TRANSIENT DIRECTIONAL RESPONSE TEST
PROCEDURES FOR AUTOMOBILES

MVMA Project 1149

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1.0 INTRODUCTION

The purpose of this report is to present the findings of a study entitled "Transient Directional Response" performed by the Highway Safety Research Institute (HSRI) of The University of Michigan for the Motor Vehicle Manufacturers Association (MVMA). This research investigation has been directed at assessing the merits of various test procedures for obtaining numerics describing the transient directional response properties of passenger cars. The types of vehicle tests examined include step steer, reverse steer, random or pseudo-random steer, and lane-change maneuvers similar to those used by various research groups and vehicle manufacturers. The results of this study are intended to be useful in aiding U.S. vehicle manufacturers in commenting on the transient response tests currently being studied by the International Standards Organization (ISO) [1]. The specific objectives of this study are to provide an understanding of (1) practical considerations involved in performing transient response tests, (2) the repeatability and interpretability of the results obtained, and (3) the ability of each test to discriminate between vehicles.

The next section (Section 2) presents detailed descriptions of the test procedures. Differences between the tests performed in this study and those specified in preliminary documents from ISO are described. Practical matters related to the conduct of each test are also discussed in Section 2.

Test results are used in Section 3 to provide an evaluation of the root-mean-square variability of the numerics obtained in repeats of each test procedure. Requirements for acquiring accurate results are presented there.

In Section 4, experimental results are examined to furnish an assessment of the discriminatory ability of the tests studied. These results were obtained using a small rear-wheel-drive vehicle (a 1980 Chevrolet Chevette) and a large front-wheel-drive passenger car (a 1980
Buick Riviera). These vehicles were tested in two loading conditions: (1) driver plus instruments and (2) a four-passenger load.

The interpretability of test results could cover a wide range of topics, depending upon the interests of the organization evaluating the results. In this study, the following two questions related to the interpretability of the results have been examined:

- How do nonlinearities in vehicle properties influence the results? and

- Is it possible to extrapolate from the results of one test to predict the results of the other tests?

These questions are addressed in Sections 5 and 6, respectively.

The report concludes with a summary of findings and recommendations pertaining to the establishment of transient directional response tests for passenger cars.
2.0 TEST PROCEDURES AND DATA PROCESSING

The proposed procedures are classified by steering input type as follows:

1) Ramp/Step Input (U.S.) [2]
2) Random Input (U.K.) [3]
3) Sinusoidal Input (Sweden) [4]

These three approaches to measurement of transient response fall into three entirely different generic categories:

1) Approximation of a step response
2) Broad-band excitation
3) Lane-change approximation

The first two categories above represent generally accepted tests of any system's dynamic performance. Response to a step input is a commonly used and conceptually simple test procedure resulting in response times to achieve various levels of change in steady state. Broad-band excitation is often used to excite a system over a frequency range of interest so that a transfer function relating output to input can be calculated. In theory, either the step function response or the transfer function provides a complete description of a linear system.

The lane-change approximation approach to this problem is peculiar to the automotive scenario and is intended to evaluate the vehicle's performance during a lateral displacement maneuver. Sweden's proposal to approximate the lane-change maneuver with a single cycle of sine wave applied by a steering machine is intended to make this procedure purely open loop, and thus unaffected by the test driver. Lane-change-like maneuvers have been used by a number of testing organizations to evaluate vehicle dynamic performance because lane-changes are considered to be challenging to the vehicle's response and representative of actual driving situations [5].
In addition to these various generic test types, there are variations within each classification. Step input tests are conducted from an initially straight path, a slightly curved path in the same direction as the input, and from a curved path in the opposite direction as the input (reverse steer). The objective of broad-band excitation can be achieved by the application of random steering pulses, as proposed by the United Kingdom, or by the test driver consciously varying steering frequency by "sweeping" through the frequency range of interest. Lane-change-like tests are conducted in both open- and closed-loop fashion. Open-loop tests involve the application of a single sine-wave-like input applied by a steering controller or test driver, while the closed-loop test involves a driver navigating a course or avoiding an obstacle. The variations on these basic test themes are nearly endless and not conducive to standardized data gathering and reporting.

In this study, tests from each of the three generic types are examined through experiments with two vehicles. The tests examined are:

1) Ramp/step input (from a straight path)
2) Reverse steer input
3) Pseudo-random input (frequency sweep)
4) Quasi-sinusoidal steer (manually applied)

Numerics are generated from time history data that correspond to those suggested by proponents of the respective procedure. These numerics are then evaluated on the basis of run-to-run repeatability, discriminatory ability and interpretability to aid in assessing of the merits of the tests.

2.1 Step Input Tests

Two variations of this test type were combined into one procedure; a ramp/step input is applied to a straight-running vehicle (ramp/step steer test), after a steady turn is achieved in response to the first input, an input of equal amplitude and opposite direction is applied (reverse-steer test). Time histories for steering angle, yaw rate, and
lateral acceleration for this procedure are shown in Figure 1. The numerics used for quantification of response are illustrated in this figure.

These procedures also provide quasi-steady-state results. True steady-turning tests require constant velocity; in these procedures, however, the throttle is held fixed and speed is lost during the maneuver. The speed loss is small enough for low-to-moderate maneuvering levels that this steady-state approximation is adequate for most purposes.

2.1.1 Ramp/Step Steer Test [2]. The objective of this test is to measure the vehicle's speed of response in a transition from straight running to steady turning. The test is designed to approximate a step input response, but the input rate is necessarily finite resulting in a ramp approach to a steady value.

The straight-running vehicle is disturbed with a short duration ramp leading to a constant steering input. Transient response numerics are used to describe the vehicle's transitional response between straight running and steady turning. The numerics generated from this test take the form of response times. To compensate for the ramp portion of the input, the response times are measured from a "reference" time corresponding to 50 percent of the steady input level, as shown in Figure 1. Response time, $t_r$, is defined as the time elapsed between the reference time and the vehicle variable's first crossing of 90 percent of its steady-state value. Peak response time, $t_p$, is that time elapsing between the reference and the variable's reaching its maximum value. These response times are determined for the yaw rate and lateral acceleration of the vehicle, as shown in Figure 1.

Tests are run with increasing levels of steering input to evaluate the vehicle's response at various levels of maneuvering severity. The test and numerics make no assumptions about the vehicle's behavior (linearity, vehicle system order, etc.) and are taken as being complete within themselves. In this manner, the entire vehicle operating range can be examined relative to its response to this particular input.
Figure 1. Step/ramp and reverse steer time histories and definitions of numerics.
The ramp/step test is performed as specified by the U.S. delegation's proposal to the ISO. The vehicle is driven in a straight path; upon reaching the test speed, the test driver abruptly turns the steering wheel and holds it against an adjustable steering angle limiter. Steering is held fixed until a quasi-steady-state condition is reached. During the maneuver, the vehicle's throttle is held fixed and a small loss of speed occurs.

This test requires a relatively large paved area be available, along with an approach sufficiently long to allow the vehicle to reach the test speed. An 800-foot-square area proved adequate for testing at 50 and 60 mph. The approach was provided by part of an oval track surrounding the test area.

Aside from instrumentation, the only necessary special hardware is a steering angle limiting system. Mechanical stops built into a steering wheel are used for steer angle control. Steering inputs are supplied by the test driver who is required to turn the steering wheel as quickly as possible to the stop. Ramp rates of 200-500 degrees per second are suggested and this seems relatively easy to achieve. In order to justify the approximation of this test to a step input test, the input must be applied quickly enough that the time elapsed during the ramp portion of the input is short compared to response time of the vehicle. (Response to a ramp input is also a valid measure of transient response but it is not desired in this test.)

2.1.2 Reverse Steer Test. In this test a vehicle negotiating a steady turn in one direction is disturbed by a ramp/step input in the opposite direction. To establish a steady turn, the vehicle is subjected to the ramp/step input as described previously. Upon reaching a quasi-steady-state condition, the steering is rapidly reversed to an equal steer angle in the opposite direction. Steering is then held constant until a new quasi-steady turn is established. With the throttle held constant, the speed of the vehicle decreases during this maneuver, but the speed loss is not large enough to alter the results markedly. Test facilities and equipment necessary for this test are identical to those required for the ramp step test.
The numerics associated with this test are similar to those for
the ramp/step. Again the ramp duration is compensated for by defining
a reference time corresponding to 50 percent of the change in steering
angle. Time elapsing between this reference time and the vehicle response
variables (yaw rate or lateral acceleration) achieving 50 percent and
90 percent of the change in steady state and its maximum value are
defined as delay time, \( t_d \), response time, \( t_r \), and peak response time, \( t_p \),
respectively. Steering angle, yaw rate and lateral acceleration time
histories for this test are shown in Figure 1, illustrating the numerics
defined.

If this test is used to approximate a step input, the rate of
change of steering angle must be high due to the large excursions neces-
sary for high level runs. A ramp rate of 200 degrees per second would
produce a ramp 0.8 seconds long for a test performed with \( \pm 80 \) degrees of
steering angle. This is on the order of twice the response time of
lateral acceleration and four times the response time for yaw rate. These
times are measured from the half-way point in the ramp. Hence, for a
200 degrees per second ramp rate, the vehicle would be responding to the
ramp portion of the input instead of the step. In previously reported
work [6], the ramp rate was selected to match (approximately) typical rates
used by drivers in closed-loop lane-change maneuvers. Possibly, the
intended use of the test results should dictate the ramp rate employed in
this test.

The vehicle's response to a ramp input is a legitimate measure of
transient response. If this test were being used to evaluate vehicle
response in a lane-change-like maneuver involving slower steering reversals,
the vehicle's response lag to a ramp input may be an important measure of
performance.

2.2 Pseudo-Random Input Test

The random input method suggested by the United Kingdom to ISO [3]
specifies a test involving disturbing a straight-running vehicle with
steering reversals of uniform amplitude and varying direction and timing.
The objective here is to excite vehicle response over the entire frequency range of interest, so its transfer function can be calculated from input and response data. To assure a wide-band input to the vehicle, the test was modified by instructing the test driver to "sweep" the frequency range, starting with very slow sine waves and gradually increasing frequency until reaching an ergonomic limit. If maximum frequency was reached before the test time expired, the driver then gradually decreased frequency to the lowest practicable frequency and started the process over. An example of this type of steering input is presented in Figure 2, the energy spectral density generated from this time history is shown in Figure 3.

The test was run at 50 mph on a straight track two lanes wide. The vehicle was driven in a nominally straight path; after data had been collected for five seconds of straight running the frequency sweep was begun. The sweep was continued for approximately 25 seconds, then the vehicle was returned to its straight path for five seconds and data collection was stopped.

Lateral space requirements for this test are minimal. Any nearly straight, smooth roadway one mile long will suffice for steering frequencies greater than .25 Hz. Lower frequencies and their attendant large lateral excursions require a relatively wide roadway. Equipment demands are limited to instrumentation.

Care must be taken to avoid large lateral accelerations while conducting this test as the data processing assumes a linear system representation of the vehicle. Operating the vehicle in its nonlinear range will degrade the quality of the results as the fit of a linear system becomes less adequate for matching the data.

2.3 Sinusoidal Steer [4]

The sinusoidal-steer test suggested to ISO by Sweden involves the use of a steering machine to apply a precise single cycle of a sine wave to the vehicle's steering wheel. Such a machine was not readily available for this test work. In lieu of a steering machine, the test driver was
Figure 2. Pseudo-random (frequency sweep) steering input.
Figure 3. Energy spectral density of pseudo-random input shown in Figure 2.
called upon to apply a quasi-sine-wave input of specified period and amplitude to the vehicle. Previous experience [7] indicated that an input closely resembling a sine wave could be generated by a driver with (1) aid from steering stops and (2) practice. Example input and response time histories are shown in Figure 4.

The basic Swedish test calls for two test conditions, one with peak first half-wave lateral acceleration of less than four m/s² (.41 g), and one with higher acceleration, both with two-second input periods. As suggested, the steering amplitudes were determined using the following formula:

\[ \delta_{sw} = \frac{66}{V_v} \cdot \frac{l}{10} \cdot N_G \cdot \sigma \]

\( \delta_{sw} \) - steering wheel angle (deg)
\( V_v \) - vehicle velocity (ft/s)
\( l \) - wheelbase (ft)
\( N_G \) - steering ratio
\( \sigma \) - is set to 2, 4, 6, etc.

Trial runs were made with increasing \( \sigma \) until the appropriate maneuvering levels were reached. Tests were run at 50 mph.

Trajectories resulting from this input approximate a lane-change maneuver and thus the test can be conducted on a wide, straight roadway of sufficient length to perform the maneuver. This type of roadway is almost universally available and this type of maneuver is routinely performed by the driving public. These factors give this test a realism and applicability that the other procedures tend to lack.

Control of the input waveform generated by the driver is not an easy task. The human controller must provide a reasonably repeatable input waveform to assure repeatable results. Considerable practice is required to generate a balanced sine-like wave of a specified period. A tolerance
Figure 4. Quasi-sinusoidal steer test time histories
of 10 percent (.1 sec) variation from the ideal half-wave period of one second was used to determine valid tests. The test rejection rate was very high. If a large number of these tests were required, the development of a steering controller would be a reasonable investment.

Transient response to this input is measured with a number of numerics quantifying first and second half-wave lags and gains and their relative magnitudes. Response time lags, τ₁ and τ₂, are calculated separately for each half-wave using the cross-correlation method and the ratio of the second half-wave lag, τ₂, to the first, τ₁, is defined as the time lag amplification, TLA. The maneuvering level is defined by the peak lateral acceleration during the first half-wave; this value is treated as the equivalent of steady-state lateral acceleration in the ramp/step test. Gains for the first and second half-waves are defined by peak response variable values over peak input values and the ratio of these gains is defined as the amplification factor, AMP. These numerics provide a very complete description of the vehicle response in this particular lane-change-like maneuver.

2.4 Vehicles and Instrumentation

Two vehicles were tested in minimum and four-passenger load conditions. The vehicles were chosen to represent different vehicle sizes and design purposes. The vehicles used were a 1980 Chevrolet Chevette, a rear-wheel drive subcompact, and a 1980 Buick Riviera, a front-wheel drive personal luxury vehicle. Specifications for these vehicles are presented in Table 1.

The vehicle minimum load condition was defined as vehicle weight plus instrumentation and driver. A four-passenger load was simulated by adding six sand bags weighing 80 pounds each. The sand bags were placed on the vehicle seats and in the footwells to provide the approximate center of gravity shift of passengers.

To aid in the performance of tests requiring steering inputs of controlled amplitude, a steering-wheel system with mechanical stops was used. Two adjustable stops were fixed to the steering wheel to provide
<table>
<thead>
<tr>
<th>Table 1. Vehicle Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1980 Buick Riviera</strong></td>
</tr>
<tr>
<td><strong>Test Weights (lb)</strong></td>
</tr>
<tr>
<td><strong>Minimum Load</strong></td>
</tr>
<tr>
<td>LF/RF</td>
</tr>
<tr>
<td>LR/RR</td>
</tr>
<tr>
<td><strong>4 Passenger Load</strong></td>
</tr>
<tr>
<td>LF/RF</td>
</tr>
<tr>
<td>LR/RR</td>
</tr>
<tr>
<td><strong>Wheelbase (in)</strong></td>
</tr>
<tr>
<td><strong>Track F/R (in)</strong></td>
</tr>
<tr>
<td><strong>Tires</strong></td>
</tr>
<tr>
<td><strong>Cold Inflation Pressure (psi)</strong></td>
</tr>
<tr>
<td><strong>Suspension</strong></td>
</tr>
<tr>
<td><strong>Front</strong></td>
</tr>
<tr>
<td><strong>Rear</strong></td>
</tr>
<tr>
<td><strong>Steering Type</strong></td>
</tr>
<tr>
<td><strong>Steering Ratio</strong></td>
</tr>
</tbody>
</table>
control on steering angle displacement to the right and left. A single body-fixed stop provides a rigid ground point for the stop mechanism. The steering wheel was also equipped with an angular displacement potentiometer and a torque-sensing hub assembly to provide steering angle and torque signals for recording.

Vehicle response variables, lateral acceleration, $a_y$, yaw rate, $r$, and roll angle, $\phi$, were measured using a stabilized platform unit. Transducer specifications are given in Table 2.

Analog signals from the transducers were filtered with a 20-Hz linear phase filter and appropriately amplified for digitizing with an analog signal conditioning unit. Data acquisition was controlled with a microprocessor-based controller. The microprocessor was controlled from a keyboard by the test driver and had the capability of zeroing transducer offsets, checking analog gains and monitoring signals from the transducers in real time for feedback to the driver. Test data was stored in random access memory during data collection. After the test was terminated, the digital data could then be recorded on a tape cartridge for future processing.

2.5 Data Processing

Digital data collected in the field was transferred to magnetic tape so data could be processed using the Michigan Terminal System (MTS) computer. Data was read from the tape, converted to engineering units, and stored in disc files prior to analysis to generate the specific test numerics.

2.5.1 Ramp/Step and Reverse Steer Tests. The raw data was filtered using a 10-Hz moving window function to reduce the effects of high frequency noise on the transducer signals. Analysis of this data was straightforward, consisting of (1) location of steering transients, (2) calculation of steady-state values, and (3) calculation of numerics. The first two steps established the reference times and levels necessary for response time determination. After determining the crossing of various
<table>
<thead>
<tr>
<th>Variable</th>
<th>Transducer</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yaw rate, r</td>
<td>Humphrey Rate Sensor</td>
<td>+60°/s</td>
</tr>
<tr>
<td>Lateral acceleration, a_y</td>
<td>Humphrey Stabilized Platform</td>
<td>+1.5 g</td>
</tr>
<tr>
<td>Roll angle, ϕ</td>
<td>Humphrey Stabilized Platform</td>
<td>+20°</td>
</tr>
<tr>
<td>Steering-wheel angle</td>
<td>Gear Driven Potentiometer</td>
<td>+360°</td>
</tr>
<tr>
<td>Steering-wheel torque</td>
<td>HSRI Torque Cell</td>
<td>+600 in-lb</td>
</tr>
<tr>
<td>Forward velocity</td>
<td>Labeco Fifth Wheel D.C. Tachometer</td>
<td>0-75 mph</td>
</tr>
</tbody>
</table>
response levels (e.g., 50 percent or 90 percent of steady-state values),
straight-line interpolation between the discrete data points was used to
estimate the response times to the nearest millisecond.

2.5.2 Pseudo-Random Input Tests. To obtain the frequency response
function describing the vehicle response to steering input, a fast Fourier
transform is applied to the time histories of input and response variables.
The transform is evaluated using the equation:

\[ X_k = X(k\Delta f) = \Delta t \sum_{n=0}^{N-1} X_n \exp(-j2\pi \frac{k n}{N}) \]

where

- \( X_k \) = Fourier component at \( f_k = k\Delta f \)
- \( X_n \) = nth data point
- \( N \) = total number of data points
- \( \Delta t \) = time between samples
- \( \Delta f = 1/(N\Delta t) \), frequency resolution
- \( k = 0,1,2,...,N-1 \)

These results are then frequency smoothed using a boxcar window averaging
together five adjacent estimates.

The transformed data is used to calculate auto-spectra, \( G_x \), cross-
spectra, \( G_{xy} \), and coherence, \( \gamma_{xy}^2 \), functions for input and output variables
using these equations:

\[ G_x(f_k) = \frac{2}{T} |X(f_k)|^2 \]

\[ G_{xy}(f_k) = \frac{2}{T} |X^*(f_k)Y(f_k)| \]

\[ \gamma_{xy}^2(f_k) = \frac{|G_{xy}(f_k)|^2}{G_x(f_k)G_y(f_k)} \]
where

\[ f_k = \text{Fourier frequency corresponding to } k\Delta f, \, k=0,1,2,\ldots N-1 \]

\[ X^* = \text{complex conjugate of } X \]

\[ Y = \text{Fourier transform of output} \]

The frequency response function \( H(f_k) \) can then be calculated by:

\[
H(f_k) = \frac{G_{xy}(f_k)}{G_x(f_k)}
\]

Breaking the frequency response function into its real and imaginary parts, \( H_R \) and \( H_I \), gain \(|H(f_k)|\), and phase, \( \phi(f_k) \), can be calculated as follows:

\[
|H(f_k)| = \sqrt{(H_R(f_k))^2 + (H_I(f_k))^2}
\]

\[
\phi(f_k) = \tan^{-1}(H_I(f_k)/H_R(f_k))
\]

The gain and phase results for frequencies up to 2.5 Hz are then plotted as a function of frequency.

Due to the transient nature of the input, it is proper to express the input frequency content in terms of its energy spectral density \( E_X \). This quantity is calculated using the relationship:

\[
E_X = T G_x
\]

where \( T = \text{total record length} \). This is only a scale factor change for a given test, but it provides a consistent measure of input energy for varying record lengths.
2.5.3 **Sinusoidal Steer Test.** The 10-Hz digital filter, used for ramp/step and reverse-steer analysis, was also applied to this data before processing. Calculations of gain and amplification numerics were accomplished by searching the data for input and output peaks for the first and second "half-waves" of the measured signals. Ratios of the first and second half-wave output-to-input peaks were defined as gains and ratios of second-to-first half-wave gains were defined as amplifications.

As suggested by the Swedish proposal, time lags were calculated for each half-wave using a cross-correlation technique. For discrete samples of data, the cross correlation between the input, \(x\), and the output, \(y\), is defined as:

\[
R_{xy}(K) = \frac{1}{N} \sum_{i=1}^{N} x_i y_{i+k}
\]

where

- \(R_{xy}(K)\) = cross correlation for a time shift of \(K\) sampling intervals
- \(N\) = number of data samples
- \(K\) = lag number set to -50 to +50

Values of this cross-correlation function were computed for time shifts of up to 50 samples (that is, up to 0.5 second at 100 Hz sampling frequency). The value of \(K\) corresponding to the largest cross-correlation magnitude is multiplied by the sampling interval to give the time lag. Time lag amplification is then calculated by ratioing the time lag for the second half-wave to the time lag for the first half-wave.
3.0 REPEATABILITY OF THE RESULTS OBTAINED IN EACH TEST PROCEDURE

Four repeats of each test were run with selected vehicles to assess the variability of the numerics calculated from the results. The test-to-test variability is expressed in terms of the root-mean-square (rms) deviation. The magnitudes of these variabilities will be examined and discussed in general terms.

3.1 Ramp/Step Variability

Table 3 presents the results of the ramp/step variability calculations for the minimum-load Chevette at 60 mph. The first observation made is that the rms deviations of the response times \( t_r \) for both yaw rate and lateral acceleration are very small with a magnitude of approximately the sampling period. Thus, these numerics are as repeatable as the resolution of the data acquisition system. The peak times, \( t_p \), show generally larger variability than the response times.

Ramp/step variability results are shown in Table 4 for the Riviera in its minimum-load condition at 50 mph. The variabilities of the response times for this vehicle are considerably larger than those for the Chevette, particularly at lower lateral acceleration levels. Peak times show similarly large variability compared to the previous data. The larger variations are believed to be due to variation in testing protocol—not to properties of the vehicle.

To minimize the variance of the ramp/step results, the steering input should resemble a step as nearly as possible. The appropriate rate of input is defined by the system being tested, the ramp must be over before the vehicle response becomes dominated by the ramp portion of the input. Correcting for the ramp duration by using a zero time corresponding to half of the input does not compensate for the alteration in vehicle behavior caused by a slow ramp. For example, Figures 5 and 6 illustrate steering-wheel angle and yaw rate time histories for the two vehicles. The time history presented in Figure 5 shows that the ramp portion of the input for the Chevette is completed very early in the vehicle response,
Table 3. Ramp/Step Variability, Chevette - Minimum Load, 60 mph.

<table>
<thead>
<tr>
<th>Nominal Steering-Wheel Angle (deg)</th>
<th>Steady Lateral Acceleration (g)</th>
<th>Response Time, $t_r$ (s)</th>
<th>Peak Time, $t_p$ (s)</th>
<th>Lateral Acceleration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean 41.4</td>
<td>.324</td>
<td>.151</td>
<td>.336</td>
<td>.322</td>
</tr>
<tr>
<td>$\sigma$ .7</td>
<td>.021</td>
<td>.007</td>
<td>.036</td>
<td>.008</td>
</tr>
<tr>
<td>$\sigma_n$ .017</td>
<td>.065</td>
<td>.045</td>
<td>.108</td>
<td>.023</td>
</tr>
<tr>
<td>Mean 50.4</td>
<td>.373</td>
<td>.142</td>
<td>.327</td>
<td>.315</td>
</tr>
<tr>
<td>$\sigma$ .1</td>
<td>.011</td>
<td>.008</td>
<td>.015</td>
<td>.005</td>
</tr>
<tr>
<td>$\sigma_n$ .001</td>
<td>.029</td>
<td>.056</td>
<td>.047</td>
<td>.015</td>
</tr>
<tr>
<td>Mean 60.1</td>
<td>.437</td>
<td>.137</td>
<td>.357</td>
<td>.326</td>
</tr>
<tr>
<td>$\sigma$ .4</td>
<td>.014</td>
<td>.010</td>
<td>.010</td>
<td>.009</td>
</tr>
<tr>
<td>$\sigma_n$ .006</td>
<td>.032</td>
<td>.076</td>
<td>.028</td>
<td>.026</td>
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<tr>
<td>Mean 69.8</td>
<td>.465</td>
<td>.127</td>
<td>.281</td>
<td>.333</td>
</tr>
<tr>
<td>$\sigma$ .2</td>
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<td>.007</td>
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<td>.011</td>
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<td>$\sigma_n$ .002</td>
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<td>.032</td>
</tr>
<tr>
<td>Nominal Steering-Wheel Angle (deg)</td>
<td>Steady Lateral Acceleration (g)</td>
<td>Yaw Rate Response Time, $t_r$ (s)</td>
<td>Yaw Rate Peak Time, $t_p$ (s)</td>
<td>Lateral Acceleration Response Time, $t_r$ (s)</td>
</tr>
<tr>
<td>-----------------------------------</td>
<td>---------------------------------</td>
<td>----------------------------------</td>
<td>-------------------------------</td>
<td>---------------------------------</td>
</tr>
<tr>
<td>Mean</td>
<td>-29.6</td>
<td>-.158</td>
<td>.213</td>
<td>.340</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>.17</td>
<td>.008</td>
<td>.036</td>
<td>.039</td>
</tr>
<tr>
<td>$\sigma_n$</td>
<td>.006</td>
<td>.051</td>
<td>.169</td>
<td>.115</td>
</tr>
<tr>
<td>Mean</td>
<td>-40.6</td>
<td>-.214</td>
<td>.213</td>
<td>.350</td>
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<tr>
<td>$\sigma$</td>
<td>.15</td>
<td>.011</td>
<td>.024</td>
<td>.023</td>
</tr>
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<td>$\sigma_n$</td>
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<td>.051</td>
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<td>.066</td>
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<tr>
<td>Mean</td>
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<td>.371</td>
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<td>$\sigma$</td>
<td>.56</td>
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<td>.009</td>
<td>.056</td>
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</tr>
<tr>
<td>Mean</td>
<td>-80.4</td>
<td>-.428</td>
<td>.194</td>
<td>.347</td>
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<tr>
<td>$\sigma$</td>
<td>.25</td>
<td>.010</td>
<td>.012</td>
<td>.012</td>
</tr>
<tr>
<td>$\sigma_n$</td>
<td>.003</td>
<td>.023</td>
<td>.062</td>
<td>.035</td>
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</tbody>
</table>
Figure 5. Ramp/step and reverse steer maneuver with short ramp duration, Chevette—minimum load, 60 mph.
Figure 6. Ramp/step and reverse steer maneuver with long ramp duration, Riviera—minimum load, 50 mph.
while Figure 6 shows an input finishing well into the vehicle response for the Riviera. The exact effect of the input ramp rate and subsequent period has not been quantified, but the potential for the distortion of the results is clearly present. Measuring the response times of the vehicle in the vicinity of the input's transition from a ramp to a constant level has an important effect on the repeatability of the results.

3.2 Reverse Steer Variability

The repeatability results for the same vehicles in the reverse steer are shown in Tables 5 and 6. In addition to the response times calculated for the ramp/step, the "delay time," from reference to 50 percent of the change in the output, is calculated. As with the ramp/step, the rms deviations calculated for the Chevette response times \( t_r \) are of the same magnitude as the time between digitizing samples. The yaw rate delay time also exhibits a very small rms deviation. Again the peak times have larger variability as does the lateral acceleration delay time.

The results for the Riviera (Table 6) executing the reverse steer show a similar trend to that for the ramp/step; i.e., larger variations in numerics than the Chevette. Yaw rate delay time shows variability in the range of the data acquisition system's resolution. This is probably a very good measure of the vehicle's delay time response to a ramp input.

3.3 Pseudo-Random Input Test

As part of the data processing (described in Section 2), the coherence function, describing the adequacy of the frequency response results, was calculated. This function is a measure of fit between the transfer function generated and the test data. As a result of its meaning, it can be used as an approximation of the repeatability of the test results for a particular vehicle. This result is illustrated in Figure 7. Coherence values and rms deviations of gain and phase for yaw rate and lateral acceleration normalized to the respective mean values are shown as a function of frequency. The coherence values are for the ensemble average of four runs, while the deviations are representative
<table>
<thead>
<tr>
<th></th>
<th>Steady Lateral Acceleration (g)</th>
<th>Delay Time, $t_d$ (s)</th>
<th>Response Time, $t_r$ (s)</th>
<th>Peak Time, $t_p$ (s)</th>
<th>Lateral Acceleration Response</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Delay Time, $t_d$ (s)</td>
</tr>
<tr>
<td><strong>Mean</strong></td>
<td>-42.0</td>
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<td>.089</td>
<td>.159</td>
<td>.328</td>
</tr>
<tr>
<td><strong>σ</strong></td>
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<td>.005</td>
<td>.006</td>
<td>.020</td>
</tr>
<tr>
<td><strong>σ_n</strong></td>
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<td>.062</td>
<td>.039</td>
<td>.060</td>
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<tr>
<td><strong>Mean</strong></td>
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<td>-.389</td>
<td>.087</td>
<td>.158</td>
<td>.331</td>
</tr>
<tr>
<td><strong>σ</strong></td>
<td>.9</td>
<td>.009</td>
<td>.005</td>
<td>.006</td>
<td>.032</td>
</tr>
<tr>
<td><strong>σ_n</strong></td>
<td>.016</td>
<td>.024</td>
<td>.054</td>
<td>.041</td>
<td>.097</td>
</tr>
<tr>
<td><strong>Mean</strong></td>
<td>-62.2</td>
<td>-.430</td>
<td>.084</td>
<td>.156</td>
<td>.306</td>
</tr>
<tr>
<td><strong>σ</strong></td>
<td>.2</td>
<td>.010</td>
<td>.006</td>
<td>.005</td>
<td>.031</td>
</tr>
<tr>
<td><strong>σ_n</strong></td>
<td>.002</td>
<td>.022</td>
<td>.067</td>
<td>.033</td>
<td>.101</td>
</tr>
<tr>
<td><strong>Mean</strong></td>
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<td>.089</td>
<td>.156</td>
<td>.324</td>
</tr>
<tr>
<td><strong>σ</strong></td>
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<td>.002</td>
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<tr>
<td><strong>σ_n</strong></td>
<td>.007</td>
<td>.028</td>
<td>.021</td>
<td>.034</td>
<td>.048</td>
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</table>
Table 6. Reverse Steer Variability, Riviera - Minimum Load, 50 mph

<table>
<thead>
<tr>
<th>Steering Wheel Angle (deg)</th>
<th>Yaw Rate Response</th>
<th>Lateral Acceleration Response</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Delay Time, $t_d$</td>
<td>Response Time, $t_r$</td>
</tr>
<tr>
<td></td>
<td>$t_d$ (s)</td>
<td>$t_r$ (s)</td>
</tr>
<tr>
<td></td>
<td>Delay Time, $t_d$</td>
<td>Response Time, $t_r$</td>
</tr>
<tr>
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<td>31.1</td>
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</tr>
<tr>
<td></td>
<td>0.12</td>
<td>0.014</td>
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<td></td>
<td>0.004</td>
<td>0.079</td>
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<tr>
<td>Mean</td>
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<tr>
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<td>0.22</td>
<td>0.004</td>
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<tr>
<td></td>
<td>0.12</td>
<td>0.011</td>
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<tr>
<td></td>
<td>0.002</td>
<td>0.031</td>
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<tr>
<td>Mean</td>
<td>79.8</td>
<td>0.463</td>
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<tr>
<td></td>
<td>0.44</td>
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<tr>
<td></td>
<td>0.006</td>
<td>0.017</td>
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</tbody>
</table>
Figure 7a. Pseudo-random input test, yaw rate transfer function variability and coherence, vehicle 1, 50 mph
Figure 7b. Pseudo-random input test, lateral acceleration transfer function variability and coherence, vehicle 1, 50 mph
of the run-to-run variation of the four tests. It can be seen here that repeatability is best when the coherence is very high, while the variability is very high for frequencies with lower coherence.

3.4 Sinusoidal Steer Variability

The repeatability for the sinusoidal steer test was evaluated in the same manner as the ramp/step and reverse steer tests. Means and rms deviations were calculated and the magnitude of the deviation relative to the mean was derived. Due to the practical problems encountered in conducting this test, only repeatability for the Chevette in its minimum loading condition was examined. The results are presented in Table 7.

All measured numerics ($\tau_1, \tau_2, \text{gain}$) show variabilities of the same relative magnitude as those for response times from the ramp/step and reverse steer tests. Time lags have a variability consistently less than the period between digitized samples. Calculated numerics relating first and second half-wave responses show slightly larger variability at higher lateral acceleration level than other numerics.
Table 7. Sinusoidal Steer Variability, Chevette - Minimum Load, 50 mph.

<table>
<thead>
<tr>
<th>$\delta_{\phi M}$ Nom. (deg)</th>
<th>$a_{yp}$ (g)</th>
<th>$\tau_1$ (s)</th>
<th>$\tau_2$ (s)</th>
<th>$G_1$ (g/deg)</th>
<th>AMP (-)</th>
<th>TLA (-)</th>
<th>$\tau_1$ (s)</th>
<th>$\tau_2$ (s)</th>
<th>$G_1$ (1/s)</th>
<th>AMP (-)</th>
<th>TLA (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
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<td>.193</td>
<td>.213</td>
<td>.0066</td>
<td>.92</td>
<td>1.11</td>
<td>.117</td>
<td>.103</td>
<td>.191</td>
<td>.94</td>
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<tr>
<td>$\sigma$</td>
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<td>.0006</td>
<td>.006</td>
<td>.006</td>
<td>-</td>
<td>.02</td>
<td>.06</td>
<td>.006</td>
<td>.006</td>
<td>.002</td>
<td>.03</td>
</tr>
<tr>
<td>$\sigma_{\phi M}$</td>
<td>-</td>
<td>.001</td>
<td>.030</td>
<td>.027</td>
<td>-</td>
<td>.022</td>
<td>.050</td>
<td>.049</td>
<td>.056</td>
<td>.012</td>
<td>.033</td>
</tr>
<tr>
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<td>.539</td>
<td>.193</td>
<td>.223</td>
<td>.0060</td>
<td>.96</td>
<td>1.16</td>
<td>.113</td>
<td>.093</td>
<td>.177</td>
<td>.98</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>-</td>
<td>.016</td>
<td>.005</td>
<td>.013</td>
<td>.0002</td>
<td>.05</td>
<td>.09</td>
<td>.005</td>
<td>.005</td>
<td>.007</td>
<td>.10</td>
</tr>
<tr>
<td>$\sigma_{\phi M}$</td>
<td>-</td>
<td>.030</td>
<td>.026</td>
<td>.057</td>
<td>.032</td>
<td>.053</td>
<td>.074</td>
<td>.044</td>
<td>.054</td>
<td>.042</td>
<td>.102</td>
</tr>
</tbody>
</table>
4.0 DISCRIMINATORY ABILITY

Implicit in the characterization of transient response using the numerics generated by the various tests is the assumption that the numerics be able to point up differences between vehicles. To assess the ability of each test to make a distinction between vehicles, results for the minimum-load Chevette and four-passenger-load Riviera are compared here, the Chevette being denoted Vehicle 1 and the Riviera, Vehicle 2. Intuitively, it would seem that the physical disparity between the two vehicles in terms of weight, wheelbase, and other fundamental parameters would have a significant influence on the relative transient response of the vehicles.

4.1 Steady-State Results

The quasi-steady-state results generated using the ramp/step test data are presented here to provide reference information on the directional response properties of the vehicles studied. Steady-state lateral acceleration and yaw rate are shown as functions of steering-wheel angle in Figure 8 for Vehicles 1 and 2, respectively. Vehicle understeer, \( K \), and gains for yaw rate and lateral acceleration, \( G_r \) and \( G_{ay} \), for the linear operating range are presented in Table 8.

In low-level maneuvers, Vehicle 1 exhibits substantially less understeer than Vehicle 2. This is accompanied by the higher gains seen for Vehicle 1. At elevated levels, Vehicle 1 shows increasing understeer while Vehicle 2 maintains nearly constant gain over the maneuvering range tested.

4.2 Ramp/Step Steer and Reverse Steer Tests

The transient response numerics for the ramp/step test, i.e., yaw rate and lateral acceleration response times, are shown in Figure 9 as a function of the steady-state lateral acceleration of the test vehicles.

The yaw rate response times for the two vehicles exhibit very similar magnitude and trends. The differences in response times at any given lateral acceleration level are within .03 seconds. Asymmetries in
Table 8. Steady-State Results, 50 mph.

<table>
<thead>
<tr>
<th>Vehicle</th>
<th>$K$ (deg/g)</th>
<th>$G_r$ (l/s)</th>
<th>$G_{av}$ (g/deg)</th>
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</thead>
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<tr>
<td>1</td>
<td>4.2</td>
<td>.196</td>
<td>.00731</td>
</tr>
<tr>
<td>2</td>
<td>9.6</td>
<td>.140</td>
<td>.00527</td>
</tr>
</tbody>
</table>
Figure 8a. Steady-state results, yaw rate vs. steering-wheel angle, 50 mph
Figure 8b. Steady-state results, lateral acceleration vs. steering-wheel angle, 50 mph
Figure 9a. Step/ramp test results, yaw rate response time, 50 mph
Figure 9b. Step/ramp test results, lateral acceleration response time, 50 mph
response are large enough to reverse the ranking of the vehicles from right to left turns. Both vehicles also show decreasing response times with lateral acceleration.

Greater discrimination is provided by the lateral acceleration response. Vehicle 1 has a significantly shorter response time in both right and left turns. In addition to shorter response times, Vehicle 1 also displays larger asymmetry. Though low-level response is comparable for left and right turns, response times increase with lateral acceleration in right turns, and the opposite is the case in left turns. Vehicle 2 exhibits near constant or slightly increasing response times as lateral acceleration level increases.

The reverse-steer results shown in Figure 10 are similar to those from the ramp/step test. Both yaw rate response time and yaw rate delay time show very little difference between the two vehicles. Yaw rate response times in the reverse steer are very close to those in the ramp/step for Vehicle 1, while those for Vehicle 2 are longer than their ramp/step counterpart. The trends are similar for the numerics for both vehicles.

For lateral acceleration response, the reverse steer results are nearly identical to those for ramp/step steer. The only significant change is in Vehicle 1's left turning response. For the reverse steer, both left and right turns are characterized by increasing response times with lateral acceleration.

Results from step-input-like tests lend themselves to direct interpretation. The response time numerics are a quantification of the system's maximum speed of response. As such, a lower limit on the time required to approach the steady-state level associated with a steering input level is established. Though this kind of maneuver is not routinely encountered in normal driving, the results acquired can be used as a rough measure of vehicle controllability.

Response time numerics also reflect the stability of a system. Significantly increasing response times at elevated maneuvering levels are indicative of decreased stability (decreased understeer) in the
Figure 10a. Reverse-steer results, yaw rate response time, 50 mph.
Figure 10 b. Reverse-steer results, lateral acceleration response time, 50 mph
Figure 10c. Reverse-steer results, yaw rate delay time, 50 mph
nonlinear regime of vehicle performance. Increasing response times can adversely affect vehicle controllability, even in a stable system, by posing a challenge to the operator's ability to successfully close the directional control loop. This information is critical to vehicle characterization pertaining to controllability.

The application of other numerics to this procedure could also be of additional help in defining the vehicle response to the step input, viz., percent overshoot of response and settling time. These numerics and a repeatable peak response time measurement would provide significant information concerning the vehicle's oscillatory behavior during this maneuver.

4.3 Pseudo-Random Input Test

The results of this test for the two test vehicles are shown in Figure 11. Gain and phase of the response variables, yaw rate and lateral acceleration, relative to steering-wheel angle, are used to quantify the vehicle response in the linear regime.

Yaw rate response for the two vehicles is seen to be majorly different in terms of gain. The shape of the gain curves is also slightly different, Vehicle 2 showing more of a peak than Vehicle 1. It is tempting to try to explain this in terms of vehicle damping ratio, but the response of the vehicle's feed forward loop on the rate of change of steering angle (that is, the derivative term in the numerator of the transfer function) confounds this simple explanation. Phase relationships for the two vehicles are nearly identical.

As with yaw rate, the largest difference observed in lateral acceleration response is again the difference in gain. Both vehicles exhibit similar trends with frequency (i.e., gradually decreasing gain with a minimum in the 1.75 to 2.0 Hz range). There is also a measurable difference in phase relationships. Vehicle 2 exhibits less phase lag than Vehicle 1. The rate of change of the phase angles is different, with Vehicle 2 having less increase in lag with frequency out to 1.25 Hz. Above this frequency, Vehicle 2 has a decreasing phase lag leading to a phase lead
Figure 11a. Pseudo-random input test results, yaw rate gain
Figure 11b. Pseudo-random input test results, yaw rate phase
Figure 11c. Pseudo-random input test results, lateral acceleration gain.
Figure 11d. Pseudo-random input test results, lateral acceleration phase
at approximately 2 Hz. Vehicle 1 has low coherence in this range (see Fig. 7) and an accurate representation of its response in this range is not possible.

This type of test can be successfully and accurately used to describe the frequency response of a linear system to (relatively) broadband excitation. Any form of nonlinearity in the system, viz., nonlinear tire force generation, steering system lash, or Coulomb friction, will degrade the estimate of the vehicle's transfer functions. The presence of nonlinear effects can in large part be ascertained by examination of the coherence function, as these effects lessen the adequacy of the linear system fit and, thus, reduce the coherence function. The results of this procedure can be used to discriminate between the frequency response characteristics of various vehicles in normal, linear range maneuvers.

4.4 Sinusoidal Steer Test

The basic sinusoidal test procedure involved only two maneuvering levels, as suggested in [4], for each vehicle. This provides a very limited amount of data on which to evaluate a vehicle's directional performance, but can still provide insight into the test procedure.

Due to the limited nature of the data collected for this procedure (one test level producing a peak lateral acceleration less than .41 g (4 m/s²) and the level corresponding to the next highest level defined by Equation (1)), the data from two conditions have been linearly interpolated to provide an estimate of the vehicle performance at the 4 m/s² level. These results for Vehicles 1 and 2 are presented in Table 9.

The yaw rate lag for Vehicle 1 is approximately .02 seconds shorter than Vehicle 2 for the first half-wave and about .01 seconds shorter for the second half-wave. This result is reflected in the relative magnitudes of the yaw rate time lag amplification; while both vehicles exhibit a lag amplification of less than 1.0, Vehicle 1 has a value of 0.89, Vehicle 2, .77. The difference in lag times is of the same magnitude as the data acquisition system's sampling interval and about twice the rms deviation observed for these numerics. Hence, the results indicate that the differences in yaw rate lag for these vehicles are not readily distinguished in these tests.
Table 9. Sinusoidal Steer Test Results, Normalized to 4 m/s² (0.41 g).

<table>
<thead>
<tr>
<th>Vehicle</th>
<th>Steering Wheel Angle (deg)</th>
<th>Lateral Acceleration Response</th>
<th>Yaw Rate Response</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\tau_1$ (s)</td>
<td>$\tau_2$ (s)</td>
</tr>
<tr>
<td>1</td>
<td>62.2</td>
<td>0.193</td>
<td>0.214</td>
</tr>
<tr>
<td>2</td>
<td>81.4</td>
<td>0.173</td>
<td>0.237</td>
</tr>
</tbody>
</table>
Lateral acceleration time lags show slightly larger differences, with Vehicle 1 having a longer lag than Vehicle 2 by .02 seconds for the first half-wave. The time lag amplification, of course, reflects the time lag observations. As with the step input tests, these vehicles seem to differ principally in transient lateral acceleration response.

Yaw rate gain and amplification also point to differences in the two vehicles' response to the single sine wave input. Contrary to the steady-state gains calculated from steady turning measurements, Vehicle 2 exhibits higher yaw rate gain (.213 vs. .190) than Vehicle 1. For the .5 Hz single sine wave, Vehicle 2 has a yaw rate gain greater than its steady-state gain, while Vehicle 1's gain is marginally lower for the sine steer than for steady state. Vehicle 2 also exhibits a larger first-to-second half-wave amplification, approximately 1.07 vs. Vehicle 1's .94.

Gain and amplification for lateral acceleration also provide bases for discriminating between these vehicles. Vehicle 1 exhibits the higher lateral acceleration gain at .0066, compared to .0054 for Vehicle 2. Again Vehicle 1's gain is lower than that measured for steady state and Vehicle 2's gain is higher than its steady-state value. Gains for both vehicles show decreasing trends with increasing lateral acceleration. As with yaw rate, the amplification of lateral acceleration is greater for Vehicle 2 at approximately 1.05, while Vehicle 1 has an amplification of about .92.

For the moderate level maneuvers considered here, the sinusoidal steer test has demonstrated its ability to produce consistent results, even with a manually applied steering input. Though the data collected was very limited, the numerics generated, particularly those relating to lateral acceleration, were able to appreciably discriminate between the two vehicles tested. The limited nature of the testing (in terms of steering level and frequency) leaves questions concerning the significance of the basic results with regard to an overall assessment of transient directional response.

This type of procedure is specifically designed to assess vehicle performance in a lane-changing maneuver. The numerics calculated here for response to the quasi-sine-wave input provide a relatively complete
description of the vehicle's performance in a particular maneuver. An extended test program, as suggested in [4], would yield information pertaining to lateral displacement maneuvers over the entire range of maneuvering levels and steering frequencies, from a gentle freeway lane change to an accident-avoidance maneuver. It is likely that at higher acceleration levels and frequencies, differences between vehicles would be greater and more significant evaluations of vehicle performance could be made.
5.0 EFFECTS OF NONLINEARITIES ON TRANSIENT RESPONSE TEST RESULTS

5.1 The Influence of Steering System Nonlinearities on the Results of the Random Steer Test

To assist in interpreting test results, a computerized model [8] of the directional dynamics of automobiles has been used to simulate vehicle tests over a wide range of lateral acceleration levels extending from the linear into the nonlinear regime of vehicle performance. The model contains yaw, sideslip, and roll degrees of freedom. This model's features include constant forward velocity, linear parameters describing suspension geometry, nonlinear tire characteristics, and a hysteretic steering system representation which approximates the lash, compliance and friction in the steering system.

The unique feature of this model is the steering system representation added in this study. The steering system is approximated using an empirical model similar to one used in [9] to model leaf springs. An example of measured data and the model approximation is shown in Figure 12. This model is defined by upper and lower boundaries with an exponential function used to describe the steering torque characteristics within the envelope of the boundaries. Steering system compliance is controlled by the slope of the boundaries, lash by the boundary spacing, and friction by the exponential function. The input to the calculation is torque, $\tau$, and the output is the change in steer angle, $\delta$.

The exponential function used in the digital calculation of $\delta_i$ (where the subscript $i$ denotes the current time step) is described by the following equation and associated definitions:

$$
\delta_i = \delta_{\text{ENV}_i} + (\delta_{i-1} - \delta_{\text{ENV}_{i-1}}) e^{-\frac{|\tau_i - \tau_{i-1}|}{\beta}}
$$

where $\delta_{\text{ENV}_i}$ is the value of the appropriate boundary of the envelope at $\tau_i$, $i-1$ denotes the previous time step, and $\beta$ is a constant chosen to fit
Figure 12. Steering system torque-deflection characteristics.
the test data. The upper or lower boundary of the envelope is selected depending upon whether \( \tau_i \) is less or greater than \( \tau_{i-1} \).

Figure 13 illustrates the principal effect of steering system nonlinearities on vehicle response to a pseudo-random steering input. The simulated vehicle with the hysteretic steering system exhibits a larger yaw rate gain than the vehicle with the linear steering system. In addition, lateral acceleration gain also shows a small increase for the vehicle with the nonlinear steering system. Steady-turning gains are essentially unaffected by these nonlinearities.

This effect was also observed in full-scale tests, as shown in Figure 14. For the increased lash condition, the Riviera's steering system was loosened to allow approximately one inch (2.5 cm) of free play at the steering wheel. Again, the vehicle with increased lash (increased nonlinearity) demonstrated a higher yaw rate gain than the baseline case. Other measurements, viz., lateral acceleration gain, yaw rate and lateral acceleration phase, steady-turning gains, showed no appreciable change with the steering system modification.

For the vehicle used in this study, free play in the steering system causes a major increase in the yaw rate gain over the range of frequencies used in rapid maneuvering on the highway. Even though the importance of free play has been demonstrated for only one vehicle, the result is profound enough to warrant special attention in future work. The fact that the computer simulation of a "typical" vehicle predicts the same type of phenomenon as the test results lends credence to the generality of the observation. Certainly, free play should be carefully controlled if meaningful results are to be obtained from random steer tests. Further research is needed to develop a better understanding of the importance of small-scale nonlinearities in the steering system.
Figure 13. Influence of steering system nonlinearities on yaw rate transfer function gain, simulation results.
Figure 14. Influence of increased steering system lash on yaw rate transfer function gain, experimental results.
5.2 Effects of Tire Nonlinearities on Transient Response Results

The computerized vehicle model was used to provide insight into the effect of tire nonlinearities on the results generated from the different test procedures. To accomplish this, the tire description approximating the measured shear force characteristics of a tire was replaced by one producing side force linearly with slip angle and constant with load. In order to provide a legitimate basis for comparison, the "cornering stiffness" type of linear tire model was selected to approximate the nonlinear model's performance in the low slip-angle regime at nominal static loads. This simulation approach (that is not possible with full-scale testing) provides a simple method of examining the effects of tire nonlinearities.

Steady turning results for the linear and nonlinear tire simulations are shown in Figure 15. Tire nonlinearities cause decreased gain at higher lateral acceleration levels as would be expected on most standard passenger vehicles. The effect of the nonlinear side force generation with load and slip angle on steady turning is relatively small for this configuration, and would suggest small influences on transient response.

Figure 16 shows the ramp/step and reverse steer results for a vehicle simulated with both linear and nonlinear tire force characteristics. The tire nonlinearities have a negligible effect on the yaw rate transient response or lateral acceleration response time. The lateral acceleration peak response times (time to peak) for both ramp/step and reverse steer inputs show a marked increased at higher lateral acceleration levels with the nonlinear tires. An ideal linear system would have constant delay, response and peak response times with any level of steering input. Other non-tire-related nonlinearities cause a small deviation from this ideal, causing all the response times to increase slightly with increasing lateral acceleration. The tire nonlinearities have a negligible effect on transient response to a ramp/step input, except for lateral acceleration time to peak.
Figure 15. Influence of tire non-linearities on steady turning performance.
Figure 16. Influence of tire nonlinearities on ramp/step and reverse steer results.
The pseudo-random input test is analyzed to provide a linear system approximation of the vehicle's frequency response and, as such, is subject to distorted results if nonlinear mechanisms are present. The gain and phase representations of yaw rate and lateral acceleration with steering amplitudes of 20 and 50 degrees (corresponding to steady-state levels of 0.2 and 0.5 g, respectively) are shown in Figure 17. The larger input amplitude results in a lower gain for both yaw rate and lateral acceleration. This is in keeping with the steady-state results of Figure 15 that show a decreasing gain with larger steering inputs. While yaw rate phase relations are nearly identical for the two input levels, the lateral acceleration response has a longer lag with the larger input. The phase lag increases more with frequency at the higher level and reaches its maximum magnitude at a lower frequency than with the lower input level.

A similar pattern is seen in the sinusoidal steer results of Figure 18. Again, the vehicle with linear tires maintains near constant gain and lag with lateral acceleration, while the nonlinear tires bring about decreased gain and increased lags at higher lateral acceleration levels.

Tire nonlinearities are known to have an effect on vehicle response at elevated lateral accelerations. These nonlinear effects are easily seen in the results of the pseudo-random and sinusoidal steer tests by changes in the gain or lag relationships of yaw rate and lateral acceleration. The ramp/step steer test, while yielding information concerning nonlinear effects on steady-turning performance, provides almost no discrimination between linear- and nonlinear-tired vehicle transient response. Only the peak response time for lateral acceleration is noticeably affected (however, this is one of the numerics that provided inconsistent results in full-scale testing).
Figure 17. Influence of increased steering amplitude on pseudo-random test results.
Influence of increased steering amplitude on Pseudo-random input test results.
Figure 18. Influence of tire nonlinearities on sinusoidal steer test results.
Figure 18 (Cont.). Influence of tire nonlinearities on sinusoidal steer test results.
6.0 INTERRELATIONSHIPS BETWEEN TESTS

The three generic types of tests are not similar. The steering inputs are not alike. The data processing procedures are entirely different. And the numerics or results obtained are not directly comparable.

Nevertheless, if the results from one of the tests could be used to infer the results of the others, then that test could be viewed as a fundamental test for transient response properties. Furthermore, in particular cases, results from one type of test may be available while the results from another type of test may be desired. Hence, the intent of this section is to provide experimentally based answers to questions concerning extrapolating from the results of one test procedure to those of another.

6.1 Limitations on the Ability to Extrapolate From One Test to Another

For a linear system, the step response and the frequency response are two classical methods for defining the system. Ideally, the principles of LaPlace or Fourier transform theory can be used to convert results from the time domain to the frequency domain or vice versa. To a certain extent, that theory can be applied to results from the ramp/step and pseudo-random input tests in order to estimate the results of one from the other.

However, the ramp/step test is employed in both the linear and nonlinear regimes of vehicle operation. (Generally, passenger cars are treated as linear systems for maneuvers requiring less than approximately 0.3 g, although free play and hysteresis in the steering system may have significant nonlinear effects on low-level steering results.) In contrast, the frequency sweep or random steering test is based on linear system theory and the input levels are chosen to remain below the threshold of significant tire shear force nonlinearities. Hence, the ramp/step test covers a broader range of vehicle operation than the frequency sweep test and the results from the frequency sweep test can be misleading if they are used to infer time history information in severe turning maneuvers exceeding approximately 0.3 g.
In addition to this rather obvious difference between the ramp/step and pseudo-random tests, there are practical differences in the range of usable frequencies involved. These differences are illustrated by the example input energy spectra for the two tests, as shown in Figure 19. The measured input spectrum for the ramp/step (plus a subsequent reverse steer) has a large amount of its energy in the frequency range from 0 to approximately 0.3 Hz, while the pseudo-random test tends to have less energy per unit bandwidth below 0.3 Hz than it does from 0.6 Hz to 1.0 Hz or above. Accordingly, the output signals (yaw rate or lateral acceleration time histories) from the ramp/step or reverse-steer tests will have less frequency content above 0.5 Hz than that attained by the corresponding responses from the pseudo-random test. In effect, the ramp/step and pseudo-random tests emphasize separate frequency bands, thereby making extrapolations of the results from one test to the other difficult in a practical situation with noise and other confounding factors present.

Given the test procedures, instrumentation, noise, and vehicles employed in this study, the use of a fast Fourier transform applied to the ramp/step signals to estimate transfer functions was only moderately worthwhile. Example results for lateral acceleration and yaw rate are shown in Figure 20. It is interesting to note that the phase characteristics obtained from both test procedures for both yaw rate and lateral acceleration compare very favorably at frequencies where the coherence is good. However, the amplitude characteristics determined from the ramp/step test do not serve as more than a rough estimate of the pseudo-random steer results. At least for the manner in which the ramp/step tests were performed in this study, the results from that test do not provide a satisfactory estimate of the vehicle's transfer function for frequencies greater than 0.4 Hz.

On the other hand, the pseudo-random input test (as performed in this study) does not contain enough low frequency information to establish steady-state gain for frequencies approaching zero. Tests for establishing the steady turning gain are being developed separately from the study of transient response [1]. Clearly, the zero frequency gain is important in studying the vehicle and its influence on the driver as an element of
Figure 19. Pseudo-random and step/ramp-reverse steer input spectra
Figure 20a. Transfer functions from pseudo-random and step/ramp-reverse steer inputs, yaw rate gain
Figure 20b. Transfer functions from pseudo-random and step/ramp-reverse steer inputs, yaw rate phase
Figure 20c. Transfer functions from pseudo-random and step/ramp-reverse steer inputs, lateral acceleration gain.
Figure 20d. Transfer functions from pseudo-random and step/ramp-reverse steer inputs, lateral acceleration phase.
a closed-loop control system. Hence, it is desirable to supplement the
pseudo-random test results with information at low frequencies in order
to specify the vehicle's open-loop transfer function.

The third type of test, that is, lane-change or sinusoidal-steer
test, is based on a particular type of maneuver that vehicles may be
required to perform to resolve traffic conflicts. The energy spectrum of
the input is limited to a band of frequencies determined by the period of
the steering activity. For example, Figure 21 shows a typical energy
spectrum for an approximately sinusoidal steering input occurring within
a two-second period. As expected, the energy spectrum for the sinusoidal
steer is characterized by a main "lobe" at 0.5 Hz and, interestingly, as
shown in Figure 21, 0.5 Hz is approximately where the ramp/step and the
pseudo-random spectra are both small and smaller than the sinusoidal steer
spectrum in this case.

Since the magnitude of the input spectrum for the quasi-sinusoidal
steer maneuver is almost zero at certain frequencies, estimates of the
transfer functions for yaw rate and lateral acceleration can be expected
to be very erratic. This phenomenon is illustrated for the amplitudes of
these transfer functions in Figures 22a and 22c. Even so, it seem sur-
prising that the amplitude results from the sine steer test do not match
those obtained from the pseudo-random test at frequencies near 0.5 Hz.
Given this situation, it is puzzling that the phase characteristics derived
from these two test procedures are approximately equivalent in the neigh-
borhood of 0.5 Hz (see Figures 22b and 22d). Nevertheless, this empirical
evidence supports the hypothesis that extrapolations between phase charac-
teristics, and therefore time lags, may be possible, but accurate gain
relationships are not to be expected.

6.2 Comparisons and Extrapolations Between Numerics

The results of the previous section indicate limitations on the
generality of any of the three generic types of test procedures. Given
that the inputs for the various test procedures emphasize different fre-
quency ranges, practical attempts to derive overall transfer functions

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Figure 21. Input spectra for pseudo-random, sinusoidal, and step/ramp-reverse steer inputs
Figure 22a. Transfer functions from pseudo-random and sinusoidal steer inputs, yaw rate gain.
Figure 22b. Transfer functions from pseudo-random and sinusoidal steer inputs, yaw rate phase.
Figure 22c. Transfer functions from pseudo-random and sinusoidal steer inputs, lateral acceleration gain.
Figure 22d. Transfer functions from pseudo-random and sinusoidal steer inputs, lateral acceleration phase.
from the time histories developed in any of the test procedures are hindered by the lack of input energy density at certain important frequencies. In particular, the time histories obtained in the quasi-sinusoidal steer and ramp/step tests are not satisfactory for accurately estimating the amplitudes of the transfer functions determined in the pseudo-random steer test.

Nevertheless, the response time lags, \( \tau_1 \) and \( \tau_2 \), from the quasi-sinusoidal steer test (as obtained in this test program) corresponds fairly closely to estimates of the response time lags derived from the transfer function measured in the pseudo-random steer procedure. Evidence supporting this statement is presented in Table 10 which shows results for lateral acceleration and yaw rate for Vehicles 1 and 2 in a minimum-load condition. The time lags \( (\tau_{0.5}) \) are estimated from the frequency sweep data using the following equation:

\[
\tau_f = \frac{\phi(f)}{f}
\]

where

- \( \tau \) is the steady-state time lag in seconds
- \( f \) is the frequency of the input signal in deg/sec
- \( \phi(f) \) is the phase angle (in degrees) of the transfer function at frequency \( f \)

(For the basic sine steer test \( f = 0.5 \) Hz, i.e., 180 deg/sec.)

Although the random-steer data pertain to steady-state responses to sinusoidal inputs while the sine-steer test only employs one cycle of a sine wave, the transient in the sine-steer results appears to have little influence on the phase or speed of the response for an input with a two-second period. (The main influence relates to the gains \( G_1 \) and \( G_2 \).) Hence, attempts to extrapolate phase or time lag information between the random- and sine-steer tests can successfully provide reasonably accurate results.
Table 10. Comparison of Sinusoidal Steer Numerics with Results from Random Steer Tests

<table>
<thead>
<tr>
<th>Sine Steer Numerics</th>
<th>Transfer Function Values at 0.5 Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_1$ (s)</td>
<td>$\phi(0.5)$ (deg)</td>
</tr>
<tr>
<td>$T_2$ (s)</td>
<td></td>
</tr>
<tr>
<td>$G_1$</td>
<td></td>
</tr>
<tr>
<td>$G_2$</td>
<td></td>
</tr>
</tbody>
</table>

Vehicle 1

<table>
<thead>
<tr>
<th>$A_y$</th>
<th>$r$</th>
<th>$T_1$</th>
<th>$T_2$</th>
<th>$G_1$</th>
<th>$G_2$</th>
<th>$\phi(0.5)$</th>
<th>$T_{0.5}$</th>
<th>$G_{0.5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.19</td>
<td>.21</td>
<td>.0066</td>
<td>.0061</td>
<td></td>
<td></td>
<td>-35.5</td>
<td>.20</td>
<td>.0077</td>
</tr>
<tr>
<td>.12</td>
<td>.10</td>
<td>.191</td>
<td>.180</td>
<td></td>
<td></td>
<td>-16.0</td>
<td>.09</td>
<td>.201</td>
</tr>
</tbody>
</table>

Vehicle 2

<table>
<thead>
<tr>
<th>$A_y$</th>
<th>$r$</th>
<th>$T_1$</th>
<th>$T_2$</th>
<th>$G_1$</th>
<th>$G_2$</th>
<th>$\phi(0.5)$</th>
<th>$T_{0.5}$</th>
<th>$G_{0.5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.17</td>
<td>.18</td>
<td>.0056</td>
<td>.0063</td>
<td></td>
<td></td>
<td>-27.0</td>
<td>.15</td>
<td>.0050</td>
</tr>
<tr>
<td>.12</td>
<td>.09</td>
<td>.202</td>
<td>.189</td>
<td></td>
<td></td>
<td>-14.2</td>
<td>.08</td>
<td>.149</td>
</tr>
</tbody>
</table>

$A_y =$ lateral acceleration
$r =$ yaw rate
In contrast, the 90-percent response times measured for Vehicles 1 and 2 do not have a consistent relationship to the response lags measured in the sine-steer test. As shown in Table 11, the time lag in lateral acceleration response is less for Vehicle 2 than for Vehicle 1 in the sine-steer test, even though the 90-percent response time for lateral acceleration is considerably longer for Vehicle 2 than for Vehicle 1. This same type of finding applies to yaw rate also, although the difference is not as noticeable. This inconsistency between the results for Vehicles 1 and 2 shows that a direct extrapolation between 90-percent response times and time lags in the sine-steer tests can lead to errors in the relative ranking of vehicle response properties—clearly, a very undesirable situation.

Furthermore, using the phase information from the random-steer test can lead to questionable estimates of the 90-percent response times and vice versa. For example, the concept of an effective time constant, $T_e$ [10,11], has been employed as a numeric for studying the yaw response of passenger cars. The quantity $T_e$ is defined as the reciprocal of the frequency (in rad/sec) at which the phase angle of the yaw rate transfer function equals -45 degrees. As illustrated in Figure 23, the phases of the yaw rate transfer functions for Vehicles 1 and 2 are nearly identical below approximately 1.1 Hz and the measured values of $T_e$ for these vehicles are identical with $T_e = 0.147$ sec. The small difference of 0.03 second discerned by the 90-percent response times measured in the ramp/step test does not show up in the effective time constant. At a frequency equal to $1/T_e$, a difference in time lag of 0.03 second is equivalent to a difference of almost 12 degrees in phase angle. Nevertheless, $T_e$ appears to provide a rough estimate of the 63-percent rise time being equal to approximately 0.7 of the value of the 90-percent response time for yaw rate.

Note that these vehicles achieve similar yaw rate phase characteristics in quite different ways, as indicated by the normalized gain versus frequency characteristics shown in Figure 24. On the other hand, the normalized lateral acceleration gains for these vehicles are very similar up to 1.75 to 2 Hz, where it is suspected that differences in the numerators of
Table 11. Comparison of 90% Response Times with Delay Times, \( \tau_2 \), and Time Constants Evaluated at 45° Phase Lag

<table>
<thead>
<tr>
<th></th>
<th>Ramp/Step 90% Response Times (At Approximately 0.2 ( g )) (sec)</th>
<th>Sinusoidal Steer, ( \tau_2 ) (2 Sec Period) (sec)</th>
<th>Random Steer, Effective Time Constant (At 45° Phase Shift) (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Vehicle 1</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( A_y )</td>
<td>0.27</td>
<td>0.21</td>
<td>0.24</td>
</tr>
<tr>
<td>( r )</td>
<td>0.17</td>
<td>0.10</td>
<td>0.145</td>
</tr>
<tr>
<td><strong>Vehicle 2</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( A_y )</td>
<td>0.345</td>
<td>0.18</td>
<td>0.20</td>
</tr>
<tr>
<td>( r )</td>
<td>0.20</td>
<td>0.09</td>
<td>0.145</td>
</tr>
</tbody>
</table>

\( A_y \) = lateral acceleration  
\( r \) = yaw rate
Figure 23. Determining effective time constant from yaw rate phase
Figure 24. Normalized yaw rate gain
the transfer functions have large influences on the results. The data shown in Figure 25 indicate that it is very difficult to get any significant lateral acceleration response from Vehicle 1 at 2 Hz. The strong "anti-resonance" at 2 Hz means that even if the input has reasonable energy density at 2 Hz, the coherence may be low due to the lack of a significant level of output signal.

Even though the lateral acceleration gain characteristics are similar up to 1.5 Hz for these two vehicles, the phase characteristics, presented in Figure 11d, indicate that the time lag for Vehicle 1 will be greater than the time lag for Vehicle 2. For instance, using -45 degree phase lag as a reference value yields estimates of lateral acceleration delay times of 0.2 second for Vehicle 2 and 0.24 second for Vehicle 1 for responses to steering inputs at frequencies less than 0.75 Hz. These delay times are not representative of the situation observed by comparing 90-percent response times. As summarized in Table 11, the 90-percent acceleration response time is much longer for Vehicle 2 than it is for Vehicle 1. Clearly, simple extrapolations between the numerics for the ramp/step test and the results of the other test procedures can be misleading with respect to the quickness of the lateral acceleration response in a particular maneuver.

Rather than showing that any one test is sufficient, the comparisons and attempts to extrapolate between numerics as presented here show that each test produces unique results that are not easily ascertained from the results of the other tests. If particular numerics (such as the 90-percent response times or the gains $G_1$ and $G_2$ for each half-wave of a lane change) are desired, the results indicate that the appropriate test specifically designed to obtain these numerics should be performed.
Figure 25. Normalized lateral acceleration gain
7.0 SUMMARY AND CONCLUSIONS

Experimental results obtained from tests involving (1) approximations to step inputs, (2) pseudo-random steering oscillations, and (3) quasi-sinusoidal steer maneuvers (approximate lane-change situations) have been examined with regard to repeatability, ability to discriminate between vehicles, and uniqueness of the results. In general, each of these three types of tests were found to be useful based on considerations of adequate repeatability and the existence of numerics that (1) discriminate between vehicles and (2) provide information that is difficult to obtain from the other tests.

Specifically, the ramp/step or reverse-steer tests performed in this study provided measures of 90-percent response times for lateral acceleration and yaw rate with rms deviations that were generally less than five percent of their mean values. The lateral acceleration response time, in particular, provided a clear discrimination between the vehicles studied. These response times were not only found to be difficult to estimate from the other tests, but attempts to extrapolate to these results could lead to false assessments of the relative ranking of vehicles.

The pseudo-random steer test can provide frequency response information with high coherence (above 0.96) for low lateral acceleration situations over the frequency range from approximately 0.3 Hz to 2.0 Hz for yaw rate and from 0.3 Hz to 1.2 Hz for lateral acceleration. Differences in gain and phase characteristics for yaw rate and lateral acceleration transfer functions are easily discernible for various vehicles. None of the other tests provide as general a description of the linear range transient performance of the automobile. Nevertheless, the results of this test should be supplemented with low frequency information (at least steady turning gain) and some information on the influence of nonlinearities at higher maneuvering levels.

The sinusoidal-steer test was performed without the use of an automatic device for applying a single cycle of a sine wave to the steering wheel. The input was supplied by the test driver. Nevertheless, the procedure used furnished satisfactory results. The time lags determined from
cross-correlations of corresponding "halves" of the input and output waveforms were very repeatable. The rms deviations were less than six percent of the mean values. The time lags from this test and the phase information from the random-steer test appeared to be related in a manner allowing simple extrapolations between these particular results. However, the gain characteristics obtained in this test are unique to it and these characteristics do vary from vehicle to vehicle.

The findings from this study only address certain questions concerning repeatability, discrimination power, and uniqueness of the results. This study does not examine the motivations or reasons for performing particular tests. Correlations with subjective ratings are not considered. However, accurate descriptions of vehicle properties as result from carefully conceived open-loop tests are needed before meaningful correlation studies can be performed. If the goal is to learn as much as possible about the transient directional response properties of a particular set of vehicles, then all three of these types of tests can contribute useful information for the evaluation and comparison of these vehicles. In this sense, no particular test is recommended as being sufficient. Rather, it is suggested that all three tests be performed if adequate space, time, and resources are available.
8.0 REFERENCES


