Development of an Efficient Computational Procedure for Evaluating Band Saw Blade Stresses

by

James E. Borchelt

A. Galip Ulsoy

and

Panos Papalambros

UM-MEAM-83-13

Department of Mechanical Engineering
and Applied Mechanics

and

The Center for Robotics and
Integrated Manufacturing

The University of Michigan
Ann Arbor, MI 48109

June 1983

UMR0417

TABLE OF CONTENTS

ABSTRACT 1

INTRODUCTION 2

THEORY 4

COMPUTATIONAL RESULTS 15

SUMMARY AND CONCLUSIONS 18

ACKNOWLEDGEMENTS 25

REFERENCES 26

LIST OF SYMBOLS 28

APPENDICES 29

A - The Total Potential Energy Expression 29

B - The Governing Differential Equations 32

C - The Variational Equations 36

D - Functional Form of the Approximating Functions . . 40

E - Ordering of the Functions Φ and ψ. When N = 3 44

F - Program Output 46

ABSTRACT

The stresses in a band saw blade are difficult to measure, and are known to be significant for band sawing performance. This report presents an approximate solution method, using the principle of minimum potential energy, for computation of blade stresses in band sawing. The method presented accounts for effects typical of bandsawing such as velocity dependent blade tension, thermal gradients, and cutting forces. A program based on the method presented is shown to be accurate and computationally efficient when compared to a standard finite element program. This stress analysis program, together with a previous program for blade vibration analysis, can form the modeling basis for design and process optimization studies in band sawing.

INTRODUCTION

The design of band saw blades and sawing assemblies is an important industrial problem. Bandsawing, as a manufacturing operation, is used extensively not only in the wood industry, but also for metal cutting in the production of many machine parts. In recent years, the attempt to decrease raw material and labor costs has led to the use of thinner blades and higher operating velocities and the design of so-called high-strain mills (1, 2). However, under these demanding production conditions blade vibration and stability problems become significant. The results of these vibrations are increased material waste, cutting inaccuracies, and poor surface quality (1, 3, 4). The situation is further complicated by the fact that the state of stress and cutting conditions affect the vibration response of the blade (4, 5, 6, 7). The nature of the process is such that experiments to study this problem are both costly and difficult. Therefore, the need for an appropriate mathematical model and computational simulation can be easily demonstrated.

From the theoretical viewpoint, the band saw vibration problem belongs to the class known as axially moving material vibrations (1, 8). This class includes vibrations occuring in magnetic and paper tapes, moving threadlines, belts, and pipes transporting fluids. All these problems are quite complicated and their study is important for subsequent design evaluations. The band saw blade problem has been studied by treating the blade as a simply supported, axially moving beam (1, 9) and as a simply supported or clamped, axially moving plate (3, 4, 7). Two numerical procedures using the classical Ritz Method and the finite element - Ritz Method were developed (10) and tested against experimental data (3, 4, 7). The comparison showed that the simply supported plate model is a valid one for representing the process. These two procedures, though useful research tools, have some limitations if they are to be used in repeated calculations. The first procedure is incomplete, since it does not include the stress calculations, while the second one is not computationally efficient. Therefore, if repeated computations are needed a new procedure must be developed having a computational advantage.

The purpose of this report is to describe the development of a computationally efficient method for calculating blade stresses, and demonstrate its performance. The method is based on the classical variational formulation of minimizing the potential energy of the system (11). The theory and development of the variational equation are introduced first, followed by a discussion of the admissible approximating functions. Computational results are then presented which show that acceptable accuracy is obtained at considerably reduced computational time as compared to a standard FEM package (MSC/NASTRAN).

THEORY

The stresses in a band saw blade arise from the band axial tension, prestressing operations, thermal gradients, and cutting forces (4, 5, 6). Blade stresses are known to be one of the most significant factors influencing band-sawing performance (3, 4, 7, 12). Since blade stresses are extremely difficult to measure in a production environment, it is desirable to develop an analytical method for their computation.

In this section a model of the typical band saw blade stress problem is presented. This model, together with the Principle of Minimum Potential Energy, is used to derive the governing differential equations and boundary conditions. An approximate solution method, which utilizes global polynomial approximating functions for the displacement fields U(x,y) and V(x,y), is also presented.

Model

As shown is Figs. 1 and 2, consider the segment of the band saw blade between supports (guides) to be an axially moving plate of thickness H, width B, length L, and constant axial velocity c. The plane stress problem, shown in Fig. 3, is formulated by specifying U(0,0) = V(0,0) = 0 and V(L,0) = 0 to eliminate rigid body translations and rotation. As described in (3, 9) the constant axial velocity c is accounted for through its effect on the axial tension force,

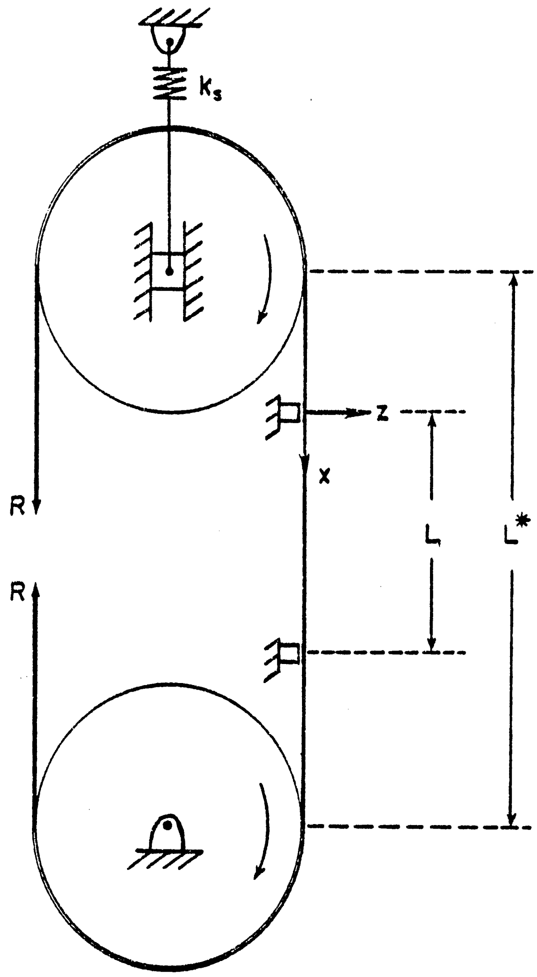

Figure 1 Idealized Model of a Band Mill

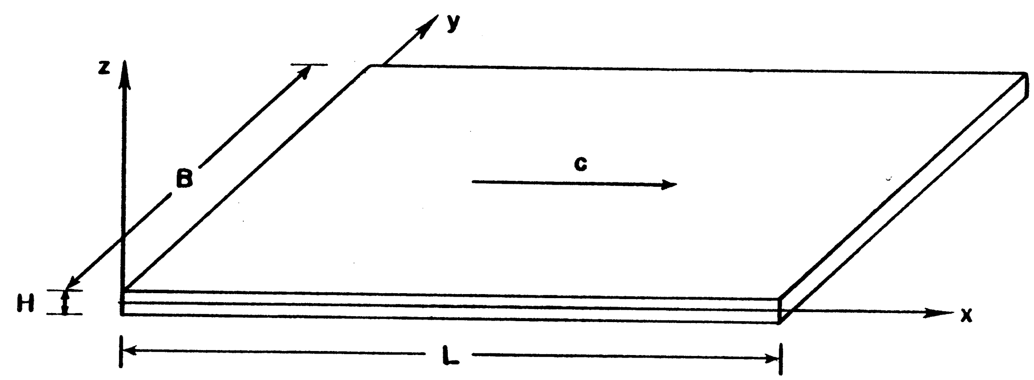

Figure 2 Idealized Model of a Band Saw Blade Between Supports as an Axially Moving Plate

$$R = R_o - \kappa\rho HBc^2 \quad (1)$$

where R_o is the band tension at $c = 0$, and $0 \leq \kappa \leq 1$ is a wheel support system constant.

The effects of the cutting force are represented by its normal (F_n) and tangential (F_t) components. These result in uniformly distributed boundary loads ($F_n/\Delta x$) and ($F_t/\Delta x$) which act at the edge $y = B$ and over a segment of the blade from $x = x_1$ to $x = x_2$. The cutting force and guides also introduce thermal effects, which are brought into the problem as a prescribed and steady temperature distribution $T(x,y)$ on the blade.

A basic assumption is that of steady cutting conditions, since R, F_n, F_t, and T are all taken to be time invariant. Although R is in practice a function of y due to wheel tilting and wheel crowning, these effects are neglected and a uniformly distributed boundary load (R/B) is assumed along the edges at $x = 0$ and $x = L$ (3, 4, 5, 6). Blade teeth and gullet effects are also neglected in this analysis.

Governing Equations

The governing differential equations and boundary conditions for the band saw blade stress problem are derived using the Principle of Minimum Potential Energy (11, 13, 14). The potential energy functional, derived in Appendix A, is given by,

$$\pi(U,V) = (H/2)\int_0^B\int_0^L \{(E/(1-\nu^2))((\partial U/\partial x)+(\partial V/\partial y))^2$$

$$+ G[((\partial U/\partial y) + (\partial V/\partial x))^2 - 4(\partial U/\partial x)(\partial V/\partial y)]$$

$$- H[(E\alpha/(1-\nu))((\partial U/\partial x) + (\partial V/\partial y) - \alpha T)T\}\, dxdy$$

$$- \int_0^B [(R/B)U]_{x=0}^{x=L} dy + \int_{x_1}^{x_2} [(F_n/\Delta x)V + (F_t/\Delta x)U]^{y=B} dx \qquad (2)$$

Setting the variation of the potential energy functional equal to zero, and integrating by parts to eliminate the derivatives with respect to x and y of δU and δV as needed, leads to the governing differential equations, (15, 16) (see Appendix B)

$$G[\nabla^2 U + ((1+\nu)/(1-\nu))(\partial((\partial U/\partial x) + (\partial V/\partial y))/\partial x)]$$
$$- (E\alpha/(1-\nu))(\partial T/\partial x) = 0 \qquad (3)$$

$$G[\nabla^2 V + ((1+\nu)/(1-\nu))(\partial((\partial U/\partial x) + (\partial V/\partial y))/\partial y)]$$
$$- (E\alpha/(1-\nu))(\partial T/\partial y) = 0 \qquad (4)$$

and the required boundary conditions,

$$\sigma_x(0,y) = [(E/(1-\nu^2))((\partial U/\partial x) + \nu(\partial V/\partial y))]^{x=0}$$
$$= (R/BH) + [(E\alpha/(1-\nu))T]^{x=0} \qquad (5)$$

$$\sigma_x(L,y) = [(E/(1-\nu^2))((\partial U/\partial x) + \nu(\partial V/\partial y))]^{x=L}$$
$$= (R/BH) + [(E\alpha/(1-\nu))T]^{x=L} \qquad (6)$$

$$\sigma_y(x,B) = [(E/(1-\nu^2))((\partial V/\partial y) + \nu(\partial U/\partial x))]^{y=B}$$
$$= -(F_n/\Delta xH)[u(x-x_1) - u(x-x_2)]$$
$$+ [(E\alpha/(1-\nu))T]^{y=B} \qquad (7)$$

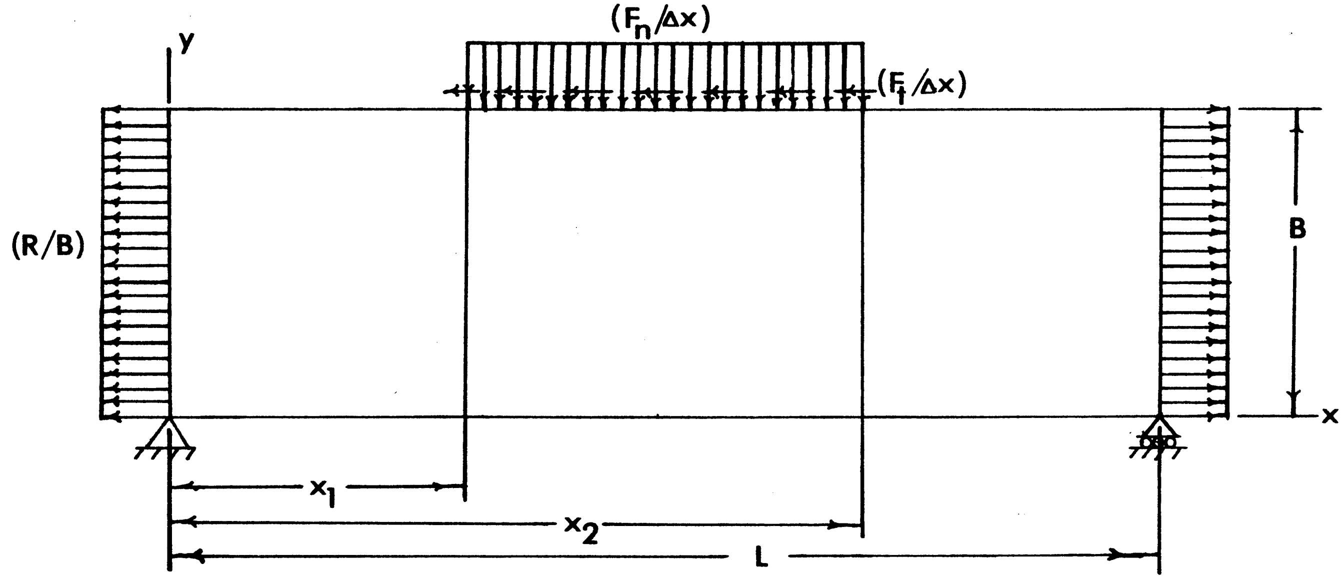

Figure 3 Boundary Loading and Support Conditions for Band Saw Plane Stress Problem

$$\sigma_y(x,0) = [(E\alpha/(1-\nu))T]^{y=0} \tag{8}$$

$$\tau_{xy}(0,y) = 0 \tag{9}$$

$$\tau_{xy}(L,y) = 0 \tag{10}$$

$$\tau_{xy}(x,0) = 0 \tag{11}$$

$$\tau_{xy}(x,B) = [G((\partial U/\partial y) + (\partial V/\partial x))]^{y=B}$$

$$= -(F_t/\Delta xH)[u(x-x_1) - u(x-x_2)] \tag{12}$$

The solution to the problem given in eqs. (3) to (12), or to the dual variational problem,

$$\delta\pi(U,V) = 0 \tag{13}$$

in general requires an approximate solution method. The finite element method (FEM) is widely used for the solution of such plane stress problems for various geometric, loading, and boundary conditions (14, 17). The advantage of the finite element approach is its generality; the disadvantage is the large cost and computation time requirements associated with general purpose programs. In the band saw blade stress problem the geometry, loading, and boundary conditions are well defined. Thus, this problem is well suited for global approximate solution methods such as the one presented below.

For the plane stress problem presented here the variational equation is found to be

$$0 = H\int\int_{(A)} \{G[\nabla^2U+((1+\nu)/(1-\nu))(\partial((\partial U/\partial x) + (\partial V/\partial y))/\partial x)]$$

$$-(E\alpha/(1-\nu))(\partial T/\partial x)\}\delta U dA$$

$$+ H \int\int_{(A)} G[\nabla^2 V+((1+\nu)/(1-\nu))(\partial((\partial U/\partial x) + (\partial V/\partial y))/\partial y)]$$

$$- (E\alpha/(1-\nu))(\partial T/\partial y)\}\delta V dA$$

$$+(EH\alpha/(1-\nu)) \int_0^B [T\delta U]_{x=0}^{x=L} dy + (EH\alpha/(1-\nu)) \int_0^L [T\delta V)_{y=0}^{y=B} dx$$

$$-(EH/(1-\nu^2)) \int_0^B [((\partial U/\partial x) + \nu(\partial V/\partial y))\delta U]_{x=0}^{x=L} dy$$

$$-(EH/(1-\nu^2)) \int_0^L [(\nu(\partial U/\partial x) + (\partial V/\partial y))\delta V]_{y=0}^{y=B} dx$$

$$-(EH/(2(1+\nu))) \int_0^L [((\partial U/\partial y) + (\partial V/\partial x))\delta U]_{y=0}^{y=B} dx$$

$$-(EH/(2(1-\nu))) \int_0^B [((\partial U/\partial y) + (\partial V/\partial x))\delta V]_{X=0}^{x=L} dy$$

$$+ \int_0^B [R\delta U]_{x=0}^{x=L} dy - \int_{x_1}^{x_2} [F_n \delta V]^{y=B} dx - \int_{x_1}^{x_2} [F_t \delta U]^{y=B} dx \qquad (14)$$

See Appendix C for the development of the variational equation (14).

Approximate Solution

To obtain an approximate solution a finite series representation of the displacement field functions is utilized,

$$U_k(x,y) = \sum_{j=1}^{k} A_j \phi_j (x,y) \qquad (15)$$

and

$$V_m(x,y) = \sum_{j=1}^{m} B_j \Psi_j (x,y) \qquad (16)$$

where ϕ_j and Ψ_j are global admissible approximating functions which satisfy the geometric (essential) boundary conditions.

Admissible Approximating Functions

Polynomial approximating functions based on Pascal's Polynomial Triangle were chosen for U_k and V_m, because the natural frequency program (10) uses polynomial approximating functions for the stress field functions. The approximating functions were also chosen so their functional form satisfied the differential equations of equilibrium (13, 17). The appropriate functional form was found to be binomial (see Appendix D). So U and V are given as

$$U_k = \sum_{n=0}^{N} A_{no}x^n + \sum_{\substack{m=1 \\ m+n}}^{N} \sum_{\substack{n=1 \\ \leq N}}^{N} A_{nm}x^n y^m + \sum_{m=1}^{N} A_{om}y^m \qquad (17)$$

$$V_m = \sum_{n=0}^{N} B_{no}x^n + \sum_{\substack{n=1 \\ m+1}}^{N} \sum_{\substack{n=1 \\ N}}^{N} B_{nm}x^n y^m + \sum_{m=1}^{N} B_{om}y^m \qquad (18)$$

Even though the functional forms U and V satisfy the differential equations of equilibrium, the geometric boundary conditions must still be satisfied.

Boundary Condition Satisfaction

The rigid body translation of the plate is eliminated in the x and y direction by setting A_{00} and B_{00} equal to zero. Eliminating the rigid body rotation, and satisfying the simply supported plate model requires the V equal zero at the point x = L and y = 0. Substituting for x and y in V yields

$$0 = B_{10}L + B_{20}L^2 + \dots + B_{N0}L^N \tag{19}$$

Solving for B_{n0} the admissible approximating function for V is found to be

$$V_m(x,y) = \sum_{n=1}^{N-1} B_{n0}(x^n - (x^N/L^{N-n})) + \sum_{m=1}^{N} \sum_{n=1}^{N} B_{nm}x^n y^m + \sum_{m=1}^{N} B_{om}y^m \tag{20}$$

The admissible approximating function for U is found to be

$$U_k(x,y) = \sum_{n=1}^{N} A_{n0}x^n + \sum_{\substack{m=1 \\ m+n \leq N}}^{N} \sum_{n=1}^{N} A_{nm}x^n y^m + \sum_{m=1}^{N} A_{om}y^m \tag{21}$$

The admissible approximating functions when substituted into the variational equation forms a set of linear symmetric equations.

Figure 4 shows the admissible approximating functions, ϕ, while Figure 5 shows the admissible approximating functions, Ψ, when N = 3. The variation of the admissible approximating functions for U and V is really a variation of the undetermined coefficients A and B. See Appendix E for the ordering of the admissible approximating functions ϕ and Ψ.

Equations in Terms of ϕ and Ψ

The approximate functions U_k and V_m are substituted into the variational expression in (14). The variation of this approximate functional is set equal to zero to obtain a system of linear symmetric equations in the undetermined coefficients $\underline{A}$ and $\underline{B}$,

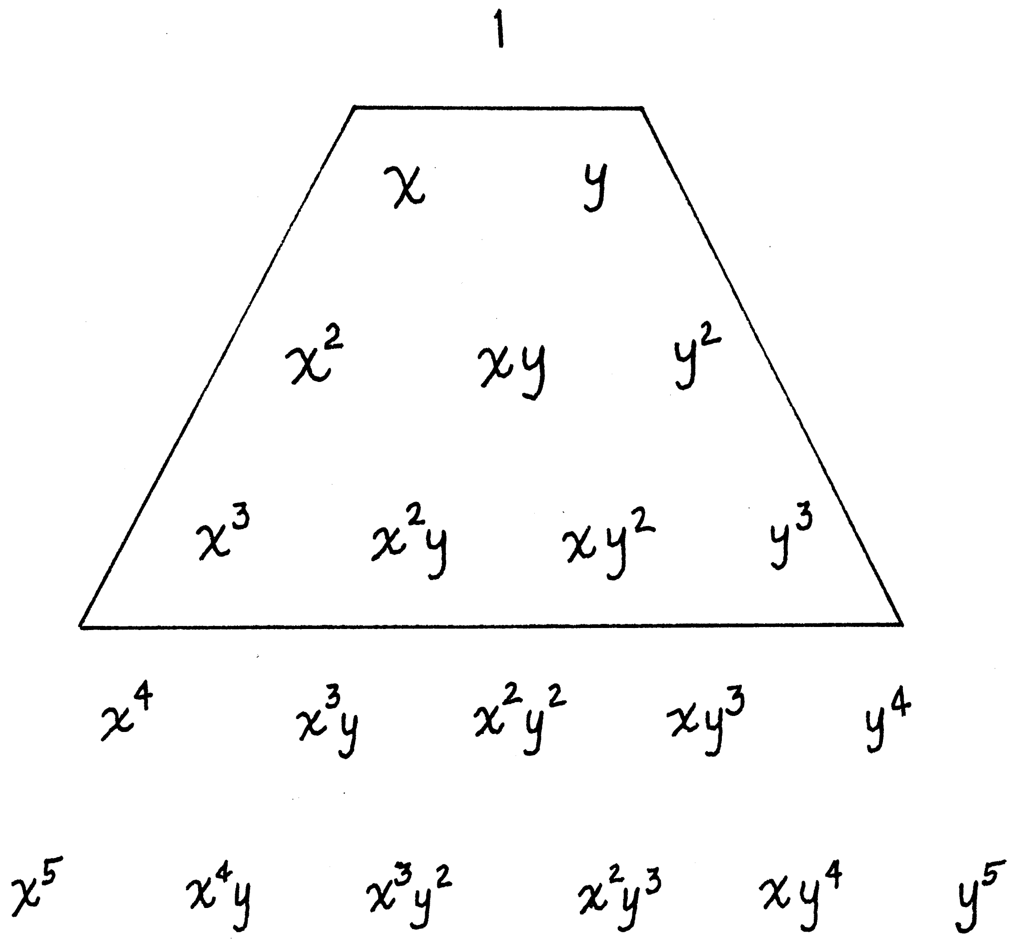

Figure 4 Admissible Approximating Functions $\phi(x_1y)$ for the Displacement Field $U(x_1y)$ when N = 3

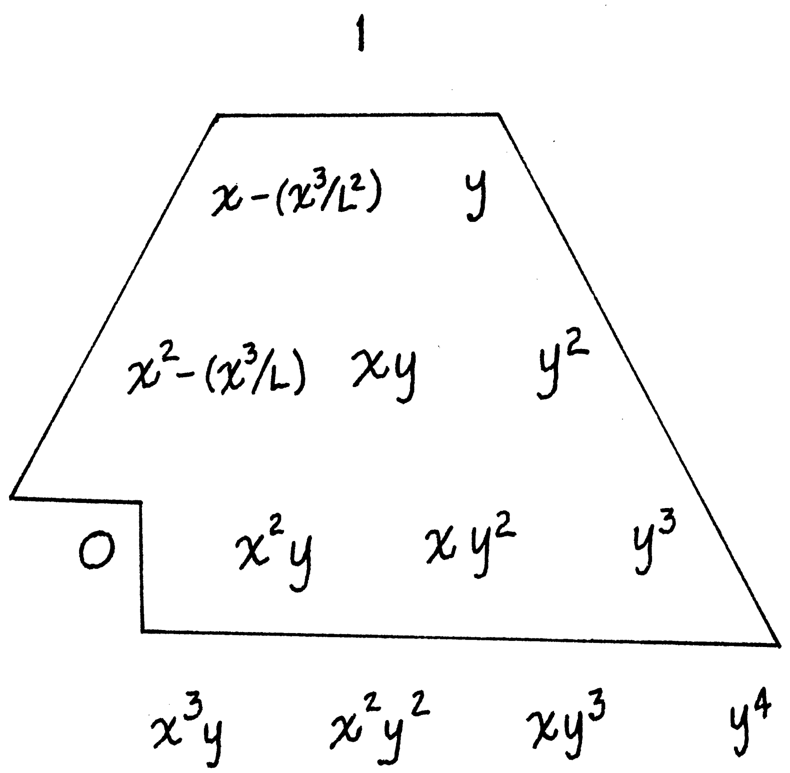

Figure 5 Admissible Approximating Functions $\psi(x_1 y)$ for the Displacement Field $V(x_1 y)$ when N = 3

$$\begin{bmatrix} \underline{K}^{\phi} & \underline{K}^{\phi\Psi} \\ \underline{K}^{\Psi\phi} & \underline{K}^{\Psi} \end{bmatrix} \begin{Bmatrix} \underline{A} \\ \underline{B} \end{Bmatrix} = \begin{Bmatrix} \underline{F}^{\phi} \\ \underline{F}^{\Psi} \end{Bmatrix} \qquad (22)$$

The submatrices, $\underline{K}^{\phi}$, $\underline{K}^{\phi\Psi}$, $\underline{K}^{\Psi\phi}$, $\underline{K}^{\Psi}$, before the integration by parts to obtain (14) are given as:

$$K^{\phi}_{ij} = -(EH/(1-\nu^2)) \int\int_{(A)} ((\partial\phi_j/\partial x)(\partial\phi_i/\partial x)+((1-\nu)/2)$$
$$(\partial\phi_j/\partial y)(\partial\phi_i/\partial y))dA \qquad (23)$$

$$K^{\phi\Psi}_{ij} = -(EH\nu/(1-\nu^2)) \int\int_{(A)} ((\partial\Psi_j/\partial y)(\partial\phi_i/\partial x)+((1-\nu)/(2\nu))$$
$$(\partial\Psi_j/\partial x)(\partial\phi_i/\partial y))dA \qquad (24)$$

$$K^{\Psi\phi}_{ij} = -(EH\nu/(1-\nu^2)) \int\int_{(A)} ((\partial\phi_j/\partial x)(\partial\Psi_i/\partial y)+((1-\nu)/(2\nu))$$
$$(\partial\phi_j/\partial y)(\partial\Psi_i/\partial x))dA \qquad (25)$$

$$K4^{\Psi}_{ij} = -(EH/(1-\nu^2)) \int\int_{(A)} ((\partial\Psi_j/\partial y)(\partial\Psi_i/\partial y)+((1-\nu)/2)$$
$$(\partial\Psi_j/\partial x)(\partial\Psi_i/\partial x))dA \qquad (26)$$

The observation is made that $\underline{K}^{\phi}$ and $\underline{K}^{\Psi}$ ((23) and (26)) are symmetric, since changing the indices i,j does not change the result of K^{ϕ}_{ij} or K^{Ψ}_{ij}. Changing the indices i,j in $K2_{ij}$ yields

$$K^{\phi\Psi}_{ji} = -(EH\nu/(1-\nu^2)) \int\int_{(A)} ((\partial\phi_j/\partial x)(\partial\Psi_i/\partial y)+((1-\nu)/(2\nu))$$
$$(\partial\phi_j/\partial y)(\partial\Psi_i/\partial x))dA \qquad (27)$$

So $\underline{K}_{ji}^{\phi\Psi} = \underline{K}_{ij}^{\Psi\phi}$ and therefore $\underline{K}^{\Psi\phi}$ is equal to $\underline{K}^{\phi\Psi}$ transposed. Since $\underline{K}^{\phi}$ and $\underline{K}^{\Psi}$ are symmetric and $\underline{K}^{\Psi\phi}$ equals $\underline{K}^{\phi\Psi}$ transposed, the matrix (22) is symmetric.

After the integration by parts

$$K_{ij}^{\phi} = (EH/(1-\nu^2))\left[\int_0^L\int_0^B (\partial^2\phi_j/\partial x^2)\phi_i dydx - \int_0^B ((\partial\phi_j/\partial x)\phi_i)\Big|_{x=0}^{x=L} dy\right]$$
$$+ (EH/(2(1+\nu)))\left[\int_0^L\int_0^B (\partial^2\phi_j/\partial y^2)\phi_i dydx - \int_0^L ((\partial\phi_j/\partial y)\phi_i)\Big|_{y=0}^{y=B} dx\right] \quad (28)$$

$$K_{ij}^{\phi\Psi} = (EH/(2(1-\nu)))\int_0^L\int_0^B (\partial^2\Psi_m/\partial x\partial y)\phi_i dydx$$
$$- (EH\nu/(1-\nu^2))\int_0^B ((\partial\Psi_j/\partial y)\phi_i)\Big|_{x=0}^{x=L} dy$$
$$- (EH/(2(1+\nu)))\int_0^L ((\partial\Psi_j/\partial x)\phi_i)\Big|_{y=0}^{y=B} dx \quad (29)$$

$$K_{ij}^{\Psi\phi} = (EH/(2(1-\nu)))\int_0^L\int_0^B (\partial^2\phi_j/\partial x\partial y)\Psi_i dydx$$
$$- (EH\nu/(1-\nu^2))\int_0^L ((\partial\phi_j/\partial x)\Psi_i)\Big|_{y=0}^{y=B} dx$$
$$- (EH/(2(1+\nu)))\int_0^B ((\partial\phi_j/\partial y)\Psi_i)\Big|_{x=0}^{x=L} dy \quad (30)$$

$$K_{ij}^{\Psi} = (EH/(1-\nu^2))\left[\int_0^L\int_0^B (\partial^2\Psi_j/\partial y^2)\Psi_i dydx - \int_0^L ((\partial\Psi_j/\partial y)\Psi_i)\Big|_{y=0}^{y=B} dx\right]$$
$$+ (EH/(2(1+\nu)))\left[\int_0^L\int_0^B (\partial^2\Psi_j/\ x^2)\Psi_i dydx - \int_0^B ((\partial\Psi_j/\partial x)\Psi_i)\Big|_{x=0}^{x=L} dy\right] \quad (31)$$

The integration by parts does not affect the symmetry of the matrix. The righthand side vector of (22) is given by:

$$F_{\phi_i} = (EH\alpha/(1-\nu))\left[\int_0^L\int_0^B (\partial T/\partial x)\phi_i dydx - \int_0^B (T\phi_i)\Big|_{x=0}^{x=L} dy\right]$$

$$- \int_0^B [R\phi_i]\Big|_{x=0}^{x=L} dy + \int_{x_1}^{x_2} [F_t\phi_i]\Big|^{y=B} dx \qquad (32)$$

$$F_{\Psi_i} = (EH\alpha/(1-\nu))\left[\int_0^L\int_0^B (\partial T/\partial y)\Psi_i dydx - \int_0^L (T\Psi_i)\Big|_{y=0}^{y=B} dx\right]$$

$$+ \int_{x_1}^{x_2} [F_n\Psi_i]\Big|^{y=B} dx \qquad (33)$$

The temperature field function, T, chosen relative to some datum point, is selected to be of binomial form since the approximating functions U_k and V_m are of binomial form. Equations (28) to (32) were used to develop the program (19) which calculates the undetermined coefficients $\underline{A}$ and $\underline{B}$.

Equation (22) is readily solved by standard methods (18) to obtain the coefficients $\underline{A}$ and $\underline{B}$, then U_k and V_m are calculated from eqs. (15) and (16). To obtain the stresses from the approximate displacement fields Hooke's Law (13), is utilized,

$$\sigma_x(x,y) = (E/(1-\nu^2))((\partial U_k/\partial x) + \nu(\partial V_m/\partial y)) + \sigma_{ox} \qquad (34)$$

$$\sigma_y(x,y) = (E/(1-\nu^2))((\partial V_m/\partial y) + \nu(\partial U_k/\partial x)) + \sigma_{oy} \qquad (35)$$

and

$$\tau_{xy}(x,y) = G((\partial U_k/\partial y) + (\partial V_m/\partial x)) + \tau_{oxy} \qquad (36)$$

where σ_{ox}, σ_{oy}, and τ_{oxy} are specified blade prestresses which are superimposed on the solution to obtain the total blade stress.

The approximate solution method presented here has been used as the basis for a computer program (19). In the next section results from this program are compared to those from a standard FEM program (MSC/NASTRAN) for a problem typical of band sawing.

COMPUTATIONAL RESULTS

Equations (14) to (21) were used to develop an efficient interactive computer program for calculating band saw blade stresses (19). Table 1 shows some of the program output for a loading case typical of bandsawing. Shown in the table is the computer feedback of the input data, displacements (U) at various points on the blade, and stress (σ_x) at various points on the blade. Also, the computer generates tables of V, σ_y, τ_{xy}, maximum principle normal stress, and temperature, all with the same out-put format. Generated with the above tables are lists of displacement field coefficients for U and V, stress field coefficients for σ_x, σ_y, and τ_{xy}, and temperature field coefficients. Of course the user has the option of specifying which tables and lists are to be displayed.

The simple polynomial approximating functions (see Appendix D) can be expected to lead to a poorly conditioned matrix in Eqn. (22) when the order (N) of the approximating functions becomes large (18). Numerical experiments were performed with the program which showed that conditioning problems become evident for $N \geq 8$. These tests were run for uniform axial tension with $F_n = F_t = \theta$ on an Amdahl 470 computer. The program was written in standard FORTRAN IV using double precision. Note that the N=8 case leads to an 87 by 87 matrix in Eqn. (22). Values of $N \leq 8$, as shown below, led to sufficiently accurate results. Therefore, alternative basis functions for representing the displacement fields were not investigated.

Two FEM models were compared with the current program based on displacements and execution time. The comparison was done for three different loading conditions: 1) normal load (F_n), 2) tangential load (F_t), and 3) normal and tangential load (F_n and F_t). Typical values of the cutting forces that occur during the bandsawing operation (4) were utilized; $F_n = 250N$ and $F_t = 500N$. Note that R = 0.

The standard FEM package used was MSC/NASTRAN. One FEM model consisted of sixty, eight-noded curved shell elements (CQUAD8), while the other FEM model consisted of sixty, four-noded membrane elements (CQDMEM). Each membrane element utilizes four overlapping constant-strain triangle elements. Figure 6 shows the FEM mesh, node numbering, and where the loads were applied. For both the normal and tangential load the two outer nodes were loaded with one-fourth of the given loads, and the middle node was loaded with one-half of the given loads.

Execution time for the FEM analysis (MSC/NASTRAN) was approximately 18.0 central processing unit (CPU) seconds for CQUAD8, and 8.0 CPU seconds for CQDMEM, while the program presented here ran all three problems in approximately 0.8 CPU seconds. Thus, the program is 22 times faster than CQUAD8 and 10 times faster than CQDMEM. This computational advantage can enable interactive or iterative use of the method as in a computer-aided design or design optimization program (19). This relative computational efficiency can also permit implementation of the analysis on a slower mini or micro computer.

The CQUAD8 model always showed a smaller displacement than the program for displacement, U, with a normal load, and for displacement, V, with a tangential load. (see Appendix F). The CQDMEM model always showed a smaller displacement than the program for displacements, U, V, when either a normal load or tangential load was applied (see Appendix F). Table 2 gives displacements for the two FEM models, and for the program with various degrees for the admissible approximating functions, when a combined normal and tangential loading is applied. As can be seen from Table 2 the program usually gives results closer to the CQUAD8, a more accurate finite element, than does the CQDMEM model for displacement, U. Because of the loading condition, the normal load effects cancel the tangential load effects on certain displacements. Thus, small displacements are seen at the edges of the band saw blade model. This cancelling effect plus the smoothing effects of the global polynomial functions accounts for the discrepency in values for displacement, V, along the edges as shown in Table 2. Note that R is typically large, alleviating these small displacement problems. Overall the results, as shown in Table 2, compare favorably with FEM results.

SUMMARY AND CONCLUSIONS

An efficient interactive computer program for calculating plane stresses in a band saw blade has been developed using the principle of minimum potential energy with polynomial approximations of the displacement fields (19). A comparision between this global approach (current program) and a standard finite element program (MSC/NASTRAN) for execution time and in-plane displacements was made for three loading conditions typical of the band sawing operation. Two different finite elements were employed CQDMEM, and the more accurate CQUAD8.

Execution time for the current program was 10 times faster than CQDMEM and 22 times faster than CQUAD8. Because of this efficiency the program can be used interactively, and/or iteratively. The program displacements were usually closer to the CQUAD8 displacements than were the CQDMEM displacements. However, when a normal load with a tangential load was applied a discrepancy was found between the program displacements and CQDMEM, CQUAD8 displacements. The force/moment cancellation, with the smoothing due to the approximation, are the reasons for this discrepancy which occurs along the edges of the plate. Except for these small displacement cases, the program is clearly sufficiently accurate and its computational efficiently makes it attractive for use as a tool in the design of band saw blades.

The method presented here for the stress problem and a previously developed method for blade vibrations (10) can be combined to form a computationally efficient package for design and process optimization in bandsawing. Further extensions to these studies can include active process control, computation of blade temperature gradients, and computation of blade prestresses due to roll tensioning or thermal tensioning.

Table 1 (a) Typical Program Output - Input Feedback

```
                       NORMAL AND TANGENTIAL LOAD DEGREE EQUAL SIX
THE DEGREE OF THE POLYNOMIAL FOR THE STRESS CALCULATION IS 6
THE DEGREE OF THE POLYNOMIAL FOR THE TEMPERATURE FIELD CALCULATION IS 0
THE LENGTH OF THE BANDSAW BLADE IS   0.100000E+01 [m]
THE WIDTH OF THE BANDSAW BLADE IS   0.250000E+00  [m]
THE THICKNESS OF THE BANDSAW BLADE IS   0.107000E-02   [m]
THE MODULUS OF ELASTICITY IS   0.207000E+12 [Pa]
POISON"S RATIO IS   0.300000E+00
THE MASS DENSITY IS   0.834600E+01 [Kg/m2]
THE VELOCITY OF THE BANDSAW BLADE IS   0.0
THE WHEEL SUPPORT COEFFICIENT IS   0.0
THE INITIAL TENSION IS   0.0
THE NORMAL CUTTING FORCE IS   0.250000E+03 [N]
THE TANGENTIAL CUTTING FORCE IS   0.500000E+03 [N]
THE STARTING X-COORDINATE FOR THE NORMAL CUTTING IS   0.487000E+00 [m]
THE ENDING X-COORDINATE FOR THE NORMAL CUTTING IS   0.513000E+00  [m]
THE STARTING X-COORDINATE FOR THE TANGENTIAL CUTTING FORCE IS   0.487000E+00 [m]
THE ENDING X-COORDINATE FOR THE TANGENTIAL CUTTING FORCE IS   0.513000E+00   [m]
```

Table 1 (b) Typical Program Output - Displacement Field $U(x_1 y)$

NORMAL AND TANGENTIAL LOAD DEGREE EQUAL SIX

DISPLACEMENT FIELD U [m]

Y COORDINATE											
0.250E+00	-.484E-05	-.434E-05	-.473E-05	-.603E-05	-.779E-05	-.944E-05	-.106E-04	-.110E-04	-.107E-04	-.102E-04	-.968E-05
0.200E+00	-.514E-05	-.493E-05	-.539E-05	-.647E-05	-.780E-05	-.894E-05	-.960E-05	-.976E-05	-.961E-05	-.952E-05	-.974E-05
0.150E+00	-.538E-05	-.534E-05	-.580E-05	-.669E-05	-.771E-05	-.850E-05	-.889E-05	-.892E-05	-.882E-05	-.886E-05	-.910E-05
0.100E+00	-.506E-05	-.555E-05	-.616E-05	-.692E-05	-.764E-05	-.811E-05	-.827E-05	-.822E-05	-.817E-05	-.825E-05	-.823E-05
0.500E-01	-.356E-05	-.546E-05	-.667E-05	-.745E-05	-.783E-05	-.786E-05	-.767E-05	-.746E-05	-.744E-05	-.761E-05	-.743E-05
0.0	0.0	-.485E-05	-.747E-05	-.855E-05	-.856E-05	-.793E-05	-.709E-05	-.651E-05	-.646E-05	-.687E-05	-.690E-05
X-COORDINATE	0.0	0.100E+00	0.200E+00	0.300E+00	0.400E+00	0.500E+00	0.600E+00	0.700E+00	0.800E+00	0.900E+00	0.100E+01

Table 1 (c) Typical Program Output - Stress Field $\sigma_x(x_1y)$

NORMAL AND TANGENTIAL LOAD DEGREE EQUAL SIX

SIGMA X [Pa]

Y COORDINATE											
0.250E+00	0.163E+07	0.234E+06	-.189E+07	-.343E+07	-.385E+07	-.317E+07	-.178E+07	-.251E+06	0.872E+06	0.125E+07	0.946E+06
0.200E+00	0.955E+06	-.202E+06	-.180E+07	-.281E+07	-.286E+07	-.209E+07	-.943E+06	0.405E+05	0.407E+06	-.169E+05	-.929E+06
0.150E+00	0.397E+06	-.471E+06	-.164E+07	-.227E+07	-.212E+07	-.138E+07	-.469E+06	0.157E+06	0.203E+06	-.274E+06	-.670E+06
0.100E+00	-.128E+07	-.125E+07	-.164E+07	-.174E+07	-.137E+07	-.708E+06	-.933E+05	0.175E+06	0.257E+05	-.192E+06	0.490E+06
0.500E-01	-.552E+07	-.337E+07	-.216E+07	-.123E+07	-.390E+06	0.245E+06	0.488E+06	0.256E+06	-.236E+06	-.278E+06	0.152E+07
0.0	-.140E+08	-.789E+07	-.372E+07	-.847E+06	0.972E+06	0.178E+07	0.159E+07	0.613E+06	-.594E+06	-.899E+06	0.159E+07
X-COORDINATE	0.0	0.100E+00	0.200E+00	0.300E+00	0.400E+00	0.500E+00	0.600E+00	0.700E+00	0.800E+00	0.900E+00	0.100E+01

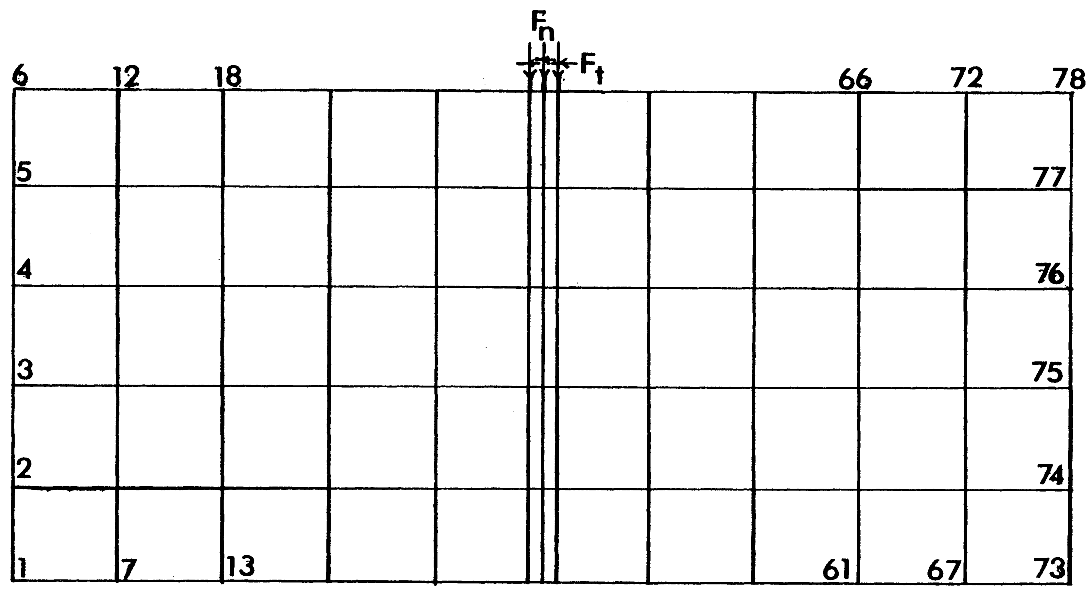

Figure 6 Finite Element Mesh and Loading for the Band Saw Plane Stress Problem

Table 2 Displacement Fields U and V for Combined Normal and Tangential Loading - A Comparison of the Method Presented Here With a Standard FEM Code.

Plate Coordinates		Displacement Field U [μm]: Results from Method Presented Here with Approx. Functions of Order N			Displacement Field U [μm]: MSC/NASTRAN Results		Displacement Field V [μm]: Results from Method Presented Here with Approx. Functions of Order N			Displacement Field V [μm]: MSC/NASTRAN Results	
X	Y	N=5	N=6	N=7	CQDMEM	CQUAD8	N=5	N=6	N=7	CQDMEM	CQUAD8
0.0	0.05	-2.49	-3.56	-4.88	-2.72	-4.46	0.0746	0.101	0.132	-0.310	-0.442
	0.15	-4.55	-5.38	-6.49	-4.03	-5.53	-0.0828	0.113	0.247	-0.453	-0.384
	0.25	-3.94	-4.84	-5.96	-3.68	-5.05	-0.257	0.181	0.716	-0.394	-0.269
0.20	0.05	-5.92	-6.67	-7.76	-5.31	-6.86	-2.66	-2.18	-2.13	-2.56	-2.74
	0.15	-5.05	-5.80	-6.93	-4.83	-6.12	-2.61	-2.19	-2.04	-2.42	-2.46
	0.25	-4.19	-4.73	-5.41	-3.89	-5.01	-2.50	-2.11	-1.87	-2.27	-2.34
0.40	0.05	-6.88	-7.83	-8.91	-6.60	-8.05	-6.04	-6.50	-6.60	-6.07	-6.65
	0.15	-6.76	-7.71	-8.92	-6.63	-8.06	-6.03	-6.44	-6.58	-6.09	-6.77
	0.25	-7.32	-7.79	-9.10	-6.61	-7.63	-5.95	-6.28	-6.43	-5.84	-6.40
0.60	0.05	-6.79	-7.67	-8.72	-6.49	-7.80	-5.93	-6.41	-5.72	-5.26	-5.71
	0.15	-7.97	-8.89	-9.84	-7.59	-8.98	-5.95	-6.47	-5.82	-5.18	-5.58
	0.25	-9.48	-10.6	-12.1	-9.28	-10.7	-5.95	-6.54	-5.98	-5.12	-5.32
0.80	0.05	-6.62	-7.44	-8.61	-6.46	-7.77	-3.12	-2.99	-2.63	-2.60	-2.76
	0.15	-8.18	-8.82	-9.96	-7.75	-9.18	-3.11	-2.92	-2.50	-2.58	-2.79
	0.25	-9.69	-10.7	-11.4	-9.05	-10.6	-3.12	-2.94	-2.38	-2.57	-2.79
1.00	0.05	-6.72	-7.43	-8.64	-6.46	-7.77	-0.0127	-0.091	0.101	0.0	0.0
	0.15	-8.00	-9.10	-9.96	-7.76	-9.17	0.00841	-0.358	0.154	0.0	0.0
	0.25	-9.79	-9.68	-11.9	-9.04	-10.6	0.00751	-0.404	0.100	0.0	0.0

ACKNOWLEDGEMENTS

The authors are pleased to acknowledge the financial support of the Center for Robotics and Integrated Manufacturing at The University of Michigan, and the Society of Manufacturing Engineers Educational Foundation. They are also greatful to Carol Bovan and Sheryll Marshall for assistance with the preparation of the manuscript.

REFERENCES

1. Ulsoy, A.G., Mote, C.D., Jr. and Szymani, R., "Principal Developments in Band Saw Vibration and Stability Research," Holtz als Rohund Werkstoff, Vol. 36, 1978, pp. 273-280.

2. Porter, A.W., "Some Engineering Considerations of High Strain Band Saws," Forest Products Journal, Vol. 21, No. 4, April 1971, pp. 24-32.

3. Ulsoy, A.G., Vibration and Stability of Band Saw Blades: A Theoretical and Experimental Study, Ph.D. Dissertation, University of California, Berkeley, 1979.

4. Ulsoy, A.G., and C.D. Mote, Jr., "Analysis of Bandsaw Vibration," Wood Science, Vol. 13, No. 1, July 1980, pp. 1-10.

5. Thunell, B., "The Stresses in a Band Saw Blade," Paperija Puu, Vol. 54, No. 11, November 1972, pp. 759-764.

6. Kirbach, E. and Bonac, T., "The Effect of Tensioning and Wheel Tilting on the Torsional and Lateral Fundamental Frequencies of Band Saw Blades," Wood and Fiber, Vol. 9, No. 4, April 1978, pp. 245-251.

7. Ulsoy, A.G., and Mote, C-D., Jr., "Vibration of Wide Band Saw Blades," Journal of Engineering for Industry, Vol. 104, No. 1, February 1982, pp. 71-78.

8. Mote, C.D., Jr., "Dynamic Stability of Axially Moving Materials," Shock and Vibration Digest, Vol. 4, No. 4, April 1972, pp. 2-11.

9. Mote, C.D., Jr., "A Study of Band Saw Vibrations," Journal of the Franklin Institute, Vol. 279, No. 6, June 1965, pp. 430-444.

10. Ulsoy, A.G., Two Computer Codes for Band Saw Vibration and Stability Analysis, University of California, Berkeley, 1978.

11. Washizu, K., Variational Methods in Elasticity and Plasticity, Pergamon Press, 1975.

12. Hermann, G., and Armenakas, A.E., "Vibration and Stability of Plates Under Initial Stress," Proceedings of the ASCE, Journal of the Engineering Mechanics Division, June 1960, pp. 65-93.

13. Lanczos, C., The Variational Principles of Mechanics, University of Toronto Press, Toronto, 1977.

14. Huebner, K.H., The Finite Element Method for Engineers, John Wiley and Sons, New York, 1975.

15. Timoshenko, S.P., and Goodier, J.N., Theory of Elasticity, 3rd Ed., McGraw-Hill Inc., New York, 1970.

16. Boley, B.A., and Weiner, J.H., Theory of Thermal Stresses, John Wiley and Sons, Inc., New York, 1960.

17. Zienkiewicz, O.C., The Finite Element Method, McGraw-Hill Inc., New York, 1977.

18. Bathe, K.J., and E.L. Wilson, Numerical Methods in Finite Element Analysis, Prentice-Hall, New Jersey, 1976.

19. Borchelt, J.E., Ulsoy, A.G., and Papalambros, P., A Computational Package for Design and Process Evaluation in Bandsawing - A User's Manual, Report No. UM-MEAM-83-14, The University of Michigan, Ann Arbor, June 1983.

20. Ugural, A.C., and Fenster, S.K., Advanced Strength and Applied Elasticity, Elsevier, New York, 1981, Chs. 2, 3.

LIST OF SYMBOLS

B:	Width of band saw blade
c:	Velocity of band saw blade
E:	Modulus of Elasticity
F_n:	Normal cutting force component
F_t:	Tangential cutting force component
G:	Modulus of Rigidity
H:	Thickness of band saw blade
k:	Number of terms in admissible approximating function for U; ((N+1)(N+2)/2)-1
L:	Length of band saw blade between guides
m:	Number of terms in admissible approximating function for Y; ((N+1)(N+2)/2)-2
N:	Highest order of polynomial approximating functions for U, V
R:	Axial tension force
R_o:	Axial tension force when c = 0
T(x,y):	Specified temperature field relative to some datum point
u:	Unit step function
U(x,y), V(x,y):	Displacement field functions
x_1, x_2:	Starting and ending point for normal and tangential cutting forces
α:	Coefficient of thermal expansion
Δx:	$\Delta x = x_2 - x_1$, length over which cutting forces act
δ:	Variation
κ:	Wheel support coefficient
ν:	Poisson's Ratio
$\phi_i(x,y)$:	Admissible approximating function for U;
$\Psi_i(x,y)$:	Admissible approximating function for V;
ρ:	Mass density
$\sigma_x(x,y)$:	Normal stress in x-direction
$\sigma_y(x,y)$:	Normal stress in y-direction
$\tau_{xy}(x,y)$:	Shear Stress
∇^2:	Laplacian $\left(\frac{\partial^2(\ )}{\partial x^2} + \frac{\partial^2(\ )}{\partial x^2}\right)$
π:	Total potential energy
A_i, B_i:	Undertermined coefficients of the admissible approximating functions for U and V

APPENDIX A

The Total Potential Energy Expression

Strain Energy

Strain-energy for a plane stress problem is given by (11, 15, 21)

$$U = (H/2E) \iint_{(A)} \{\sigma_x^2+\sigma_y^2-2\nu\sigma_x\sigma_y+2(1+\nu)\tau_{xy}^2\}dA \qquad \text{(A-1)}$$

However, for the approach used in this problem the strain-energy should be expressed in terms of the displacement field functions U and V, and the temperature distribution, T.

Stresses in Terms of U, V and T

The relationship between stresses and displacements are (15, 20)

$$\sigma_x = (E/(1-\nu^2))((\partial U/\partial x)+\nu(\partial V/\partial y)) - (E\alpha/(1-\nu))T \qquad \text{(A-2)}$$

$$\sigma_y = (E/(1-\nu^2))(\nu(\partial U/\partial x)+(\partial V/\partial y))-(E\alpha/(1-\nu))T \qquad \text{(A-3)}$$

$$\tau_{xy} = G((\partial U/\partial y) + (\partial V/\partial x)) \qquad \text{(A-4)}$$

Substituting (A-2) to (A-4) into (A-1) yields

$$U = (H/2E) \iint_{(A)} \{(E^2/(1-\nu^2))[(\partial U/\partial x)^2+2\nu(\partial U/\partial x)(\partial V/\partial y) + (\partial V/\partial y)^2]-(2E^2\alpha/(1-\nu))[(\partial U/\partial x)+(\partial V/\partial y)]T + (2E^2\alpha^2/(1-\nu))T^2+2(1+\nu)G^2[(\partial U/\partial y)+(\partial V/\partial x)]^2\}\, dA \qquad \text{(A-5)}$$

Adding zero by adding and subtracting $2(\partial U/\partial x)(\partial V/\partial y)$ and substituting $E = 2G(1+\nu)$, then cancelling E/E yields

$$U = (H/2) \iint_{(A)} \{(E/(1-\nu^2))[(\partial U/\partial x)+(\partial V/\partial y)]^2$$

$$+ G[((\partial U/\partial y)+(\partial V/\partial x))^2-4(\partial U/\partial x)(\partial V/\partial y)]\} \, dA$$

$$- H \iint_{(A)} \{(E\alpha/(1-\nu))[(\partial U/\partial x)+(\partial V/\partial y)]T\} \, dA$$

$$+ H \iint_{(A)} \{(E\alpha^2/(1-\nu))T^2\} \, dA \qquad \text{(A-6)}$$

Equation (A-6) is the strain-energy in terms of the displacement fields U and V, and the known temperature distribution T.

Work Done By External Loads

The equation for the work done by the external loads is

$$W = \int_0^B [RU]_{x=0}^{x=L} dy - \int_{x_1}^{x_2} [F_n V]^{y=B} dx - \int_{x_1}^{x_2} [F_t U]^{y=B} dx \qquad \text{(A-7)}$$

Total Potential Energy Equation

The total potential energy of the plate is a linear combination of the strain-energy, which includes thermal energy, and work done by the external loads.

$$\pi = U - W \qquad \text{(A-8)}$$

Substituting for U and W yields

$$\pi = (H/2) \iint_{(A)} \{(E/(1-\nu^2))[(\partial U/\partial x)+(\partial V/\partial y)]^2$$

$$+ G[((\partial U/\partial y)+(\partial V/\partial x))^2-4(\partial U/\partial x)(\partial V/\partial y)]\} \, dA$$

$$+ H \iint_{(A)} \{(E\alpha/(1-\nu))[-((\partial U/\partial x)+(\partial V/\partial y))+\alpha T]T\} \, dA$$

$$- \int_0^B [RU]_{x=0}^{x=L} dy + \int_{x_1}^{x_2} [F_n V]^{y=B} dx + \int_{x_1}^{x_2} [F_t U]^{y=B} dx \qquad \text{(A-9)}$$

Equation (A-9) is the potential energy equation used in the derivation of the variational equation.

APPENDIX B

The Governing Differential Equations

Facts from the Theory of Elasticity

The governing differential equations are developed by substituting Hooke's Law for stresses into the differential equations of equilibrium. Then the relations between strains and displacements are used to obtain the governing differential equations in terms of U and V.

Differential Equations of Equilibrium

In a plane stress problem, the differential equations of equilibrium must be satisfied for the plate to be in static equilibrium (15, 20). The differential equations of equilibrium with a temperature gradient are

$$(\partial\sigma_x/\partial x) + (\partial\tau_{xy}/\partial y) - (E\alpha/(1-\nu))(\partial T/\partial x) = 0 \qquad \text{(B-1)}$$

$$(\partial\sigma_y/\partial y) + (\partial\tau_{xy}/\partial x) - (E\alpha/(1-\nu))(\partial T/\partial y) = 0 \qquad \text{(B-2)}$$

The stresses, σ_x, σy, and τ_{xy}, are related to strains by the generalized Hooke's Law.

Generalized Hooke's Law

In a plane stress problem, Hooke's Law gives the following relations between stresses and displacements (15, 20).

$$\sigma_x = (E/(1-\nu^2))((\partial U/\partial x) + \nu(\partial V/\partial y)) \qquad \text{(B-3)}$$

$$\sigma_y = (E/(1-\nu^2))(\nu(\partial U/\partial x) + (\partial V/\partial y)) \qquad \text{(B-4)}$$

$$\tau_{xy} = G((\partial U/\partial y) + (\partial V/\partial x)) \qquad \text{(B-5)}$$

Governing Differential Equations

The governing differential equations are found by substituting (B-3) to (B-5) into (B-1) and (B-2), yields

$$(\partial((E/(1-\nu^2))((\partial U/\partial x) + \nu(\partial V/\partial y)))/\partial x)$$
$$+ (\partial(G((\partial U/\partial y) + (\partial V/\partial x)))/\partial y) - (E\alpha/(1-\nu))(\partial T/\partial x) = 0 \quad \text{(B-6)}$$

$$(\partial((E/(1-\nu^2))(\nu(\partial U/\partial x) + (\partial V/\partial y)))/\partial y)$$
$$+ (\partial(G(\partial U/\partial y) + (\partial V/\partial x)))/\partial x) - (E\alpha/(1-\nu))(\partial T/\partial y) = 0 \quad \text{(B-7)}$$

Carrying out the differentiation and factoring out $E/(1-\nu^2)$ and G, then substituting $E = 2G(1+\nu)$ yields

$$(2G/(1-\nu))((\partial^2U/\partial x^2) + \nu(\partial^2V/\partial x\partial y)))$$
$$+ G((\partial^2U/\partial y^2) + (\partial^2V/(\partial x\partial y))) - (E\alpha/(1-\nu))(\partial T/\partial x) = 0 \quad \text{(B-8)}$$

$$(2G/(1-\nu))(\nu(\partial^2U/\partial x\partial y)) + (\partial^2V/\partial y^2))$$
$$+ G((\partial^2U/(\partial x\partial y) + (\partial^2V/\partial x^2)) - (E\alpha/(1-\nu))(\partial T/\partial y) = 0 \quad \text{(B-9)}$$

Factoring out G and rearranging terms and substituting $2/(1-\nu) = 1 + (1+\nu)/(1-\nu)$ and $2\nu/(1-\nu) + 1 = (1+\nu)/(1-\nu)$ yields

$$G\{(1+(1+\nu)/(1-\nu))(\partial^2U/\partial x^2) + (\partial^2U/\partial y^2) +((1+\nu)/(1-\nu))$$
$$(\partial^2V/(\partial x\partial y))\} - (E\alpha/(1-\nu))(\partial T/\partial x) = 0 \quad \text{(B-10)}$$

$$G\{((1+\nu)/(1-\nu))(\partial^2U/\partial x\partial y)) + (1+(1+\nu)/(1-\nu))(\partial^2V/\partial y^2)$$
$$+ (\partial^2V/\partial x^2) - (E\alpha/(1-\nu))(\partial T/\partial y) = 0 \quad \text{(B-11)}$$

Rearranging terms so those multiplied by the Laplacian, $\nabla^2 = \partial(\ \ )/\partial x^2+\partial(\ \ )/\partial y^2$, and those multiplied by $(1+\nu)/(1-\nu)$ are grouped together, then factoring out the common differentiation from the terms multiplied by $(1+\nu)/(1-\nu)$ yields

$$G\{\nabla^2U+((1+\nu)/(1-\nu))(\partial((\partial U/\partial x)+(\partial V/\partial y))/\partial x)$$
$$- (E\alpha/(1-\nu))(\partial T/\partial x) = 0 \quad \text{(B-12)}$$

$$G\{\nabla^2 V+((1+\nu)/(1-\nu))(\partial((\partial U/\partial x)+(\partial V/\partial y))/\partial y)\}$$
$$-\ (E\alpha/(1-\nu))(\partial T/\partial y) \ = \ 0 \qquad \text{(B-13)}$$

Equations (B-12) and (B-13) are the governing differential equations for a plane stress problem with a known termperature distribution.

APPENDIX C

The Variational Equation

Variation of U and V

The variational equation is derived using algebraic expressions for U and V substituted into the total potential energy equation.

$$U = U + \delta U$$
$$V = V + \delta V \qquad \text{(C-1)}$$

Variational Equation Before Integration by Parts

Substituting (C-1) and (C-2) and their derivatives into the total potential energy equation. Then eliminating the higher orders of the variation, $(\partial\delta U/\partial x)^2$, $(\partial\delta U/\partial x)(\partial\delta V/\partial y)$, $(\partial\delta V/\partial y)^2$, $(\partial\delta U/\partial y)^2$, and $(\partial\delta V/\partial x)^2$, since the variations are small. The resulting equation is

$$\delta\pi = H\iint_{(A)} \{(E/(1-\nu^2))[((\partial U/\partial x)+(\partial V/\partial y))(\partial\delta U/\partial x)$$
$$+((\partial U/\partial x)+(\partial V/\partial y))(\partial\delta V/\partial x)]+G[((\partial U/\partial y)+(\partial V/\partial x))(\partial\delta U/\partial y)$$
$$+((\partial U/\partial y)+(\partial V/\partial x))(\partial\delta V/\partial x)-2((\partial U/\partial x)(\partial\delta V/\partial y)+$$
$$(\partial V/\partial y)(\partial\delta U/\partial x))]\}\, dA - H\iint_{(A)}\{(E\alpha/(1-\nu))((\partial\delta U/\partial x)+$$
$$(\partial\delta V/\partial y))T\}\, dA - \int_0^B [R\delta U]_{x=0}^{x=L} dy + \int_{x_1}^{x_2} [F_n\delta V]^{y=B} dx$$
$$+\int_{x_1}^{x_2} [F_t\delta U]^{y=B} dx \qquad \text{(C-2)}$$

Rearranging terms, factoring out $(\partial\delta U/\partial x)$, $(\partial\delta U/\partial y)$, $(\partial\delta V/\partial x)$, and $(\partial\delta V/\partial y)$, and substituting $G = (E/(2(1+\nu)))$ yields

$$\delta\pi = H \iint_{(A)} \{(E/(1-\nu^2))[((\partial U/\partial x)+\nu(\partial V/\partial y))(\partial\delta U/\partial x)$$
$$+ (\nu(\partial U/\partial x)+(\partial V/\partial y))(\partial\delta V/\partial y)]+G[((\partial U/\partial y)+(\partial V/\partial x))(\partial\delta U/\partial y)$$
$$+ ((\partial U/\partial y)+(\partial V/\partial x))(\partial\delta V/\partial x)]\} \, dA$$
$$- (EH\alpha/(1-\nu)) \iint_{(A)} [((\partial\delta U/\partial x) + (\partial\delta V/\partial y))T] \, dA$$
$$- \int_0^B [R\delta U]_{x=0}^{x=L} dy + \int_{x_1}^{x_2} [F_n\delta V]^{y=B} dx + \int_{x_1}^{x_2} [F_t\delta U]^{y=B} dx \qquad \text{(C-3)}$$

Variational Equation

Integrating by parts to eliminate the derivatives of the variations yields

$$\delta\pi = (EH/(1-\nu^2)) \{\int_0^B [((\partial U/\partial x)+\nu(\partial V/\partial y))\delta U]_{x=0}^{x=L} dy$$
$$- \iint_{(A)} ((\partial^2 U/\partial x^2)+\nu(\partial^2 V/(\partial x\partial y)))\delta U \, dA\}$$
$$+ (EH/(1-\nu^2))\{\int_0^L [(\nu(\partial U/\partial x) + (\partial V/\partial y))\delta V]_{y=0}^{y=B} dx$$
$$- \iint_{(A)} (\nu(\partial^2 U/(\partial x\partial y)) + (\partial^2 V/\partial y^2))\delta V \, dA$$
$$+ GH \{ \int_0^L [((\partial U/\partial y) + (\partial V/\partial x))\delta U]_{y=0}^{y=B} dx - \iint_{(A)} ((\partial^2 U/\partial y^2)$$
$$+ (\partial^2 V/(\partial x\partial y))\delta U \, dA\} + GH\{\int_0^B [((\partial U/\partial y) + (\partial V/\partial x))\delta V]_{x=0}^{x=L} dy$$
$$- \iint_{(A)} ((\partial^2 U/(\partial x\partial y)) + (\partial^2 V/\partial x^2))\delta V \, dA\}$$
$$-(EH\alpha/(1-\nu))\{\int_0^B [T\delta U]_{x=0}^{x=1} dy - \iint_{(A)} (\partial T/\partial x)\delta U \, dA \}$$
$$-(EH\alpha/(1-\nu))\{\int_0^L [T\delta V]_{y=0}^{x=B} dx - \iint_{(A)} (\partial T/\partial y)\delta V \, dA \}$$
$$- \int_0^B [R\delta U]_{x=0}^{x=L} dy + \int_{x_1}^{x_2} [F_n\delta V]^{y=B} dx + \int_{x_1}^{x_2} [F_t\delta U]^{y=B} dx \qquad \text{(C-4)}$$

Substituting $(E/(1-\nu^2)) = G(1+((1+\nu)/(1-\nu)))$, rearranging terms, and substituting $\nabla^2 = \partial^2(\ \ )/\partial x^2 + \partial^2(\ \ )/\partial y^2$ yields

$$\delta\pi = -(EH/1-\nu^2)) \int_0^B [((\partial U/\partial x) + \nu(\partial V/\partial y))\delta V]_{x=0}^{x=L} dy$$

$$-(EH/(1-\nu^2)) \int_0^L [(\nu(\partial U/\partial x)+(\partial V/\partial y))\delta V]_{y=0}^{y=B} dx$$

$$-GH \int_0^L [((\partial U/\partial y)+(\partial V/\partial x))\delta U]_{y=0}^{y=B} dx - GH \int_0^B [((\partial U/\partial y)+(\partial V/\partial x))\delta V]_{x=0}^{x=L} dy$$

$$+ \int_0^B [R\delta U]_{x=0}^{x=L} dy - \int_{x_1}^{x_2} [F_n \delta V]^{y=B} dx - \int_{x_1}^{x_2} [F_t \delta U]^{y+B} dx$$

$$+ H \iint_{(A)} \{G[\nabla^2 U+((1+\nu)/(1-\nu))(\partial((\partial U/\partial x)+(\partial V/\partial y))/\partial x)]$$

$$- (E\alpha/(1-\nu))(\partial T/\partial x)\delta\}U\, dA + (EH\alpha/(1-\nu)) \int_0^B [T\delta U]_{x=0}^{x=L} dy$$

$$+ H \iint_{(A)} \{G[\nabla^2 V+((1+\nu)/(1-\nu))(\partial((\partial U/\partial x)+(\partial V/\partial y))/\partial y)]$$

$$- (E\alpha/(1-\nu))(\partial T/\partial y)\}\delta V dA + (EH\alpha/(1-\nu)) \int_0^L [T\delta V]_{y=0}^{y=B} dx$$

(C-5)

Since the plate remains in equilibrium, the variation of the total potential energy, $\delta\pi$, must equal zero. The admissible approximating functions for U and V are substituted into (C-5) to form a system of linear equations.

APPENDIX D

Functional Form of the Approximating Functions

Satisfaction of Equilibrium Equations

The differential equations given in Appendix A((1A) and 2A)) are not satisfied by the assumed stress functions used in the vibration program (10). The functional form used is given by (10)

$$\text{Stress} = \sum_{m=0}^{N} \sum_{n=0}^{N} S_{n+m} x^n y^m \qquad \text{(D-1)}$$

The above statement will be verified for the case when N=3, and no temperature field present.

Let

$$\sigma_x = n_o+n_1x+n_2x^2+n_3x^3+n_4xy+n_5x^2y+n_6x^3y+n_7xy^2$$
$$+ n_8x^2y^2+n_9x^3y^2+n_{10}xy^3+n_{11}x^2y^3+n_{12}x^3y^3+n_{13}y$$
$$+ n_{14}y^2+n_{15}y^3 \qquad \text{(D-2)}$$

$$y = \mu_o+\mu_1x+\mu 2x^2+\mu_3x^3+\mu_4xy+\mu_5x^2y+\mu_6x^3y+\mu_7xy^2+\mu_8x^2y^2$$
$$+ \mu_9x^3y^2+\mu_{10}xy^3+\mu_{11}x^2y^3+\mu_{12}x^3y^3+\mu_{13}y+\mu_{14}y^2+\mu_{15}y^3$$
$$\text{(D-3)}$$

$$\tau_{xy} = \tau_o+\tau_1x+\tau_2x^2+\tau_3x^3+\tau_4xy+\tau_5x^2y+\tau_6x^3y+\tau_7xy^2$$
$$+ \tau_8x^2y^2+\tau_9x^3y^2+\tau_{10}xy^3+\tau_{11}x^2y^3+\tau_{12}x^3y^3+\tau_{13}y+\tau_{14}y^2+\tau_{15}y^3$$
$$\text{(D-4)}$$

Substituting (D-2) to (D-4) into the differential equations of equilibrium and gathering like terms together yields

$$(n_1+\tau_{13})+(2n+\tau_4)x+(3n_3+\tau_t)x^2+(2n_5+2\tau_7)xy$$
$$+ (3n_0+2\tau_8)x^2y+(2n_8+3\tau_{10})xy^2+(3n_9+3\tau_{11})x^2y^2$$
$$+ (n_4+2\tau_{14})y+(n_7+3\tau_{15})y^2+2n_{11}xy^3+3n_{12}x^2y^3+n_{10}y^3$$
$$+ \tau_0x^3+2\tau_9x^3y+3\tau_{12}x^3y^2 = 0 \qquad \text{(D-5)}$$

and

$$(\tau_1+\mu_{13})+(2\tau_2+\mu_4)x+(3\tau_3+\mu_5)x^2+(2\tau_t+2\mu_7)xy$$
$$+ (3\tau_6+2\mu_8)x^2y+(2\tau_8+3\mu_{10})xy^2+(3\tau_9+3\mu_{11})x^2y^2$$
$$+ (\tau_4+2\mu_{14})y+(\tau_7+3\mu_{15})y^2+2\tau_{11}xy^3+3\tau_{12}x^2y^3$$
$$+ \tau_{10}y^3+\mu_0x^3+2\mu_9x^3y+3\mu_{12}x^3y^2 = 0 \qquad \text{(D-6)}$$

But the terms in (D-5) and (D-6) must add up to zero. So n_{10}, n_{11}, n_{12}, τ_6, τ_9, τ_{10}, τ_{11}, τ_{12}, μ_6, μ_9, and μ_{12} must equal zero, since they are constant coefficients. However, τ_9, τ_{11}, τ_{10} and τ_6 equal zero, thus μ_8, μ_{11}, n_8, and n_9 must be set equal to zero. So the final functional forms of σ_x, σ_y, and τ_{xy} are

$$\sigma_x = n_0+n_1x+n_2x^2+n_3x^3+n_4xy+N_5x^2y+N_6x^3y+n_7xy^2$$
$$+ n_{13}y+N_{14}y^2+n_{15}y^3 \qquad \text{(D-7)}$$

$$\sigma_y = \mu_0+\mu_1x+\mu_2x^2+\mu_3x^3+\mu_4xy+\mu_5x^2+y\mu_7xy^2$$
$$+ \mu_{10}xy^3+\mu_{13}y+\mu_{14}y^2+\mu_{15}y^3 \qquad \text{(D-8)}$$

$$\tau_{xy} = \tau_0+\tau_1x+\tau_2x^2+\tau_3x^3+\tau_4xy+\tau_5x^2y+\tau_7xy^2$$
$$+ \tau_8x^2y^2+\tau_{13}y+\tau_{14}y^2+\tau_{15}y^3 \qquad \text{(D-9)}$$

Determination of U and V

Using Hooke's Law, the functional forms of U and V can be determine.

To satisfy the equation for σ_x (D-7)

$$U_1 = A_0+A_1x+A_2x^2+A_3x^3+A_4x^4+A_5xy+A_6x^2y+A_7x^3y$$
$$+ A_8xy^2+A_9x^2y^2+A_{10}xy^3+A_{11}x^4y+f(y) \qquad \text{(D-10)}$$

and

$$V_1 = g(x)+B_5xy+B_6x^2y+B_7x^3y+B_8xy^2+B_9x^2y^2$$
$$+ B_{10}xy^3+B_{11}y+B_{12}y^2+B_{13}y^3+B_{14}y^4+B_{15}x^3y^2 \qquad \text{(D-11)}$$

To satisfy the equation for σ_j (D-8)

$$U_2 = A_0+A_1x+A_2x^2+A_3x^3+A_4x^4+A_5xy+A_6x^2y+A_7x^3y + A_8xy^2+A_9x^2y^2+A_{10}xy^3+A_{12}x^2y^3+f(y) \quad \text{(D-12)}$$

and

$$V_2 = g(x)+B_5xy+B_6x^2y+B_7x^3y+B_8xy^2+B_9x^2y^2 + B_{10}xy^3+B_{11}y+B_{12}y^2+B_{13}y^3+B_{14}y^4+B_{16}xy^4 \quad \text{(D-13)}$$

But U_1 must equal U_2, and V_1 must equal V_2, therefore A_{11}, A_{12}, B_{15}, and B_{16} must equal zero.

The functions for U and V must also satisfy the shear stress relationship given by Hooke's Law.

$$\tau_{xy} = G((\partial U/\partial y)+(\partial V/\partial x)) = G[(A_5x+A_6x^2+A_7x^3+g'(x)) + (2A_8+2B_6)xy+(2A_9+3B_7)x^2y+(3A_{10}+2B_9)xy^2 + (f'(y)+B_5y+B_8y^2+B_{10}y^3)] \quad \text{(D-14)}$$

Thus, $g'(x) = B_1+B_2x+B_3x^2+B_4x^3$ (D-15)

and $f'(y) = A_{11}+A_{12}y+A_{13}y^2+A_{14}y^3$ (D-16)

So $g(x) = B_0+B_1x+\frac{B_2}{2}x^2 + \frac{B_3}{3}x^3 + \frac{B_4}{4}x^4$ (D-17)

and $f(y) = A_0 + A_{11}y + A_{12}y^2 + \frac{A_{13}}{3}y^3 + \frac{A_{14}}{4}y^4$ (D-18)

Hence, the final functional form is found to be

$$U = A_0+A_1x+A_2x^2+A_3x^3+A_4x^4+A_5xy+A_6x^2y+A_7x^3y + A_8xy^2+A_9x^2y^2+A_{10}xy^3+A_{11}y+A_{12}y^2+A_{13}y^3+A_{14}y^4 \quad \text{(D-19)}$$

and

$$V = B_0+B_1x+B_2x^2+B_3x^3+B_4x^4+B_5xy+B_6x^2y+B_7x^3y + B_8xy^2+B_9x^2y^2+B_{10}xy^3+B_{11}y+B_{12}y^2+B_{13}y^3+B_{14}y^4 \quad \text{(D-20)}$$

The observation is made that U and V are of binomial form. The degree of the polynomial approximating function for the temperature field should be at most one degree less than the degree of the polynomial used for U and V.

Ordering of Functions ϕ and ψ

$\phi_1 = x$	$\psi_1 = x - (x^3/L^2)$
$\phi_2 = x^2$	$\psi_2 = x^2 - (x^3/L)$
$\phi_3 = x^3$	$\psi_3 = xy$
$\phi_4 = xy$	$\psi_4 = x^2y$
$\phi_5 = x^2y$	$\psi_5 = xy^2$
$\phi_6 = xy^2$	$\psi_6 = y$
$\phi_7 = y$	$\psi_7 = y^2$
$\phi_8 = y^2$	$\psi_8 = y^3$
$\phi_9 = y^3$	

APPENDIX F

Program Output

a) Displacements for program when N=5,6,7

b) Displacements for finite element runs (CQDMEM, CQUAD8)

<u>NOTE</u> - In finite element runs subcase 1 is a tangential load, and subcase 2 is a normal load.

NORMAL LOAD DEGREE EQUAL FIVE

```
THE DEGREE OF THE POLYNOMIAL FOR THE STRESS CALCULATION IS 5
THE DEGREE OF THE POLYNOMIAL FOR THE TEMPERATURE FIELD CALCULATION IS 0
THE LENGTH OF THE BANDSAW BLADE IS   0.100000E+01
THE WIDTH OF THE BANDSAW BLADE IS   0.250000E+00
THE THICKNESS OF THE BANDSAW BLADE IS   0.107000E-02
THE MODULUS OF ELASTICITY IS   0.207000E+12
POISON"S RATIO IS   0.300000E+00
THE MASS DENSITY IS   0.834600E+01
THE VELOCITY OF THE BANDSAW BLADE IS   0.0
THE WHEEL SUPPORT COEFFICIENT IS   0.0
THE INITIAL TENSION IS   0.0
THE NORMAL CUTTING FORCE IS   0.250000E+03
THE TANGENTIAL CUTTING FORCE IS   0.0
THE STARTING X-COORDINATE FOR THE NORMAL CUTTING IS   0.487000E+00
THE ENDING X-COORDINATE FOR THE NORMAL CUTTING IS   0.513000E+00
```

NORMAL LOAD DEGREE EQUAL FIVE

DISPLACEMENT FIELD U

Y COORDINATE											
0.250E+00	0.140E-04	0.141E-04	0.132E-04	0.117E-04	0.975E-05	0.765E-05	0.554E-05	0.362E-05	0.208E-05	0.119E-05	0.132E-05
0.200E+00	0.114E-04	0.114E-04	0.108E-04	0.992E-05	0.882E-05	0.765E-05	0.648E-05	0.538E-05	0.445E-05	0.386E-05	0.385E-05
0.150E+00	0.898E-05	0.892E-05	0.864E-05	0.830E-05	0.796E-05	0.765E-05	0.733E-05	0.700E-05	0.665E-05	0.637E-05	0.631E-05
0.100E+00	0.639E-05	0.641E-05	0.650E-05	0.675E-05	0.715E-05	0.765E-05	0.814E-05	0.855E-05	0.880E-05	0.889E-05	0.890E-05
0.500E-01	0.348E-05	0.373E-05	0.429E-05	0.518E-05	0.634E-05	0.765E-05	0.896E-05	0.101E-04	0.110E-04	0.116E-04	0.118E-04
0.0	0.0	0.702E-06	0.187E-05	0.350E-05	0.548E-05	0.765E-05	0.982E-05	0.118E-04	0.134E-04	0.146E-04	0.153E-04
X-COORDINATE	0.0	0.100E+00	0.200E+00	0.300E+00	0.400E+00	0.500E+00	0.600E+00	0.700E+00	0.800E+00	0.900E+00	0.100E+01

NORMAL LOAD DEGREE EQUAL FIVE

DISPLACEMENT FIELD V

-.209E-05	-.716E-05	-.127E-04	-.174E-04	-.206E-04	-.217E-04	-.206E-04	-.174E-04	-.127E-04	-.716E-05	-.209E-05
-.181E-05	-.729E-05	-.129E-04	-.176E-04	-.207E-04	-.218E-04	-.207E-04	-.176E-04	-.129E-04	-.729E-05	-.181E-05
-.148E-05	-.736E-05	-.130E-04	-.176E-04	-.207E-04	-.217E-04	-.207E-04	-.176E-04	-.130E-04	-.736E-05	-.148E-05
-.109E-05	-.735E-05	-.131E-04	-.176E-04	-.206E-04	-.216E-04	-.206E-04	-.176E-04	-.131E-04	-.735E-05	-.109E-05
-.610E-06	-.727E-05	-.131E-04	-.176E-04	-.204E-04	-.214E-04	-.204E-04	-.176E-04	-.131E-04	-.727E-05	-.610E-06
0.0	-.707E-05	-.130E-04	-.174E-04	-.201E-04	-.211E-04	-.201E-04	-.174E-04	-.130E-04	-.707E-05	-.678E-20
0.0	0.100E+00	0.200E+00	0.300E+00	0.400E+00	0.500E+00	0.600E+00	0.700E+00	0.800E+00	0.900E+00	0.100E+01

X-COORDINATE

NORMAL LOAD DEGREE EQUAL SIX

THE DEGREE OF THE POLYNOMIAL FOR THE STRESS CALCULATION IS 6

THE DEGREE OF THE POLYNOMIAL FOR THE TEMPERATURE FIELD CALCULATION IS 0

THE LENGTH OF THE BANDSAW BLADE IS 0.100000E+01

THE WIDTH OF THE BANDSAW BLADE IS 0.250000E+00

THE THICKNESS OF THE BANDSAW BLADE IS 0.107000E-02

THE MODULUS OF ELASTICITY IS 0.207000E+12

POISON"S RATIO IS 0.300000E+00

THE MASS DENSITY IS 0.834600E+01

THE VELOCITY OF THE BANDSAW BLADE IS 0.0

THE WHEEL SUPPORT COEFFICIENT IS 0.0

THE INITIAL TENSION IS 0.0

THE NORMAL CUTTING FORCE IS 0.250000E+03

THE TANGENTIAL CUTTING FORCE IS 0.0

THE STARTING X-COORDINATE FOR THE NORMAL CUTTING IS 0.487000E+00

THE ENDING X-COORDINATE FOR THE NORMAL CUTTING IS 0.513000E+00

NORMAL LOAD DEGREE EQUAL SIX

DISPLACEMENT FIELD U

0.139E-04	0.139E-04	0.133E-04	0.119E-04	0.996E-05	0.763E-05	0.530E-05	0.332E-05	0.195E-05	0.132E-05	0.132E-05
0.114E-04	0.113E-04	0.109E-04	0.100E-04	0.891E-05	0.763E-05	0.635E-05	0.523E-05	0.440E-05	0.395E-05	0.382E-05
0.897E-05	0.886E-05	0.863E-05	0.832E-05	0.798E-05	0.763E-05	0.728E-05	0.694E-05	0.663E-05	0.640E-05	0.629E-05
0.637E-05	0.643E-05	0.648E-05	0.669E-05	0.710E-05	0.763E-05	0.816E-05	0.857E-05	0.878E-05	0.883E-05	0.889E-05
0.345E-05	0.382E-05	0.424E-05	0.503E-05	0.621E-05	0.763E-05	0.905E-05	0.102E-04	0.110E-04	0.114E-04	0.118E-04
0.0	0.826E-06	0.174E-05	0.320E-05	0.524E-05	0.763E-05	0.100E-04	0.121E-04	0.135E-04	0.144E-04	0.153E-04
0.0	0.100E+00	0.200E+00	0.300E+00	0.400E+00	0.500E+00	0.600E+00	0.700E+00	0.800E+00	0.900E+00	0.100E+01

X-COORDINATE

NORMAL LOAD DEGREE EQUAL SIX

DISPLACEMENT FIELD V

-.209E-05	-.735E-05	-.127E-04	-.176E-04	-.211E-04	-.223E-04	-.211E-04	-.176E-04	-.127E-04	-.735E-05	-.209E-05
-.201E-05	-.742E-05	-.129E-04	-.178E-04	-.212E-04	-.225E-04	-.212E-04	-.178E-04	-.129E-04	-.742E-05	-.201E-05
-.181E-05	-.748E-05	-.130E-04	-.178E-04	-.212E-04	-.224E-04	-.212E-04	-.178E-04	-.130E-04	-.748E-05	-.181E-05
-.142E-05	-.747E-05	-.130E-04	-.178E-04	-.211E-04	-.223E-04	-.211E-04	-.178E-04	-.130E-04	-.747E-05	-.142E-05
-.815E-06	-.740E-05	-.131E-04	-.178E-04	-.209E-04	-.221E-04	-.209E-04	-.178E-04	-.131E-04	-.740E-05	-.815E-06
0.0	-.726E-05	-.130E-04	-.176E-04	-.206E-04	-.217E-04	-.206E-04	-.176E-04	-.130E-04	-.726E-05	0.0
0.0	0.100E+00	0.200E+00	0.300E+00	0.400E+00	0.500E+00	0.600E+00	0.700E+00	0.800E+00	0.900E+00	0.100E+01

X-COORDINATE

NORMAL LOAD DEGREE EQUAL SEVEN

```
THE DEGREE OF THE POLYNOMIAL FOR THE STRESS CALCULATION IS 7

THE DEGREE OF THE POLYNOMIAL FOR THE TEMPERATURE FIELD CALCULATION IS 0

THE LENGTH OF THE BANDSAW BLADE IS   0.100000E+01

THE WIDTH OF THE BANDSAW BLADE IS   0.250000E+00

THE THICKNESS OF THE BANDSAW BLADE IS   0.107000E-02

THE MODULUS OF ELASTICITY IS   0.207000E+12

POISON"S RATIO IS   0.300000E+00

THE MASS DENSITY IS   0.834600E+01

THE VELOCITY OF THE BANDSAW BLADE IS   0.0

THE WHEEL SUPPORT COEFFICIENT IS   0.0

THE INITIAL TENSION IS   0.0

THE NORMAL CUTTING FORCE IS   0.250000E+03

THE TANGENTIAL CUTTING FORCE IS   0.0

THE STARTING X-COORDINATE FOR THE NORMAL CUTTING IS   0.487000E+00

THE ENDING X-COORDINATE FOR THE NORMAL CUTTING IS   0.513000E+00
```

NORMAL LOAD DEGREE EQUAL SEVEN

DISPLACEMENT FIELD U

0.139E-04	0.140E-04	0.134E-04	0.121E-04	0.101E-04	0.773E-05	0.538E-05	0.339E-05	0.206E-05	0.147E-05	0.153E-05
0.116E-04	0.114E-04	0.109E-04	0.101E-04	0.902E-05	0.773E-05	0.645E-05	0.534E-05	0.453E-05	0.406E-05	0.390E-05
0.907E-05	0.898E-05	0.874E-05	0.844E-05	0.810E-05	0.773E-05	0.737E-05	0.703E-05	0.673E-05	0.649E-05	0.640E-05
0.648E-05	0.655E-05	0.659E-05	0.681E-05	0.722E-05	0.773E-05	0.825E-05	0.866E-05	0.888E-05	0.892E-05	0.899E-05
0.359E-05	0.392E-05	0.433E-05	0.513E-05	0.632E-05	0.773E-05	0.915E-05	0.103E-04	0.111E-04	0.115E-04	0.119E-04
0.0	0.886E-06	0.185E-05	0.334E-05	0.537E-05	0.773E-05	0.101E-04	0.121E-04	0.136E-04	0.146E-04	0.155E-04
0.0	0.100E+00	0.200E+00	0.300E+00	0.400E+00	0.500E+00	0.600E+00	0.700E+00	0.800E+00	0.900E+00	0.100E+01

X-COORDINATE

NORMAL LOAD DEGREE EQUAL SEVEN

DISPLACEMENT FIELD V

Y COORDINATE											
0.250E+00	-.246E-05	-.746E-05	-.130E-04	-.179E-04	-.213E-04	-.225E-04	-.213E-04	-.179E-04	-.130E-04	-.746E-05	-.246E-05
0.200E+00	-.227E-05	-.757E-05	-.131E-04	-.180E-04	-.214E-04	-.226E-04	-.214E-04	-.180E-04	-.131E-04	-.757E-05	-.227E-05
0.150E+00	-.202E-05	-.765E-05	-.132E-04	-.180E-04	-.214E-04	-.226E-04	-.214E-04	-.180E-04	-.132E-04	-.765E-05	-.202E-05
0.100E+00	-.158E-05	-.767E-05	-.132E-04	-.180E-04	-.213E-04	-.225E-04	-.213E-04	-.180E-04	-.132E-04	-.767E-05	-.158E-05
0.500E-01	-.922E-06	-.762E-05	-.132E-04	-.179E-04	-.211E-04	-.223E-04	-.211E-04	-.179E-04	-.132E-04	-.762E-05	-.922E-06
0.0	0.0	-.751E-05	-.132E-04	-.177E-04	-.208E-04	-.219E-04	-.208E-04	-.177E-04	-.132E-04	-.751E-05	-.252E-27
X-COORDINATE	0.0	0.100E+00	0.200E+00	0.300E+00	0.400E+00	0.500E+00	0.600E+00	0.700E+00	0.800E+00	0.900E+00	0.1C0E+01

SUBCASE 2

D I S P L A C E M E N T   V E C T O R

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
1	G	0.0	0.0	0.0	0.0	0.0	0.0
2	G	2.507330E-06	-4.985575E-07	0.0	0.0	0.0	0.0
3	G	4.705719E-06	-8.646942E-07	0.0	0.0	0.0	0.0
4	G	6.768399E-06	-1.085101E-06	0.0	0.0	0.0	0.0
5	G	8.832042E-06	-1.175461E-06	0.0	0.0	0.0	0.0
6	G	1.100549E-05	-1.179077E-06	0.0	0.0	0.0	0.0
7	G	4.276369E-07	-5.578896E-06	0.0	0.0	0.0	0.0
8	G	2.751675E-06	-5.667511E-06	0.0	0.0	0.0	0.0
9	G	4.817328E-06	-5.731830E-06	0.0	0.0	0.0	0.0
10	G	6.768634E-06	-5.763741E-06	0.0	0.0	0.0	0.0
11	G	8.738170E-06	-5.758534E-06	0.0	0.0	0.0	0.0
12	G	1.084451E-05	-5.718638E-06	0.0	0.0	0.0	0.0
13	G	1.170260E-06	-1.011174E-05	0.0	0.0	0.0	0.0
14	G	3.124732E-06	-1.021657E-05	0.0	0.0	0.0	0.0
15	G	4.909376E-06	-1.025942E-05	0.0	0.0	0.0	0.0
16	G	6.621262E-06	-1.025035E-05	0.0	0.0	0.0	0.0
17	G	8.372044E-06	-1.019454E-05	0.0	0.0	0.0	0.0
18	G	1.027305E-05	-1.009596E-05	0.0	0.0	0.0	0.0
19	G	2.278678E-06	-1.385727E-05	0.0	0.0	0.0	0.0
20	G	3.760367E-06	-1.401274E-05	0.0	0.0	0.0	0.0
21	G	5.094786E-06	-1.408658E-05	0.0	0.0	0.0	0.0
22	G	6.383842E-06	-1.408281E-05	0.0	0.0	0.0	0.0
23	G	7.736039E-06	-1.400375E-05	0.0	0.0	0.0	0.0
24	G	9.258221E-06	-1.385443E-05	0.0	0.0	0.0	0.0
25	G	3.801281E-06	-1.653092E-05	0.0	0.0	0.0	0.0
26	G	4.663434E-06	-1.674265E-05	0.0	0.0	0.0	0.0
27	G	5.387424E-06	-1.686708E-05	0.0	0.0	0.0	0.0
28	G	6.077683E-06	-1.691094E-05	0.0	0.0	0.0	0.0
29	G	6.841807E-06	-1.686269E-05	0.0	0.0	0.0	0.0
30	G	7.841295E-06	-1.669426E-05	0.0	0.0	0.0	0.0
31	G	5.504260E-06	-1.760559E-05	0.0	0.0	0.0	0.0
32	G	5.670502E-06	-1.786587E-05	0.0	0.0	0.0	0.0
33	G	5.766261E-06	-1.806144E-05	0.0	0.0	0.0	0.0
34	G	5.850820E-06	-1.823950E-05	0.0	0.0	0.0	0.0
35	G	5.946265E-06	-1.841581E-05	0.0	0.0	0.0	0.0
36	G	6.186074E-06	-1.854332E-05	0.0	0.0	0.0	0.0
37	G	5.832553E-06	-1.762400E-05	0.0	0.0	0.0	0.0
38	G	5.832554E-06	-1.789031E-05	0.0	0.0	0.0	0.0
39	G	5.832554E-06	-1.808221E-05	0.0	0.0	0.0	0.0
40	G	5.832555E-06	-1.826373E-05	0.0	0.0	0.0	0.0
41	G	5.832555E-06	-1.847127E-05	0.0	0.0	0.0	0.0
42	G	5.832556E-06	-1.883390E-05	0.0	0.0	0.0	0.0
43	G	6.160849E-06	-1.760559E-05	0.0	0.0	0.0	0.0
44	G	5.994606E-06	-1.786587E-05	0.0	0.0	0.0	0.0
45	G	5.898848E-06	-1.806144E-05	0.0	0.0	0.0	0.0
46	G	5.814289E-06	-1.823950E-05	0.0	0.0	0.0	0.0
47	G	5.718845E-06	-1.841581E-05	0.0	0.0	0.0	0.0
48	G	5.479035E-06	-1.854332E-05	0.0	0.0	0.0	0.0
49	G	7.863827E-06	-1.653092E-05	0.0	0.0	0.0	0.0
50	G	7.001675E-06	-1.674265E-05	0.0	0.0	0.0	0.0

D I S P L A C E M E N T   V E C T O R

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
51	G	6.277685E-06	-1.686708E-05	0.0	0.0	0.0	0.0
52	G	5.587426E-06	-1.691094E-05	0.0	0.0	0.0	0.0
53	G	4.823302E-06	-1.686269E-05	0.0	0.0	0.0	0.0
54	G	3.823814E-06	-1.669426E-05	0.0	0.0	0.0	0.0
55	G	9.386430E-06	-1.385727E-05	0.0	0.0	0.0	0.0
56	G	7.904742E-06	-1.401274E-05	0.0	0.0	0.0	0.0
57	G	6.570323E-06	-1.408658E-05	0.0	0.0	0.0	0.0
58	G	5.281267E-06	-1.408281E-05	0.0	0.0	0.0	0.0
59	G	3.929070E-06	-1.400375E-05	0.0	0.0	0.0	0.0
60	G	2.406889E-06	-1.385443E-05	0.0	0.0	0.0	0.0
61	G	1.049485E-05	-1.011174E-05	0.0	0.0	0.0	0.0
62	G	8.540377E-06	-1.021657E-05	0.0	0.0	0.0	0.0
63	G	6.755733E-06	-1.025943E-05	0.0	0.0	0.0	0.0
64	G	5.043846E-06	-1.025035E-05	0.0	0.0	0.0	0.0
65	G	3.293065E-06	-1.019454E-05	0.0	0.0	0.0	0.0
66	G	1.392061E-06	-1.009596E-05	0.0	0.0	0.0	0.0
67	G	1.123747E-05	-5.578898E-06	0.0	0.0	0.0	0.0
68	G	8.913434E-06	-5.667513E-06	0.0	0.0	0.0	0.0
69	G	6.847781E-06	-5.731832E-06	0.0	0.0	0.0	0.0
70	G	4.896475E-06	-5.763744E-06	0.0	0.0	0.0	0.0
71	G	2.926939E-06	-5.758536E-06	0.0	0.0	0.0	0.0
72	G	8.205985E-07	-5.718641E-06	0.0	0.0	0.0	0.0
73	G	1.166511E-05	0.0	0.0	0.0	0.0	0.0
74	G	9.157779E-06	-4.985571E-07	0.0	0.0	0.0	0.0
75	G	6.959390E-06	-8.646936E-07	0.0	0.0	0.0	0.0
76	G	4.896710E-06	-1.085100E-06	0.0	0.0	0.0	0.0
77	G	2.833067E-06	-1.175460E-06	0.0	0.0	0.0	0.0
78	G	6.596205E-07	-1.179076E-06	0.0	0.0	0.0	0.0

SUBCASE 2

D I S P L A C E M E N T   V E C T O R

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
1	G	0.0	0.0	0.0	0.0	0.0	0.0
2	G	3.227071E-06	-8.135539E-07	0.0	0.0	0.0	0.0
3	G	5.847140E-06	-1.361505E-06	0.0	0.0	0.0	0.0
4	G	8.259087E-06	-1.663885E-06	0.0	0.0	0.0	0.0
5	G	1.066377E-05	-1.777958E-06	0.0	0.0	0.0	0.0
6	G	1.317292E-05	-1.791558E-06	0.0	0.0	0.0	0.0
7	G	6.708742E-07	-7.066489E-06	0.0	0.0	0.0	0.0
8	G	3.489708E-06	-6.999286E-06	0.0	0.0	0.0	0.0
9	G	5.954273E-06	-6.983005E-06	0.0	0.0	0.0	0.0
10	G	8.247177E-06	-6.990944E-06	0.0	0.0	0.0	0.0
11	G	1.055089E-05	-6.984998E-06	0.0	0.0	0.0	0.0
12	G	1.301688E-05	-6.949593E-06	0.0	0.0	0.0	0.0
13	G	1.613303E-06	-1.210939E-05	0.0	0.0	0.0	0.0
14	G	3.870186E-06	-1.225332E-05	0.0	0.0	0.0	0.0
15	G	6.003665E-06	-1.228049E-05	0.0	0.0	0.0	0.0
16	G	8.045908E-06	-1.224062E-05	0.0	0.0	0.0	0.0
17	G	1.011590E-05	-1.215948E-05	0.0	0.0	0.0	0.0
18	G	1.237238E-05	-1.203485E-05	0.0	0.0	0.0	0.0
19	G	2.912077E-06	-1.642526E-05	0.0	0.0	0.0	0.0
20	G	4.652667E-06	-1.660574E-05	0.0	0.0	0.0	0.0
21	G	6.212399E-06	-1.669227E-05	0.0	0.0	0.0	0.0
22	G	7.741110E-06	-1.666979E-05	0.0	0.0	0.0	0.0
23	G	9.362101E-06	-1.655384E-05	0.0	0.0	0.0	0.0
24	G	1.113716E-05	-1.638112E-05	0.0	0.0	0.0	0.0
25	G	4.744418E-06	-1.945996E-05	0.0	0.0	0.0	0.0
26	G	5.734194E-06	-1.971803E-05	0.0	0.0	0.0	0.0
27	G	6.565459E-06	-1.987035E-05	0.0	0.0	0.0	0.0
28	G	7.348452E-06	-1.992044E-05	0.0	0.0	0.0	0.0
29	G	8.239515E-06	-1.981652E-05	0.0	0.0	0.0	0.0
30	G	9.472547E-06	-1.951479E-05	0.0	0.0	0.0	0.0
31	G	6.735187E-06	-2.059914E-05	0.0	0.0	0.0	0.0
32	G	6.907055E-06	-2.090367E-05	0.0	0.0	0.0	0.0
33	G	7.007207E-06	-2.112587E-05	0.0	0.0	0.0	0.0
34	G	7.111500E-06	-2.133413E-05	0.0	0.0	0.0	0.0
35	G	7.174574E-06	-2.159218E-05	0.0	0.0	0.0	0.0
36	G	7.507501E-06	-2.179173E-05	0.0	0.0	0.0	0.0
37	G	7.086012E-06	-2.062035E-05	0.0	0.0	0.0	0.0
38	G	7.086012E-06	-2.092936E-05	0.0	0.0	0.0	0.0
39	G	7.086013E-06	-2.114815E-05	0.0	0.0	0.0	0.0
40	G	7.086013E-06	-2.136323E-05	0.0	0.0	0.0	0.0
41	G	7.086014E-06	-2.158804E-05	0.0	0.0	0.0	0.0
42	G	7.086014E-06	-2.220692E-05	0.0	0.0	0.0	0.0
43	G	7.436840E-06	-2.059914E-05	0.0	0.0	0.0	0.0
44	G	7.264972E-06	-2.090367E-05	0.0	0.0	0.0	0.0
45	G	7.164819E-06	-2.112587E-05	0.0	0.0	0.0	0.0
46	G	7.060527E-06	-2.133413E-05	0.0	0.0	0.0	0.0
47	G	6.997454E-06	-2.159218E-05	0.0	0.0	0.0	0.0
48	G	6.664526E-06	-2.179173E-05	0.0	0.0	0.0	0.0
49	G	9.427607E-06	-1.945996E-05	0.0	0.0	0.0	0.0
50	G	8.437832E-06	-1.971803E-05	0.0	0.0	0.0	0.0

SUBCASE 2

D I S P L A C E M E N T   V E C T O R

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
51	G	7.606567E-06	-1.987035E-05	0.0	0.0	0.0	0.0
52	G	6.823575E-06	-1.992044E-05	0.0	0.0	0.0	0.0
53	G	5.932512E-06	-1.981652E-05	0.0	0.0	0.0	0.0
54	G	4.699479E-06	-1.951479E-05	0.0	0.0	0.0	0.0
55	G	1.125995E-05	-1.642526E-05	0.0	0.0	0.0	0.0
56	G	9.519359E-06	-1.660576E-05	0.0	0.0	0.0	0.0
57	G	7.959628E-06	-1.669227E-05	0.0	0.0	0.0	0.0
58	G	6.430917E-06	-1.666979E-05	0.0	0.0	0.0	0.0
59	G	4.809925E-06	-1.655384E-05	0.0	0.0	0.0	0.0
60	G	3.034866E-06	-1.638112E-05	0.0	0.0	0.0	0.0
61	G	1.255872E-05	-1.210939E-05	0.0	0.0	0.0	0.0
62	G	1.030184E-05	-1.225332E-05	0.0	0.0	0.0	0.0
63	G	8.168361E-06	-1.228050E-05	0.0	0.0	0.0	0.0
64	G	6.126119E-06	-1.224062E-05	0.0	0.0	0.0	0.0
65	G	4.056126E-06	-1.215948E-05	0.0	0.0	0.0	0.0
66	G	1.799652E-06	-1.203485E-05	0.0	0.0	0.0	0.0
67	G	1.350115E-05	-7.066491E-06	0.0	0.0	0.0	0.0
68	G	1.068232E-05	-6.999288E-06	0.0	0.0	0.0	0.0
69	G	8.217754E-06	-6.983008E-06	0.0	0.0	0.0	0.0
70	G	5.924850E-06	-6.990946E-06	0.0	0.0	0.0	0.0
71	G	3.621134E-06	-6.985001E-06	0.0	0.0	0.0	0.0
72	G	1.155146E-06	-6.949595E-06	0.0	0.0	0.0	0.0
73	G	1.417203E-05	0.0	0.0	0.0	0.0	0.0
74	G	1.094496E-05	-8.135534E-07	0.0	0.0	0.0	0.0
75	G	8.324887E-06	-1.361504E-06	0.0	0.0	0.0	0.0
76	G	5.912940E-06	-1.663884E-06	0.0	0.0	0.0	0.0
77	G	3.508254E-06	-1.777958E-06	0.0	0.0	0.0	0.0
78	G	9.991072E-07	-1.791557E-06	0.0	0.0	0.0	0.0

TANGENTIAL LOAD DEGREE EQUAL FIVE

THE DEGREE OF THE POLYNOMIAL FOR THE STRESS CALCULATION IS 5

THE DEGREE OF THE POLYNOMIAL FOR THE TEMPERATURE FIELD CALCULATION IS 0

THE LENGTH OF THE BANDSAW BLADE IS 0.100000E+01

THE WIDTH OF THE BANDSAW BLADE IS 0.250000E+00

THE THICKNESS OF THE BANDSAW BLADE IS 0.107000E-02

THE MODULUS OF ELASTICITY IS 0.207000E+12

POISON"S RATIO IS 0.300000E+00

THE MASS DENSITY IS 0.834600E+01

THE VELOCITY OF THE BANDSAW BLADE IS 0.0

THE WHEEL SUPPORT COEFFICIENT IS 0.0

THE INITIAL TENSION IS 0.0

THE NORMAL CUTTING FORCE IS 0.0

THE TANGENTIAL CUTTING FORCE IS 0.500000E+03

THE STARTING X-COORDINATE FOR THE TANGENTIAL CUTTING FORCE IS 0.487000E+00

THE ENDING X-COORDINATE FOR THE TANGENTIAL CUTTING FORCE IS 0.513000E+00

TANGENTIAL LOAD DEGREE EQUAL FIVE

DISPLACEMENT FIELD U

-.179E-04	-.174E-04	-.174E-04	-.174E-04	-.171E-04	-.163E-04	-.150E-04	-.134E-04	-.118E-04	-.107E-04	-.111E-04
-.160E-04	-.155E-04	-.155E-04	-.157E-04	-.158E-04	-.157E-04	-.152E-04	-.144E-04	-.134E-04	-.126E-04	-.126E-04
-.135E-04	-.134E-04	-.137E-04	-.142E-04	-.147E-04	-.151E-04	-.153E-04	-.152E-04	-.148E-04	-.144E-04	-.143E-04
-.103E-04	-.111E-04	-.120E-04	-.129E-04	-.138E-04	-.147E-04	-.154E-04	-.159E-04	-.162E-04	-.163E-04	-.163E-04
-.597E-05	-.835E-05	-.102E-04	-.118E-04	-.132E-04	-.145E-04	-.158E-04	-.168E-04	-.176E-04	-.182E-04	-.185E-04
0.0	-.477E-05	-.818E-05	-.108E-04	-.128E-04	-.147E-04	-.163E-04	-.179E-04	-.192E-04	-.204E-04	-.212E-04
0.0	0.100E+00	0.200E+00	0.300E+00	0.400E+00	0.500E+00	0.600E+00	0.700E+00	0.800E+00	0.900E+00	0.100E+01

X-COORDINATE

TANGENTIAL LOAD DEGREE EQUAL FIVE

DISPLACEMENT FIELD V

0.183E-05	0.653E-05	0.102E-04	0.129E-04	0.147E-04	0.153E-04	0.147E-04	0.127E-04	0.954E-05	0.571E-05	0.209E-05
0.162E-05	0.661E-05	0.103E-04	0.130E-04	0.147E-04	0.153E-04	0.147E-04	0.128E-04	0.976E-05	0.584E-05	0.181E-05
0.140E-05	0.669E-05	0.104E-04	0.130E-04	0.146E-04	0.152E-04	0.147E-04	0.129E-04	0.991E-05	0.590E-05	0.149E-05
0.112E-05	0.670E-05	0.105E-04	0.130E-04	0.145E-04	0.151E-04	0.146E-04	0.129E-04	0.997E-05	0.588E-05	0.109E-05
0.685E-06	0.660E-05	0.104E-04	0.129E-04	0.143E-04	0.149E-04	0.145E-04	0.128E-04	0.995E-05	0.578E-05	0.598E-06
0.0	0.630E-05	0.102E-04	0.127E-04	0.141E-04	0.146E-04	0.142E-04	0.127E-04	0.985E-05	0.558E-05	-.102E-19
0.0	0.100E+00	0.200E+00	0.300E+00	0.400E+00	0.500E+00	0.600E+00	0.700E+00	0.800E+00	0.900E+00	0.100E+01

X-COORDINATE

TANGENTIAL LOAD DEGREE EQUAL SIX

THE DEGREE OF THE POLYNOMIAL FOR THE STRESS CALCULATION IS 6

THE DEGREE OF THE POLYNOMIAL FOR THE TEMPERATURE FIELD CALCULATION IS 0

THE LENGTH OF THE BANDSAW BLADE IS 0.100000E+01

THE WIDTH OF THE BANDSAW BLADE IS 0.250000E+00

THE THICKNESS OF THE BANDSAW BLADE IS 0.107000E-02

THE MODULUS OF ELASTICITY IS 0.207000E+12

POISON"S RATIO IS 0.300000E+00

THE MASS DENSITY IS 0.834600E+01

THE VELOCITY OF THE BANDSAW BLADE IS 0.0

THE WHEEL SUPPORT COEFFICIENT IS 0.0

THE INITIAL TENSION IS 0.0

THE NORMAL CUTTING FORCE IS 0.0

THE TANGENTIAL CUTTING FORCE IS 0.500000E+03

THE STARTING X-COORDINATE FOR THE TANGENTIAL CUTTING FORCE IS 0.487000E+00

THE ENDING X-COORDINATE FOR THE TANGENTIAL CUTTING FORCE IS 0.513000E+00

TANGENTIAL LOAD DEGREE EQUAL SIX

DISPLACEMENT FIELD U

-.188E-04	-.183E-04	-.180E-04	-.180E-04	-.177E-04	-.171E-04	-.159E-04	-.143E-04	-.127E-04	-.115E-04	-.110E-04
-.166E-04	-.162E-04	-.162E-04	-.165E-04	-.167E-04	-.166E-04	-.159E-04	-.150E-04	-.140E-04	-.135E-04	-.136E-04
-.143E-04	-.142E-04	-.144E-04	-.150E-04	-.157E-04	-.161E-04	-.162E-04	-.159E-04	-.155E-04	-.153E-04	-.154E-04
-.114E-04	-.120E-04	-.126E-04	-.136E-04	-.147E-04	-.157E-04	-.164E-04	-.168E-04	-.170E-04	-.171E-04	-.171E-04
-.702E-05	-.928E-05	-.109E-04	-.125E-04	-.140E-04	-.155E-04	-.167E-04	-.177E-04	-.185E-04	-.191E-04	-.192E-04
0.0	-.568E-05	-.921E-05	-.118E-04	-.138E-04	-.156E-04	-.171E-04	-.186E-04	-.200E-04	-.213E-04	-.222E-04
0.0	0.100E+00	0.200E+00	0.300E+00	0.400E+00	0.500E+00	0.600E+00	0.700E+00	0.800E+00	0.900E+00	0.100E+01

X-COORDINATE

TANGENTIAL LOAD DEGREE EQUAL SIX

DISPLACEMENT FIELD V

0.227E-05	0.699E-05	0.106E-04	0.132E-04	0.148E-04	0.153E-04	0.145E-04	0.127E-04	0.979E-05	0.603E-05	0.165E-05
0.218E-05	0.708E-05	0.107E-04	0.133E-04	0.148E-04	0.153E-04	0.147E-04	0.129E-04	0.997E-05	0.609E-05	0.161E-05
0.192E-05	0.717E-05	0.108E-04	0.133E-04	0.148E-04	0.153E-04	0.147E-04	0.130E-04	0.100E-04	0.608E-05	0.145E-05
0.152E-05	0.723E-05	0.109E-04	0.132E-04	0.146E-04	0.152E-04	0.147E-04	0.130E-04	0.101E-04	0.601E-05	0.118E-05
0.916E-06	0.720E-05	0.109E-04	0.131E-04	0.144E-04	0.149E-04	0.145E-04	0.130E-04	0.101E-04	0.590E-05	0.724E-06
0.0	0.697E-05	0.107E-04	0.128E-04	0.141E-04	0.146E-04	0.143E-04	0.129E-04	0.101E-04	0.570E-05	-.271E-19
0.0	0.100E+00	0.200E+00	0.300E+00	0.400E+00	0.500E+00	0.600E+00	0.700E+00	0.800E+00	0.900E+00	0.100E+01

X-COORDINATE

TANGENTIAL LOAD DEGREE EQUAL SEVEN

THE DEGREE OF THE POLYNOMIAL FOR THE STRESS CALCULATION IS 7

THE DEGREE OF THE POLYNOMIAL FOR THE TEMPERATURE FIELD CALCULATION IS 0

THE LENGTH OF THE BANDSAW BLADE IS 0.100000E+01

THE WIDTH OF THE BANDSAW BLADE IS 0.250000E+00

THE THICKNESS OF THE BANDSAW BLADE IS 0.107000E-02

THE MODULUS OF ELASTICITY IS 0.207000E+12

POISON"S RATIO IS 0.300000E+00

THE MASS DENSITY IS 0.834600E+01

THE VELOCITY OF THE BANDSAW BLADE IS 0.0

THE WHEEL SUPPORT COEFFICIENT IS 0.0

THE INITIAL TENSION IS 0.0

THE NORMAL CUTTING FORCE IS 0.0

THE TANGENTIAL CUTTING FORCE IS 0.500000E+03

THE STARTING X-COORDINATE FOR THE TANGENTIAL CUTTING FORCE IS 0.487000E+00

THE ENDING X-COORDINATE FOR THE TANGENTIAL CUTTING FORCE IS 0.513000E+00

TANGENTIAL LOAD DEGREE EQUAL SEVEN

DISPLACEMENT FIELD U

-.199E-04	-.195E-04	-.188E-04	-.189E-04	-.192E-04	-.188E-04	-.175E-04	-.154E-04	-.134E-04	-.126E-04	-.134E-04
-.177E-04	-.176E-04	-.174E-04	-.176E-04	-.180E-04	-.179E-04	-.172E-04	-.161E-04	-.153E-04	-.148E-04	-.145E-04
-.156E-04	-.154E-04	-.157E-04	-.163E-04	-.170E-04	-.173E-04	-.172E-04	-.169E-04	-.167E-04	-.166E-04	-.164E-04
-.129E-04	-.130E-04	-.138E-04	-.150E-04	-.161E-04	-.170E-04	-.175E-04	-.179E-04	-.181E-04	-.183E-04	-.183E-04
-.846E-05	-.104E-04	-.121E-04	-.137E-04	-.152E-04	-.166E-04	-.179E-04	-.190E-04	-.198E-04	-.201E-04	-.205E-04
0.0	-.704E-05	-.109E-04	-.131E-04	-.146E-04	-.162E-04	-.181E-04	-.201E-04	-.216E-04	-.225E-04	-.235E-04
0.0	0.100E+00	0.200E+00	0.300E+00	0.400E+00	0.500E+00	0.600E+00	0.700E+00	0.800E+00	0.900E+00	0.100E+01

X-COORDINATE

TANGENTIAL LOAD DEGREE EQUAL SEVEN

DISPLACEMENT FIELD V

0.317E-05	0.731E-05	0.111E-04	0.135E-04	0.148E-04	0.155E-04	0.153E-04	0.138E-04	0.106E-04	0.620E-05	0.256E-05
0.279E-05	0.751E-05	0.111E-04	0.134E-04	0.148E-04	0.156E-04	0.155E-04	0.139E-04	0.107E-04	0.629E-05	0.237E-05
0.226E-05	0.769E-05	0.111E-04	0.133E-04	0.148E-04	0.157E-04	0.156E-04	0.139E-04	0.107E-04	0.637E-05	0.217E-05
0.170E-05	0.784E-05	0.111E-04	0.132E-04	0.147E-04	0.157E-04	0.155E-04	0.139E-04	0.106E-04	0.639E-05	0.174E-05
0.105E-05	0.791E-05	0.111E-04	0.130E-04	0.145E-04	0.155E-04	0.154E-04	0.137E-04	0.106E-04	0.637E-05	0.102E-05
0.0	0.770E-05	0.109E-04	0.127E-04	0.142E-04	0.153E-04	0.152E-04	0.136E-04	0.105E-04	0.628E-05	0.542E-19
0.0	0.100E+00	0.200E+00	0.300E+00	0.400E+00	0.500E+00	0.600E+00	0.700E+00	0.800E+00	0.900E+00	0.100E+01

X-COORDINATE

SUBCASE 1

D I S P L A C E M E N T   V E C T O R

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
1	G	0.0	0.0	0.0	0.0	0.0	0.0
2	G	-5.232930E-06	1.887678E-07	0.0	0.0	0.0	0.0
3	G	-8.445949E-06	4.227292E-07	0.0	0.0	0.0	0.0
4	G	-1.079703E-05	6.316384E-07	0.0	0.0	0.0	0.0
5	G	-1.277797E-05	7.597533E-07	0.0	0.0	0.0	0.0
6	G	-1.468831E-05	7.849263E-07	0.0	0.0	0.0	0.0
7	G	-4.309937E-06	4.621875E-06	0.0	0.0	0.0	0.0
8	G	-6.739766E-06	4.918928E-06	0.0	0.0	0.0	0.0
9	G	-9.032383E-06	4.940240E-06	0.0	0.0	0.0	0.0
10	G	-1.102260E-05	4.900929E-06	0.0	0.0	0.0	0.0
11	G	-1.279966E-05	4.862989E-06	0.0	0.0	0.0	0.0
12	G	-1.450865E-05	4.827358E-06	0.0	0.0	0.0	0.0
13	G	-7.042819E-06	7.388468E-06	0.0	0.0	0.0	0.0
14	G	-8.431728E-06	7.661135E-06	0.0	0.0	0.0	0.0
15	G	-9.954144E-06	7.789749E-06	0.0	0.0	0.0	0.0
16	G	-1.145340E-05	7.833138E-06	0.0	0.0	0.0	0.0
17	G	-1.286311E-05	7.839985E-06	0.0	0.0	0.0	0.0
18	G	-1.416021E-05	7.821811E-06	0.0	0.0	0.0	0.0
19	G	-9.062339E-06	9.218256E-06	0.0	0.0	0.0	0.0
20	G	-9.956799E-06	9.450880E-06	0.0	0.0	0.0	0.0
21	G	-1.098612E-05	9.598223E-06	0.0	0.0	0.0	0.0
22	G	-1.206957E-05	9.672211E-06	0.0	0.0	0.0	0.0
23	G	-1.312644E-05	9.705655E-06	0.0	0.0	0.0	0.0
24	G	-1.405086E-05	9.728570E-06	0.0	0.0	0.0	0.0
25	G	-1.068673E-05	1.047067E-05	0.0	0.0	0.0	0.0
26	G	-1.126420E-05	1.066921E-05	0.0	0.0	0.0	0.0
27	G	-1.193308E-05	1.078311E-05	0.0	0.0	0.0	0.0
28	G	-1.271148E-05	1.081285E-05	0.0	0.0	0.0	0.0
29	G	-1.359087E-05	1.079793E-05	0.0	0.0	0.0	0.0
30	G	-1.445581E-05	1.084977E-05	0.0	0.0	0.0	0.0
31	G	-1.200530E-05	1.125543E-05	0.0	0.0	0.0	0.0
32	G	-1.227930E-05	1.145265E-05	0.0	0.0	0.0	0.0
33	G	-1.261269E-05	1.157095E-05	0.0	0.0	0.0	0.0
34	G	-1.312290E-05	1.160496E-05	0.0	0.0	0.0	0.0
35	G	-1.399121E-05	1.156512E-05	0.0	0.0	0.0	0.0
36	G	-1.564567E-05	1.158997E-05	0.0	0.0	0.0	0.0
37	G	-1.223118E-05	1.133159E-05	0.0	0.0	0.0	0.0
38	G	-1.242645E-05	1.153454E-05	0.0	0.0	0.0	0.0
39	G	-1.270042E-05	1.166444E-05	0.0	0.0	0.0	0.0
40	G	-1.315423E-05	1.173195E-05	0.0	0.0	0.0	0.0
41	G	-1.397528E-05	1.173723E-05	0.0	0.0	0.0	0.0
42	G	-1.577284E-05	1.166916E-05	0.0	0.0	0.0	0.0
43	G	-1.246087E-05	1.139185E-05	0.0	0.0	0.0	0.0
44	G	-1.257318E-05	1.159668E-05	0.0	0.0	0.0	0.0
45	G	-1.278785E-05	1.174181E-05	0.0	0.0	0.0	0.0
46	G	-1.318336E-05	1.184270E-05	0.0	0.0	0.0	0.0
47	G	-1.393220E-05	1.188928E-05	0.0	0.0	0.0	0.0
48	G	-1.542261E-05	1.173216E-05	0.0	0.0	0.0	0.0
49	G	-1.383051E-05	1.128790E-05	0.0	0.0	0.0	0.0
50	G	-1.349610E-05	1.148354E-05	0.0	0.0	0.0	0.0

SUBCASE 1

D I S P L A C E M E N T   V E C T O R

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
51	G	-1.327743E-05	1.163229E-05	0.0	0.0	0.0	0.0
52	G	-1.317518E-05	1.173243E-05	0.0	0.0	0.0	0.0
53	G	-1.316423E-05	1.174780E-05	0.0	0.0	0.0	0.0
54	G	-1.310537E-05	1.157031E-05	0.0	0.0	0.0	0.0
55	G	-1.522592E-05	9.933048E-06	0.0	0.0	0.0	0.0
56	G	-1.437207E-05	1.008675E-05	0.0	0.0	0.0	0.0
57	G	-1.364910E-05	1.018046E-05	0.0	0.0	0.0	0.0
58	G	-1.299490E-05	1.021374E-05	0.0	0.0	0.0	0.0
59	G	-1.231892E-05	1.016994E-05	0.0	0.0	0.0	0.0
60	G	-1.149067E-05	1.002536E-05	0.0	0.0	0.0	0.0
61	G	-1.630909E-05	7.510225E-06	0.0	0.0	0.0	0.0
62	G	-1.500336E-05	7.615629E-06	0.0	0.0	0.0	0.0
63	G	-1.386079E-05	7.665543E-06	0.0	0.0	0.0	0.0
64	G	-1.279379E-05	7.668547E-06	0.0	0.0	0.0	0.0
65	G	-1.169334E-05	7.623071E-06	0.0	0.0	0.0	0.0
66	G	-1.044013E-05	7.526596E-06	0.0	0.0	0.0	0.0
67	G	-1.705061E-05	4.282191E-06	0.0	0.0	0.0	0.0
68	G	-1.537614E-05	4.371140E-06	0.0	0.0	0.0	0.0
69	G	-1.395799E-05	4.436992E-06	0.0	0.0	0.0	0.0
70	G	-1.265308E-05	4.471437E-06	0.0	0.0	0.0	0.0
71	G	-1.132907E-05	4.468271E-06	0.0	0.0	0.0	0.0
72	G	-9.866484E-06	4.428967E-06	0.0	0.0	0.0	0.0
73	G	-1.747905E-05	0.0	0.0	0.0	0.0	0.0
74	G	-1.562048E-05	4.984764E-07	0.0	0.0	0.0	0.0
75	G	-1.406999E-05	8.644026E-07	0.0	0.0	0.0	0.0
76	G	-1.265387E-05	1.084705E-06	0.0	0.0	0.0	0.0
77	G	-1.123542E-05	1.174962E-06	0.0	0.0	0.0	0.0
78	G	-9.706278E-06	1.178557E-06	0.0	0.0	0.0	0.0

SUBCASE 1

D I S P L A C E M E N T   V E C T O R

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
1	G	0.0	0.0	0.0	0.0	0.0	0.0
2	G	-7.686057E-06	3.710601E-07	0.0	0.0	0.0	0.0
3	G	-1.122142E-05	8.740303E-07	0.0	0.0	0.0	0.0
4	G	-1.379107E-05	1.279894E-06	0.0	0.0	0.0	0.0
5	G	-1.600145E-05	1.484845E-06	0.0	0.0	0.0	0.0
6	G	-1.822604E-05	1.522896E-06	0.0	0.0	0.0	0.0
7	G	-6.441022E-06	6.321515E-06	0.0	0.0	0.0	0.0
8	G	-8.679926E-06	6.660274E-06	0.0	0.0	0.0	0.0
9	G	-1.157023E-05	6.417124E-06	0.0	0.0	0.0	0.0
10	G	-1.389645E-05	6.217731E-06	0.0	0.0	0.0	0.0
11	G	-1.594350E-05	6.113153E-06	0.0	0.0	0.0	0.0
12	G	-1.797100E-05	6.047188E-06	0.0	0.0	0.0	0.0
13	G	-9.330203E-06	9.203653E-06	0.0	0.0	0.0	0.0
14	G	-1.072926E-05	9.513754E-06	0.0	0.0	0.0	0.0
15	G	-1.238649E-05	9.737055E-06	0.0	0.0	0.0	0.0
16	G	-1.416498E-05	9.780226E-06	0.0	0.0	0.0	0.0
17	G	-1.583387E-05	9.757401E-06	0.0	0.0	0.0	0.0
18	G	-1.737927E-05	9.693526E-06	0.0	0.0	0.0	0.0
19	G	-1.140789E-05	1.134636E-05	0.0	0.0	0.0	0.0
20	G	-1.240884E-05	1.159569E-05	0.0	0.0	0.0	0.0
21	G	-1.356343E-05	1.177168E-05	0.0	0.0	0.0	0.0
22	G	-1.479736E-05	1.189024E-05	0.0	0.0	0.0	0.0
23	G	-1.596964E-05	1.196551E-05	0.0	0.0	0.0	0.0
24	G	-1.693958E-05	1.197364E-05	0.0	0.0	0.0	0.0
25	G	-1.312575E-05	1.285936E-05	0.0	0.0	0.0	0.0
26	G	-1.378891E-05	1.306498E-05	0.0	0.0	0.0	0.0
27	G	-1.453261E-05	1.317172E-05	0.0	0.0	0.0	0.0
28	G	-1.541139E-05	1.315413E-05	0.0	0.0	0.0	0.0
29	G	-1.644203E-05	1.307381E-05	0.0	0.0	0.0	0.0
30	G	-1.709841E-05	1.311700E-05	0.0	0.0	0.0	0.0
31	G	-1.456007E-05	1.375207E-05	0.0	0.0	0.0	0.0
32	G	-1.487600E-05	1.396931E-05	0.0	0.0	0.0	0.0
33	G	-1.519951E-05	1.410156E-05	0.0	0.0	0.0	0.0
34	G	-1.567164E-05	1.413231E-05	0.0	0.0	0.0	0.0
35	G	-1.655309E-05	1.398787E-05	0.0	0.0	0.0	0.0
36	G	-1.883079E-05	1.400863E-05	0.0	0.0	0.0	0.0
37	G	-1.479920E-05	1.383490E-05	0.0	0.0	0.0	0.0
38	G	-1.503446E-05	1.405976E-05	0.0	0.0	0.0	0.0
39	G	-1.528805E-05	1.420137E-05	0.0	0.0	0.0	0.0
40	G	-1.571020E-05	1.427029E-05	0.0	0.0	0.0	0.0
41	G	-1.644281E-05	1.426557E-05	0.0	0.0	0.0	0.0
42	G	-1.908893E-05	1.417664E-05	0.0	0.0	0.0	0.0
43	G	-1.504434E-05	1.389902E-05	0.0	0.0	0.0	0.0
44	G	-1.519096E-05	1.412871E-05	0.0	0.0	0.0	0.0
45	G	-1.538442E-05	1.428412E-05	0.0	0.0	0.0	0.0
46	G	-1.572186E-05	1.439133E-05	0.0	0.0	0.0	0.0
47	G	-1.647262E-05	1.452205E-05	0.0	0.0	0.0	0.0
48	G	-1.858217E-05	1.432622E-05	0.0	0.0	0.0	0.0
49	G	-1.662713E-05	1.375418E-05	0.0	0.0	0.0	0.0
50	G	-1.624229E-05	1.399237E-05	0.0	0.0	0.0	0.0

SUBCASE 1

D I S P L A C E M E N T   V E C T O R

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
51	G	-1.595462E-05	1.418165E-05	0.0	0.0	0.0	0.0
52	G	-1.580502E-05	1.434093E-05	0.0	0.0	0.0	0.0
53	G	-1.579447E-05	1.440601E-05	0.0	0.0	0.0	0.0
54	G	-1.539706E-05	1.419071E-05	0.0	0.0	0.0	0.0
55	G	-1.831951E-05	1.218421E-05	0.0	0.0	0.0	0.0
56	G	-1.729121E-05	1.236858E-05	0.0	0.0	0.0	0.0
57	G	-1.642107E-05	1.248375E-05	0.0	0.0	0.0	0.0
58	G	-1.560387E-05	1.250836E-05	0.0	0.0	0.0	0.0
59	G	-1.469137E-05	1.241319E-05	0.0	0.0	0.0	0.0
60	G	-1.357170E-05	1.222970E-05	0.0	0.0	0.0	0.0
61	G	-1.962208E-05	9.312997E-06	0.0	0.0	0.0	0.0
62	G	-1.807565E-05	9.458998E-06	0.0	0.0	0.0	0.0
63	G	-1.665046E-05	9.487405E-06	0.0	0.0	0.0	0.0
64	G	-1.530646E-05	9.445842E-06	0.0	0.0	0.0	0.0
65	G	-1.392368E-05	9.362940E-06	0.0	0.0	0.0	0.0
66	G	-1.235641E-05	9.242027E-06	0.0	0.0	0.0	0.0
67	G	-2.057620E-05	5.671970E-06	0.0	0.0	0.0	0.0
68	G	-1.845697E-05	5.604653E-06	0.0	0.0	0.0	0.0
69	G	-1.669262E-05	5.586344E-06	0.0	0.0	0.0	0.0
70	G	-1.509770E-05	5.591974E-06	0.0	0.0	0.0	0.0
71	G	-1.349082E-05	5.584890E-06	0.0	0.0	0.0	0.0
72	G	-1.172221E-05	5.549004E-06	0.0	0.0	0.0	0.0
73	G	-2.124900E-05	0.0	0.0	0.0	0.0	0.0
74	G	-1.871939E-05	8.134718E-07	0.0	0.0	0.0	0.0
75	G	-1.679757E-05	1.361065E-06	0.0	0.0	0.0	0.0
76	G	-1.508419E-05	1.662729E-06	0.0	0.0	0.0	0.0
77	G	-1.337786E-05	1.776209E-06	0.0	0.0	0.0	0.0
78	G	-1.156777E-05	1.789916E-06	0.0	0.0	0.0	0.0