. If it is three or larger binary data will be both read in at the beginning and saved at the end of the run.

The last four numbers (30,60,17,30) are used to speed the convergence of the field calculations on the first cycle. The initial field set up by the program is a linear field between the lower and upper electrodes. These numbers define a region, usually in front of the cathode, which is to be multiplied by 0.5. Since this area would eventually be depressed by space charge, multiplying by 0.5 accelerates this process.

The first two of these numbers are the left and right I coordinates, and the last two the lower and upper J coordinates of this region.

The remaining three numbers on this card are not used and can be left blank.

Card 2, Format 8E9.2

400.0 1000.0 0.016 0.1 0.002 1.0E-11 0.0001

The second data card contains scaling for the gun and additional convergence requirements. The first number (400.0) is the expected current density from the cathode, this value must be estimated from the geometry of the gun. Errors of 2 or 3 in this magnitude should cause little difficulty in execution of the program and will quickly be corrected by the program.

The second number (1000.0) is the scaling voltage for this gun. This will normally be the highest voltage used in the gun although it does not have to be. All other voltages read in (see below) by the program must be normalized to this value.

The next number (0.016) is the magnetic field in webers per square meter.

The fourth and fifth numbers determine the scaling of the gun on the voltage matrix. The fourth number (0.1) is the length of one matrix square in scaled units. In this case one scaled unit corresponds to 10 matrix squares. The fifth number (0.002) is the physical length in meters that corresponds to 1.0 units. In this case 2 mm corresponds to 1.0 scaled units, all lengths read in or printed out must be normalized to 2 mm in this case.

The time constant for the trajectory calculations (10^{-11}) is given by the sixth constant on this card. The constant is a little large for

this example because of the large electrode spacings. For most guns this would be between 10^{-12} and 10^{-11} .

The seventh number (0.0001) is the convergence condition on the field calculation. Whenever the difference between two consecutive calculations at a single point falls below this value the point is considered converged. This requirement can be made somewhat smaller but the additional accuracy attained usually does not justify doing this, in any case this should not be made less than 3×10^{-5} .

The eighth number on this card is not used by the program.

Card 3, Format 2I4,4F12.5

30 40 2.9000001 3.9 0.3 1.0

This card tells the program where the cathode is located along JL (see below) and some of the conditions along the cathode. The first two numbers (30, 40) give the approximate left- and right-hand coordinates of the cathode. The left-hand coordinate must be either exactly at the cathode edge or the first matrix point to the left of the cathode. The right side need only be the nearest matrix point to the edge of the cathode.

The third and fourth numbers (2.9, 3.9) are the exact right- and left-hand edges of the cathode in scaled units. Note that it is not necessary that the cathode end or start exactly at a matrix point although the cathode must lie along a matrix row. The left edge coordinate is expressed as 2.9000001 because of possible round off errors in the computer.

The last two numbers (0.3, 1.0) are the relaxing conditions on the current density calculations. Since the conditions along the cathode are likely to be somewhat erratic during the first few cycles, it was found desirable to place slowly relaxing conditions on the variations in current density along the cathode. On each cycle the size of the variation allowed is increased starting with a 10 percent variation (1.0 - 0.1) on the first

This matrix (JL) is made up of the coordinates of the lower anodes of the gun. This group will consist of $NX/20$ (in this case $120/20$) cards up to a maximum of 6 cards. NX should always be a multiple of 20 in order to keep the handling of this and the following matrices easy. Again it must be remembered that the matrix starts in the lower left-hand corner with the coordinate 1,1 and not 0,0 so that on this and the following matrices the number at each point is one larger than the number of squares between it and the origin.

For this particular gun the matrix consists of the surface defined by A-B-C-D (at $J = 15$) and then jumps to the surface E-F ($J = 1$). Only the section between B and C, which is to be used as the cathode, must of necessity lie along a horizontal matrix line, all other surfaces may bend or jump as the gun geometry demands.

JU

```
43 43 43 43 43 43 43 43 43 43 43 43 43 43 43 43 43 43 43 43
43 43 43 43 43 43 43 43 43 43 43 43 43 43 43 43 43 43 43 43
43 43 43 43 43 43 43 43 43 43 43 43 43 43 43 57 57 57 57
57 57 57 57 57 57 57 57 57 57 57 57 57 57 57 57 57 57 57 57
57 57 57 57 57 57 57 57 57 57 57 57 57 57 57 57 57 57 57 57
57 57 57 57 57 57 57 57 57 57 57 57 57 57 57 57 57 57 57 57
```

Where the first matrix (JL) defined the lower anodes (including the cathode), this matrix (JU) defines the first electrodes encountered on the matrix above the lower anodes. The matrix follows the surface G-H ($J = 43$) and then jumps to the surface K-L ($J = 57$).

For this and most other crossed-field guns this will effectively cover most of the geometry, in this case only the region K-M-N-P has been missed.

JC, JD

```

0 0 0 0 0 0 0
0
46 46 46 46 46 46 46 46 46 46 46 46 46 46 46
0
0
0 0 0 0 0 0 0
0 0 0 0 0 0 0
0
57 57 57 57 57 57 57 57 57 57 57 57 57 57 57
0
0
0 0 0 0 0 0 0

```

These matrices give the upper and lower limits of any region within the gun geometry not covered between JL and JU. For this gun only the region K-M-N-P is affected. The first matrix JC gives the lower limits, i.e., the surface N-P, of this region. The first 41 points are zero, from N to P it is 46, and zero the remaining points, only where this matrix is non-zero will the calculations be made.

JD defines the upper limits of this region and also must be zero at all points except between M and K (where it is 57 in this case).

JBU, JBL

```

16 17 18 19 20 21 22 23 24 25 26
27 28 29 30 30 30 30 30 30 30 30 30 30 30 30 30 30 30 30 30
30 30 30 30 30 30 30 30 30 30 30 30 30 30 30 30 30 30 30 30
30 30 30 30 30 30 30 30 30 30 30 30 30 30 30 30 30 30 30 30
30 30 30 30 30 30 30 30 30 30 30 30 30 30 30 30 30 30 30 30

17 17 17 17 17 17 17 17 17 17 17
18 19 20 21 22 23 24 25 26 26 26 26 26 26 26 26 26 26 26
26 26 26 26 26 26 26 26 26 26 26 26 26 26 26 26 26 26 26
26 26 26 26 26 26 26 26 26 26 26 26 26 26 26 26 26 26 26
26 26 26 26 26 26 26 26 26 26 26 26 26 26 26 26 26 26 26

```

These two matrices give approximate initial borders (upper and lower) for the beam. It is not necessary that these borders be particularly accurate, although a reasonable guess as to the beam shape should be made. These borders are used for two things, first they define the region where

the initial space charge is to be set up and second they are used to check for convergence of the program after each execution cycle.

JBU, the upper border, is made up of the J coordinates along the line B-Q-R, and is zero from A to B. JBL consists of the surfaces B-C-S-T and is also zero from A to B.

Electrode Voltages, Format 2I4,F12.4

1	56	0.25	}	JU
57	120	0.625		
57	120	0.625		
1	56	0.0	}	JL
57	120	-0.375		
57	120	-0.375		
42	56	0.25		JC
42	56	0.625		JD

Voltages are set on the anodes defined by JU, JL, JC and JD in the following manner. For each voltage to appear on the anode a card with the lowest and highest I coordinates and the (normalized) voltage to be set is made up. This voltage will then be set at all the matrix points along the anode defined by I and the proper anode matrix.

The first three cards allow the setting of voltages along the surface defined by JU, notably the surface G-H (I = 1 to I = 56) and the surface K-L (I = 57 to 120). Since only two voltages are set the third card is merely a duplicate of the preceeding one. The next three cards set the voltages along JL and are very similar to the cards for JU. It should be noted that the cathode voltage must be zero in any case.

The remaining two cards set the voltages along JC and JD respectively. These are made up in the same manner (with the two I limits and the the proper voltage) as the previous cards.

Voltages on Verticle Lines, Format I4/(3I4,6E10.3)

2								
46	51	41	0.29	0.33	0.37	0.41	0.45	0.49
52	57	41	0.53	0.57	0.61	0.62	0.625	0.625

The number on the first card specifies the number of cards to follow with the $3I4,6E10.3$ format. The information on the latter cards consists of the lowest J coordinate, the highest J coordinate, the column (I) in which the numbers are to be stored, and the six (normalized) voltages. The two J coordinates must specify exactly six numbers, i.e., the highest J must exceed the lowest by exactly 5.

In this example an approximately uniform field is set up along the surface N-M, this being the only gap between two electrodes (other than the ends at $I = 1$ and 120) for this particular gun.

The only electrode voltages that have not now been set by the above methods are those at the end (H-P) of the cathode accelerating anode.

Voltages at Individual Points, Format $I4/(2I4,F12.4$

8		
56	44	0.25
56	45	0.25
55	45	0.25
55	3	0.11
55	5	0.05
55	7	0.035
55	9	0.024
55	11	0.018

The first card specifies the number of cards to follow. The remaining cards give the I and J coordinates of points on the voltage matrix and the values to set at these points.

For Fig. II.1 the need for the first points (56, 44 and 56, 45) is fairly obvious, the third point (55, 45) requires some explanation. As was stated earlier the voltage matrix is solved first in a coarse matrix (consisting of odd-numbered points of the voltage matrix). For this calculations are made in every odd-numbered column (I) of the matrix using odd-numbered points in column I and in $I-2$ and $I+2$. This means that

during calculations in a column (such as 57) next to an anode (H-P in column 56) matrix points in back of the anode surface (in this case $I = 55, J = 45$) will be used. Since the surface N-M falls along an odd-numbered column (41) this problem does not occur here.

Also, since the surface D-E is in an even-numbered column (56), voltages in column 55 will be used in a similar manner. In this case, however, the problem is not so serious since these points are on an electrode which is at zero potential to begin with. The program would execute properly if these points were left at zero, but the convergence of the field calculation will be speeded if the gradient of this surface is taken into account. The numbers shown here were selected with some hindsight, but an examination of the gun layout should enable one to compute approximately what they should be.

The next page presents a complete listing of the data for this gun.

APPENDIX III. RDCHK-WRCHK

The subroutine RDCHK (A,N) reads binary cards into an erasable buffer region and transmits up to twenty words per card (according to the word count) to a backward block A until N data words have been read and checksummed. The SCARDS subroutine is used to read one card image from the input tape into the erasable storage from which the transmission takes place.

The subroutine WRCHK (A,N) transmits N words from a backward storage block A in groups of a maximum of twenty words into an erasable buffer region and forms the checksummed, serialized card image to be written on a punch tape. SPUNCH writes the contents of the erasable region on an output tape as a punch record.

Since the subroutine SCARDS and SPUNCH are University of Michigan input-output routines, one would have to use similar routines with RDCHK and WRCHK or replace the calls for them by 709/90 channel commands.

These subroutines are written in UMAP (University of Michigan Assembly Program) which is similar, but not identical, to FAP.

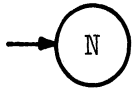
Listings of the subroutines appear on the following pages.

	LAS	WBUFF+1		RDCHK
	TRA	RTT1		RDCHK
	TRA	RBIN6		RDCHK
RTT1	CLA	RTT		RDCHK
	ADD	=1		RDCHK
	STO	RTT		RDCHK
RBIN6	TIX	RBIN1,4,**		RDCHK
REDUN	NZT	RTT		RDCHK
	TRA	RB4		RDCHK
	PRINT			RDCHK
	FMT	FORM1		RDCHK
	IOP	RTT		RDCHK
	ENDIO			RDCHK
RB4	AXT	**,4		RDCHK
RB2	AXT	**,2		RDCHK
RB1	AXT	**,1		RDCHK
	TRA	3,4		RDCHK
BDSWT	PRINT			RDCHK
	FMT	FORM2		RDCHK
	ENDIO			RDCHK
	TRA	REDUN		RDCHK
EOF	CALL	SYSTEM		RDCHK
BDCRD	PRINT			RDCHK
	FMT	FORM4		RDCHK
	ENDIO			RDCHK
	CALL	SYSTEM		RDCHK
*	FORMAT	SPECIFICATIONS		RDCHK
FORM1	BCD	//S10,I3,45H CHECKSUM ERROR(S) DURING BINARY READ-IN *****	*RDCHK	
FORM2	BCD	///S10,47HDECIMAL CARD TERMINATED BINARY READ-IN *****	*RDCHK	
FORM4	BCI	*,1H6,44HSORRY BUT THE CARD ORDER IS WRONG *****	*RDCHK	
WBASE	PZE	WBUFF+4		RDCHK
SWT	PZE			RDCHK
RTT	PZE			RDCHK
	ERAS	26		RDCHK
WBUFF	ERAS	0		RDCHK
	END			RDCHK

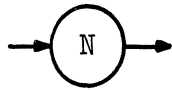
	ARS	1		WRCH
	TZE	NEXT		WRCH
	SLW	TAB,2		WRCH
	TRA	*+4		WRCH
NEXT	CAL	BZERO		WRCH
	SLW	TAB,2		WRCH
	TIX	BACK,2,1		WRCH
	CAL	TAB-1		WRCH
	ALS	12		WRCH
	ORA	TAB-2		WRCH
	ORA	ID2		WRCH
	SLW	ID+1		WRCH
	CAL	TAB-3		WRCH
	ALS	12		WRCH
	ORA	TAB-4		WRCH
	ALS	12		WRCH
	SLW	ID+2		WRCH
*				WRCH
*	OUTPUT SECTION			WRCH
*				
	SXA	*+3,4		WRCH
	CALL	SPUNCH		
	BLK	WBUFF,,27		WRCH
	AXT	** ,4		WRCH
	TIX	WBIN1,4,20	CARD LOOP	WRCH
WB1	AXT	** ,1		WRCH
WB2	AXT	** ,2		WRCH
WB4	AXT	** ,4		WRCH
	TRA	3,4		
*				WRCH
*	RESET SERIALIZATION TO ZERO			WRCH
*				WRCH
SERIAL	CAL	BZERO		WRCH
	SLW	TAB-1		WRCH
	SLW	TAB-2		WRCH
	SLW	TAB-3		WRCH
	SLW	TAB-4		WRCH
	TRA	1,4		WRCH
*				WRCH
*	TABLES AND STORAGE			WRCH
*				WRCH
ID1	OCT	101040104004		
ID2	OCT	0		WRCH
	OCT	1000		WRCH
	OCT	1000		WRCH
	OCT	1000		WRCH
	OCT	1000		WRCH
TAB	SYN	*		WRCH
BZERO	OCT	1000		WRCH
TWT	PZE	0,0,20		WRCH
W79	OCT	5K8		WRCH
	ERAS	25		WRCH
WBUFF	ERAS	0		WRCH
ID	SYN	WBUFF+24		WRCH
	END			WRCH

APPENDIX IV. FLOW CHARTS FOR CEMFEG

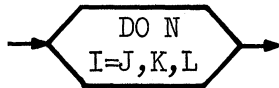
The following figures present detailed flow charts for the program. The following symbols are used:



A transfer point; if N is a number (e.g., 5) it corresponds to an actual statement number in the program, if it is a letter it is a break only in the flow chart.



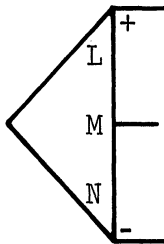
N is a statement number and there is a transfer to this point in the program.



The beginning of a FORTRAN DO loop through statement number N for I starting at J, ending at L, incremented by M. If M is omitted it is assumed equal to 1.



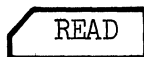
N is a statement number corresponding to the last statement of a DO loop.



A FORTRAN IF statement; a transfer is made to L, M or N as X is greater than, equal to, or less than zero, respectively.



An operation of the program; includes subroutine calls, tape manipulation and binary card reading.



Means input from cards by READ INPUT TAPE calls.



Printed output.

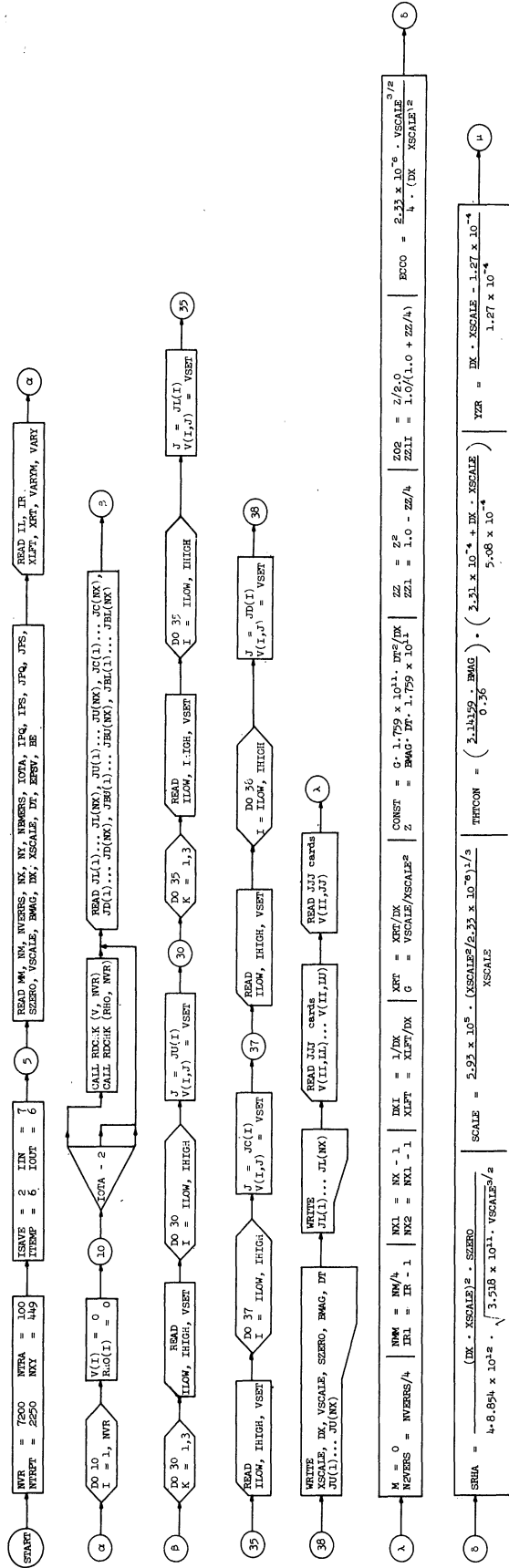


FIG. IV.1 INITIAL CONDITIONS, INPUT DATA, CONSTANTS.

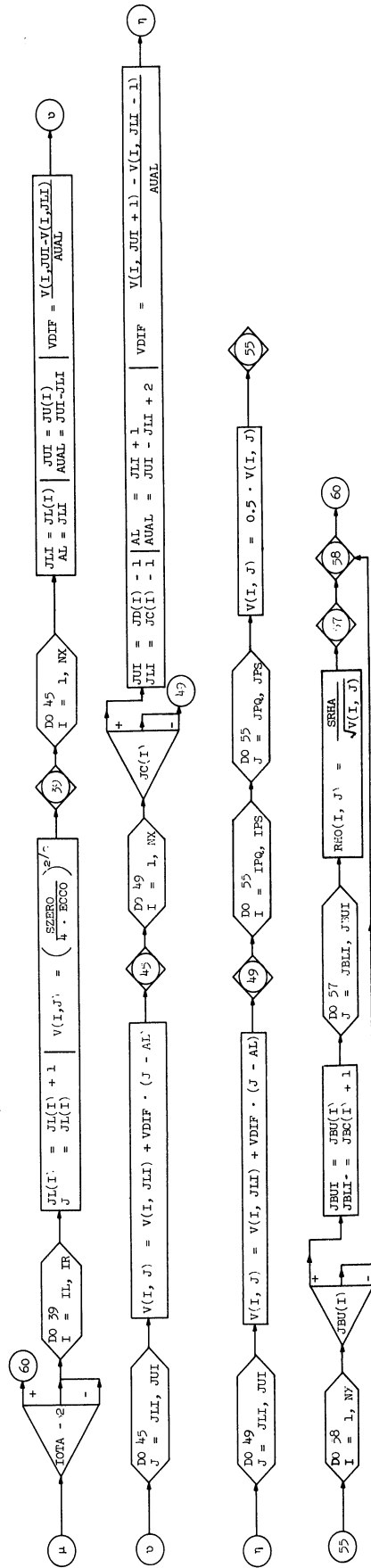
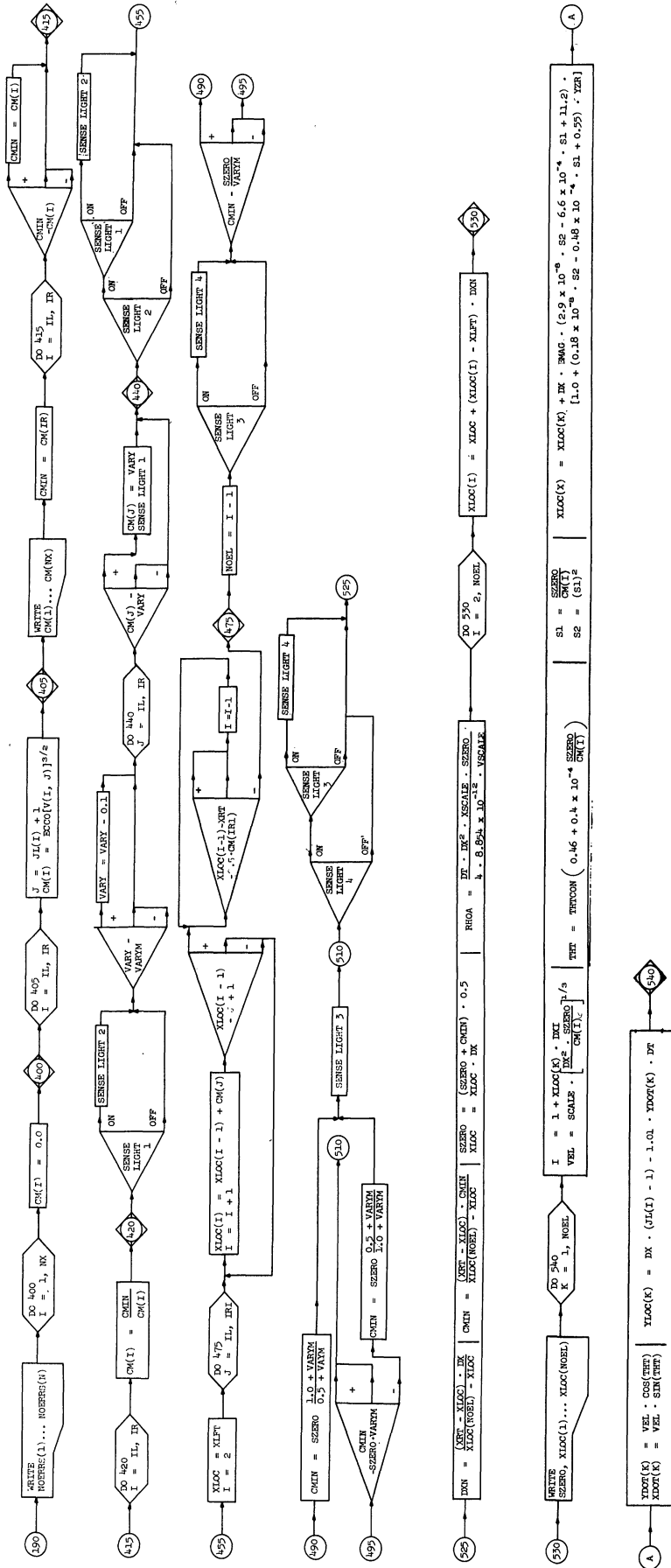


FIG. IV.2 INITIAL VOLTAGE AND SPACE-CHARGE SETUP.



IV.4 CURRENT DENSITIES AND NEW TRAJECTORY STARTING POINTS.

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