Use of Momentum Balance in Calibrating Orifices for Flow of Gases

J. J. MARTIN and V. R. PABBI

University of Michigan, Ann Arbor, Michigan

A method of determining the absolute calibration of a gas-flow orifice without the use of gas holders or any comparative device is described. The method is based on the application of the momentum balance, as well as the energy balance, to the flow of the gas. The application requires the measurement of pressures on the face of the orifice in addition to the usual pressure-drop measurements along the axis of flow.

Orifice coefficients determined by the force-momentum principle are shown to agree within an average deviation of 1.4% with those determined by other standard techniques. Also the application of the force-momentum principle demonstrates clearly why orifice coefficients are much less than unity.

The absolute calibration of an orifice for gas-flow measurement usually requires somewhat awkward and bulky equipment, particularly if the flow rates are high. This is due to the fact that large volumes of gas must be collected and measured at constant pressure. To avoid building a large gas holder with a constant pressure control system, one is inclined to calibrate an orifice by comparison with some other device which has previously been given an absolute calibration. The reliability of the comparison calibration therefore depends upon the accuracy with which the original absolute calibration was made. This may not always be completely satisfactory to the user of an orifice, who would prefer his own absolute calibration.

This study was undertaken to develop a procedure for calibrating a gas-flow orifice by measuring pressure only. In the conventional treatment of an orifice an energy balance is applied to the flowing stream. In the following treatment emphasis is placed on the application of the momentum balance and on the determination of the forces which are exerted on the flowing stream.

THEORY

Figure 1 is a diagrammatic sketch of the flow through a circular thin-plate orifice of standard design (*1*). Point 1 is taken at least one pipe diameter upstream, where the flow is not appreciably affected by the orifice; point 2 is in the plane of the upstream face of the orifice opening itself; point 3 is at the vena contracta, the narrowest section of the rapidly moving stream; and point 4 is taken several pipe diameters downstream, where maximum pressure recovery has been obtained.

For flow of a gas where the pressure drop across the orifice is not more than one-tenth the absolute upstream pressure, experimental studies show that the following assumptions are reasonable. (1) There is almost negligible friction between points 1 and 3; the bulk of the friction loss occurs between points 3 and 4 during the irreversible expansion of the moving stream. (2) The pressure at the vena contracta point 3 is little different from the pressure immediately on the downstream side of the orifice plate. For a freely discharging orifice, such as one in which there is no pipe downstream from the orifice (that is discharge is directly to a large chamber), this assumption is exact. (3) The density of the gas may be assumed constant at the average pressure and temperature between points 1 and 3, since for small pressure drops the density does not vary much. (4) The pressure exerted backward by the upstream face of the orifice plate is uniform over the plate and equal to P_1. This assumption will later be modified slightly.

With the above assumptions one may write momentum, energy, and mass balances for the fluid between points 1 and 3:

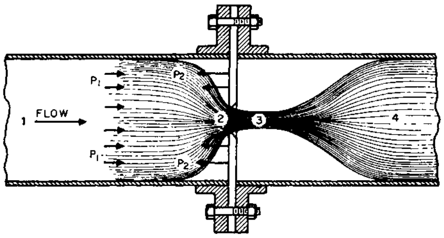
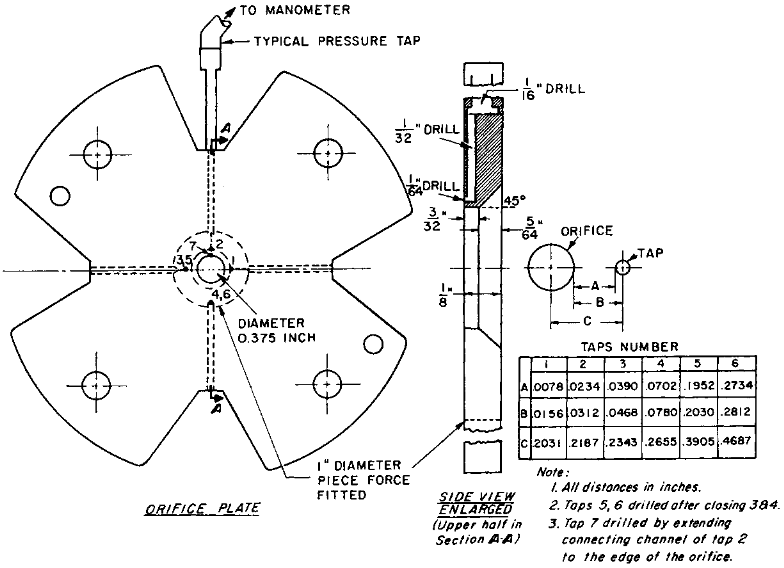


Fig. 1. Diagrammatic sketch of flow through a thin-plate orifice.



	1	2	3	4	5	6
A	.0078	.0234	.0390	.0702	.1952	.2734
B	.0156	.0312	.0468	.0780	.2030	.2812
C	.2031	.2187	.2343	.2655	.3905	.4687

Fig. 2. Experimental orifice plate.

TABLE 1

Ratio $\frac{\text{orifice diameter}}{\text{pipe diameter}}$	K, experimental (Amer. Soc. Mech. Engrs.)	Equation (6) $K = \frac{1}{2\sqrt{1-\frac{A_2}{A_1}}}$
0	——	0.5
0.125	0.6015	0.503
0.4	0.609	0.545
0.5	0.625	0.577
0.6	0.652	0.625

TABLE 2

Run				Pressure differential = in. of water			
C*→	0.2031	0.2187	0.2343	0.2655	0.3905	0.4687	01.033
1	24.8	27.95	29.0	30.0	——	——	31.2
2	17.6	19.85	20.7	21.3	22.1	22.15	22.1
3	11.6	13.2	13.7	14.1	14.6	14.65	14.7
4	23.8	25.8	26.9	27.6	28.5	28.5	28.6
5	5.2	5.8	6.0	6.1	6.12	6.27	6.4
6	19.35	22.2	22.9	23.05	——	——	23.6
7	15.22	17.15	17.9	18.4	——	——	19.2
8	11.75	13.27	13.95	14.2	14.6	14.65	14.7
9	5.0	5.55	5.9	6.0	6.12	6.27	6.35
10	12.15	13.85	14.30	14.75	14.70	14.92	15.30

* C = Distance between tap and orifice in inches shown in Figure 2. Average air temperature = 82°F. Average barometeric pressure = 29.9 in. Hg.

Momentum balance

$$\frac{u_1\rho A_1}{g_c}(u_3-u_1) = P_1A_1 - P_1(A_1-A_2) - P_3A_2 = A_2(P_1-P_3) \qquad (1)$$

Energy balance

$$\frac{u_1^2}{2g_c} + \frac{P_1}{\rho} = \frac{u_3^2}{2g_c} + \frac{P_3}{\rho} \qquad (2)$$

Mass balance

$$u_1A_1\rho = u_2A_2\rho = u_3A_3\rho \qquad (3)$$

It is to be noted in Equation (1) that the pressure applies over the whole orifice area and not merely over the cross-sectional area at point 3, in line with the second assumption.

From Equations (1) and (3), u_1 may be eliminated to give

$$u_3^2 = \frac{u_2^2A_2^2}{A_1^2} + \frac{(P_1-P_3)^2g_c^2}{u_2^2\rho^2} + \frac{2(P_1-P_3)g_cA_2}{\rho A_1} \qquad (4)$$

In the same manner Equations (2) and (3) may be combined to give

$$u_3^2 = \frac{u_2^2A_2^2}{A_1^2} + \frac{(P_1-P_3)2g_c}{\rho} \qquad (5)$$

If now the last two equations are equated to eliminate u_3, there results after simplification

$$u_2 = \frac{1}{2\sqrt{1-\frac{A_2}{A_1}}}\sqrt{\frac{2g_c(P_1-P_3)}{\rho}} \qquad (6)$$

Experimental values of the coefficient in the equation

$$u_2 = K\sqrt{\frac{2g_c(P_1-P_3)}{\rho}} \qquad (7)$$

are given for orifice Reynolds numbers above 10,000. These may be compared with the theoretical prediction of Equation (6), as shown in Table 1.

It is seen that although the theory predicts the order of magnitude of K, and indeed why K is nowhere near unity, still the agreement with experimental values is not sufficiently good to recommend the use of the theoretical K's for accurate measurement of flow. It was suspected that the major discrepancy was due to the fourth assumption above. The backward pressure of the orifice plate would hardly be uniform all the way from the edge of the pipe to the edge of the orifice opening. It would seem reasonable that this pressure would be equal to P_1 at the edge of the pipe but would fall rapidly in the immediate vicinity of the orifice.

If the average pressure along the upstream side of the orifice plate is taken as something less than P_1, say 0.9 P_1, the momentum balance Equation (1) becomes

$$\frac{u_1\rho A_1}{g_c}(u_3-u_1) = P_1A_1 - 0.9\,P_1(A_1-A_2) - P_3A_2 = A_2(P_1-P_3) + 0.1\,P_1(A_1-A_2) \qquad (8)$$

Since A_1-A_2 is positive, the right side of (8), which is the net force acting on the fluid, is greater than $A_2(P_1-P_3)$ alone. Therefore it is said that the net force is $m\,A_2(P_1-P_3)$, where $m > 1$. If this quantity is used for the right side of (8), and Equations (2) and (3) are combined with it in the same manner as before, the result is

$$u_2 = \frac{m}{2\sqrt{1-\frac{mA_2}{A_1}}}\sqrt{\frac{2g_c(P_1-P_3)}{\rho}} = K\sqrt{\frac{2g_c(P_1-P_3)}{\rho}} \qquad (9)$$

To use Equation (9) it is necessary to evaluate the quantity m. The following describes the experimental deter-

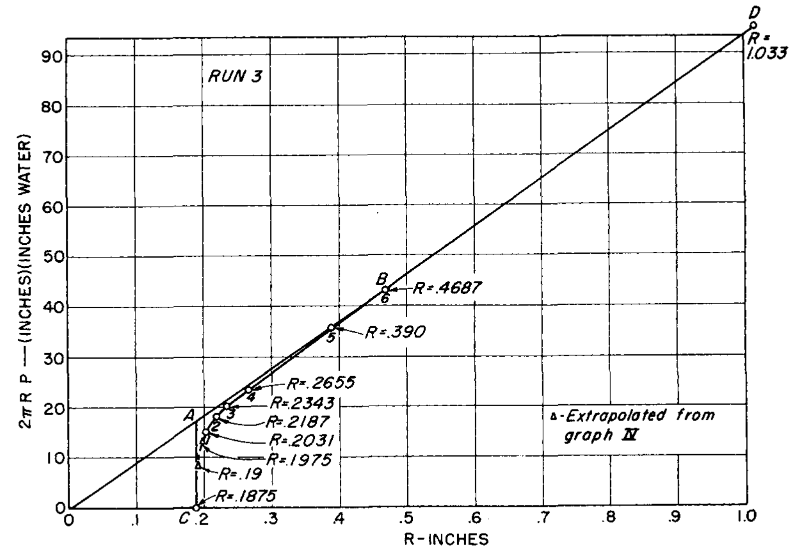


Fig. 3. Data for run 3 to give resisting force of orifice plate.

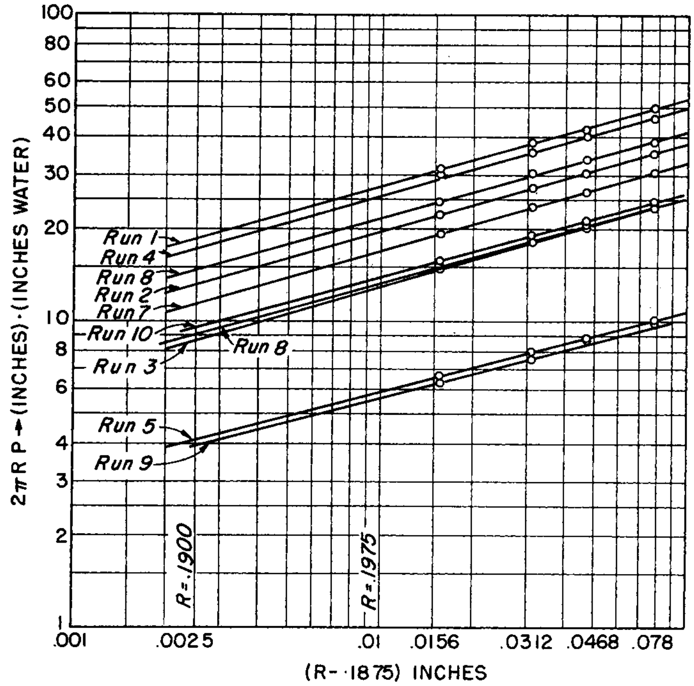


Fig. 4. Extrapolation of orifice-plate pressures to points closer to orifice radius.

mination of m and subsequently the coefficient calculated from it.

EXPERIMENTAL WORK

The orifice plate was constructed according to specifications (*1*) to permit comparison with the experimental values of K. A ⅛-in. brass plate was used with a ⅜-in. orifice hole drilled in the center, as shown in Figure 2. The hole was beveled at an angle of 45 deg. on the downstream side so that the cylindrical portion along the axis of flow was 3/64-in. long.

On the upstream face of the orifice four small holes, 1/64-in. diameter, were drilled to a depth of 1/32-in. at varying distances from the orifice opening. These were connected to small holes running parallel to the plane of the plate, permitting pressure communication to outside manometers. The distances between the pressure-tap holes in the upstream face and the orifice are shown in the table on Figure 2. It is noted that the hole closest to the orifice has its edge only 0.0156 in. from the edge of the orifice. Getting the pressure hole this close to the orifice posed a very diffcult problem. Also accurate drilling was required to make each hole perfectly circular and to meet it properly with the 1/32-in. hole parallel to the plate. This was done by building the orifice plate in two parts. A center section of 1-in. diameter was prepared with its 1/64-in. pressure holes and 1/32-in. connecting holes. The outer section was made with 1/16-in. holes leading to the manometer tubing connections. This section had a hole in center just 1 in. in diameter, into which the center section was force fitted and soldered. In addition to the four pressure taps shown, two extra holes were drilled later to get better spacing of pressure data across the upstream face of the orifice.

The finished orifice plate was mounted between standard flanges in a 2-in. schedule 40 pipe. The calming section of pipe preceding the orifice was 4 ft. long. The orifice was fed air from a compressed air line and discharged directly into the room. The pressure taps were connected to manometer tubes which could measure up to 32 in. of water pressure drop with a precision of ± 0.05 in.

DATA

Pressure readings at various taps are presented in Table 2 for ten runs. A separate run with tap 7 right inside the orifice opening showed the pressure there to be that of the room.

RESULTS

Discharge coefficients evaluated from Equation (9) are summarized in Table 3 and compared with those taken from the American Society of Mechanical Engineers report. The method of calculation of K in Equation (9) can be explained by reference to Figure 3 which for run 3 plots $2\pi RP$ vs. R. The area under a curve on this plot constitutes a force according to the integral

$$F = \int_{R_1}^{R_2} 2\pi RPdR \qquad (10)$$

The area under the straight line, *OABD*, represents the total force at the upstream position pushing the fluid toward the orifice. The area under *ABD* is the force from the orifice plate opposing the flow if the pressure over the plate were uniformly the same as the upstream pressure. The area under the curve, *CBD*, is the actual measured force of the orifice plate opposing the flow. The area of the triangle, *OACO*, is the net force accelerating the fluid in the idealized case where the pressure on the orifice plate is uniform and equal to the upstream pressure. The area of the irregular shape, *OABCO*, is the net force determined experimentally, and it is clearly seen how this force is greater than that in the idealized case.

It is noted on Figure 3 that the experimental pressure points across the orifice plate extend down to a radius of R = 0.2031 in. To obtain pressures closer to the orifice radius of 0.1875 in., where the pressure must go to zero, the quantity $2\pi RP$ was plotted vs. R − 0.1875 on logarithmic coordinates as shown in Figure 4. For all runs the curves on this graph yielded straight lines, so that extrapolation to points closer to the orifice opening was justified. In this manner the pressures at R = 0.1975 and 0.1900 in. were obtained for use on Figure 3.

In the specific case of run 3 the area *OABCO* was found to be 1.890 units, while *OACO* gave 1.595 units. This made m = 1.890/1.595 = 1.185. Using this in Equation (9) with $A_2/A_1 = (0.375/2.067)^2$ = 0.0329 one determines K as 0.604.

The Reynolds number in the orifice

TABLE 3

Run	Reynolds number in orifice	K from equation (9)	K from R. (*1*)	Percentage deviation $\left(\frac{K_9 - K_{A.S.M.E.}}{K_{A.S.M.E.}}\right)$
1	70,400	0.616	0.598	+3.01
2	59,000	0.596	0.599	−0.50
3	48,500	0.604	0.601	+0.50
4	62,400	0.607	0.599	+1.34
5	32,000	0.593	0.602	−1.49
6	61,000	0.587	0.598	−1.84
7	55,000	0.609	0.600	+1.50
8	48,500	0.597	0.601	−0.67
9	32,000	0.600	0.602	−0.33
10	49,500	0.617	0.601	+2.66

was calculated in the usual way as $D_2 \rho u_2 / \mu$. Air was assumed to behave as an ideal gas at the low pressures employed in this work.

DISCUSSION AND CONCLUSIONS

The agreement between the measured orifice coefficients according to the American Society of Mechanical Engineers report and those determined by use of the momentum balance is considered quite good. It would be desirable in a more complete study to try other ratios of orifice to pipe diameter. However it is believed that the agreement in this experiment demonstrates the applicability of the momentum balance and the assumptions employed. The study shows rather graphically why the orifice coefficient is in the neighborhood of 0.6.

If an orifice is to be used to measure gas flow and there is no convenient way to make a calibration, the technique of measuring a few pressures on the upstream face of the orifice plate should prove useful. The most serious drawback would be the mechanical problem of making pressure taps close to the orifice opening. However this problem can be solved as shown here.

NOTATION

A = area
D = diameter
F = force
g_c = conversion factor = 32.17 (lb.-mass/lb.-force) (ft./sec.^{2})
K = orifice discharge coefficient
m = ratio of net force in actual case to that in idealized case
P = pressure
R = radius
u = velocity
μ = viscosity

Subscripts

1 = upstream position
2 = orifice opening
3 = vena contracta

LITERATURE CITED

1. Am. Soc. Mech. Engrs., "Fluid Meters, Their Theory and Application," 4 ed. (1937); Am. Soc. Mech. Engrs., "Flow Measurement," Part 5 (1940).

Manuscript received June 17, 1959; revision received September 28, 1959; paper accepted October 1, 1959. Paper presented at A.I.Ch.E. Kansas City meeting.

The Mechanics of Vertical Moving Fluidized Systems: IV. Application to Batch-Fluidized Systems with Mixed Particle Sizes

ROBERT F. HOFFMAN, LEON LAPIDUS, and J. C. ELGIN
Princeton University, Princeton, New Jersey

The present paper extends the previous investigations from this laboratory on ideal fluidized systems to a system which is somewhat nonideal. Mixtures of different but well-defined glass spheres are fluidized by water to ascertain whether the principles developed for a single particle size still hold. The analysis indicates that the ideal prediction method gives a reasonable representation of the batch-expansion curves for mixed sizes.

In a previous publication from this laboratory (*3*) a detailed theoretical analysis was presented for predicting the behavior of all types of vertical moving fluidized systems. The basic postulate of this development was the proposal that a simple unique relationship exists between the slip velocity and the holdup for any system. The slip velocity is defined as the relative velocity between the particles and the fluid and can be represented mathematically as

$$V_s = \text{slip velocity}$$
$$= \frac{V_f'}{\epsilon} - \frac{V_d'}{1-\epsilon} \qquad (1)$$

In accordance with the theory, the holdup can be calculated for any mode of fluidization once the relationship between the slip velocity and the holdup has been determined. Figure 1 gives a typical relationship.

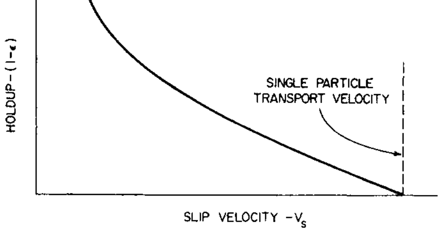


Fig. 1. Typical holdup slip velocity for an ideal batch fluidized bed.

The generalized theory has however been proved only for ideal systems (*1, 11, 12, 14*). An ideal system is one of uniform rigid spheres fluidized with a liquid having a density not too different from that of the particles. Such a system was termed *particulate fluidization* by Wilhelm and Kwauk (*16*).

The present investigation was undertaken to study the behavior of a

TABLE 1. PHYSICAL PROPERTIES OF GLASS SPHERES USED

Particle	U.S. screen	Average diameter, in.	Average deviation from average diameter, %	Density, lb./cu. ft.
1	40	0.0183	4.1	157.6
3	60	0.0106	4.5	155.2
6	170	0.00382	3.8	152.0
7	140	0.00475	4.9	154.0
8	80	0.00752	3.5	153.9