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LINEAR PROGRAMMING in SHIP DESIGN.

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SYNOPSIS

In this paper an attempt is made to study the relative economics of a group of similar merchant ships while in their embryonic stage of preliminary design in order to determine the optimum ship.

Linear programming is proposed as a method of analysis, and a digital computer is suggested and expected to be used for the solution of the final equations of the problem at hand. The equations developed herein are general, in order to cover as many phases and variations of a ship's design and operation as possible. Thus, for each particular design problem, these expressions should be modified to correspond to the requirements and specifications set forth by the shipowner requesting that design.

No attempt is made here to translate the problem into a computer language for the final computations.

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OUTLINE

The work is presented in a sequence which is dictated by the logical execution of the steps necessary to fulfill the purpose of this paper.

The main objective of this analysis is to determine one ship Δ_j which will meet two major requirements. The ship should have characteristics that:

1. will meet the owner's specifications concerning deadweight.
2. will render itself the most profitable ship possible.

Actually, the second requirement is implied in the first. Nevertheless, we can clarify matters considerably if we distinguish between them and label the first one the Design part and the second the Economics part of our analysis. Moreover, it is obvious that both parts are inter-related since they have the ship characteristics as a common variable. But for a specified deadweight, the Design part governs, because the economically optimal characteristics might not satisfy the demands for the storage of the deadweight. Therefore, the Design part should be treated first. However since some theory and explanations will be necessary, the subject matter is presented in the following order.

In the first section, linear programming is explained and the necessary general equations are derived.

In the second section, the Design part and the use of the computer are introduced, and the accumulation of data is described.

In the last section, the Economics part is used to determine the optimum ship according to equations derived in the first part and to results obtained in the second.

General remarks are occasionally made.

INTRODUCTION

A naval architect who is confronted with the design of a new ship can use a few basic equations and arrive at a set of characteristics for a ship which will meet the owner's specified requirements. However, by varying (within limits) one or more of these characteristics, and there is no reason as to why he could not or should not, he can obtain a new set of dimensions defining another ship which will again meet the initial owner's requirements. Obviously then, he can propose many designs, a whole group of ships that can do the same job. Assuming now that all these ships are feasible as far as their construction and operations are concerned, which ship should he propose to be built as the best one? Analyzing the group of all the different feasible ships, he should be able to choose one as the optimum, judged by certain criterion either of economics or performance, or both. Supposing that he is to design a merchant ship whose economics is more important than performance, stability characteristics, for example, the optimum vessel from his group of ships will be that which will meet the architect's economic criterion to the highest degree. Now, what technique can he employ, and what economic criterion can he use to arrive at his choice of the optimum ship?

Of course, a long-hand detailed economic analysis based on some criterion, the capital recovery factor, for instance, can always be carried out which will at the end produce the optimum ship. However, considering the number of feasible ships of the proposed group, this process is tedious and involves a great amount of paper work. Therefore, it is desirable that another method be employed for carrying out this detailed analysis. The new method should give satisfactory results with a minimum amount of work and cost.

One of the techniques that might answer these demands is linear programming. This method, as applied to the design, construction and operations of the ships, is not in itself simple. Nevertheless, it can be used to analyze the shipping business accurately and efficiently, because it can be programmized and put through a digital computer which will give the proper answer after a reasonable amount of time and, as will be shown later, with a reasonable cost. Considering these advantages of linear programming, that is, its accuracy and wide range of analysis, the amount of time, and the comparatively small amount of manual work and cost required, we shall develop linear programming in such a way that it can be used in the field of Naval Architecture.

PART I

1. LINEAR PROGRAMMING

Linear programming, in general, as the name implies treats only linear functions, and is a mathematical method which can be used to maximize or minimize a given function whose variables are subject to a set of constraints in the form of inequalities. A typical mathematical problem solved by linear programming can be stated as follows:

Maximize or minimize the function

$$f = c_1X_1 + c_2X_2 + c_3X_3 + \cdots + c_nX_n \qquad (1)$$

subject to

$$\begin{aligned} a_{11}X_1 + a_{12}X_2 + a_{13}X_3 + \cdots + a_{1n}X_n &\leq \varepsilon_1 \\ a_{21}X_1 + a_{22}X_2 + a_{23}X_3 + \cdots + a_{2n}X_n &\leq \varepsilon_2 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots & \qquad (2) \\ a_{m1}X_1 + a_{m2}X_2 + a_{m3}X_3 + \cdots + a_{mn}X_n &\leq \varepsilon_m \end{aligned}$$

where

all c's, a's and ε's are ~~constraints~~ constants

and X's are the variables.

Weak inequalities are acceptable. (2)*

The mathematical solution and proof of linear programming will not be given here as being outside the scope of this paper. However, it should be noted at this point, that one condition for this problem to have a solution is that all variables included in equation (1) and the set of inequalities (2) should be non-negative, that is, they can assume any positive value or else be zero.

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* Numbers in parentheses refer to bibliography at the end of this paper.

In economics, linear programming is used to determine the optimum production conditions and output levels pertinent to free enterprise systems and purely competitive firms. Whenever, there is a variety of processes that a firm can use to produce a certain good, linear programming analysis is the best adaptable one for determining the most economical process and the most profitable, the optimum, level of output of that good.

The method is primarily based on the neoclassical theory of the firm, that is, the marginal analysis of the firm, and in most cases ignores the quality of the product, assuming it to be the same for the outputs of all the available processes. More about this analysis will be said later, when some of the concepts involved will be defined and explained. At this point, we may add that for an economics problem equation (1) is usually the criterion of profitability of a process and the system of inequalities (2) expresses the input constraints as is explained later. We shall proceed with the introduction of the method in Naval Architecture by defining some related concepts of linear programming.

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2. DEFINITIONS

A "process" is defined as a particular method for manufacturing a certain product by using definite quantities of a number of inputs. The distinction between processes of a production program is based on the fact that both the quantities of each input used in the production of one unit of output, and the number of inputs utilized in each process are characteristic of that process. This leads to the definition of the "unit productivity" of each input designated by a_{ij}, as the quantity of the i input absorbed in the production of one unit of output of the j process. Thus, knowing the process j we know a_{ij}, and vice versa.

In Naval Architecture, however, and in this paper, the above definition of a process should and will be modified. We define as a process, any ship j which will have definite characteristics for a certain displacement Δ_j. Here the total displacement of a ship has been chosen as the output, each long ton being the unit of that output. Hence, the unit productivities will be expressed as the amount of the inputs absorbed in the production of one long ton of total displacement. Since linear programming determines not only the optimum process but also the optimum level of its output, we shall manipulate the total displacement of a group of ships and determine the optimum one to satisfy the owner's requirements. Later in the paper, we shall elaborate further on our choice of the displacement as an output.

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3. DESIGN CONSIDERATIONS

Any production program implies that there is substitution of inputs between processes, but there is no such substitution between the inputs of the same process. This implication has a number of consequences in our analysis, the most important of them being the fact that we cannot compare two identical ships, one having a geared turbine installation and the other a diesel, in order to determine the merits and profitability of each installation. This is so because the two identical ships with different machinery units are two different products, two different outputs. The fact, however, that these two ships are considered as two different products, suggests the following method of comparison, if at all desired, between the diesel and the steam turbine. We can set up two linear programs, one for each ship, having among their inputs, the first a steam turbine unit productivity and the second a diesel. Then following the steps outlined in this paper, we can determine the optimum ship with the diesel installation and the optimum one with the steam turbine. These two ships will probably be of different characteristics. Next, we can compare these two ships by using an economic criterion, like the capital recovery factor, and determine which of the two is the most profitable. This same method can be followed when other minor features are to be investigated and compared. With the same token, we cannot compare a steel ship with an identical aluminum one. However, we will always arrive at the optimal ship, in cases like the ones mentioned above, by using two or more linear programs, since we are really talking about two different products or outputs, as far as linear programming is concerned.

4. THE MODEL

With this background, we can now list some of the variables and the parameters of linear programming, as defined and related in a Naval Architecture economics problem of designing and operating a merchant ship.

Assume that the problem involves the analysis of combining i different inputs and producing j different ships.

VARIABLES

I_j = the number of dollars invested by the owner in his j ship.

c_j = the number of dollars' worth of cost incurred annually by the owner in his j ship.

p_j = the number of dollars' worth of net profit earned annually by the owner in his j ship.

σ_j = the number of physical units of pay-load capacity rented annually by the owner in his j ship.

x_{ij} = the number of physical units of the i input used by a shipyard for constructing the owner's j ship.

PARAMETERS

a_{ij} = the unit productivity of the i input defined as the number of physical units of the i input absorbed per physical unit of total displacement of the j ship, a parameter not to be manipulated.

Π_i = the price of a unit of the i input, not to be manipulated.

Π_o = the price of a unit of the pay-load capacity, cargo rate, not to be manipulated.

Δ_j = the number of physical units of total displacement of the j ship, a parameter to be manipulated.

τ_j = the number of round trips of the j ship per year.

Then for each ship we can write the following equations.

1. Investment equation: (Construction or manufacturing costs).

$$I_j = \sum_{i=1}^{m} (\Pi_i x_{ij}) \quad \text{for } j = 1, 2 \ldots m. \tag{3}$$

Later this equation is somewhat modified in order to correspond to the shipyard bill for the construction of the vessel. The shipyard bill is a part of the cost breakdown which is widely used today.

2. Cost equation: (Average annual operating costs).

$$c_j = \tau_j \left(\sum f(SHP)_j + \sum f(\Delta)_j + C \right) \quad j = 1, 2, \ldots n \tag{4}$$

where C covers miscellaneous expenses.

3. Income equation: (Average annual income).

$$\text{Income} = \Pi_o \sigma_j \tag{5}$$

$$\sigma_j = C_c (\Delta - \Delta_{light}) \quad j = 1, 2, \ldots n \tag{6}$$

where C_c is the capacity coefficient.

4. Profits equation:

$$p_j = \Pi_o \sigma_j - c_j \quad j = 1, 2, \ldots n \tag{7}$$

5. Input-output equation:

$$x_{ij} = a_{ij} \Delta_j \quad \begin{array}{l} i = 1, 2, \ldots m \\ j = 1, 2, \ldots n \end{array} \tag{8}$$

These basic equations will later enable us to formulate the equations necessary for our analysis. We shall start with our input relationships.

5. INPUTS

As inputs we shall consider the structural material of the ship, all measured in long tons, as shown in TABLE 1, below.

TABLE 1 Inputs.

Input item	Subscript	Designation
Construction Steel	1	W_s
Hull Outfitting Materials	2	W_o
Hull Engineering Materials	3	W_e
Machinery Materials	4	W_m

It is important to emphasize that the man-hours and the machine-hours which will be required for the construction of the j ship should be included here as inputs. However, in order to reduce the number of our equations and therefore the number of solutions for a given number of displacements to be analyzed, we omit these two items from the group of inputs, and we include their costs instead in the cost of the four inputs above as $ per ton of each material.

6. INPUT CONSTRAINTS

According to equation (4) the optimum ship j will absorb a quantity $a_{ij}\Delta_j$ of each input i. The utilized amount of any input, however, is limited for at least two reasons.

First, there might be practical limitations. For instance, the amount of steel required to construct a ship a mile long might be purchased, but we just cannot build a ship a mile long. Hence, we have to limit the amount of steel to be used to some reasonable quantity, determined, with a good margin, from similar existing ships.

Second, there might be "fixed plant" limitations. If, for example, there is only one welding machine in a shipyard, we cannot count but to use only one machine-hour per hour. Or if the size of a dry dock limits the size of a ship that can be built in it, it limits the amount of the inputs. If, further, the steel mills are on strike, the available quantity of steel that a shipyard can purchase is perhaps only 5,000 long tons, in spite of the fact that the shipyard might have a pending construction contract calling for 10,000 tons of steel.

In our study, although we can assume that a shipyard can employ at any time all the needed man-power, supply the machinery, and purchase the necessary quantities of each input, we are still forced to place an upper limit on all our inputs. Of course, a naval architect does not carry out a cost study for a shipyard, when designing a ship, in order to worry about the yard's practices. Nevertheless, in this type of study, we should concern ourselves with shipyard techniques and capacities so that we may take full advantage of them when we limit the inputs.

Since these limits should not necessarily be exact, experience is a good guide to the naval architect. That is, if he has a "hunch" that the optimum ship will require, say about 5,000 tons of steel he can safely assign 7,000 tons as an upper limit for the steel input. Actually, he may, in this case, assign with his pleasure, the amount of 200,000 tons of steel. This will not affect his analysis, as will be explained in the discussion of the disposal process, because he may very well assume that the shipyard can sell the extra steel with no loss at all on its part in the transactions. Actually, the shipyard never bought the steel that the architect worries about.

Thus, we may say that for the optimum ship, the weight of steel, hull outfitting, hull engineering, and machinery should be less or at most equal to the available or specified quantities of W_s, W_o, W_e, W_m, respectively. Equation (8) then becomes:

$$\begin{aligned} a_{11}\Delta_1 + a_{12}\Delta_2 + \cdots + a_{1j}\Delta_j &\leqslant W_s \\ a_{21}\Delta_1 + a_{22}\Delta_2 + \cdots + a_{2j}\Delta_j &\leqslant W_o \\ a_{31}\Delta_1 + a_{32}\Delta_2 + \cdots + a_{3j}\Delta_j &\leqslant W_e \\ a_{41}\Delta_1 + a_{42}\Delta_2 + \cdots + a_{4j}\Delta_j &\leqslant W_m \end{aligned} \qquad (9)$$

The inequalities of system (9) are the input constraints to which the criterion of profitability is subject.

7. DISPOSAL PROCESS

For a geometric solution of linear programming, the inequalities expressing the input constraints are very easy to handle. However, for an algebraic solution these inequalities complicate matters considerably, and it would be desirable to have them transformed into equalities. The introduction of the disposal processes will make such a transformation possible.

The disposal process can be defined as a process which has one input and no output at all. Let this process be designated by δ (delta). Then, delta represents the quantity of the i input disposed of as waste or idling material not utilized for the production of the optimum quantity of the output. It is equal to the difference of the initially specified quantity of the i input and the amount absorbed by the output of the optimum process.

Generally, it is assumed that the disposal process has only a certain cost attached to it, and no profit. The cost is taken to be equal to the value of the input to be disposed of, and is given by the expression:

$$C_{(n+i)} = \Pi_i \, x_{i(n+i)} \qquad (10)$$

If linear programming is applied to a firm with a fixed plant, the disposal process might include idling man-power or machinery, and in this case the cost of this process should be carefully considered. In our problem, however, we can assume that a shipyard can employ only the needed man-power and purchase exactly the necessary amount (with an allowance for scrap and losses) of the input materials to be utilized for the construction of the optimum ship. Therefore, we can ignore the cost of this disposal process entirely, and we only use the concept for the sake of the equalities.

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8. INPUT-OUTPUT EQUATIONS AND RELATIONS

Introducing the disposal process δ_i into system (9) we obtain the following equations:

$$
\begin{aligned}
a_{11}\Delta_1 + a_{12}\Delta_2 + \cdots + a_{1j}\Delta_j + \delta_1 &= W_s \\
a_{21}\Delta_1 + a_{22}\Delta_2 + \cdots + a_{2j}\Delta_j + \delta_2 &= W_o \\
a_{31}\Delta_1 + a_{32}\Delta_2 + \cdots + a_{3j}\Delta_j + \delta_3 &= W_e \\
a_{41}\Delta_1 + a_{42}\Delta_2 + \cdots + a_{4j}\Delta_j + \delta_4 &= W_m
\end{aligned}
\qquad (11)
$$

where

a_{ij} = unit productivity of the i input,

δ_i = disposed quantity of the i input in long tons,

Δ_j = output, the total displacement, to be manipulated.

i = 1, 2, 3, 4.

j = 1, 2, ... j'.

From mathematics, in addition to the conditions stated previously, the system (11), in order to have a solution, should be a nondegenerate one.

When linear programming is applied in Naval Architecture, it has a peculiar and important characteristic which should be brought out at this point. This characteristic is the relation between the output and the unit productivity of each input. Usually, when the level of output for each process is specified, the quantities of each input absorbed by that output are also specified. For example, if a television set requires only one amplifier, the unit productivity of amplifiers is one (1) and a simple statement that a firm produces one thousand television sets per month automatically means that a firm uses one thousand amplifiers per month. In Naval Architecture, however, a similar statement is almost meaningless, because by knowing the displacement, that is the output, we do not know the quantity of the inputs absorbed by that displacement unless we know the characteristics of the ship. For instance, the steel and the machinery weights of a ship depend on the cubic number and the shaft horsepower of that ship. By knowing only the total displacement of that ship, we have no idea what these two items will be at all.

At this point, we can draw an important conclusion, that is, when we calculate the unit productivity a_{ij} in the next section, we must express them in terms of ship characteristics rather than total displacements.

A consequence of the relationships between the total displacement and the ship construction materials will be taken up in section 10 . This consequence affects the method of solution of equation (11) and will be better understood if the expressions for the individual unit productivities are derived first.

9. CALCULATIONS OF UNIT PRODUCTIVITIES

In order to solve system (11), we must compute the unit productivity a_{ij} for each of the four inputs.

9.1 For the First Input, a_{ij}

Considering the first input, the construction steel, a_{ij} is the number of tons of steel per ton of total displacement.

By definition then,

$$a_{ij} = \frac{\text{Weight of steel}}{\text{Displacement}} = \frac{W_s}{\Delta} \qquad (12)$$

The steel weight of a ship is conveniently calculated for our purposes by the cubic number method. A similar ship is selected and the steel weight coefficient is calculated. Then the hull steel weight of the proposed ship is computed and corrected for any differences in dimensions and construction between the proposed ship and the existing similar one. For the calculations we can use either an overall weight coefficient or one for the main hull and one for the superstructure, depending on the degree of similarity between the two ships, and also on the accuracy desired. For our comparative purposes of the similar ships, however, an overall steel weight coefficient will give satisfactory results. It should be emphasized that Telfer's or any other method of steel weight calculations could be used equally well.

Hence, the steel weight of the hull is:

$$W_s = C_s \times C_{C_b} \times C_{L/D} \times \frac{LBD}{100} \qquad (13)$$

where

C_s = steel weight coefficient determined from a similar ship called basis ship.

C_{c_b} = correction factor for block coefficient differences.

$C_{L/D}$ = correction factor for L/D ratio differences.

The last two correction factors are:

$$C_{C_b} = \frac{(1 + 0.5\,C_b)_p}{(1 + 0.5\,C_b)_b}$$

and

$$C_{L/D} = \sqrt{\frac{(L/D)_p}{(L/D)_b}}$$

Where subscripts p and b refer to proposed design and basis ship respectively.

The displacement Δ is given by:

$$\Delta = \frac{L B d C_b}{35}$$

Substituting Δ we obtain:

$$a_{ij} = \frac{C_s \times C_{C_b} \times C_{L/D} \times \dfrac{L B D}{100}}{\dfrac{L B d C_b}{35}}$$

$$= \frac{35\, C_s \times C_{C_b} \times C_{L/D} \times L \times B \times D}{100\, L B d C_b}$$

For the j ship:

$$a_{ij} = \frac{35\, C_s \dfrac{(1 + 0.5\,C_b)_{pj}}{(1 + 0.5\,C_b)_b} \times \sqrt{\dfrac{(L/D)_{pj}}{(L/D)_b}} \times L_j B_j D_j}{100\, L_j B_j d_j C_{bj}} \qquad (14)$$

Now if we use the following designations for the j ship of a family:

$$\frac{L_j}{B_j} = k_{1j}$$

$$\frac{L_j}{D_j} = k_{2j}$$

$$\frac{L_j}{d_j} = k_{3j}$$

equation (14) becomes:

$$a_{1j} = \frac{35\, C_s \dfrac{(1+0.5\,C_b)_{Pj}}{(1+0.5\,C_b)_b} \times \sqrt{\dfrac{(L/D)_{Pj}}{(L/D)_b}} \times \dfrac{L_j^3}{k_{1j}\, k_{2j}}}{100 \times \dfrac{L^3_j}{k_{1j}\, k_{3j}}\, C_{bj}}$$

or

$$a_{1j} = \frac{0.35\, C_s\, k_{3j}\, \dfrac{(1+0.5\,C_b)_{Pj}}{(1+0.5\,C_b)_b} \times \sqrt{\dfrac{(L/D)_{Pj}}{(L/D)_b}}}{k_{2j}\, C_{bj}} \qquad (15)$$

Equation (15), hence, is the expression of the unit productivity of the first input for any ship j.

Equation (15) will be applicable when the structure of the ship is to be built only out of steel. In cases where it is desirable to use aluminum for the superstructure, we should include aluminum as a fifth input, with a unit productivity of a_{5j}. The weight of aluminum to be absorbed by any ship j will be a function of the cubic number of the superstructure of that ship. Assuming similar superstructures for all ships, we can calculate a coefficient for the aluminum weight from a basis ship, and express the quantity of aluminum to be absorbed by a ship j as a function of that coefficient and the cubic number of superstructure of the j ship. In fact, this breakdown could be used for steel ships also, where more accuracy is desired.

9.2 For the Second Input, a_{2j}

Considering next the second input, the hull outfitting materials, by definition we have:

$$a_{2j} = \frac{\text{Weight of hull outfitting materials}}{\text{Total displacement}} = \frac{W_o}{\Delta} \quad (16)$$

The weight of hull outfitting is assumed to be a function of the cubic number and the total number of persons aboard the ship, or:

$$W_o = C_o \times \frac{LBD}{100} + C_{10} P_t \quad (17)$$

where

C_o = outfitting weight coefficient, determined from a basis ship.

C_{10} = an estimated coefficient giving the outfitting weight per person, and

P_t = total number of persons aboard the ship.

The total number of persons aboard the ship includes both passengers and crew. The number of passengers N_p for a particular family of ships, can be expressed in terms of their displacement.(10) By assigning a certain number of passengers to a prototype ship, we can assume that N_{pj} for any other ship of the same family will vary in some fashion with L, like λ^3, for instance. For a passenger-cargo ship, the variation of the number of passengers with the total displacement-$L \propto \Delta$-will be similar to the one shown in Fig. 1.(10) Since the relation will be linear, we can write:

$$N_{pj} = C_2 (\Delta_j)$$

Fig. 1 Number of passengers for passenger and passenger-cargo vessels

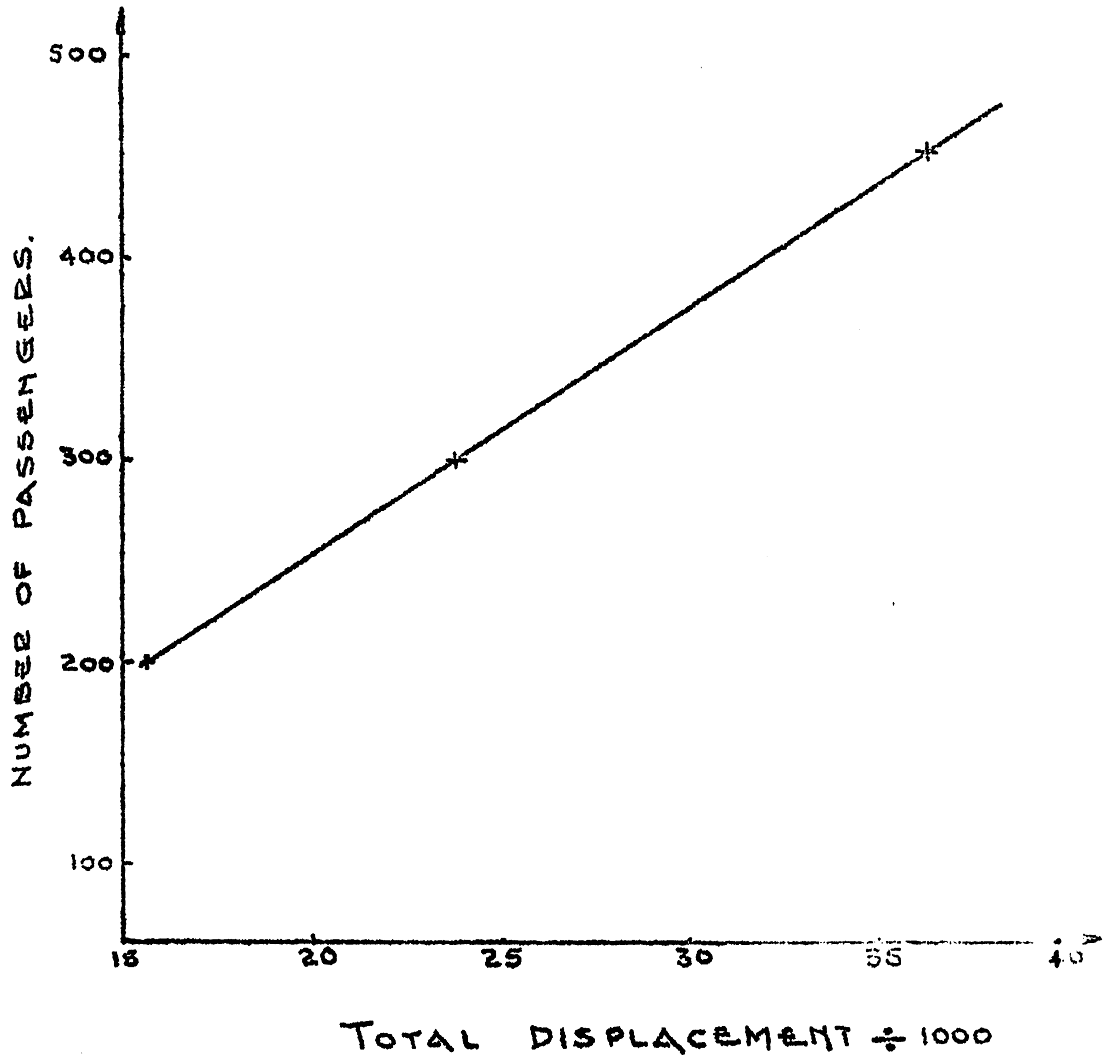

FIG. 1

Fig. 1 , shows the variation of the number of passengers with the total displacement for a particular family of passenger-cargo ships.(1o)

By substitution, the previous equation becomes:

$$N_{pj} = C_2 \frac{L_j^3 \; C_{bj}}{35 \; K_{1j} \; K_{3j}}$$

where C_2 should be computed from the curve of N_p versus Δ.

The number of crew N_c, on the other hand, depends on the S H P of the ship, the deck area, and the number of passengers aboard. N_c then includes, (a) engine hands N_e, (b) deck hands N_d, (c) staff N_o, and (d) stewards N_s, or

$$N_{cj} = N_{ej} + N_{dj} + N_{oj} + N_{sj} \qquad (18)$$

The number of hands of each department, in the form of an equation, can be obtained as follows:

For the engine room,

$$N_e = f_1 \text{ (S H P)},$$

For the deck,

$$N_d = f_2 \text{ (L x B)},$$

For the staff,

$$N_o = f_3 (N_p), \text{ and}$$

For the stewards,

$$N_s = f_4 (N_p).$$

Appropriate functions f_1, f_2, f_3, and f_4 are found from data on existing similar ships, and for passenger and passenger-cargo vessels the corresponding graphs are shown in Figures 2, 3, 4, and 5.

For illustrative purposes, these graphs were programmized and fed into the IBM 704 computer of the University of Michigan, with the instructions that the computer will derive equations to fit each curve. The results are shown next.

Fig. 2 Engine room hands for passenger and passenger-cargo vessels

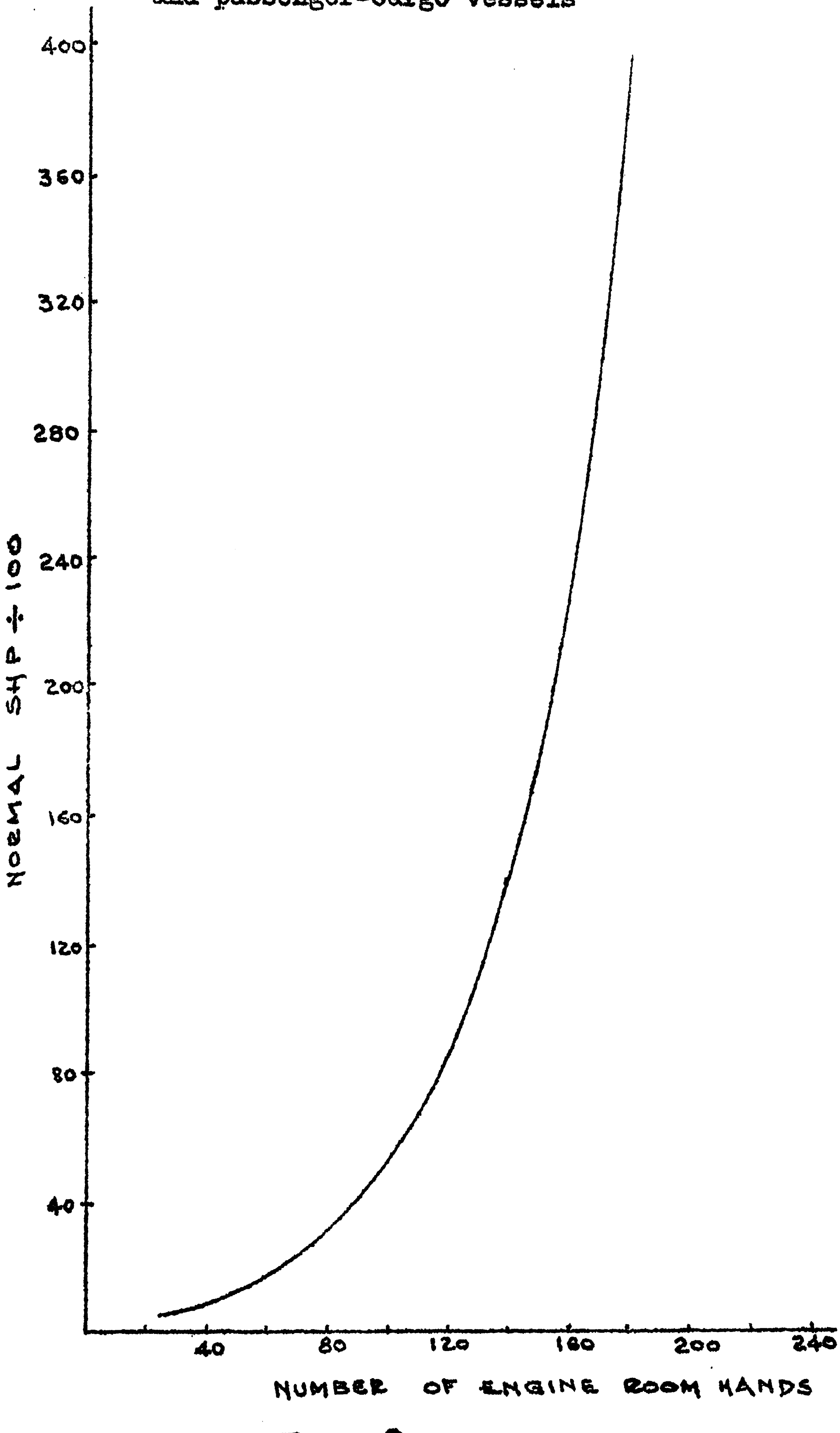

FIG. 2.

Fig. 3. Deck hands for passenger and passenger-cargo vessels.[10]

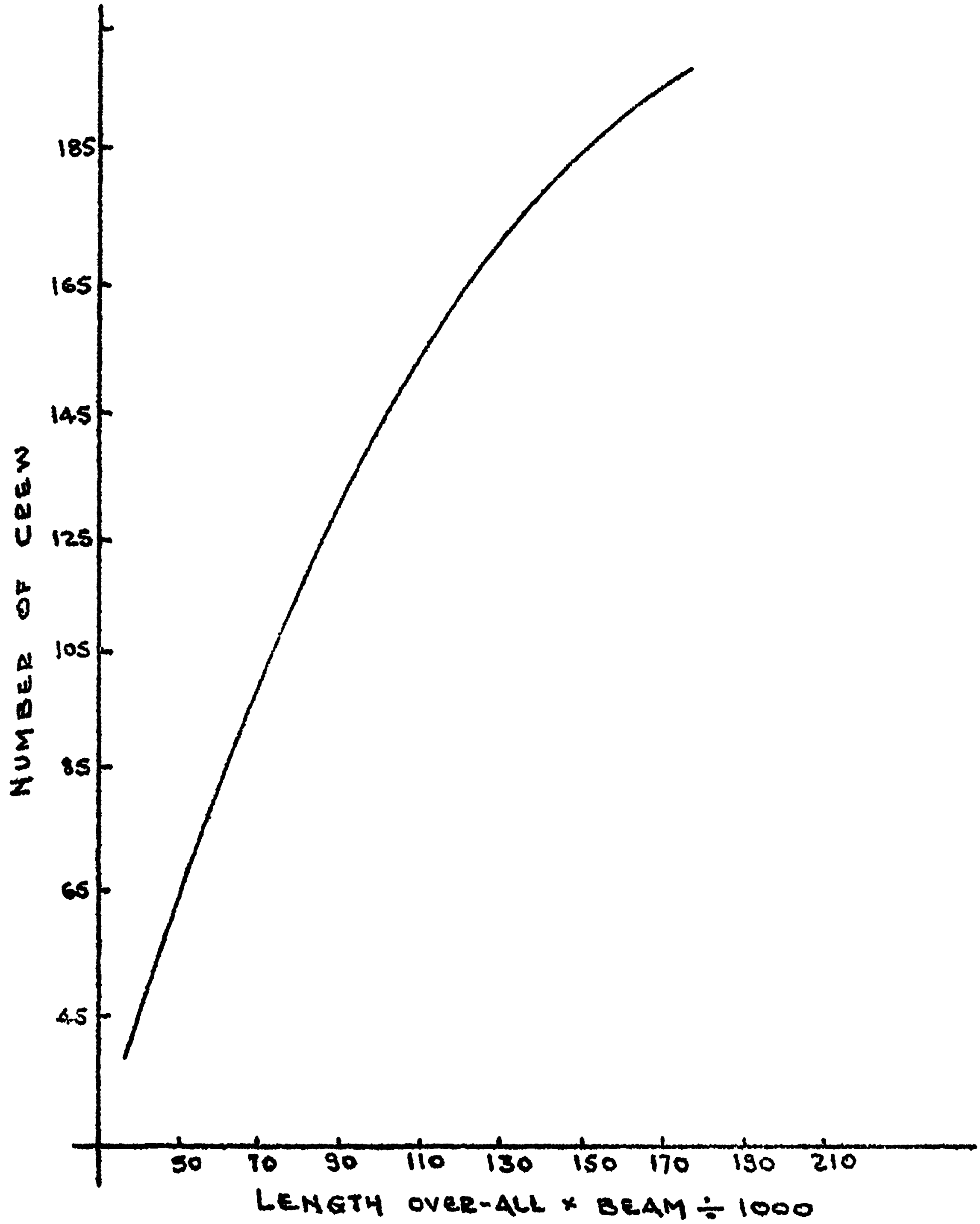

FIG. 3

FIG 4. NUMBER OF STAFF FOR PASSENGER AND PASSENGER-CARGO VESSELS.[10]

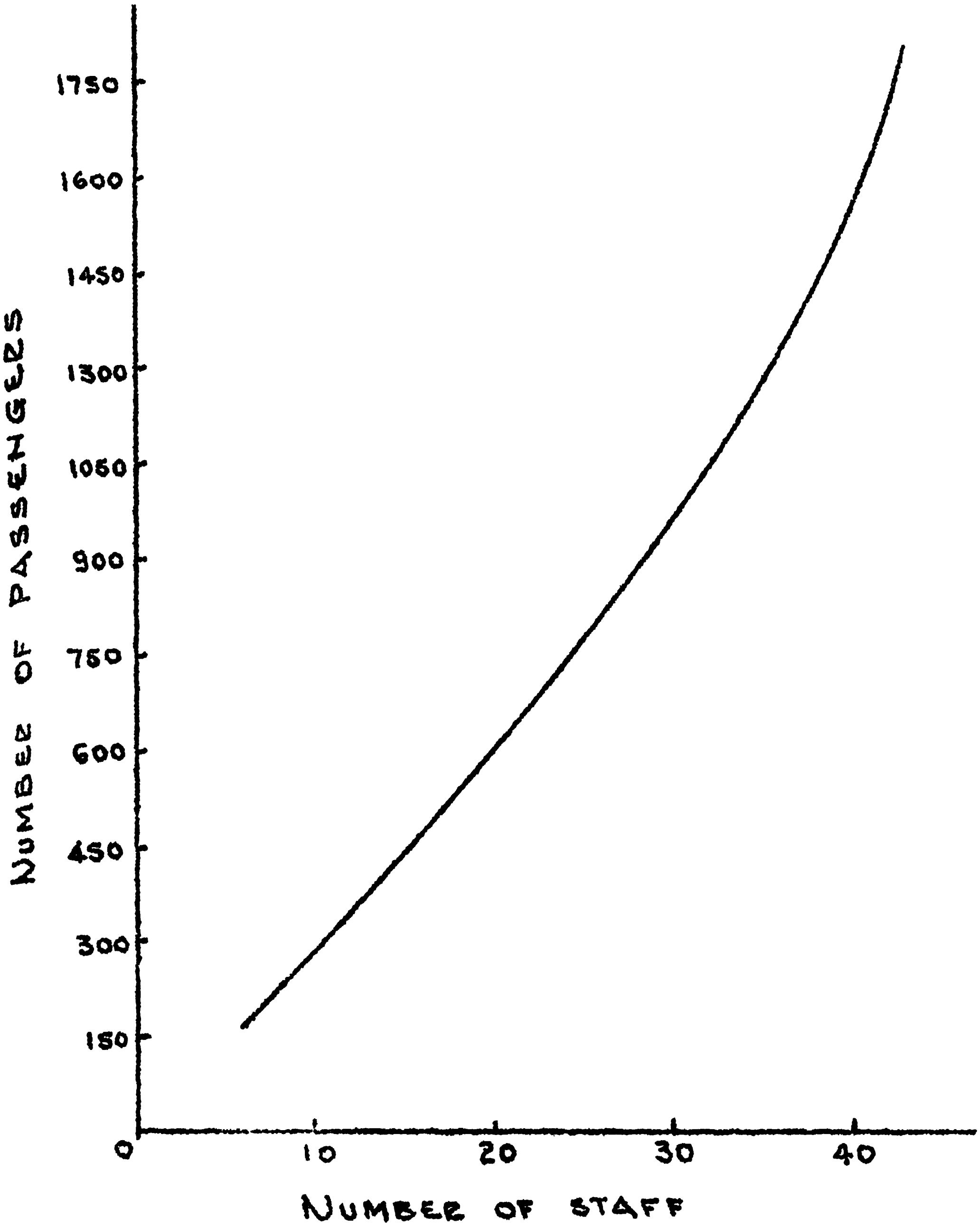

FIG. 4.

FIG. 5. Number of stewards for passenger and passenger-cargo vessels.[10]

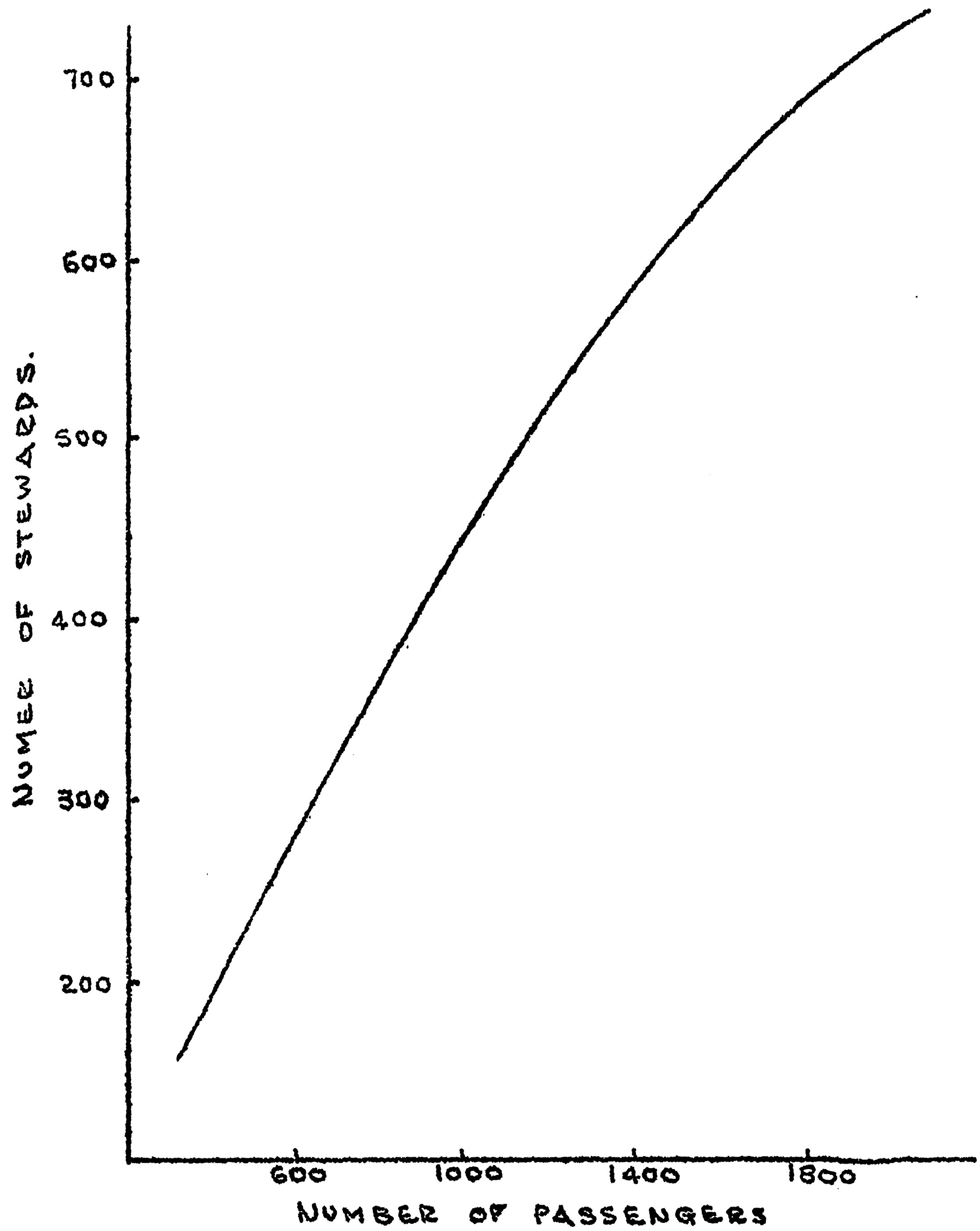

FIG. 5.

The equations which the computer produced are the following:

$$N_e = -123.48 + 58.3(SHP)^{0.25} - 2.55(SHP)^{0.5} \qquad (19)$$

$$N_d = -97.6 + 23.3(L \times B)^{0.5} - 3.85 \times 10^{6}(L \times B)^{-4} \qquad (20)$$

$$N_o = -22.73 + 1270.75(N_p)^{-1} - 4 \times 10^{-13}(N_p)^{4} + 1.66(N_p)^{0.5} \qquad (21)$$

$$N_s = -518 + 3451(N_p)^{-0.5} + 26.6(N_p)^{0.5} \qquad (22)$$

Returning to equation (17), we can write:

$$W_o = C_o \times \frac{LBD}{100} + C_{10}(N_p + N_c)$$

or

$$W_o = C_o \frac{L^3}{100 k_1 k_2} + C_{10}(N_p + N_e + N_d + N_o + N_s)$$

Upon substitution, and for the j ship, this becomes:

$$W_o = \frac{C_o L_j^3}{100 k_{1j} k_{2j}} + C_{10}\left[C_2 \frac{L_j^3 C_{bj}}{35 k_{1j} k_{3j}} + f_1(SHP)_j + f_2(L_j \times B_j) + f_3\left(C_2 \frac{L_j^3 C_{bj}}{35 k_{1j} k_{3j}}\right) + f_4\left(C_2 \frac{L_j^3 C_{bj}}{35 k_{1j} k_{3j}}\right)\right]$$

Finally according to equation (16) $a_{2j} = \frac{W_o}{\Delta}$. Expressing Δ in terms of dimensional characteristics and dividing each term separately, we obtain:

$$a_{2j} = \frac{0.35 C_o k_{3j}}{k_{2j} C_{bj}} + \frac{35 C_{10} C_2 k_{1j} k_{3j}}{L_j^3 C_{bj}}\left(\frac{L_j^3 C_{bj}}{35 k_{1j} k_{3j}}\right) + \frac{C_{10} k_{1j} k_{3j}}{L_j^3 C_{bj}} \times f_1(SHP)_j + \frac{C_{10} k_{1j} k_{3j}}{L_j^3 C_{bj}} \times f_2\left(\frac{L_j^2}{k_{1j}}\right) + \frac{C_{10} k_{1j} k_{3j}}{L_j^3 C_{bj}} \times f_3\left(\frac{C_2 L_j^3 C_{bj}}{35 k_{1j} k_{3j}}\right) + \frac{C_{10} k_{1j} k_{3j}}{L_j^3 C_{bj}} \times f_4\left(\frac{C_2 L_j^3 C_{bj}}{35 k_{1j} k_{3j}}\right) \qquad (23)$$

Cancellation of equal characteristics in some of the above terms, has been purposely omitted in order to save the generality of the equation, for C_{10} might not always be a constant coefficient. Note also, that in some cases, of ship design, some or all of the categories of crew hands might be kept constant without serious error. The terms, then, of the final equation are somewhat simplified.

Equation (23), thus, is the unit productivity a_{2j} of the second input, that of the hull outfitting materials.

****

9.3 For the Third Input, a_{3j}

For the unit productivity of the third input, the hull engineering materials, by definition we have:

$$a_{3j} = \frac{\text{Hull engineering materials}}{\text{Total displacement}} = \frac{W_e}{\Delta} \qquad (24)$$

The weight of hull engineering materials is assumed to be a function of the cubic number of the ship, or

$$W_e = C_e \times \frac{LBD}{100}$$

where C_e = hull engineering weight coefficient, determined from a basis ship.

Substituting the expressions for the engineering weight and the displacement in equation (24), we obtain:

$$a_{3j} = \frac{0.35\, C_e\, L_j\, B_j\, D_j}{L_j\, B_j\, d_j\, C_{bj}}$$

or

$$a_{3j} = \frac{0.35\, C_e\, D_j}{d_j\, C_{bj}}$$

which becomes

$$a_{3j} = \frac{0.35\, C_e\, k_{3j}}{k_{2j}\, C_{bj}} \qquad (25)$$

Equation (25) gives the unit productivity of the third input, the hull engineering weight.

****

9.4 For the Fourth Input, a_{4j}

For the machinery materials we define a_{4j} as the total machinery weight per ton of total displacement. This machinery weight could include not only the main propulsion units, but also all auxilliaries such as equipment for hotel services, refrigeration, heating and ventilation, etc. The latter group depends, of course, on the number of persons aboard the ship and may be accounted for by the use of a coefficient. Hence,

$$a_{4j} = \frac{\text{Weight of machinery}}{\text{Total displacement}} + C_4 P_t \qquad (26)$$

where C_4, a coefficient, is to be empirically determined, see TABLE 2.

P_t is the number of persons served.

The weight of machinery is a function of the shaft horsepower required for the propulsion of the ship. As such, it is quite accurately given by:

$$W_m = 242 \sqrt{\frac{SHP}{1000}} \qquad (27)$$

or

$$W_m = 165 + 91 \times \frac{BHP}{1000} \qquad (28)$$

TABLE 2. Average Fuel Consumption Rates [10]

1. Passenger ships:

For propulsion purposes	0.48 lbs/S H P hour
For hotel services	1.29 lbs/S H P hour

2. Passenger-cargo vessels:

For propulsion purposes	Varies with S H P
For hotel services	1.25 lbs/person hour
For refrigeration	1 lb/1000 ft.^{3} hour
For cargo handling in ports	11 lbs/hatch hour

3. General cargo ships:

For propulsion purposes	0.52 lbs/S H P hour
For others	Same as passenger-cargo ships

4. Tankers:

For propulsion purposes	0.52 lbs/S H P hour

5. Ore carriers:

For propulsion purposes	0.52 lbs/S H P hour

UNIT WEIGHT OF MACHINERY FOR HOTEL SERVICES.

C_4 = 0.185 to 0.159 tons/person varying with size of plant.

depending on the type of machinery to be installed. Equation (27) gives the weight of a steam turbine installation, whereas equation (28) gives that of a diesel. For the moment, let us say that

$$W_m = C' \times f_s(S\ H\ P)$$

where C' could be a coefficient for equation (27) or (28).

The S H P, next, is a function of the effective horsepower, E H P, and will vary for each hull. With an appropriate propulsive coefficient, (P.C.) we can write:

$$S\ H\ P = (P.C.)E\ H\ P$$

where (P.C.) will depend on the type and hull of the analyzed ship.

The E H P should now be considered. The experimental results of model tests are the only sources from which data for the E H P calculations can be obtained. The two main test data generally used are those of Taylor for twin screw vessels and those of Series-60 for single screw hulls. For each design problem the resistance data pertinent to the design hull should be used. For either twin or single screw vessels the E H P is given by:

$$E\ H\ P = C_5 \times S \times V_k^3 \times C_t \qquad (29)$$

where

C_5 = a constant = $0.00438 \times \rho$,

S = the wetted surface of the vessel, and

C_t = the total resistance coefficient as determined from the model test data.

The wetted surface, using Taylor's expression is:

$$S = C_s \times \sqrt{\Delta \cdot L_{WL}} = C_s \cdot \sqrt{\Delta (L - L')} \qquad (30)$$

where

C_s = the wetted surface coefficient, and

$L' = (L_{WL} - L)$, assumed constant for all ships.

C_s varies with the mid-ship section coefficient C_m and the beam-draft ratio. For our analysis, however, without serious error we can assume a value of $C_m = 0.95$, which is an average for the hulls used for merchant ships, and express C_s as a function of the beam-draft ratio. Again for illustrative purposes, the values for wetted surface coefficients at $C_m = 0.95$ and different beam to draft ratios, were read off(12) and plotted as shown in Fig. 6. The faired points were then programmized according to the stepwise regression method, and equation (31) was obtained.

$$C_s = 2.45 + 0.0014\left(\frac{B}{d}\right)^4 + 4.48\left(\frac{B}{d}\right)^{-4} \qquad (31)$$

The calculation of C_t, on the other hand, is one of the most tedious steps of this analysis. Since everything has to be programmized for the computer in the form of equations, C_t must be expressed as a function of some variables for each family of ships, so that the computer can calculate the appropriate value of C_t, for each ship of that family. The total resistance coefficient C_t, following Froude's method, is the sum of the friction resistance coefficient C_f, the residual resistance coefficient C_r, and a flat roughness allowance, or

$$C_t = C_f + C_r + C_a. \qquad (32)$$

C_a is taken as 4×10^{-4} in accordance with the ATTC 1947 recommendations.

FIG. 6. Wetted surface coefficient C. at $C_m = 0.95$ (12)

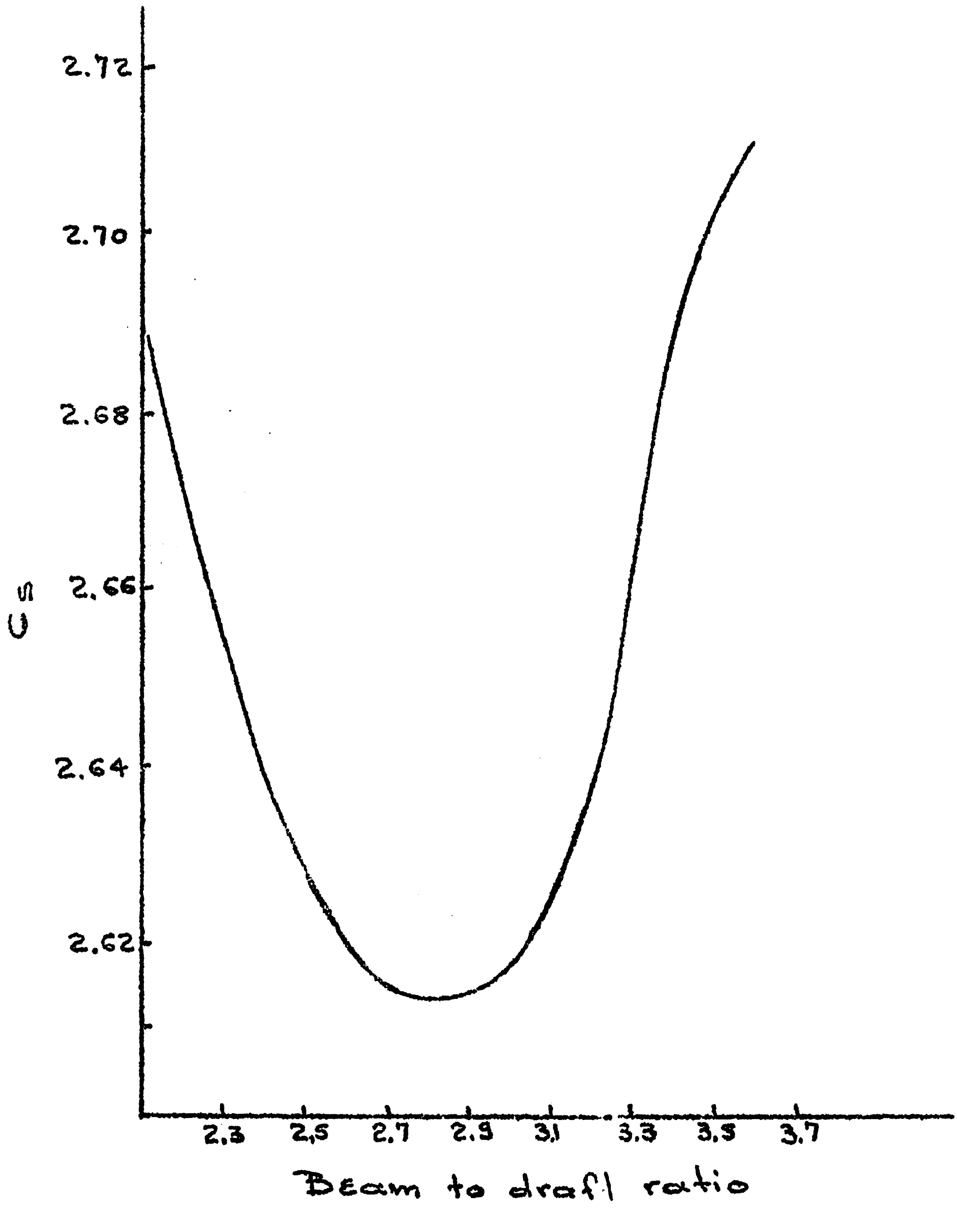

FIG. 6.

C_f in the form of an equation can be obtained by any formulation, and for this work that of Prandtl and von Kármán has been chosen because of its simplicity. Hence,

$$C_f = 0.072 \left(\frac{v \cdot L_{WL}}{\nu}\right)^{-\frac{1}{5}}$$

or

$$C_f = 0.072 \left[\frac{1.689 V_k (L+L')}{\nu}\right]^{-\frac{1}{5}} \qquad (33)$$

where

v = the speed of the ship in feet per second,

V_k = the speed of the ship in knots,

L_{WL} = the length of the load waterline in feet,

L' = an estimated difference between L_{WL} and L, constant for all ships of a family, and

ν = the kinematic viscosity of either salt or fresh water.

Now for the determination of C_r, in order to be more specific in our discussion, let us distinguish two cases, e.g., that of a single and that of a twin screw hull. We shall carry our the analysis for only twin screw vessels using Taylor's data, with the understanding that the general statements made about twin screw ships hold true for single screw also. Of course, Series - 60 resistance data have been plotted differently than Taylor's. However, both systems are equivalent, and therefore the difference in plotting does not prevent us from deriving similar equations for both cases.

Hence, let us attempt to calculate C_r for twin screw vessels. Obviously, C_r depends on many variables, namely, the beam to draft ratio B/d, the block or the longitudinal coefficient C_l, the speed to length ratio , and the volumetric coefficient C . Our answer must be in the form of an equation such as:

$$C_r = f_6\left(\frac{B}{d}, C_l, \frac{V_k}{\sqrt{L}}, C_\nabla\right)$$

Unfortunately, up to the present time, no single equation has been derived to give values of C_r for all possible values of all the variables, even within the ranges covered by tests. Nevertheless, this is not an impossibility. In fact, concurrently with this paper, this author has undertaken the task of deriving such an equation. It is hoped that the result will be shortly available for use. Although, that problem is outside the scope of this work, it is felt that a brief description of the method used for the derivation of the said desired equation might be of some benefit to this analysis, and so a short outline is in order.

The equation is derived by a digital computer with the data programmized according to the steps of either the "simulation" method or the "stepwise regression with simple learning" method, as outlined in reference 4. The data are first grouped as shown in TABLE 3. Then they are transferred to computer cards and fed into the computer with the proper controlling statements. The solution will be an equation of the form of a polynomial involving a certain small percentage of error, which for our purposes will be entirely insignificant. It should be stated that simulation and stepwise regression programs for problems involving up to

TABLE 3 Data for Computer Cards

POINT	x_1	x_2	x_3	x_4	Y
	$\frac{B}{d}$	C_ℓ	$\frac{V_k}{\sqrt{L}}$	C_∇	C_2
0001	2.25	0.48	0.50	0.0010	0.00021
0002	----	----	----	------	-------

0NN'N"	3.00	----	----	------	-------
----					
nn'n"n"	3.75	----	----	------	-------
----					

60 variables have already been set up.[4] It should be further emphasized that the equation so obtained will be valid for any value of any of the variables within the limits used as data for TABLE 3.

If for any reason the method proposed above is not accepted or desirable, a different method, the following, is suggested. For reasons to be explained in the next section, a number of displacements will be known from the computing work of the Design part, before the actual calculations of the residual resistance will be necessary. Knowing these displacements which will correspond to definite sets of characteristics, we can follow the steps outlined below.

It is recommended that the variation of C_r with the beam-draft ratio be neglected and all resistance coefficients be calculated at a chosen B/d ratio which will be an average for the type and size of ships considered. This procedure will, of course, add to the uncertainties of the problem, but it is not easy to calculate C_r for each B/d since both the beam and the draft are variables. Taking the B/d variation into consideration would result in a considerable amount of manual work which might not justify the accuracy obtained by including this variation. The steps then involved might be:

1. Make a table showing the feasible displacements selected by the computer.

2. Assuming a constant mid-ship section coefficient, list the longitudinal coefficients corresponding to each displacement.

3. For each displacement calculate the volumetric coefficient, C_∇ .

4. From the tables of Taylor's data[6] read off the residual resistance coefficient for each C_∇ and longitudinal coefficient C_l at different speed-length ratios.

5. Plot these C_r versus speed-length ratio at constant C_∇ and C_l as shown in Fig. 7.

The maximum number of graphs necessary will be equal to the number of feasible displacements multiplied by the number of the assigned longitudinal coefficients C_1.

6. Use a digital computer to derive the equations of all these curves and properly identify each equation in order to be used again as a computer's input. As an example of this step, the computer found that the equation of the curve of Fig. 7 is:

$$C_r = 0.264 + 0.8134\left(\frac{V_k}{\sqrt{L}}\right)^4$$

Either of these two steps can be used. However, it is felt that the general equation derived or to be derived, according to the first procedure, possesses definite advantages, for instance, requires less work once derived, and as such will be the one used in this paper.

Assuming that we have completed the derivation of the equation for C_r, we can proceed with the derivation of the expression for a_{4j}.

Thus

$$a_{4j} = \frac{c' \cdot f_5(SHP)_j}{\Delta_j} + C_4 P_{tj}$$

or

$$a_{4j} = \frac{35\, C' \cdot f_5(P.C. \cdot C_5 \cdot S_j \cdot V_{kj}^3 \times C_{tj})}{L_j B_j d_j C_{bj}} + C_4 P_{tj}$$

$$\doteq \frac{35\, C' f_5(P.C.\, C_5\, S_j V_{kj}^3 C_{tj})\, k_{1j} k_{3j}}{L_j^3 C_{bj}} + C_4 P_{tj}$$

FIG. 7. Residual Resistance coefficient $C_r^{(10)}$ for $C_l = 0.6$, $C_\nabla = 2 \times 10^{-3}$, $\frac{B}{d} = 3.00$

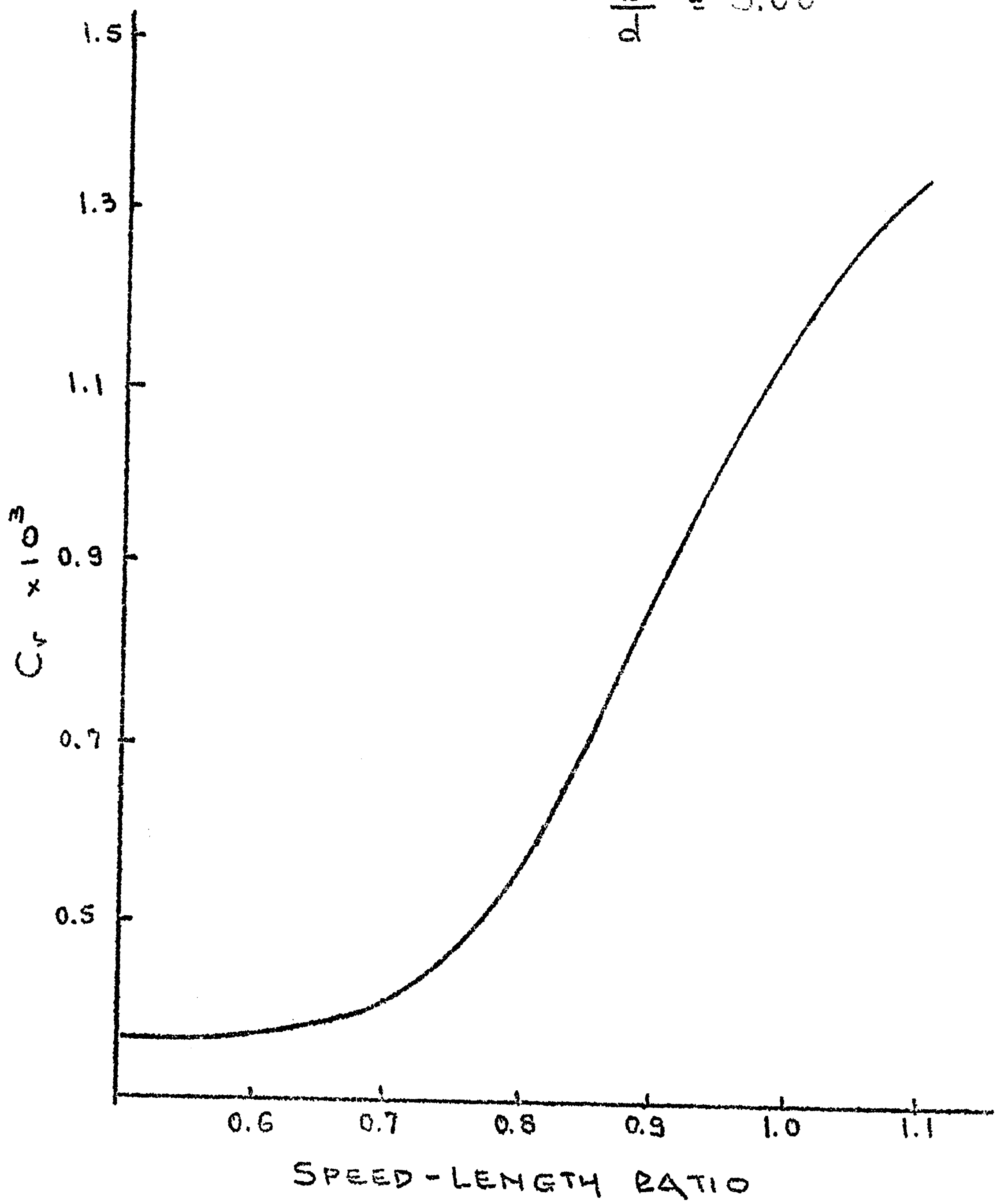

FIG. 7.

Further substitution of the functions of the wetted surface S, the total resistance coefficient C_t, and the total number of persons served P_t, yields:

$$a_{4j} = \frac{35\,C' f_s \left[P.C. \times C_s \times f'\left(\frac{B}{d}\right) \sqrt{\frac{L_j^3\, C_{bj} (L_j + L_j')}{35\, k_{1j}\, k_{3j}}} \times V_{kj}^3 \left(0.072 \left(\frac{1.689\, V_{kj} (L_j + L_j')}{\nu} \right)^{-\frac{1}{5}} \right] k_{1j}\, k_{3j}}{L_j^3\ C_{bj}}$$

$$+ \frac{35\,C' f_s \left[P.C. \times C_s \times f'\left(\frac{B}{d}\right) \sqrt{\frac{L_j^3\, C_{bj} (L_j + L_j')}{35\, k_{1j}\, k_{3j}}} \times V_{kj}^3 \times \left\{ f_6\left(\frac{B}{d}, C_\ell, \frac{V_k}{\sqrt{L}}, C_\nabla\right)_j + C_a \right\} \right] k_{1j}\, k_{3j}}{L_j^3\ C_{bj}}$$

$$+ C_4 \left[C_2 \frac{L_j^3\, C_{bj}}{35\, k_{1j}\, k_{3j}} + f_1 (SHP)_j + f_2 (L_j \cdot B_j) + f_3\left(C_2 \frac{L_j^3\, C_{bj}}{35\, k_{1j}\, k_{3j}} \right) + \right.$$

$$\left. + f_4\left(C_2 \frac{L_j^3\ C_{bj}}{35\, k_{1j}\, k_{3j}} \right) \right] \qquad (34)$$

Equation (34) gives the unit productivity of the machinery material and applies to both steam turbines as well as diesels, provided the proper value of C' and form of the function f_s will be used for each case. Also, equation (34) has been derived for twin screw ships, but as was stated earlier, a similar equation can be easily derived for single screw hulls, from data based on Series - 60.

10. A COMMENT ON UNIT PRODUCTIVITIES

N ow that we have calculated the expressions for the unit productivities of all inputs, we can better understand the relation of each input with the output, that is, the total displacement. We have previously stated that if we know only the displacement without knowing its ship characteristics we do not know the unit productivities, in other words, do not know the process for each ship. But by knowing the characteristics, we automatically know the displacement, and the unit productivities. This statement has led us to the derivation of the unit productivity equations as a function of the dimension of each ship. However, a_{4j} cannot be calculated independently of the displacement, since C_r depends on the volumetric coefficient C_∇ of each ship. Hence, this reverses the order of our thinking, in this case we have to calculate the displacement first and then compute the respective unit productivity or process involved. Obviously, the consequence of this is that in solving the equations (11) of linear programming, we don't determine the optimum level of the output but rather the optimum process. According then to out definition of process as being any ship j of definite characteristics, our linear programming will really produce the optimum dimensions of the ship and consequently the optimum displacement. That is, here we solve for a_{ij} instead of Δ_j. This unorthodox method of solving linear programming equations is imposed on our problem because of the relation between a_{4j} and displacement, that is because of the necessity to know Δ_j before we can calculate a_{4j}, the unit productivity of the machinery materials. This is one reason why the total displacement has been chosen as the output. Another reason is that with the displacement as output we can easily calculate all unit productivities in tons of the materials per ton of displacement. It would be difficult, if at all possible, to compute tons of steel per knot, for instance, if we wanted to manipulate the speed. Besides there is no need to manipulate any of the ship's characteristics, since all of them are the optimum ones for the optimum ship, when only the total displacement is being manipulated as the output. Therefore, the next necessary step in our analysis is the development of the displacements of a number of families of ships which will meet the initial owner's requirements. This development is taken up in PART 2, the Design part.

11. FINAL FORM OF THE INPUT-OUTPUT EQUATIONS

Having, thus, derived the expressions for the unit productivities, we substitute them in system (11) and obtain:

$$1. \quad \sum_{j=1}^{n}\left(\frac{0.35\, C_s\, k_{3j}\, \frac{(1+0.5\,C_b)_{pj}}{(1+0.5\,C_b)_b} \times \sqrt{\frac{(L/D)_{pj}}{(L/D)_b}}}{k_{2j}\, C_{bj}}\right) + \delta_1 = W_s \qquad (35)$$

$$2. \quad \sum_{j=1}^{n}\left(\frac{0.35\, C_o k_{3j}}{k_{2j}\, C_{bj}} + \frac{35\, C_1 C_2 k_{1j} k_{3j}}{L^3_j\, C_{bj}} \times \frac{L^3_j\, C_{bj}}{35\, k_{1j} k_{3j}} + \right.$$

$$+ \frac{C_1 k_{1j} k_{3j}}{L^3_j\, C_{bj}} \times f_1(SHP)_j + \frac{C_1 k_{1j} k_{3j}}{L^3_j\, C_{bj}} \times f_2\left(\frac{L^2_j}{k_{1j}}\right) +$$

$$\left. + \frac{C_1 k_{1j} k_{3j}}{L^3_j\, C_{bj}} \times f_3\left(C_2 \frac{L^3_j\, C_{bj}}{35\, k_{1j} k_{3j}}\right) + \frac{C_1 k_{1j} k_{3j}}{L^3_j\, C_{bj}} \times f_4\left(C_2 \frac{L^3_j\, C_{bj}}{35\, k_{1j} k_{3j}}\right)\right) +$$

$$+ \delta_2 = W_o \qquad (35)$$

$$3. \quad \sum_{j=1}^{n} \left(\frac{0.35\, C_e\, K_{3j}}{K_{2j}\, C_{bj}} \right) + \delta_3 = W_e \qquad (35)$$

$$4. \quad \sum_{j=1}^{n} \frac{35\, C' f_s \left[P.C. \times C_s f'\left(\frac{B}{d}\right) \sqrt{\frac{L_j^3 C_{bj} (L_j + L')}{35\, K_{1j}\, K_{3j}}} \times V_{kj}^3 \left(0.072 \left(\frac{1.689\, V_{kj} (L_j + L')}{\nu} \right)^{-\frac{1}{5}} \right] K_{1j}\, K_{3j}}{L_j^3\, C_{bj}} +$$

$$+ \frac{35\, C' f_s \left[P.C. \times C_s f'\left(\frac{B}{d}\right) \sqrt{\frac{L_j^3 C_{bj} (L_j + L')}{35\, K_{1j}\, K_{3j}}} \times V_{kj}^3 \times \left\{ f_6\left(\frac{B}{d}, C_\ell, \frac{V_k}{\sqrt{L}}, C_v\right)_j + C_a \right\} \right] K_{1j}\, K_{3j}}{L_j^3\, C_{bj}} +$$

$$+ C_4 \left[C_2 \frac{L_j^3\, C_{bj}}{35\, K_{1j} K_{3j}} + f_1 (SHP)_j + f_2\left(\frac{L_j^2}{K_{1j}}\right) + f_3\left(C_2 \frac{L_j^3\, C_{bj}}{35\, K_{1j}\, K_{3j}}\right) + \right.$$

$$\left. + f_4\left(C_2 \frac{L_j^3\, C_{bj}}{35\, K_{1j}\, K_{3j}}\right) \right] + \delta_4 = W_m \qquad (35)$$

System (35) is the final form of the equations of our input-output relations to be programmized for the computer.

12. BBASIC AND FEASIBLE SOLUTIONS

Our system (35) of the input constraint equations obviously has no unique solution, since the number of the proposed ships is much greater than the number of our equations. Nevertheless, from mathematics we know that a system of (m) equations in (n + m) unknowns will normally have a number of unique solutions given by :

$$\binom{n+m}{n} = \frac{(n+m)!}{n!\,m!}$$

This number represents the possible combinations of (m) unknowns left after we set arbitrarily any (n) number of unknowns equal to zero. The solutions so obtained are called "basic" solutions.

Some of the basic solutions might involve negative values for the output Δ_j, and/or the disposal process δ_j. Not accepting these solutions leaves us with the so-called "basic feasible" solutions, defined as solutions involving no more than (m) unknowns and giving non-negative values for the output and the disposal process. We thus have one constraint to be placed upon our solutions. Two more constraints follow.

PART II
THE DESIGN.

13. INTRODUCTION--STATEMENT OF THE PROBLEM

Up to this point, we have considered the theory of linear programming and have derived all the equations which will enable us to design any ship j. The meaning of the word "design", as used in this paper, should be clarified to avoid any misunderstanding. By "design" we mean the prediction of the optimum ship having a definite set of characteristics. The arrangements, accommodations and other minor details and items are not considered here, but are left entirely to the initiative of the architect. He should, however, follow as closely as he can the basis ship from which he will pick all his different coefficients and other data. In this part, we shall describe the procedure for the development of a number of ships by assigning to each one of them a different set of dimensions.

The essence of this step will be better understood if we assume that we are actually carrying out a real design problem. Thus, suppose that a shipowner has specified, as is usual, the amount and kind of deadweight to be transported, and the trade route. Hence, we know only the total deadweight and the maximum allowable operating draft, and we now venture to design the most profitable ship to meet these requirements. Furthermore, we assume that special funds have been allotted for the project to be solved by a digital computer.

In order to carry out this design, we decide to hold only the specified deadweight constant and vary all the other dimensions. Then, we break the solution down to logical steps to be programmized and fed into the computer. One of the possible arrangements of these steps follows in TABLE 3.

****

TABLE 3. Simplified flow diagram.

STEP No.	VARIABLES	CONTROLLING STATEMENTS
1. Assume:	$L_1 \quad L_2 \quad \dots \quad L_n$	$L_n = (\alpha + n\varepsilon_1)$ where $n = 1, 2, \dots N$ and ε_1 to be chosen.
2. Assign:	$d_1 \quad d_2 \quad \dots \quad d_f$	$d_f = (\psi + f\varepsilon_2)$ where $f = 1, 2, \dots F$ and ε_2 to be chosen.
3. Calculate:	$\frac{L_1}{d_1} \quad \frac{L_1}{d_2} \quad \dots \quad \frac{L_1}{d_f}$	Discard $\frac{L}{d} < \beta$ and $\frac{L}{d} > \gamma$
4. Calculate:	$L_1 d_1 \quad L_1 d_2 \dots \quad L_1 d_f$	
5. Assign:	$C_{b_1} \quad C_{b_2} \quad \dots \quad C_{b_u}$	$C_{bu} = (\delta + u\varepsilon_3)$ where $u = 1, 2, \dots U$ and ε_3 to be chosen.
6. Assume:	$C_m \quad C_m \quad \dots \quad C_m$	C_m to be properly chosen
7. Calculate:	$C_{\ell_1} \quad C_{\ell_2} \quad \dots \quad C_{\ell_u}$	$C_\ell = \frac{C_b}{C_m}$
8. Calculate:	$L_1 d_1 C_{b_1} \quad \quad L_1 d_1 C_{bu}$	
9. Assign:	$D_1 \quad D_2 \quad \dots \quad D_g$	$D_g = (\varphi + g\varepsilon_4)$ where $g = 1, 2, \dots G$ and ε_4 desired increment.

	D_1	D_2	...	D_g	
10. Calculate:	$\frac{L_1}{D_1}$	$\frac{L_1}{D_2}$	...	$\frac{L_1}{D_g}$	Discard $\frac{L}{D} < \eta$ and $\frac{L}{D} > \theta$
11. Assign:	B_1	B_2	...	B_q	$B_q = (\sigma + q\,\varepsilon_5)$ where $q = 1, 2, \ldots Q$ and ε_5 desired increment.
12. Calculate:	$\frac{L_1}{B_1}$	$\frac{L_1}{B_2}$	...	$\frac{L_1}{B_q}$	Discard $\frac{L}{B} < \lambda$ and $\frac{L}{B} > \mu$
13. Calculate:	$\frac{B_1}{d_1}$	$\frac{B_2}{d_1}$	...	$\frac{B_q}{d_1}$	Discard $\frac{B}{d} < \omega$ and $\frac{B}{d} > \xi$
14. Assign:	V_{k1}	V_{k2}	...	V_{kz}	$V_{kz} = (\rho + z\,\varepsilon_6)$ where $z = 1, 2, \ldots Z$ and ε_6 to be chosen.
15. Calculate:	$\frac{V_{k1}}{\sqrt{L_1}}$	$\frac{V_{k2}}{\sqrt{L_1}}$	...	$\frac{V_{kz}}{\sqrt{L_1}}$	Discard $\frac{V_k}{\sqrt{L}} < \nu$ and $\frac{V_k}{\sqrt{L}} > \pi$
16. Assign:	$\frac{1}{35}$	$\frac{1}{35}$		$\frac{1}{35}$	
17. Calculate:	$\frac{L_1 B_1 d_1 C_{b1}}{35} = \Delta_{fnqz}$				= TOTAL SEA WATER DIS-PLACEMENT.

14. EXPLANATION OF THE CONTROLLING STATEMENTS

The development of the ships of each family is carried out according to the steps of TABLE 3. First, we consider the deadweight specified by the owner. Its amount and nature will give us a good idea what the length of the ship will about be, plus or minus, say 50 feet. If we cannot guess, we can analyze a group of similar ships and see what the variation of length with deadweight is for that type of ship.(3) With this tentative length as a basis, we decide to vary the length between perpendiculars of the ship every ε_1 feet in the interval of α (alpha) feet to $(\alpha + n\varepsilon_1)$. The draft is then varied as desired and the L/d ratio is calculated. To this ratio, we impose the constraint that L/d should be neither less than β nor greater than γ, where the limits β and γ will depend on the deadweight. This type of controlling statements is used through step 17. The ε's (epsilon) are the increaments by which we wish to vary each dimension. The subscripts to each variable characteristic are used for identification purposes and the highest value at each working subscript represents the number of the variabes to be assigned within a specified interval.

The intervals are dictated by the amount and kind of deadweight, and their selection in each design case is guided by experience and analysis of similar ships, whenever necessary. The constraints, finally, which are imposed on each ratio of dimensions are desirable for the following reasons:

First, they limit the dimensions to values that will assure adequate stability and seaworthiness of the ships.

Second, They reduce the number of the ships to be further analyzed, by discarding the ones which will be unsatisfactory, either too big or too small to carry the desired deadweight, or unstable or too stable.

The reduction in the number of ships will result in saving of computer time and will make available more computer storage cells, whose number might be critical considering the computations involved in out problem. This magnetic storage problem is taken up in the next section.

****

15. COMPUTER STORAGE CAPACITY

The IBM 704 computer has a core storage capacity of 8192 words, which include coded instructions for each step of the solution, and also data and results. In addition, three magnetic tapes could possibly be used for storage with a capacity of 250,000 words each, and four magnetic drums, each with a capacity of a little over 2,000 words. Since most of our preliminary computations of the early parts must be stored in order to be available whenever needed in the later steps, we may easily run out of storage cells. However, in cases where it is desirable to use a great number of variables whose results will be more than can be stored in the computer, we can always overcome this difficulty by breaking the problem down into separate smaller parts, as necessary. For example, we can carry out the Design part first, including the output constraints, and the solution of linear programming next. Or we may even analyze all the ships of the same length for each length separately. Of course, we can always use only a small number of variable characteristics, if the demand for accuracy permits so.

The toll which we will pay for greater analytical accuracy by using a larger number of variables that will necessitate the employment of magnetic tapes and drums, will be increased cost due to the increase of the computer-hours for the solution; tapes and drums increase the number of computer-hours required. Therefore, for each problem the desired accuracy should be balanced with an allowable cost, and the number of variables should be determined according to that compromise.

****

16. OUTPUT CONSTRAINTS

The displacements Δ_{fnquz} of the large number of ships which the computer has defined according to our instructions in steps 1 to 17, bear no direct relation to the originally specified deadweight. Up to this point, the deadweight has been used only as a guide for the selection of the ranges within which we decided to vary the ship characteristics and their ratios. At this point, we recall that our design should meet the draft as well as the deadweight specification. The draft has already been incorporated in step 2 above, hence, all our ships have already complied with its requirement. Therefore, a correlation of each Δ_{fnquz} with the deadweight should be in order. From the many Δ_{fnquz}, we should retain only those which will be able to carry the desired deadweight.

The deadweight, furthermore, imposes two constraints on each Δ_{fnquz}. First, each ship should have enough buoyancy to carry the exact weight of the cargo, but neither more nor less. Second, each ship should have enough hold volume to store the exact cargo cubage, but neither more nor less. If a ship of Δ_{fnquz} characteristics satisfies both of the constraints, it should be retained and further analyzed from the economics point of view. If a ship satisfies only one or none of these constraints, it must be dropped out of the rest of the study. Let us consider the two deadweight constraints more rigorously. Since it is obvious that both of them spring up from the nature and the amount of the loaded cargo, we shall start with a consideration of the related topics of stowage factor and capacity coefficient.

****

17. STOWAGE FACTOR, f_s

The stowage factor is the specific volume of the cargo to be transported and is expressed in ft^3/ton. The importance in the design of a new ship should be considered first, since the factor greatly influences the economics of the ship. The "full and down" condition is unquestionably the optimum one to sail a ship and it, being the design condition, should be considered here. For a requested design, it is not only the cargo to be transported that should determine the optimum dimensions, as is the stowage factor in connection with the cargo deadweight that should specify the hold volume of the ship and, thus, indirectly her dimensions. The optimum ship for carrying ores, for instance, will not be the optimum one for the transport of automobiles or other light goods. Excessive hold space or turning down cargo for not having sufficient hold volume in a ship are conditions which should be avoided, or at least minimized. This can be well achieved in the design stage of that ship, by selecting an appropriate stowage factor based on sound assumptions and careful analysis of available experience. It is, therefore, quite proper for us to include and use in our analysis, the stowage factor as one of the most important criteria for the successful operations of a ship.

The stowage factor is introduced into the problem at hand as a constraint to which the displacement Δ [illegible] is subject before its use in the economic criterion.

* * * * * * * * * *

18. THE CAPACITY COEFFICIENT, C_c

The total hold space of a ship required for a specified cargo deadweight of a known stowage factor should be further analysed with respect to the different cargo availabilities at the various ports of call. These availabilities expressed as a percentage of the design cargo deadweight of the ship are called the "capacity coefficient" for any particular type of vessel. For a general cargo ship which sails both ways of a round trip "full and down", the capacity coefficient is For a tanker or ore carrier it is 0.5. The capacity coefficient is as important a criterion as the stowage factor itself.

only it is more of an economic nature. Therefore, after the optimum hold volume has been determined by stowage factor considerations, it should be studied economically taking into account the capacity coefficient.

At a first glance it might seem that the larger the cargo space, the more profitable the ship will be. But the larger space might possibly mean larger wasted space for a limited available amount of cargo, and hence, wasted operating expenses. It is therefore, apparent that for a given trade route there is an optimum hold space for the available amount of cargo. Thus, the inclusion of the capacity coefficient is necessary and as such it is applied in the economic criterion of our analysis, and in particular, in the calculation of the average annual income of the ship.

****

19. OUTPUT CONSTRAINT #1

This constraint, which will be step 18, springs up from the fact that the buoyancy of any ship as given by $\frac{L\,B\,d\,C_b}{35}$ should be equal to the total weight of the ship. From our Design part, we obtained these buoyancies, and we must now balance them with the total weight of the respective ships. The weight of each ship, furthermore, is equal to the light ship plus the deadweight. The light ship in turn is equal to the sum of the weights of construction steel, hull outfitting, hull engineering, and machinery. The deadweight, on the other hand, is constant as specified by the owner.

Thus, for this constraint, we can write:

$$\Delta_{fnquz} = \Delta_j = \frac{L_n \cdot B_q \cdot d_f \cdot C_{bu}}{35} = \Delta_{light} + DWT$$

or

$$\frac{L_n B_q d_f C_{bu}}{35} = \left(a_{1j}\Delta_j + a_{2j}\Delta_j + a_{3j}\Delta_j + a_{4j}\Delta_j\right) + DWT \quad (36)$$

Since the unit productivities in equation (36) must be substituted by their expressions in terms of ship characteristics, as we have explained, equation (36) becomes:

$$\frac{L_n B_q d_f C_{bu}}{35} = \left[\frac{0.35\, C_s\, k_{3j} \frac{(1+0.5C_b)_{pj}}{(1+0.5C_b)_b} \times \sqrt{\frac{(k_{2j})_p}{(k_2)_b}}}{k_{2j}\; C_{bj}} \Delta_j + \right.$$

$$+ \left\{ \frac{0.35\, C_o k_{3j}}{k_{2j} C_{bj}} + \frac{35\, C_1 C_2 k_{1j} k_{3j}}{L^3_j\; C_{bj}} \times \frac{L^3_j\; C_{bj}}{35\, k_{1j}\, k_{3j}} + \right.$$

$$+ \frac{C_1 k_{1j}\, k_{3j}}{L^3_j\; C_{bj}} \times f_1 (SHP)_j + \frac{C_1 k_{1j} k_{3j}}{L^3_j C_{bj}} \times f_2\left(\frac{L^2_j}{k_{1j}}\right) +$$

$$+ \frac{C_1 k_{1j}\, k_{3j}}{L^3_j\; C_{bj}} \times f_3\left(C_2 \frac{L^3_j\; C_{bj}}{35\, k_{1j}\, k_{3j}}\right) +$$

$$+ \left. \frac{C_1 k_{1j} k_{3j}}{L^3_j\; C_{bj}} \times f_4\left(C_2 \frac{L^3_j\; C_{bj}}{35\, k_{1j}\, k_{3j}}\right) \right\} \Delta_j +$$

Continued on next page.

$$+ \frac{0.35\, C_e\, K_{3j}}{K_{2j}\, C_{bj}}\, \Delta_j \quad +$$

$$+\left\{ \frac{35\, C' f_s \left[P.C. \cdot C_s f'\left(\frac{B}{d}\right) \sqrt{\frac{L_j^3 C_{bj}(L_j + L')}{35\, K_{1j}\, K_{3j}}} \cdot V_{kj}^3 \left(0.072 \left(\frac{1.688\, V_{kj}(L_j + L')}{\nu}\right)^{-\frac{1}{5}}\right]\, K_{1j}\, K_{3j} \right.}{L_j^3\, C_{bj}} \right. +$$

$$+ \frac{35\, C' f_s \left[P.C. \cdot C_s f'\left(\frac{B}{d}\right) \sqrt{\frac{L_j^3 C_{bj}(L_j + L')}{35\, K_{1j}\, K_{3j}}} \cdot V_k^3 \left\{ f_6\left(\frac{B}{d}, C_p, \frac{V_k}{\sqrt{L}}, C_r\right)_j + C_a \right\}\right] K_{1j}\, K_{3j}}{L_j^3\, C_{bj}} +$$

$$+ C_4 \left[C_2 \frac{L_j^3\, C_{bj}}{35\, K_{1j}\, K_{3j}} + f_1\,(SHP)_j + f_2\left(\frac{L_j^2}{K_{1j}}\right) + f_3\left(C_2 \frac{L_j^3\, C_{bj}}{35\, K_{1j}\, K_{3j}}\right) + \right.$$

$$\left. \left. + f_4\left(C_2 \frac{L_j^3\, C_{bj}}{35\, K_{1j}\, K_{3j}}\right)\right] \right\} \Delta_j + DWT$$

Equation (36) is the first constraint that all the feasible ships should comply with.

20. OUTPUT CONSTRAINT #2

Now that we have assured enough buoyancy for each ship to carry the deadweight, we must make sure that every ship to be further analyzed will have enough hold volume to satisfy the stowage factor f.s. viz, thus impose another constraint on the remaining displacements, which will be step 19. Unfortunately, there is no unique formulation of this constraint. First of all, we need an expression giving the hold volume for a type of ships in terms of their dimensions. Each type of ship, therefore, will have a different expression. [illegible] method for calculating the hold volume can always be used. In general, we can say that the equations will be of the form:

$$\text{BALE CAPACITY} = f'\left(\frac{LBD}{100}, \text{[illegible]}\right)$$

Professor Benford suggests the following [illegible] expressions for the hold volume of general [illegible] with and without refrigeration.

$$\text{BALE CAPACITY} = 88.5 \times \frac{LBD}{100} + C \text{ [illegible]}$$

and

$$\text{BALE CAPACITY} = \text{[illegible]} \times \frac{LBD}{100} + C \text{ [illegible]}$$

where [illegible]

Finally, any expression can be used [illegible] seems satisfactory to the naval architect. In case [illegible] no satisfactory expressions are available, we can always analyze carefully a number of similar ships, and derive that equation empirically by feeding the "real" points of the hold [illegible] versus the cubic number, let us say, of each ship, into a digital computer which will give us the equation desired. It should be pointed out that we must find the hold volume as a function of the ship characteristics, and not the total displacement.

Assuming that a satisfactory expression has been found, the second output constraint will be of the form:

$$f'\left(\frac{LBD}{100}, C_b\right)_j = f_s \times DWT \qquad (37)$$

Equation (37) is the second constraint that our remaining ships should comply with.

This step ends our Design part, since by now we have selected and stored the characteristics of all the feasible ships and discarded the unsatisfactory ones. Anyone of these ships could be built to satisfy the specifications of the owner, but the profitability of each ship has yet to be tested. The comparison of the profitability of all the ships is described in the third part of this paper, the Economics, which follows.

PART III

ECONOMIC ASPECTS

21. INTRODUCTION

After the completion of step 19, to be sure, we have retained only the ships of displacement Δ_j that will meet satisfactorily both of the owner's requirements. Now it remains to see which one of these ships will be the most profitable. In order to do that, we select an appropriate criterion of profitability , and accordingly, we compare all ships so that we can choose the one shich will meet our criterion to the highest degree. The economic criterion follows next.

****

22. CRITERION OF PROFITABILITY

The criterion of profitability which is most suitable to a linear programming analysis, since it can be expressed in linear form, is the present worth criterion. In the form of an equation, this criterion can be stated as follows:

$$\text{P.W.} = \frac{\text{Present worth of annual income}}{\text{Initial investment} + \text{Present worth of annual operating expenses.}}$$

or

$$\text{P.W.} = \text{Present worth of annual income} - (\text{Initial investment} + \text{Present worth of annual operating expenses.}) \quad (38)$$

Only the second expression, which is linear, will be used in our analysis.

As can be seen, this method judges profitability by comparing the investment and the present worth of the average future annual costs with the (as of now) value of all the future income of the ship. It should be emphasized here that the average annual income and operating expenses of the ship are assumed to be constant for the whole life expectancy of the ship.

The difference between the present worth of the annual income and total investment and expenses becomes a dollar measure of profitability. Thus a positive difference measures the ship's income potential over and above the cost of capital and ship's operating expenses.

One disadvantage of this criterion, though, is the fact that the dollar value of the difference between present worth of earnings and investment bears no direct relationship to the magnitude of the original investment. Therefore, it should not be used for a comparison of two entirely different investments, that of a ship, for example, and an airplane. In our analysis, however, where only ships are compared and the difference in alternative investments is not pronounced, it is felt that the present worth criterion will produce satisfactory results.

****

23. MAXIMIZATION OF THE CRITERION OF PROFITABILITY

Using the present worth method, the naval architect should choose as optimum the ship which has the largest positive present worth value. Our analysis, therefore, consists in maximizing the expression for our criterion of profitability, subject to the constraints of our four inputs. The calculations of the terms involved and their coefficients are shown next.

****

24. CALCULATION OF THE INVESTMENT

The term investment as used here refers to the so-called shipyard bill, and is the sum of money which the owner will pay a shipyard for the construction of a ship. Investment is synonymous with the initial cost of the ship. As such, it includes the cost of the materials, the costs of direct labor and overhead, miscellaneous expenses, as well as the shipyard's nominal profits and charges for insurance and drydocking. All these cost items can be grouped in different ways. However, in order to simplify our equations, the following form of the shipyard bill will be used as shown in TABLE 4.

TABLE 4. The Shipyard Bill

Item	Equation	Units
Steel cost	$\pi_1 a_{1j} \Delta_j$	\$/long ton
Hull outfitting cost	$\pi_2 a_{2j} \Delta_j$	\$/long ton
Hull engineering cost	$\pi_3 a_{3j} \Delta_j$	\$/long ton
Machinery cost	$\pi_4 a_{4j} \Delta_j$	\$/long ton
Miscellaneous costs	$\pi_5 \Delta_j$	

Sub-total = $\pi_1 a_{1j} \Delta_j + \pi_2 a_{2j} \Delta_j + \pi_3 a_{3j} \Delta_j + \pi_4 a_{4j} \Delta_j + \pi_5 \Delta_j$

Shipyard profits @ 10% of sub-total
Insurance @ 0.5% of sub-total
Owner's extra costs @ 0% of sub-total
Dry-dock $\pi_6 \Delta_j$

Therefore, the total shipyard bill is:

$$(1.105+C)\left(n_1 a_{1j}\Delta_j + n_2 a_{2j}\Delta_j + n_3 a_{3j}\Delta_j + n_4 a_{4j}\Delta_j + n_5\Delta_j\right) + n_6\Delta_j$$

It should be pointed out that n_1, n_2, n_3 and n_4, the cost per long ton of input material includes the cost of the respective material plus the appropriate charges for the associated direct and overhead labor. The miscellaneous costs, which include launching, trials and delivery, are expressed as $n_6\Delta_j$ where n_6 is a suitable charge per ton of total displacement for each type of ship to be designed. The shipyard profits and insurance are accounted for as a flat percentage of the sub-total. Dry-docking is given by $n_7\Delta_j$ where n_7 is an estimated charge per ton of total displacement. Lastly, the owner's extra costs, for champagne, are included as a percentage for the sub-total. C to be analytically determined.

In summary, according to equation (3) and our shipyard bill breakdown, the investment is:

$$I = \left[(1.105+C)\left(n_1 a_{1j} + n_2 a_{2j} + n_3 a_{3j} + n_4 a_{4j} + n_5\right) + n_6\right]\Delta_j \qquad (39)$$

* * * * *

TABLE 5. Unit Costs[10]

Item	Unit costs in $/ton of material		
	Tankers*	Passenger Ships	Passenger-Cargo Ships
Hull steel	608.00	700.00	625.00
Hull outfitting & engineering	3,057.80	3,060.00	3,060.00
Machinery	3,856.50	3,865.50**	3,806.00

* Unit costs for general cargo ships and ore carriers are the same as those for tankers.

** This figure applies to S H P up to 30,000 and increases to $6,4880.00 per ton at 400,000 S H P. Also see reference 3, Fig. 30, pp. 808.

The unit costs of the items above, include both the cost of each material and the associated labor costs for fabrication and erection.

All costs are based on the 1957 value of the dollar. See Fig. 8.

Fig. 8 below shows the relative U. S. shipbuilding costs since 1946, drawn from data supplied by the Maritime Administration; ([illegible] Files).

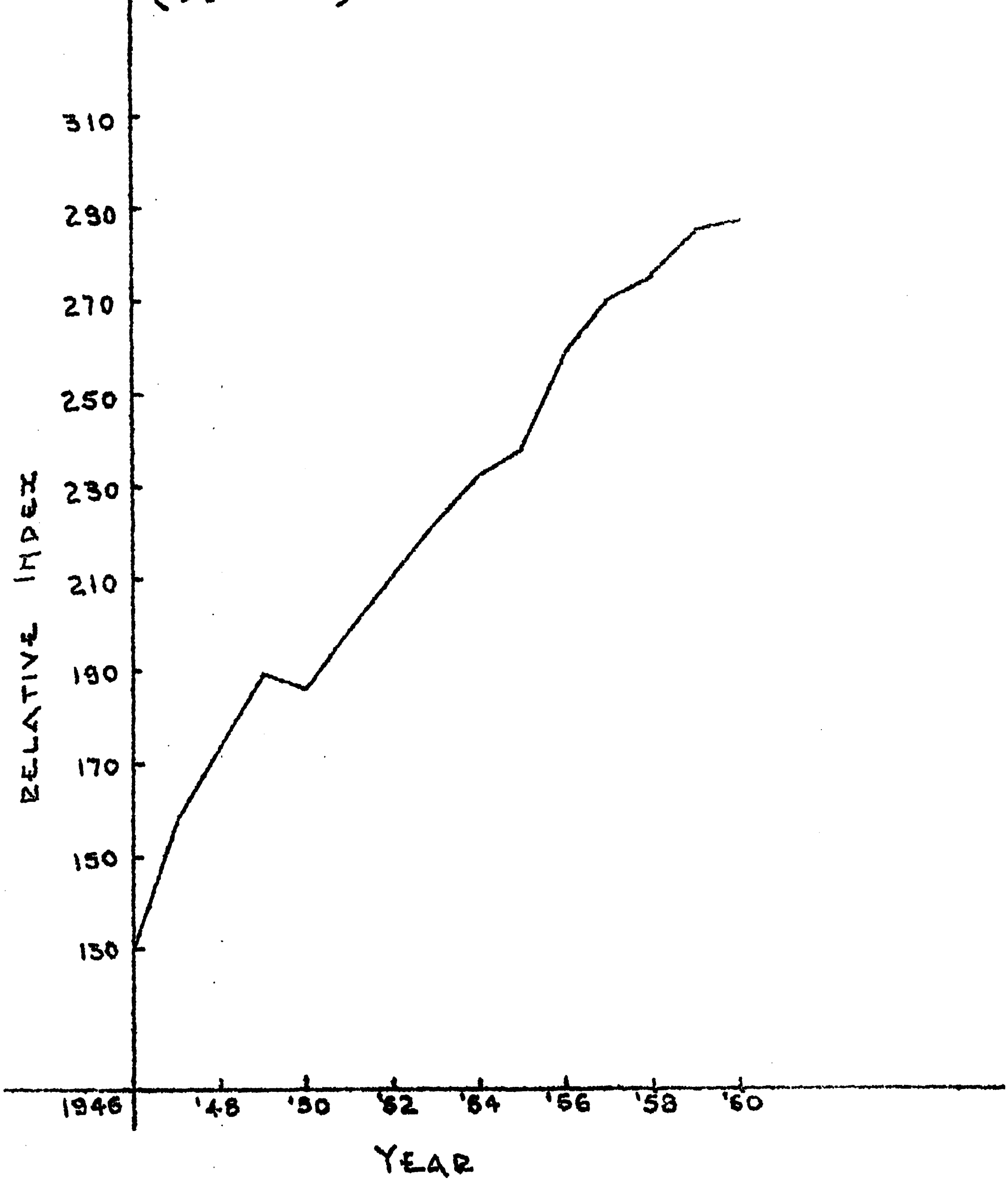

FIG. 8.

All points are plotted as of January 1st of each year.

25. CALCULATION OF THE PRESENT WORTH OF THE AVERAGE ANNUAL INCOME

If R be the average annual income and P its present worth, then

$$P = R\left[\frac{(1+i)^n - 1}{(1+i)^n + 1}\right] \qquad (40)$$

where

i = the annual rate of interest, and

n = the life expectancy of the ship in years.

The interest rate and the life expectancy of the ship are easily establish for each design problem.

The average annual income R is given by:

$$R = 2 \times \tau \times C_c \times \left[\Pi_o \times \sigma + \Pi_7 \times N_p\right] \qquad (41)$$

where

τ = the number of round trips of the ship per year,

C_c = the cargo coefficient to be determined from the type of ship and the owner's experience,

Π_o = the average cargo rate based only on the cargo weight. In case a light cargo, say 40 ft^3/ton or more is to be transported, its charge, usually based on its volume, should be converted on the basis of its weight,

σ = the cargo deadweight in long tons, the pay-load,

Π_7 = the average price of all passenger tickets, and

N_p = the average number of passengers carried on each trip.

The number of round trips per year of each ship will not be the same for all ships, since their speed will vary. Nevertheless, we can assume that all ships of the group will be out of service for an equal number of days, let us say d_o, each year and write:

$$\tau = \frac{(365 - d_o) \times 24}{2 \times \frac{s}{V_k} + H_p} \qquad (42)$$

where

s = the average distance in nautical miles of the trade route, and

H_p = an estimated average number of hours spent in each port of call.

Furthermore, the cargo deadweight is equal to the total displacement minus the light ship, fuel oil--including the reserve fuel--and also the weight of all persons aboard, their effects, subsistence, stores, fresh water, swimming pool and others.

In our problem, however, the total deadweight is assumed known, and therefore, the pay-load σ will be equal to that deadweight minus the items named above and included in the deadweight. Hence:

$$\sigma = DWT - \left[W_f - \sum (\text{Items}) \right] \qquad (43)$$

where

W_f = the weight of the fuel, including the reserve fuel, and

(Items) = refer to the weights of all persons aboard, their effects, subsistence, fresh water, swimming pool water and others.

It is seen that the weight of all the items is a function of the number of persons aboard, and as such, we may express it as a constant C_5, including all unit weights--see TABLE 6--, times the function which gives the total number of persons, that is, equations of page 38.

The weight of total fuel W_f, in turn is:

$$W_f = \left(1 + \frac{s}{125\, V_k}\right)\left(F.R. \cdot \frac{s}{V_k} \cdot SHP\right) \qquad (44)$$

where

$\frac{s}{125\, V_k}$ = 1/5 x (days at sea one way), an average reserve fuel factor.

For the j ship, substitution of equations (41), (42), (43) and (44) into (40) yields:

$$P = \left[\frac{(1+i)^n - 1}{(1+i)^n + 1}\right]\left[2 \times C_c \left(\eta_0 \sigma_j + \eta_7 N_{pj}\right) \tau_j\right]$$

$$= \frac{(1+i)^n - 1}{(1+i)^n + 1}\left\{2 \times C_c\left[\eta_0\left(DWT - \left\{W_{fj} - \sum(\text{Items})\right\}\right) + \right.\right.$$

$$\left.\left. + \eta_7 C_2 \frac{L_j^3 C_{bj}}{35 k_{1j} k_{3j}}\right] \frac{(362 - d_0) \times 24}{2 \times \frac{s}{V_{kj}} + H_p}\right\}$$

or finally:

$$P = \frac{(1+i)^n - 1}{(1+i)^n + 1}\left\{2 C_c\left[\eta_0\left\{DWT - \left[\left(1 + \frac{s}{125 V_{kj}}\right)\left(F.C. \times \frac{s}{V_{kj}} SHP_j\right) - \sum(\text{Items})\right]\right\} + \right.\right.$$

$$\left.\left. + \eta_7 C_2 \frac{L_j^3 \; C_{bj}}{35 k_{1j} \; k_{3j}}\right] \frac{(365 - d_0) \times 24}{2 \times \frac{s}{V_{kj}} + H_p}\right\} \qquad (44a)$$

TABLE 6. Average Deadweights

Item	Deadweight
Passengers and effects	250 pounds/person
Crew and effects	325 pounds/person
Baggage	200 pounds/passenger
Stores	10 pounds/person day
Fresh water	44 gallons/person day

26. CALCULATION OF THE PRESENT WORTH OF THE ANNUAL OPERATING COSTS

It is customary, in ship cost studies, to consider the following groups of operating expenses:

I. Vessel expenses.

II. Cargo handling expenses.

III. Port charges.

IV. Miscellaneous expenses.

In more detail, each of these groups includes:

I. Vessel expenses:

1. Crew wages.
2. Fuel cost.
3. Maintenance and repairs.
4. Stores and supplies.
5. Subsistence.
6. Insurance
7. Capital costs.
8. Miscellaneous costs.

II. Cargo handling expenses:

9. Wharfage cost.
10. Receiving clerks and checkers.
11. Stevedoring.
12. Watchmen.
13. Dunnage.
14. Insurance.
15. Miscellaneous costs.

III. Port charges:

16. Pilotage.

17. Customs.

18. Immigration fees.

19. Tonnage tax.

20. Miscellaneous expenses.

IV. Miscellaneous costs:

All these expenses depend, of course, on many factors, but all can be expressed in terms of the number of round trips of the ship per year, and the size of the ship. Let us consider each item separately and derive the equations whose sum will give the average annual expenses of the ship j.

****

26.1 Item 1--Crew Wages

This cost item includes the annual wages paid to all the crew members, that is, to the staff, engine hands, deck hands and stewards. Hence,

$$c_1 = 12\left(n_8 N_o + n_9 N_e + n_{10} N_d + n_{11} N_s\right) \qquad (45)$$

where

n_8, n_9, n_{10}, n_{11} are average annual salaries of the officers, engine and deck hands, and stewards, respectively.

The average monthly salary of each category will depend, of course, on the type and size of the vessel and the contracts of each shipowner with the labor unions, and also his obligations to the country of registry of the vessel. However, a survey of current practices will easily yield the average salaries. Besides, one can always find published data on the matter in the literature. For instance, reference 10, gives the following data:

Average monthly base wage for:

1. Deck and engine hands = \$353.00
2. Staff and stewards = \$280.00

These wages are based on the 1957 value of the dollar.

For the j ship, by substitution equation (45) becomes:

$$c_1 = 12\left[\eta_8 f_3 C_2 \frac{L_j^3 C_{bj}}{35 K_{1j} K_{3j}} + \eta_9 f_1 (SHP)_j + \eta_{10} f_2 (L_j \cdot B_j) + \eta_{11} f_4 C_2 \frac{L_j^3 C_{bj}}{35 K_{1j} K_{3j}}\right] \qquad (46)$$

where the functions f_1, f_2, f_3, and f_4 must be determined for each particular type of ship as before. Again, any of these functions may be taken as constant, or one function may be combined with another according to the design or if this will simplify the calculations without seriously affecting the accuracy of the result.

26.2 Item 2—Fuel Cost

The fuel cost per year is a function of the round trips τ and the S H P of the ship, and for the j ship, it is obtained from the relation:

$$c_{2j} = 2 \cdot \tau \cdot \Pi_{12} \left(V.T._j \cdot F.R. \cdot SHP_j \right)$$

where

Π_{12} = cost of fuel expressed in dollars per long ton.

τ = number of round trips of the ship per year.

V.T. = voyage time (one way) in hours.

F.R. = Fuel rate for both the main propulsion plant and the auxilliaries, in long tons per S H P per hour.

In all cases where the speed of the vessel is not held constant, the voyage time will be a variable of the problem in the form s/V_k, where s, given in nautical miles, should be the average one way distance of the trade route, and V_k the speed of the vessel in knots. Hence:

$$c_{2j} = 2 \cdot \frac{(365 - d_o)24}{2 \frac{s}{V_{kj}} + H_p} \cdot \Pi_{12} \left(\frac{s}{V_{kj}} \cdot F.R. \cdot SHP_j \right) \qquad (47)$$

Note that the reserve fuel is not included in the above equation, although if desirable, we could add the reserve fuel factor as given in equation (44).

****

26.3 Item 3—Maintenance and Repair Costs

The total cost of this item can be broken down to that of the machinery and that of the main hull. The cost of maintenance and repairs of the machinery will be a function of the S H P , and that of the main hull, a function of the total displacement. Appropriate functions can be derived from data on similar ships as furnished by ship operating companies.[3] Hence, for the j ship, the annual average maintenance and repair cost over the whole life of the ship will be:

$$c_{3j} = f_7(SHP)_j + f_8(\Delta)_j \tag{48}$$

where the functions f_7 and f_8 must be derived.

On the other hand, this cost item can be computed as a percentage of the total investment. TABLE 7 gives the cost for maintenance and repairs for the whole lifetime of the ship, for different types of ships, as a percentage of the initial investment. For our analysis, either formulation can be used.

TABLE 7 Maintenance and Repair Costs[10]

Type of Ship	Percent of Initial Investment
General cargo	21
Passenger-cargo	24
Passenger	27
Tanker	25
Ore carrier	22

26.4 Item 4--Stores and Supplies

The cost of stores and supplies is usually calculated as a function of the number of crew. Fig. 9 shows the variation of this cost item with the total number of crew, for passenger and passenger-cargo combination vessels. Thus, we can write:

$$c_{4j} = f_9(N_c)_j \qquad (49)$$

The function f_9 must be derived. For the curve of Fig. 9 for only the cost per day, the computer gave the following equation:

$$c_4 = -516.3 + 79.12 \times 10^6 N_c^{-3} - 8.14 \times 10^{-4} N_c^2 - 4.8 \times 10^{-10} N_c^4 + 62.9 N_c^{0.5}$$

In the last equation, N_c must be substituted by the functions giving the number of crew of each crew category in terms of ship characteristics. These functions for passenger and passenger cargo vessels have been given on page 28 and will not be repeated here.

****

26.5 Item 5--Subsistence Cost

The subsistence costs depend entirely on the number of persons P_t aboard the ship. For each type of ship, TABLE 8 gives the average cost per meal-day per person. Hence, we can write:

$$c_5 = C_6 \times P_t$$

$$= C_6 (N_p + N_c) \quad \text{in \$ per day.}$$

COSTS OF STORES, SUPPLIES AND EQUIPMENT FOR PASSENGER AND PASSENGER-CARGO VESSELS.[10]

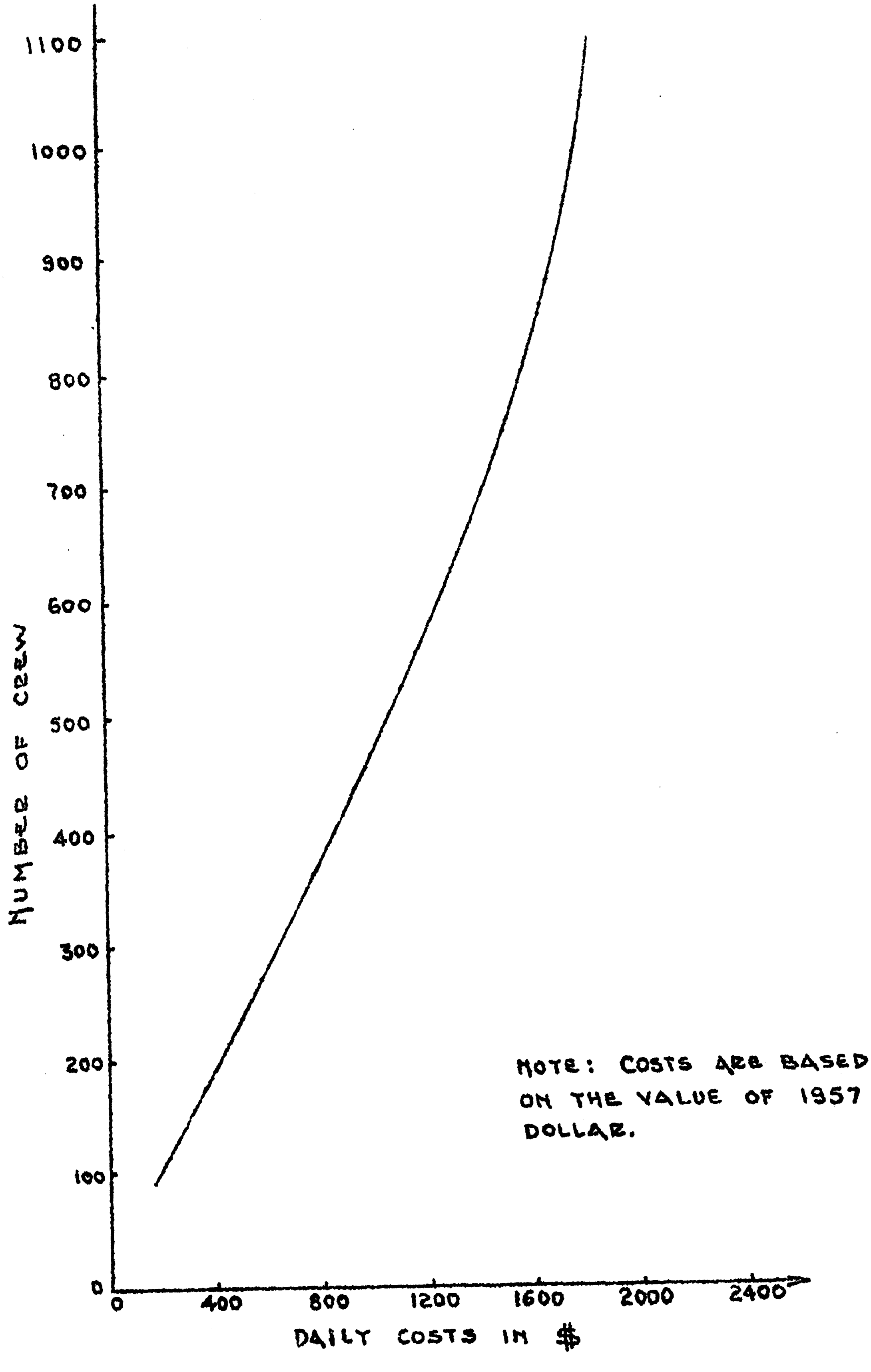

FIG. 9.

Assuming that the ship j will be operating d_1 days each year, we obtain:

$$c_{5j} = d_{1j} \times O_6 (N_p + N_c)_j \qquad (50)$$

In the last equation, the appropriate functions of N_p and N_c in terms of the characteristics of the ships should be used. It should be stated that the subsistence costs vary for different ship operators and types of vessels.

TABLE 8 Subsistence Costs*

Type of ship	$/meal-day person
Passenger	3.50
Passenger-cargo	3.00
General cargo	2.00
Tanker	2.00
Ore carrier	2.00

* Figures are based on the 1957 value of the dollar.(10)

****

26.6 Item 6--Insurance

The average annual insurance for each ship, whether American or foreign built, can be expressed in terms of the initial investment, or:

$$o_{6j} = f_{10} (I)_j \qquad (51)$$

where f_{10} should be empirically determined for current insurance rates for similar ships. The expression for I, as given by equation (39), should be substituted in equation (51).

Professor Benford in reference 3, suggests the following equations for tankers:

a. For American built vessels:

$$c_6 = 5,000 + 0.012\ (I)$$

b. For foreign built vessels:

$$c_6 = 4,000 + 0.015\ (I)$$

Any formulation can be used.

****

26.7 Item 7: Capital Costs

This cost item includes depreciation (based on nominal 20 year life) taken as 5% of the initial investment, and interest taken as about 3% of the investment, or

$$c_{7j} = 0.08\ (I)_j \qquad (52)$$

where I_j should be substituted by its value as given by equation (39).

****

26.8 Item 8: Miscellaneous Costs

This item might include medical examination, expense accounts, transportation, postage and other incidentals. For our purpose, we can assume that this cost will be the same for all ships being analyzed, and assign an appropriate constant value to it. Hence,

$$c_8 = C_7 \qquad (53)$$

where C_7 is to be empirically determined.

26.9 Group II. Cargo Handling Expenses

All cost items of this group, item 9 to 15 inclusive, are functions of the deadweight to be carried. Since we assume that the deadweight is the same for all ships we include all these cost items as a lump sum. Hence,

$$C_{(9-15)j} = 2\ \tau_j\ \eta_{13}\ C_c \times DWT \qquad (54)$$

Where η_{13} is to be appropriate for the kind of cargo and C_c is the capacity coefficient. For τ_j equation (42) should be substituted into (54)

****

26.10 Group III. Cost Item 16: Pilotage

Since the pilotage charge depends on the draft of the ship, assuming that the j ship sails always "full and down", we can write:

$$c_{16j} = 2\ \tau_j \times \eta_{14}\ d_j \qquad (55)$$

where η_{14} is to be assigned for each type of ship. Again equation (42) should be used for τ_j .

****

26.11. Items 17, 18 and 20.

These costs are functions of either the deadweight or the number of passengers or both, depending on the type of ship.

For a cargo ship j:

$$c_{(17,18,20)j} = 2\ \tau_j\ \eta_{15}\ C_c \times DWT \qquad (56)$$

For a passenger ship j:

$$c_{(17,18,20)j} = 2\ \tau_j\ \eta_{16}\ N_{pj} \qquad (57)$$

where η_{15} and η_{16} are appropriate charges, and τ_j and N_{pj} in both equations should be substituted by their respective expressions.

****

26.12 Item 19. Tonnage Tax.

This item is based on the tonnage of each ship and will vary both with the size of the ship and the canals of the trade route. This cost might be expressed in terms of the total displacement of each ship j, or

$$c_{19j} = 2\tau_j \eta_{17} \Delta_j \qquad (58)$$

where η_{17} is to be determined.

For example, port charges for tankers and ore carriers can be given in terms of port-days per year. For a tanker this variation is given in Fig. 10.[10]
For this curve the computer gave the following equation:

$$c = -22.68 + 3\times10^{-6} d_2^2 - 4\times10^{-2} d_2$$

where d_2 is the number of port-days.

FIG. 10. Port Charges for Tankers.[10]

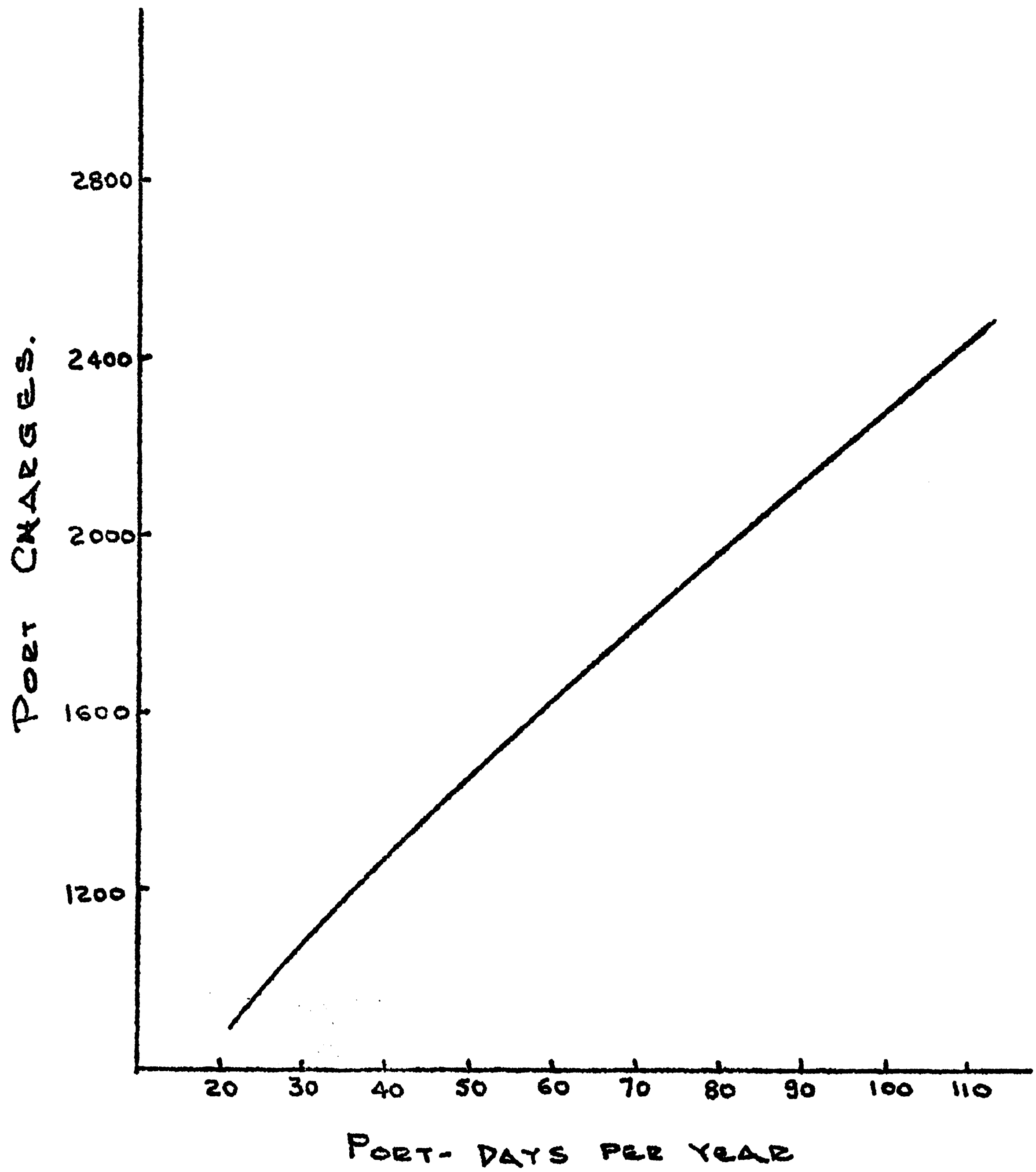

FIG. 10.

Having, thus, computed the average operating costs c_x their present worth P.W., will be:

$$P.W. = \left[\frac{(1+i)^n - 1}{(1+i)^n + 1}\right] \sum_{x=1}^{20} c_x \qquad (59)$$

Equation (59) then should be substituted in the expression of our economic criterion, equation (38).

****

27. THE PROFITABILITY EQUATION.

Substituting equations (39), (44a), and (59) into equation (38) we obtain:

$$
\begin{aligned}
P.W. = {} & \frac{(1+i)^n - 1}{(1+i)^n + 1} \Bigg\{ 2\, C_c \Bigg[\pi_0 \Bigg\{ DWT - \Big[\Big[\Big(1 + \frac{s}{125\, V_{kj}}\Big)\Big(F.R.\ \frac{s}{V_{kj}}\, SHP_j\Big) - \\
& - \sum(\text{Items})\Big]\Big]\Bigg\} + \pi_7\, C_2\, \frac{L_j^3\, C_{bj}}{35\, K_{1j}\, K_{3j}} \Bigg] \frac{(365 - d_0)\, 24}{2\, \frac{s}{V_{kj}} + H_p} \Bigg\} - \\
& - \Big[(1.105 + c)(\pi_1 \alpha_{1j} + \pi_2 \alpha_{2j} + \pi_3 \alpha_3 + \pi_4 \alpha_{4j} + \pi_5) + \pi_6\Big] \frac{L_j^3\, C_{bj}}{35\, K_{1j}\, K_{3j}} - \\
& - \frac{(1+i)^n - 1}{(1+i)^n + 1} \sum_{x=1}^{20} c_x \qquad (60).
\end{aligned}
$$

All the terms of this equation are either coefficients determined empirically or variables to be assigned and handled by the digital computer, as instructed. Therefore, care must be taken to express all terms of equation (60) only as functions of the ship characteristics used in the Design part as shown in TABLE 3.

28. OBTAINING THE OPTIMUM SHIP.

The optimum ship is determined as follows: The system of equations (35) will have four equations with a number of terms which will be equal to the retained displacements all of definite characteristics. Each ship will be identifiable by a working subscript j. Application of the criterion of profitability, equation (60), will single out the term j which will be the optimum ship having characteristics designated by the subscripts f,n,a,u,z.

29. EVALUATION OF THE METHOD.

If this method is to have any value and be of any practical usefulness, it must have at least the following distinctions:

1. It must be satisfactorily accurate.
2. It must be easy to apply, and
3. It must be comparatively inexpensive.

In order to evaluate the method with respect to the first requirement, we consider the two possible sources of errors, the computer, and the naval architect.

The digital computer, as far as the solution of the equations is concerned, is exact to more decimal points that we can use. In contrast, when the computer is used to derive equations, as suggested, from either "raw" or faired points, is erratic. The percentage of error introduced by the computer will depend on the amount of data supplied for each curve and also the nature of each curve, its continuity, sharp changes of slope etc. However, since all our curves are continuous, the very small percentage of error that can be introduced at this point is negligible and the results quite satisfactory. Besides, the amount of this error can be controlled and kept to a minimum by the naval architect, if he supplies sufficient amount of data.

Actually, it is the naval architect who controls the accuracy not only of the computer but of the whole method. In particular, it is not the amount of data that will introduce the error as is the accuracy of these data. Since most of our coefficients are obtained from analyses of empirical information supplied by ship-operating companies and shipyards, the accuracy and the handling of this information by the naval architect will be the main source of the errors. Obviously, then, care and persistance will improve the results and produce dependable answers. Therefore, this method is as accurate as the data supplied and, in general, is as or more accurate than any other relative method of long-hand calculations, since the computer eliminates the probability and possibility of human errors involved in the other methods.

With respect to the second requirements, that of the ease to apply the method, some comforting remarks can and should be made, because the method is not really as complicated as it may appear. True, there are many steps and long equations involved, but the use of the computer can make this method the easiest one to apply for the design of a new ship, even easier than the artistic one of pointing the thumb. Most of the work is handled by the computer and all of it can be programmized step by step, once and for all. Namely, the resistance calculations can be programmized and the equation of the residual resistance coefficient derived and stored for future use. Then the controlling statements for a very general problem including any design, can again be programmized both for the design and the economics parts, so that this program can be permanently ready to use for any particular problem.

Moreover, no curves have to be drawn for the derivation of any function or computation of any coefficient. This can be done by the computer if we directly feed in the empirical data, raw points. Against the argument that some of the points so obtained might carry more weight in decision making, why not use two or more points of lesser weight at one place instead of one point carrying the total weight? Use judgment to decide on the importance of information represented by each point, and the number of equivalent points that can be used instead of that single point, and let the computer plot the curve, if desired, and derive its equation simultaneously. Therefore, the only work required on the naval architect's part will be the careful collection and analysis of a group of similar ships supplying dependable information, and the decision which he has to make about values and limits involved in the controlling statements of the Design part.

Lastly, the cost for carrying out a design according to this method can favorably be compared with that of any other method of carrying out an economic analysis of the same design starting from scratch. In the first place, the use of a digital computer provides this method with the big asset that all possible combinations of the variable ship characteristics can be considered, and that for the optimum ship each one of these characteristics will be the optimum one, otherwise the ship would not be called optimum. Furthermore, the optimum ship as determined by this method is not based on the profitability or performance of any other ship in existence which ship itself might very well not be the optimum one for its design specifications. The empirical data used in this program are general, applied to all ships of the same type, and reflect or are interpretations of the rules for building and manning these ships, and indicate practices and current trends and laws common to all these ships for their operation. It is believed that these advantages cover factors which should not be overlooked in the design of a ship, and should be taken into consideration when a compromise is to be made between the result and the cost of a design.

After all, the cost of a design carried out by linear programming, is not that high. It includes the man-hours of the naval architect for selecting the inputs and programmizing the steps, and the machine-hours of the computer. As far as the architect's work is concerned, once a general program is set up the design of any ship can be carried out within one month. If a program is not already available, one man's hours of each working day for two months will be more than sufficient to determine the optimum ship. On the other hand, an IBM 704 computer whose cost is $350.00 an hour can handle all the work of a usual design problem within three hours. Of course, as stated previously there will always be a compromise to be made between the desired dependability of the results and the costs required, but for the same results this method is far less expensive than any other method.

Although this method is accurate, easy to apply and inexpensive, it does have some limitations. In the first place, for its execution it generally requires that a man know both Naval Architecture and computer techniques. Then, the optimum ship is determined to the as-of-now concept of the money value. Or could it be otherwise? Could the change of the money value constitute a great advantage for a programmized design while being a limitation to all the other design methods? Could a computer store past and current business trends and, after projecting them into the future to any desired date, incorporate the results of that projection in the prediction of the optimum ship? This indeed is possible and could easily be done. As a step, it is not included in this paper since it is a programmizing step, lying outside of the scope of this work. However, it should not be thought of as an impossibility because it really is an advantage of the use of the computer in ship design.

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30. SUMMARY OF THE APPLICATION STEPS.

In summary, for carrying out a design request, generally the following procedure may be used.

Consider the specifications thoroughly.

Collect carefully a number of ships to which the vessel to be designed will be similar. Analyze these ships and along with other relative information tabulate the data necessary for the computation of the various coefficients and functions of the operations of the ships.

Select one ship, the basis ship, out of this group, and determine the structural coefficients.

According to the demanded accuracy of the result and the information gathered thus far, decide on the number and range of the variable dimensions to be used in the Design part.

Programmize:

1. Data on coefficients and functions.
2. Unit productivity.
3. The Design steps.
4. Input-out equations, and
5. Criterion of profitability.

****

Generally, for a paper of this nature, it is important that an example be included as worked out according to the method proposed in the work. Due to school work, however, and other time limitations this has not been possible for this paper, but it is planned that such an example be included in another complementary paper which is to follow shortly.

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BIBLIOGRAPHY

1. Brems, H. Output, employment, capital and growth. New York: Harper & Brothers, 1958.

2. Dorfman, R. Application of linear programming to the theory of the firm. Berkley and Los Angeles: The University of California Press, 1951.

3. Benford, H. Engineering economy in tanker design. Transactions, The Society of Naval Architects and Marine Engineers, Vol. 65, 1957.

4. Westervelt, F. H. Automatic system simulation Programming. Ph.D Dissertation, University of Michigan, Ann Arbor, 1960.

5. Dr. Todd, F. H. Stuntz, G.R., and Dr. Pien, P. C. Series 60-the effect upon resistance and power of variations in ship proportions. Transactions, The Society of Naval Architects and Marine Engineers, Vol. 65, 1957.

6. Gertler, M. A reanalysis of the original test data for the Taylor Standard Series. David Taylor Model Basin, Washington 25, D.C., 1954.

7. Telfer, E. V. The structural weight similarity of ships. Transactions, North East Coast Institution of Engineers and Shipbuilders, Vol. 72, 1955-56.

8. Broad, R. Outfit estimating coefficients for ships. Thesis, University of Michigan, Ann Arbor, 1956.

9. Kari, A. Design and cost estimating. Technical Press, 1948.

10. Conkling, D. L. and others, Economics of nuclear and conventional ships. United States Atomic Energy Commission, Washington, D. C., 1958.

11. Arnott, D. Design and construction of steel merchant ships. New York; Society of Naval Architects and Marine Engineers, 1955.

12. Saunders, H. E. Hydrodynamics in ship design. Vol II New York: Society of Naval Architects and Marine Engineers, 1957.