# Vibrational Analysis of Peptides, Polypeptides, and Proteins. II. $\beta$-Poly(L-alanine) and $\beta$-Poly(Lalanylglycine) 

W. H. MOORE and S. KRIMM, Harrison M. Randall Laboratory of Physics and Biophysics Research Division, Institute of Science and Technology, University of Michigan, Ann Arbor, Michigan 48109

## Synopsis


#### Abstract

The normal vibration frequencies of poly(1,-alanine) and poly(L-alanylglycine) in the antiparallel chain-pleated sheet structure have been calculated, using the force field for polyglycine I from the previous paper (Biopolymers 15, 2439-2464) plus additional force constants for the methyl group. The agreement with observed ir and Raman bands is very good. This substantiates the excellent transferability of the force field, since polyglycine I was shown to have a rippled-sheet structure. The amide I and amide II mode splittings are very well accounted for by transition dipole coupling, showing that subtle structural differences are sensitively manifested through this mechanism.


## INTRODUCTION

The conformational structure of proteins, which are important biological molecules, can be elucidated by understanding the structure and vibrational spectra of model compounds, in particular synthetic polypeptides. This is possible because the ir and Raman spectra of poly(L-amino acids) are known to be sensitive to conformational change. In order to better correlate vibrational spectra with amino acid sequence and conformation, considerable research effort has been invested in the study of synthetic polypeptides. N-Methyl acetamide, the simplest molecule with the peptide backbone has been investigated in detail. ${ }^{1-3}$ X-Ray diffraction, ir, and Raman data have also provided extensive information relevant to an understanding of the structure of more complicated polypeptides. Despite this, a detailed understanding of the vibrational spectra of polypeptides has still to be achieved.

In this paper we continue our analysis of the vibrational spectra of $\beta$ polypeptides, in particular poly(L-alanine) [(Ala) $)_{n}$ ], and poly(L-alanylglycine) [(Ala-Gly) ${ }_{n}$ ]. The ir spectra of (Ala) $)_{n}$ in the region $3300-800 \mathrm{~cm}^{-1}$ have been recorded and published. ${ }^{4}$ Fraser et al. ${ }^{5}$ have reported on the X-ray structure and ir spectra of (Ala-Gly) ${ }_{n}$. Far ir spectra ${ }^{6-8}$ as well as Raman spectra from several sources ${ }^{9,10}$ are available on these polypeptides. These data, in conjunction with our studies on polyglycine I $\left[(\mathrm{Gly} \mathrm{I})_{n}\right]^{11,12}$ form the basis for the development of a transferable valence force field that
can be used to calculate and study the normal vibrational modes of $\beta$ polypeptides with alkyl side chains.

The normal vibration calculations presented in this paper are based on two previously published force fields, the Jakeš and Krimm ${ }^{3}$ valence force field for the amide group and the Moore and Krimm ${ }^{12}$ valence force field for (Gly I) $)_{n}$. If force fields and associated normal vibration calculations are to be applicable in the study of tertiary structure of more complicated synthetic polypeptides and proteins, they must be systematically derived. By this we mean that the portion of the force field which describes the peptide backbone should be consistent from one polypeptide to another and essentially independent of the nature of the side chain. It is, however, necessary to consider differences in the force field that exist due to differences in residue-backbone interactions, as well as those due to the conformation of the polymer being studied. In our studies of $\beta$ polypeptides we have considered only those that have alkyl side chains and whose tertiary structure is the antiparallel chain-pleated sheet, except for (Gly I) ${ }_{n}$, which has an antiparallel chain-rippled sheet structure. ${ }^{12}$

For an initial force field we used the already available force field for the polyglycines ${ }^{11,12}$ combined with the force field for branched hydrocarbons. ${ }^{13}$ Available ir and Raman data for (Gly I) $n$, (Ala-Gly) $n$, and (Ala) $n$ were used as the basis for a best least squares refinement of the force field. This force field was subsequently applied in a normal vibration calculation of $\beta$ -poly(L-valine) and poly( $\mathrm{L}-\alpha$-amino- $n$-butyric acid) (manuscript in preparation), and has given good agreement with experiment.

## EXPERIMENTAL DATA

All of the ir and some of the Raman data used in this work were taken from the literature. ${ }^{4-10}$ The Raman data used for (Ala-Gly) ${ }_{n}$ have not been previously published, and were obtained independently in this laboratory by Hsu (unpublished results) and Johnston (unpublished results). The Johnston data were recorded on a Spex Ramalog spectrometer and cover the region from $3300 \mathrm{~cm}^{-1}$ to $800 \mathrm{~cm}^{-1}$. The spectrometer used by Hsu has been described elsewhere. ${ }^{14}$ These two spectra agree well; however, when discrepancies exist, values that were used in the refinement are indicated. Subsequent to the refinement, Frushour and Koenig ${ }^{15}$ published the Raman spectrum of (Ala-Gly) ${ }_{n}$. Whenever possible their data have been incorporated.

## CRYSTAL STRUCTURES

The sheet structure used for (Ala) ${ }_{n}$ was taken from the work of Arnott et al., ${ }^{16}$ who give $a=4.73 \AA$ and $b=6.89 \AA$ ( $3.445 \AA$ per residue); the intersheet distance is $c=5.27 \AA .{ }^{17}$ As described previously, ${ }^{12}$ the relative axial shift between two adjacent chains in the same sheet is determined from band splittings arising from transition dipole coupling. The inter-

TABLE I
Structural Parameters of Crystalline Poly (L-alanine) and Poly (L-alanylglycine)

sheet organization corresponds to unit cell $c$ of Ref. 16, where we have taken $\Delta Z=0$, according to Colonna-Cesari et al. ${ }^{17}$

The structure used for (Ala-Gly) ${ }_{n}$ consists of antiparallel chain-pleated sheets in which all glycyl residues are extended from one side of the sheet and all alanyl residues from the other. ${ }^{5}$ The axial translation per residue is $3.475 \AA$ and the interchain distance within a sheet is $4.71 \AA . .^{17}$ The sheets are packed such that all like residues face one another. This packing results in different sheet spacings between glycyl-glycyl and alanyl-alanyl contacts: $c($ Gly-Gly $)=3.79 \AA$ and $c($ Ala-Ala $)=5.08 \AA .{ }^{17}$

These and other structural parameters are collected together in Table I.

## VIBRATIONAL ANALYSIS

## Internal Coordinates and Corresponding Symmetry Coordinates

The conventional definitions for internal coordinates for the peptide chain are adhered to. With respect to each of the hydrogen bonds, an intermolecular $\mathrm{H} \ldots \mathrm{O}$ stretching coordinate, two torsion coordinates, and two bending coordinates are used: $\Delta r(\mathrm{H} \cdots \mathrm{O}), \Delta \tau(\mathrm{C}=\mathrm{O} \cdots \mathrm{H}), \Delta \tau(\mathrm{N}-\mathrm{H} \cdots \mathrm{O})$,

TABLE II
Internal Coordinates of Poly(L-alanylglycine) and Poly(L-alanine)

|  | Group A |
| :---: | :---: |
| $R_{1}=\Delta r\left(\mathrm{C}^{\alpha}-\mathrm{C}\right)$ | $R_{16}=\Delta \theta\left(\mathrm{C}^{\alpha}-\mathrm{N}-\mathrm{H}\right)$ |
| $R_{2}=\Delta r(\mathrm{C}-\mathrm{N})$ | $R_{17}=\Delta \theta\left(\mathrm{N}-\mathrm{C}^{\alpha}-\mathrm{H}\right)$ |
| $R_{3}=\Delta r\left(\mathrm{~N}-\mathrm{C}^{\alpha}\right)$ | $R_{18}=\Delta \theta\left(\mathrm{N}-\mathrm{C}^{\alpha}-\mathrm{H}^{\alpha}\right)$ |
| $R_{4}=\Delta r(\mathrm{C}=\mathrm{O})$ | $R_{19}=\Delta \theta\left(\mathrm{C}-\mathrm{C}^{\alpha}-\mathrm{H}\right)$ |
| $R_{\text {s }}=\Delta r(\mathrm{~N}-\mathrm{H})$ | $R_{20}=\Delta \theta\left(\mathrm{C}-\mathrm{C}^{\alpha}-\mathrm{H}^{\alpha}\right)$. |
| $R_{6}=\Delta r\left(\mathrm{C}^{\alpha}-\mathrm{H}\right)$ | $R_{21}=\Delta \theta\left(\mathrm{H}-\mathrm{C}^{\alpha}-\mathrm{H}^{\alpha}\right)$ |
| $R_{\gamma}=\Delta r\left(\mathrm{C}^{\alpha}-\mathrm{H}^{\alpha}\right)$ | $R_{22}=\Delta \gamma_{\\|}(\mathrm{C}=\mathrm{O} \cdots \mathrm{H})$ |
| $R_{8}=\Delta r(\mathrm{H} \cdots \mathrm{O})$ | $R_{23}=\Delta \gamma_{\\|}(\mathrm{N}-\mathrm{H} \cdots \mathrm{O})$ |
| $R_{9}=\Delta r\left(\mathrm{H}^{\alpha} \ldots \cdot \mathrm{H}^{\alpha}\right)$ | $R_{24}=\Delta \gamma_{\perp}(\mathrm{C}=\mathrm{O} \cdots \mathrm{H})$ |
| $R_{10}=\Delta \theta\left(\mathrm{C}^{\alpha}-\mathrm{C}-\mathrm{N}\right)$ | $R_{25}=\Delta \gamma_{1}(\mathrm{~N}-\mathrm{H} \cdots \mathrm{O})$ |
| $R_{11}=\Delta \theta\left(\mathrm{C}-\mathrm{N}-\mathrm{C}^{\alpha}\right)$ | $R_{26}=\Delta \omega\left(\mathrm{O}=\mathrm{C}<\mathrm{C}^{\alpha}\right)$ |
| $R_{12}=\Delta \theta\left(\mathrm{N}-\mathrm{C}^{\alpha}-\mathrm{C}\right)$ | $R_{27}=\Delta \omega\left(\mathrm{H}-\mathrm{N}<_{\mathrm{C} \alpha}^{\mathrm{C}}\right)$ |
| $R_{13}=\Delta \theta\left(\mathrm{C}^{\alpha}-\mathrm{C}=\mathrm{O}\right)$ | $R_{28}=\Delta \tau\left(\mathrm{C}^{\alpha}-\mathrm{C}\right)$ |
| $R_{14}=\Delta \theta(\mathrm{N}-\mathrm{C}=\mathrm{O})$ | $R_{29}=\Delta \tau(\mathrm{C}-\mathrm{N})$ |
| $R_{15}=\Delta \theta(\mathrm{C}-\mathrm{N}-\mathrm{H})$ | $R_{30}=\Delta \tau\left(\mathrm{N}-\mathrm{C}^{\alpha}\right)$ |
|  | Group B |
| $R_{1}=\Delta r\left(\mathrm{C}^{\alpha}-\mathrm{C}\right)$ | $R_{21}=\Delta \theta\left(\mathrm{N}-\mathrm{C}^{\alpha}-\mathrm{C}^{\beta}\right)$ |
| $R_{2}=\Delta r(\mathrm{C}-\mathrm{N})$ | $R_{22}=\Delta \theta\left(\mathrm{C}-\mathrm{C}^{\alpha}-\mathrm{H}^{\alpha}\right)$ |
| $R_{3}=\Delta r\left(\mathrm{~N}-\mathrm{C}^{\alpha}\right)$ | $R_{23}=\Delta \theta\left(\mathrm{C}-\mathrm{C}^{\alpha}-\mathrm{C}^{\beta}\right)$ |
| $R_{4}=\Delta r(\mathrm{C}=\mathrm{O})$ | $R_{24}=\Delta \theta\left(\mathrm{H}^{\alpha}-\mathrm{C}^{\alpha}-\mathrm{C}^{\beta}\right)$ |
| $R_{\text {s }}=\Delta r(\mathrm{~N}-\mathrm{H})$ | $R_{25}=\Delta \theta\left(\mathrm{C}^{\alpha}-\mathrm{C}^{\beta}-\mathrm{H}\right)$ |
| $R_{6}=\Delta r\left(\mathrm{C}^{\alpha}-\mathrm{H}^{\alpha}\right)$ | $R_{26}=\Delta \theta\left(\mathrm{C}^{\alpha}-\mathrm{C}^{\beta}-\mathrm{H}\right)$ |
| $R_{7}=\Delta r\left(\mathrm{C}^{\beta}-\mathrm{H}\right)$ | $R_{27}=\Delta \theta\left(\mathrm{C}^{\alpha}-\mathrm{C}^{\beta}-\mathrm{H}\right)$ |
| $R_{8}=\Delta r\left(\mathrm{C}^{\beta}-\mathrm{H}\right)$ | $R_{28}=\Delta \theta\left(\mathrm{H}-\mathrm{C}^{\beta}-\mathrm{H}\right)$ |
| $R_{9}=\Delta r\left(\mathrm{C}^{\beta}-\mathrm{H}\right)$ | $R_{29}=\Delta \theta\left(\mathrm{H}-\mathrm{C}^{\beta}-\mathrm{H}\right)$ |
| $R_{10}=\Delta r\left(\mathrm{C}^{\alpha}-\mathrm{C}^{\beta}\right)$ | $R_{30}=\Delta \theta\left(\mathrm{H}-\mathrm{C}^{\beta}-\mathrm{H}\right)$ |
| $R_{11}=\Delta r(\mathrm{H} \cdots \mathrm{O})$ | $R_{31}=\Delta \theta(\mathrm{C}=\mathrm{O} \cdots \mathrm{H})$ |
| $R_{12}=\Delta r\left(\mathrm{H}^{\alpha} \cdots \cdot \mathrm{H}^{\alpha}\right)$ | $R_{32}=\Delta \theta(\mathrm{N}-\mathrm{H} \cdots \mathrm{O})$ |
| $R_{13}=\Delta \theta\left(\mathrm{C}^{\alpha}-\mathrm{C}-\mathrm{N}\right)$ | $R_{33}=\Delta \omega\left(\mathrm{O}=\mathrm{C}<\mathrm{C}^{\alpha}\right)$ |
| $R_{14}=\Delta \theta\left(\mathrm{C}-\mathrm{N}-\mathrm{C}^{\alpha}\right)$ | $R_{34}=\Delta \omega\left(\mathrm{H}-\mathrm{N}<\mathrm{C}^{\alpha}\right)$ |
| $R_{15}=\Delta \theta\left(\mathrm{N}-\mathrm{C}^{\alpha}-\mathrm{C}\right)$ | $R_{35}=\Delta \tau\left(\mathrm{C}^{\alpha}-\mathrm{C}\right)$ |
| $R_{16}=\Delta \theta\left(\mathrm{C}^{\alpha}-\mathrm{C}=\mathrm{O}\right)$ | $R_{36}=\Delta \tau(\mathrm{C}-\mathrm{N})$ |
| $R_{17}=\Delta \theta(\mathrm{N}-\mathrm{C}=\mathrm{O})$ | $R_{3,}=\Delta \tau\left(\mathrm{N}-\mathrm{C}^{\alpha}\right)$ |
| $R_{18}=\Delta \theta(\mathrm{C}-\mathrm{N}-\mathrm{H})$ | $R_{38}=\Delta \tau\left(\mathrm{C}^{\alpha}-\mathrm{C}^{\beta}\right)$ |
| $R_{19}=\Delta \theta\left(\mathrm{C}^{\alpha}-\mathrm{N}-\mathrm{H}\right)$ | $R_{39}=\Delta \tau(\mathrm{C}=0)$ |
| $R_{20}=\Delta \theta\left(\mathrm{N}-\mathrm{C}^{\alpha}-\mathrm{H}^{\alpha}\right)$ | $R_{40}=\Delta \tau(\mathrm{N}-\mathrm{H})$ |

$\Delta \theta(\mathrm{C}=\mathrm{O} \cdots \mathrm{H})$, and $\Delta \theta(\mathrm{N}-\mathrm{H} \cdots \mathrm{O})$. Within the peptide group the internal coordinates for out-of-plane bending, $\Delta \omega$, are defined as in Abe and Krimm. ${ }^{11}$ The definition of the coordinates associated with internal rotation follows the convention of Fukushima and Miyazawa. ${ }^{18}$ In Tables II and III we give these internal coordinates and their associated local symmetry coordinates, respectively. In these tables (Ala-Gly) ${ }_{n}$ requires both group A and B coordinates, whereas only group B coordinates are needed for a description of (Ala) ${ }_{n}$.

TABLE III
Local Symmetry Coordinates for One Peptide Group of Crystalline Poly(L-alanylglycine) and Poly(L-alanine)

Group A

| $S_{1}=R_{3}$ | $\mathrm{N}-\mathrm{C}^{\alpha}$ stretch |
| :---: | :---: |
| $S_{2}=R_{1}$ | $\mathrm{C}^{\alpha}-\mathrm{C}$ stretch |
| $S_{3}=R_{2}$ | $\mathrm{C}-\mathrm{N}$ stretch |
| $S_{4}=R_{4}$ | $\mathrm{C}=\mathrm{O}$ stretch |
| $S_{5}=R_{5}$ | $\mathrm{N}-\mathrm{H}$ stretch |
| $S_{6}=\left(R_{6}+R_{7}\right) / 2^{1 / 2}$ | $\mathrm{CH}_{2}$ symmetric stretch |
| $S_{7}=\left(R_{6}-R_{7}\right) / 2^{1 / 2}$ | $\mathrm{CH}_{2}$ asymmetric stretch |
| $S_{8}=\left(2 R_{11}-R_{15}-R_{16}\right) / 6^{1 / 2}$ | $\mathrm{C}-\mathrm{N}-\mathrm{C}^{\alpha}$ deformation |
| $S_{9}=\left(5 R_{12}-R_{17}-R_{18}-R_{21}-R_{19}-R_{20}\right) / 30^{1 / 2}$ | $\mathrm{N}-\mathrm{C}^{\alpha}-\mathrm{C}$ deformation |
| $S_{10}=\left(2 R_{10}-R_{14}-R_{13}\right) / 6^{1 / 2}$ | $\mathrm{C}^{\alpha}-\mathrm{C}-\mathrm{N}$ deformation |
| $S_{11}=\left(R_{14}-R_{13}\right) / 2^{1 / 2}$ | $\mathrm{C}=\mathrm{O}$ ip bend ${ }^{\text {a }}$ |
| $S_{12}=\left(R_{16}-R_{15}\right) / 2^{1 / 2}$ | $\mathrm{N}-\mathrm{H}$ ip bend |
| $S_{13}=\left(4 R_{21}-R_{17}-R_{18}-R_{19}-R_{20}\right) / 20^{1 / 2}$ | $\mathrm{CH}_{2}$ bend |
| $S_{14}=\left(R_{17}+R_{18}-R_{19}-R_{20}\right) / 2$ | $\mathrm{CH}_{2}$ wag |
| $S_{15}=\left(R_{1,}-R_{18}-R_{19}+R_{20}\right) / 2$ | $\mathrm{CH}_{2}$ twist |
| $S_{16}=\left(R_{17}-R_{18}+R_{19}-R_{20}\right) / 2$ | $\mathrm{CH}_{2}$ rock |
| $S_{17}=R_{26}$ | $\mathrm{C}=\mathrm{O}$ op bend ${ }^{\text {a }}$ |
| $S_{18}=R_{27}$ | $\mathrm{N}-\mathrm{H}$ op bend |
| $S_{19}=R_{30}$ | $\mathrm{N}-\mathrm{C}^{\alpha}$ torsion |
| $S_{20}=R_{28}$ | $\mathrm{C}^{\alpha}-\mathrm{C}$ torsion |
| $\mathrm{S}_{21}=R_{29}$ | $\mathrm{C}-\mathrm{N}$ torsion |
| $S_{22}=R_{8}$ | H - O stretch |
| $S_{23}=R_{9}$ | $\mathrm{H}^{\alpha} \cdots \mathrm{H}^{\alpha}$ stretch |
| $S_{24}=R_{22}$ | $\mathrm{C}=\mathbf{O} \cdots \mathrm{H}$ ip bend |
| $S_{25}=R_{23}$ | $\mathrm{N}-\mathrm{H} \cdots \mathrm{O}$ ip bend |
| $S_{26}=R_{24}$ | $\mathrm{C}=\mathrm{O} \cdots \mathrm{H}$ op bend |
| $S_{27}=R_{25}$ | $\mathrm{N}-\mathrm{H} \cdot \mathrm{O}$ op bend |
| Group B |  |
| $S_{1}=R_{3}$ | $\mathrm{N}-\mathrm{C}^{\alpha}$ stretch |
| $S_{2}=R_{1}$ | $\mathrm{C}^{\alpha}$-C stretch |
| $S_{3}=R_{2}$ | $\mathrm{C}-\mathrm{N}$ stretch |
| $\mathrm{S}_{4}=R_{4}$ | $\mathrm{C}=\mathrm{O}$ stretch |
| $S_{5}=R_{5}$ | N-H stretch |
| $S_{6}=R_{10}$ | $\mathrm{C}^{\alpha}-\mathrm{C}^{\beta}$ stretch |
| $S_{7}=R_{6}$ | $\mathrm{C}^{\alpha}-\mathrm{H}^{\alpha}$ stretch |
| $S_{8}=\left(R_{7}+R_{8}+R_{9}\right) / 3^{1 / 2}$ | $\mathrm{CH}_{3}$ symmetric stretch |
| $S_{9}=\left(R_{7}-R_{8}\right) / 2^{1 / 2}$ | $\mathrm{CH}_{3}$ asymmetric stretch 1 |
| $S_{10}=\left(2 R_{9}-R_{7}-R_{8}\right) / 6^{1 / 2}$ | $\mathrm{CH}_{3}$ asymmetric stretch 2 |
| $S_{11}=\left(2 R_{13}-R_{16}-R_{17}\right) / 6^{1 / 2}$ | $\mathrm{C}^{\alpha}-\mathrm{C}-\mathrm{N}$ deformation |
| $S_{12}=\left(R_{16}-R_{17}\right) / 2^{1 / 2}$ | $\mathrm{C}=\mathrm{Oip}$ bend |
| $S_{13}=\left(2 R_{14}-R_{18}-R_{19}\right) / 6^{1 / 2}$ | $\mathrm{C}-\mathrm{N}-\mathrm{C}^{\alpha}$ deformation |
| $S_{14}=\left(R_{18}-R_{19}\right) / 2^{1 / 2}$ | $\mathrm{N}-\mathrm{H}$ ip bend |
| $S_{15}=\left(R_{15}+R_{21}+R_{23}-R_{20}-R_{22}-R_{24}\right) / 6^{1 / 2}$ | $\mathrm{N}-\mathrm{C}^{\alpha}-\mathrm{C}$ deformation |
| $S_{16}=\left(2 R_{15}-R_{21}-R_{23}\right) / 6^{1 / 2}$ | $C^{\beta}$ bend 1 |
| $S_{17}=\left(R_{21}-R_{23}\right) / 2^{1 / 2}$ | $\mathrm{C}^{\beta}$ bend 2 |
| $S_{18}=\left(2 R_{24}-R_{20}-R_{22}\right) / 6^{1 / 2}$ | $\mathrm{H}^{\alpha}$ bend 1 |
| $S_{19}=\left(R_{20}-R_{22}\right) / 2^{1 / 2}$ | $\mathrm{H}^{\alpha}$ bend 2 |
| $S_{20}=\left(2 R_{28}-R_{29}-R_{30}\right) / 6^{1 / 2}$ | $\mathrm{CH}_{3}$ asymmetric bend 1 |
| $S_{21}=\left(R_{29}-R_{30}\right) / 2^{1 / 2}$ | $\mathrm{CH}_{3}$ asymmetric bend 2 |

TABLE III (continued)

| $S_{22}=\left(2 R_{25}-R_{26}-R_{27}\right) / 6^{1 / 2}$ | $\mathrm{CH}_{3}$ rock 1 |
| :--- | :--- |
| $S_{23}=\left(R_{26}-R_{27} / 2^{1 / 2}\right.$ | $\mathrm{CH}_{3}$ rock 2 |
| $S_{24}=\left(R_{28}+R_{29}+R_{30}-R_{25}-R_{26}-R_{27}\right)$ | CH symmetric bend |
| $S_{25}=R_{33}$ | $\mathrm{C}=\mathrm{O}$ op bend |
| $S_{26}=R_{34}$ | $\mathrm{~N}-\mathrm{H}$ op bend |
| $S_{27}=R_{3}$ | $\mathrm{~N}-\mathrm{C}^{\alpha}$ torsion |
| $S_{28}=R_{35}$ | $\mathrm{C}-\mathrm{C}$ torsion |
| $S_{29}=R_{36}$ | $\mathrm{C}-\mathrm{N}$ torsion |
| $S_{30}=R_{38}$ | $\mathrm{C}-\mathrm{C}^{\beta}$ torsion |
| $S_{31}=R_{31}$ | $\mathrm{C}=\mathrm{O} \cdots \mathrm{H}$ ip bend |
| $S_{32}=R_{32}$ | $\mathrm{~N}-\mathrm{H} \cdots \mathrm{O}$ ip bend |
| $S_{33}=R_{11}$ | $\mathrm{H} \cdots \mathrm{O}$ stretch |
| $S_{34}=R_{12}$ | $\mathrm{H} \alpha \cdots \mathrm{H} \alpha$ stretch |
| $S_{35}=R_{39}$ | $\mathrm{C=O}=\mathrm{O}$ torsion |
| $S_{36}=R_{40}$ | $\mathrm{~N}-\mathrm{H}$ torsion |

${ }^{\text {a }}$ Here ip denotes vibrations in the plane of the amide group and op denotes vibrations out of the plane of the amide group.

## Optical Activity and Selection Rules

For the optically active modes of $\beta$-(Ala $)_{n}$, the symmetry species and selectlon rules are those defined in Ref. 12 for the antiparallel chain-pleated sheet. There are 30 ir inactive normal modes in the $A$ symmetry group, and the $B_{1}, B_{2}$, and $B_{3}$ species each have 29 ir active modes. The $B_{1}$ modes display parallel dichroism with respect to the chain axis, while the $B_{2}$ and $B_{3}$ modes both have perpendicular dichroism. All modes are Raman active. Crystalline (Ala-Gly) $n$, with $C_{2}$ symmetry, has 50 and 49 normal modes of vibration in the $A^{\prime}$ and $A^{\prime \prime}$ symmetry groups, respectively. All modes are both ir and Raman active.

## Transition Dipole Coupling

Amide I Modes. In order to explain the observed splittings in amide I it is necessary to incorporate the effects of transition dipole coupling. ${ }^{12}$ The results on (Ala) ${ }_{n}$ were obtained in conjunction with those for (Gly I) ${ }_{n}$. As explained previously, ${ }^{12}$ we require that from one polypeptide to another the location of the transition dipole center and its orientation with respect to the $\mathrm{C}=\mathrm{O}$ bond remain unchanged. Therefore, as in (Gly I) ${ }_{n}$, the center is $0.868 \AA$ from the carbon atom along the $\mathrm{C} \rightarrow \mathrm{O}$ direction and the transition dipole is tilted $20^{\circ}$ with respect to the $\mathrm{C}=\mathrm{O}$ bond towards the $\mathrm{N} \rightarrow \mathrm{C}$ direction.

Using the approach described previously, ${ }^{12}$ the change in potential energy due to two interacting transition dipoles, $\partial \mu / \partial S$, is represented by

$$
\begin{equation*}
\Delta V_{s t}=\frac{1}{\epsilon}\left|\frac{\partial \mu_{s}}{\partial S_{s}}\right|\left|\frac{\partial \mu_{t}}{\partial S_{t}}\right| \cdot \frac{(\cos \alpha-3 \cos \beta \cos \gamma)}{\left|R_{s t}\right|^{3}} \Delta S_{s} \Delta S_{t} \tag{1}
\end{equation*}
$$

These energy terms (in $\mathrm{cm}^{-1}$ ) for various values of $s$ and $t$ represent the coefficients, $D_{s t}$, in the perturbation equation:

TABLE IV
Values of Ratios [Eqs. (3)] as a Function of $\Delta b$ for Crystalline Poly(L-alanine) and Poly(L-alanylglycine)

|  | $(A l a)_{n}$ |  | $(\text { Ala })_{n}:$ One Sheet |  | $(\text { Ala-Gly })_{n}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\Delta b, \AA$ | Ratio 1 | Ratio 2 | Ratio 1 | Ratio 2 | Ratio 1 | Ratio 2 |
| 0.0 | -0.146 | +0.854 | +0.019 | +1.019 | -0.298 | +0.702 |
| -0.1 | -0.261 | +0.739 | -0.090 | +0.902 | -0.385 | +0.615 |
| -0.2 | -0.357 | +0.642 | -0.201 | +0.799 | -0.466 | +0.533 |
| -0.3 | -0.439 | +0.561 | -0.291 | +0.709 | -0.534 | +0.464 |
| -0.4 | -0.502 | +0.487 | -0.371 | +0.629 | -0.595 | +0.404 |
| -0.5 | -0.576 | +0.429 | -0.442 | +0.558 | -0.650 | +0.350 |
| -0.6 | -0.633 | +0.367 | -0.504 | +0.496 | -0.698 | +0.302 |
| -0.7 | -0.687 | +0.313 | -0.562 | +0.438 | -0.745 | +0.255 |

$$
\text { Observed: (Ala })_{n}-\text { Ratio } 1=-0.40, \text { Ratio } 2=0.60
$$

$(\text { Ala-Gly })_{n}-$ Ratio $1=-0.51$, Ratio $2=0.48$

$$
\begin{equation*}
\nu\left(\delta, \delta^{\prime}\right)=\nu_{0}+\sum_{s, t=0}^{1} D_{s t} \cos (s \delta) \cos \left(t \delta^{\prime}\right) \tag{2}
\end{equation*}
$$

where each $D_{s t}$ represents a summation over similar terms within the chosen sphere of interaction, and the $D_{00}$ term is included in $\nu_{0}$. In order to obtain the relative axial shift between chains, we compare, as for (Gly I) $n,{ }^{12}$ calculated and observed values of the following ratios:

$$
\begin{align*}
& \text { ratio } 1=\frac{X_{01}+X_{11}}{X_{10}-X_{01}}=\frac{\nu(0,0)-\nu(0, \pi)}{\nu(0, \pi)-\nu(\pi, 0)-2 c_{1}} \\
& \text { ratio } 2=\frac{X_{10}+X_{11}}{X_{10}-X_{01}}=\frac{\nu(0,0)-\nu(\pi, 0)-2 c_{1}}{\nu(0, \pi)-\nu(\pi, 0)-2 c_{1}} \tag{3}
\end{align*}
$$

where the $X$ 's are the geometric factors in Eq. (1).
The calculated values of the ratios, obtained from the $X$ 's, are given in Table IV for (Ala) ${ }_{n}$ and (Ala-Gly) $)_{n}$. As for (Gly I) $n,{ }^{12}$ the range of interaction was $\pm 35 \AA$ within a sheet and $\pm 3$ sheets. (For comparison, we also give the results for only one sheet of (Ala) $)_{n}$.) The observed values of the ratios, obtained from the right side of Eq. (3), will depend slightly on the value of $c_{1}$, which is itself dependent on the force field. This constant is small, however, and, considering reasonable variations in the force field, we find that $0 \leq c_{1} \leq 1.0 \mathrm{~cm}^{-1}$; for (Ala) $)_{n}$, then, the value of ratio 1 is -0.40 and the value of ratio 2 is 0.60 . This implies that $\Delta b=-0.27 \AA$. Similarly, for (Ala-Gly) ${ }_{n}$ the values of ratios 1 and 2 are -0.51 and 0.48 , respectively, implying that $\Delta b=-0.25 \AA$. With these values of $\Delta b$ the amide I frequencies have been calculated, and these and the interaction constants are given in Table V, with the results for (Gly I) $n_{n}$ included for comparison. It will be seen that the agreement with the observed frequencies is very good, particularly in predicting that the $\nu(0, \pi)$ frequency is almost $20 \mathrm{~cm}^{-1}$ higher in (Ala-Gly) ${ }_{n}$ than in (Gly I) ${ }_{n}$.

Amide II Modes. The effects of transition dipole coupling on the

TABLE V
Amide I Frequencies and Interaction Constants (in $\mathrm{cm}^{-1}$ ) for Crystalline Poly(L-alanine), Poly(L-alanylglycine), and Polyglycine I

|  | (Ala) ${ }_{n}$ |  | (Ala-Gly) $n$ |  | (Gly I) $n$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Obsd | Calcd | Obsd | Calcd | Obsd | Calcd |
| $\nu(0,0)$ | 1669 | 1669 | 1665 | 1665 | 1674 | 1674 |
| $\nu(0, \pi)$ | 1695 | 1695 | 1702 | 1702 | 1685 | 1685 |
| $\nu(\pi, 0)$ | 1630 | 1630 | 1630 | 1630 | 1636 | 1637 |
| $\nu(\pi, \pi)$ |  | 1701 | 1693 | 1694 |  | 1690 |
| $D_{10}$ |  | 8.40 |  | 10.88 |  | 7.91 |
| $D_{01}$ |  | -24.20 |  | -25.60 |  | -16.10 |
| $D_{11}$ |  | 11.20 |  | 6.83 |  | 10.55 |
| $D_{00}$ |  | -1.66 |  | -3.90 |  | -6.10 |
| $\nu_{0}{ }^{\text {a }}$ |  | 1673.6 |  | 1672.9 |  | 1671.6 |
| $\Delta \mu_{\mathrm{eff}}{ }^{\text {b }}$ |  | 0.370 D |  | 0.370 D |  | 0.348 D |
| $\Delta b$ |  | -0.27 $\AA$ |  | -0.25 A |  | $0.00 \AA$ |

frequencies of the amide II vibrations in (Ala) $)_{n}$ and (Gly I) $n$ were reported previously. ${ }^{12}$ We have also incorporated these effects into our analysis of (Ala-Gly) ${ }_{n}$. As in (Gly I) $n$, we have taken the center of the transition dipole moment to be along the $\mathrm{N}-\mathrm{H}$ bond, $0.6 \AA$ away from the hydrogen atom. The angle that the amide II transition moment makes with the $\mathrm{C}=\mathrm{O}$ bond is similarly taken to be $68^{\circ}$. We calculate the magnitude of the transition dipole moment of amide II from the ratio of the intensity of the amide II band to that of the amide I band ( 0.532 ) and the effective transition moment of amide $\mathrm{I}, \Delta \mu_{\text {eff. }}$. Since amide I for (Ala) ${ }_{n}$ has $\Delta \mu_{\text {eff }}=0.370 \mathrm{D}$, and this value is the same for (Ala) ${ }_{n}$ and (Ala-Gly) $n$, we use for amide II a $\Delta \mu_{\text {eff }}=0.269$. The results of these transition dipole coupling calculations are given in Table VI, and agree well with observations.

## Amide A and B

It has been established ${ }^{12}$ that in the antiparallel chain-pleated sheet structure of homopolypeptides the only combinations of amide II bands that can possibly participate in Fermi resonance with the unperturbed $\mathrm{N}-\mathrm{H}$ stretching mode to produce the observed amide A and amide B bands are:

$$
\begin{equation*}
\nu(0,0)_{\mathrm{II}}+\nu(\pi, 0)_{\mathrm{II}} \rightarrow \nu(\pi, 0)_{\mathrm{B}} \tag{4}
\end{equation*}
$$

or

$$
\begin{equation*}
\nu(0, \pi)_{\mathrm{II}}+\nu(\pi, \pi)_{\mathrm{II}} \rightarrow \nu(\pi, 0)_{\mathrm{B}} \tag{5}
\end{equation*}
$$

In the previous paper ${ }^{12}$ we showed how to calculate the unperturbed amide A and B frequencies from the frequencies and relative intensities of the observed bands. The method was applied to (Ala) $)_{n}$ and gave $\nu_{a}{ }^{0}=3242$

TABLE VI
Amide II Frequencies and Interaction Constants (in $\mathrm{cm}^{-1}$ ) for Crystalline Poly(L-alanine), Poly(L-alanylglycine), and Polyglycine I

|  |  | ) $n$ | (Ala- | ly $)_{n}$ |  | $\mathrm{y} \mathrm{I})_{n}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Obsd | Calcd | Obsd | Calcd | Obsd | Calcd |
| $\nu(0,0)$ | 1538 (R) | 1534 |  | 1540 | 1515 (R) | 1515 |
| $\nu(0, \pi)$ | 1524 (ir) | 1523 | 1535 (R, ir) | 1533 | 1517 (ir) | 1516 |
| $\nu(\pi, 0)$ | 1555 (R) | 1555 | 1560 (R, ir) | 1570 |  | 1563 |
| $\nu(\pi, \pi)$ |  | 1587 |  | 1602 |  | 1588 |
| $D_{10}$ |  | -15.77 |  | -20.60 |  | -24.07 |
| $D_{01}$ |  | -4.95 |  | -6.82 |  | -6.43 |
| $D_{11}$ |  | 11.04 |  | 10.34 |  | 6.13 |
| $D_{00}$ |  | -1.70 |  | 6.00 |  | 2.90 |
| $\nu_{0}{ }^{\text {a }}$ |  | 1550.0 |  | 1560.8 |  | 1545.8 |
| $\Delta \mu_{\text {eff }}{ }^{\text {b }}$ |  | 0.269D |  | 0.269 D |  | 0.254 D |

and $\nu_{b}{ }^{0}=3109$. Using this approach for (Ala-Gly) $)_{n}$ gives $\nu_{a}{ }^{0}=3258$ and $\nu_{n}{ }^{0}=3108$. In (Ala-Gly) the $\nu(\pi, \pi)$ frequency is predicted very high (1602 $\mathrm{cm}^{-1}$ ), making the combination,

$$
\nu(0, \pi)_{\mathrm{II}}+\nu(\pi, \pi)_{\mathrm{II}} \rightarrow \nu(\pi, 0)_{\mathrm{B}}
$$

i.e.,

$$
1533+1602 \rightarrow 3135 \mathrm{~cm}^{-1}
$$

an unlikely one for the unperturbed amide B frequency. On the other hand, the combination

$$
\nu(0,0)_{\mathrm{II}}+\nu(\pi, 0)_{\mathrm{II}} \rightarrow \nu(\pi, 0)_{\mathrm{B}}
$$

i.e.,

$$
1540+1570 \rightarrow 3110
$$

agrees well with the predicted $\nu_{b}{ }^{0}=3108 \mathrm{~cm}^{-1}$.

## DISCUSSION

## Force Field

The force constants used to calculate the normal modes of (Ala) ${ }_{n}$ and (Ala-Gly) ${ }_{n}$ are those associated with the glycine residue, which are unchanged from the ones reported previously for (Gly I) ${ }_{n}$, ${ }^{12}$ plus force constants associated with the alanyl residue. The latter are given in Table VII.

No least squares refinement of $\mathrm{N}-\mathrm{H}$ or $\mathrm{C}-\mathrm{H}$ stretching force constants was attempted. The reported values of these force constants were adjusted to produce the best agreement with observation in (Gly I) ${ }_{n}$, (Ala-Gly) $n_{n}$, and $\alpha$ - and $\beta$-(Ala) ${ }_{n}$. Modes of (Ala) $)_{n}$ with $A, B_{1}$, and $B_{2}$ symmetry were all

TABLE VII
Force Constants for Alanyl Residue

| Force Constant ${ }^{\text {a }}$ | Value |
| :---: | :---: |
| 1. $f\left(\mathrm{C}^{\alpha}-\mathrm{C}^{\beta}\right)$ | $4.387{ }^{\text {b }}$ |
| 2. $f(\mathrm{~N}-\mathrm{H})$ | $5.674{ }^{\text {c }}$ |
| 3. $f\left(\mathrm{C}^{\alpha}-\mathrm{H}^{\alpha}\right)$ | $4.493{ }^{\text {d }}$ |
| 4. $f\left(\mathrm{C}^{\beta}-\mathrm{H}\right)$ | $4.800{ }^{\text {c }}$ |
| 5. $f\left(\mathrm{NC}^{\alpha} \mathrm{C}\right)$ | $0.595{ }^{\text {d }}$ |
| 6. $f\left(\mathrm{NC}^{\alpha} \mathrm{H}^{\alpha}\right)$ | 0.739 d |
| 7. $f\left(\mathrm{NC}^{\alpha} \mathrm{C}^{\beta}\right)$ | $1.030{ }^{\text {d }}$ |
| 8. $f\left(\mathrm{CC}^{\alpha} \mathrm{C}^{\beta}\right)$ | $1.050{ }^{\text {d }}$ |
| 9. $f\left(\mathrm{C}^{\beta} \mathrm{C}^{\alpha} \mathrm{H}^{\alpha}\right)$ | $0.642^{\text {d }}$ |
| 10. $f\left(\mathrm{C}^{\alpha} \mathrm{C}^{\beta} \mathrm{H}\right)$ | 0.669 d |
| 11. $f\left(\mathrm{HC}^{\beta} \mathrm{H}\right)$ | $0.524{ }^{\text {d }}$ |
| 12. $f\left(\mathrm{C}^{\alpha}-\mathrm{C}^{\beta}\right.$ tor $)$ | $0.110^{\text {e }}$ |
| 13. $f(\mathrm{H} \cdot \mathrm{O})$ | $0.150{ }^{\text {c }}$ |
| 14. $f\left(\mathrm{H}^{\alpha} \cdots \mathrm{H}^{\alpha}\right)$ | 0.011 [( Ala $)_{n}$ ] |
| 15. $f\left(\mathrm{H}^{\alpha} \ldots \mathrm{H}^{\alpha}\right)$ | 0.014 [( Ala -Gly $)_{n}$ ] |
| 16. $f\left(\mathrm{C}^{\alpha} \mathrm{C}, \mathrm{C}^{\alpha} \mathrm{C}^{\beta}\right)$ | $0.101^{\text {b }}$ |
| 17. $\left.f\left(\mathrm{NC}^{\alpha}, \mathrm{C}^{\alpha} \mathrm{C}^{\beta}\right) \quad\right\}$ | 0.101 b |
| 18. $f\left(\mathrm{C}^{\beta} \mathrm{H}, \mathrm{C}^{\beta} \mathrm{H}\right)$ | $-0.071^{\text {c }}$ |
| 19. $f\left(\mathrm{C}^{\alpha} \mathrm{C}, \mathrm{NC}^{\alpha} \mathrm{H}^{\alpha}\right)$ | $0.056^{\text {d }}$ |
| 20. $f\left(\mathrm{NC}^{\alpha}, \mathrm{CC}^{\alpha} \mathrm{H}^{\alpha}\right)$ | 0.056 |
| 21. $f\left(\mathrm{C}^{\alpha} \mathrm{C}, \mathrm{NC}^{\alpha} \mathrm{C}^{\beta}\right)$ | $0.010^{\text {c }}$ |
| 22. $f\left(\mathrm{NC}^{\alpha}, \mathrm{CC}^{\alpha} \mathrm{C}^{\beta}\right)$ | $0.010^{\text {c }}$ |
| 23. $f\left(\mathrm{C}^{\alpha} \mathrm{C}, \mathrm{CC}^{\alpha} \mathrm{C}^{\beta}\right)$ |  |
| 24. $f\left(\mathrm{NC}^{\alpha}, \mathrm{NC}^{\alpha} \mathrm{C}^{\beta}\right)$ | $0.417{ }^{\text {b }}$ |
| 25. $f\left(\mathrm{C}^{\alpha} \mathrm{C}^{\beta}, \mathrm{NC}^{\alpha} \mathrm{C}^{\beta}\right)$ | $0.417{ }^{\text {b }}$ |
| $\text { 26. } f\left(\mathrm{C}^{\alpha} \mathrm{C}^{\beta}, \mathrm{CC}^{\alpha} \mathrm{C}^{\beta}\right)$ |  |
| 27. $f\left(\mathrm{C}^{\alpha} \mathrm{C}, \mathrm{C}^{\beta} \mathrm{C}^{\alpha} \mathrm{H}^{\alpha}\right)$ |  |
| 28. $f\left(\mathrm{NC}^{\alpha}, \mathrm{C}^{\beta} \mathrm{C}^{\alpha} \mathrm{H}^{\alpha}\right)$ | 0.079 b |
| 29. $f\left(\mathrm{C}^{\alpha} \mathrm{C}^{\beta}, \mathrm{NC}^{\alpha} \mathrm{H}^{\alpha}\right)$ | 0.079 |
| 30. $f\left(\mathrm{C}^{\alpha} \mathrm{C}^{\beta}, \mathrm{CC}^{\alpha} \mathrm{H}^{\alpha}\right) \quad$ |  |
| 31. $f\left(\mathrm{NC}^{\alpha}, \mathrm{NC}^{\alpha} \mathrm{H}^{\alpha}\right)$ | $0.217{ }^{\text {d }}$ |
| 32. $\left.f\left(\mathbf{C}^{\alpha} \mathrm{C}^{\beta}, \mathrm{C}^{\beta} \mathrm{C}^{\alpha} \mathrm{H}^{\alpha}\right)\right\}$ | $0.328{ }^{\text {d }}$ |
| 33. $f\left(\mathrm{C}^{\alpha} \mathrm{C}^{\beta}, \mathrm{C}^{\alpha} \mathrm{C}^{\beta} \mathrm{H}\right)$ ) | $0.328{ }^{\text {d }}$ |
| 34. $f\left(\mathrm{CNC}^{\alpha}, \mathrm{NC}^{\alpha} \mathrm{H}^{\alpha}\right)$ | $0.100^{\text {d }}$ |
| 35. $f\left(\mathrm{NC}^{\alpha} \mathrm{C}, \mathrm{NC}^{\alpha} \mathrm{H}^{\alpha}\right)$ |  |
| 36. $f\left(\mathrm{NC}^{\alpha} \mathrm{C}, \mathrm{CC}^{\alpha} \mathrm{H}^{\alpha}\right)$ |  |
| 37. $f\left(\mathrm{NC}^{\alpha} \mathrm{C}^{\beta}, \mathrm{NC}^{\alpha} \mathrm{H}^{\alpha}\right)$ | -0.031b |
| 38. $f\left(\mathrm{CC}^{\alpha} \mathrm{C}^{\beta}, \mathrm{CC}^{\alpha} \mathrm{H}^{\alpha}\right)$ \} | -0.031 ${ }^{\text {b }}$ |
| 39. $f\left(\mathrm{CC}^{\alpha} \mathrm{C}^{\beta}, \mathrm{C}^{\beta} \mathrm{C}^{\alpha} \mathrm{H}^{\alpha}\right)$ |  |
| 40. $f\left(\mathrm{NC}^{\alpha} \mathrm{C}^{\beta}, \mathrm{C}^{\beta} \mathrm{C}^{\alpha} \mathrm{H}^{\alpha}\right)$ ) |  |
| 41. $f\left(\mathrm{NC}^{\alpha} \mathrm{C}^{\beta}, \mathrm{CC}^{\alpha} \mathrm{C}^{\beta}\right)$ ) |  |
| 42. $f\left(\mathrm{NC}^{\alpha} \mathrm{C}, \mathrm{NC}^{\alpha} \mathrm{C}^{\beta}\right)$ ) | $-0.041^{\text {b }}$ |
| 43. $f\left(\mathrm{NC}^{\alpha} \mathrm{C}, \mathrm{CC}^{\alpha} \mathrm{C}^{\beta}\right)$ ) |  |
| 44. $f\left(\mathrm{C}^{\alpha} \mathrm{NH}, \mathrm{NC}^{\alpha} \mathrm{H}^{\alpha}\right)$ | $0.011^{\text {d }}$ |
| 45. $f\left(\mathrm{C}^{\alpha} \mathrm{NH}, \mathrm{NC}^{\alpha} \mathrm{C}^{\beta}\right)$ | $0.020^{\text {d }}$ |
| 46. $f\left(\mathrm{NC}^{\alpha} \mathrm{H}^{\alpha}, \mathrm{CC}^{\alpha} \mathrm{H}^{\alpha}\right)$ | $0.310^{\text {d }}$ |
| 47. $\left.f\left(\mathrm{NC}^{\alpha} \mathrm{H}^{\alpha}, \mathrm{C}^{\beta} \mathrm{C}^{\alpha} \mathrm{H}^{\alpha}\right)\right\}$ | $0.043{ }^{\text {d }}$ |
| 48. $\left.f\left(\mathrm{CC}^{\alpha} \mathrm{H}^{\alpha}, \mathrm{C}^{\beta} \mathrm{C}^{\alpha} \mathrm{H}^{\alpha}\right)\right\}$ | 0.043 |
| 49. $f\left(\mathrm{C}^{\alpha} \mathrm{C}^{\beta} \mathrm{H}, \mathrm{C}^{\alpha} \mathrm{C}^{\beta} \mathrm{H}\right)$ | -0.045 ${ }^{\text {c }}$ |
| 50. $f\left(\mathrm{NC}^{\alpha} \mathrm{C}^{\beta}, \mathrm{C}^{\alpha} \mathrm{C}^{\beta} \mathrm{H}\right)^{t}$ | -0.049 ${ }^{\text {d }}$ |

TABLE VII (continued)

| Force Constanta | Value |
| :---: | :---: |
| 51. $f\left(\mathrm{NC}^{\alpha} \mathrm{C}^{\beta}, \mathrm{C}^{\alpha} \mathrm{C}^{\beta} \mathrm{H}\right)^{g}$ | -0.014 d |
| 52. $\left(\mathrm{C}^{\alpha} \mathrm{C}^{\beta} \mathrm{H}, \mathrm{C}^{\beta} \mathrm{C}^{\alpha} \mathrm{H}\right)^{t}$ | $0.122^{\mathrm{d}}$ |
| 53. $f\left(\mathrm{C}^{\alpha} \mathrm{C}^{\beta} \mathrm{H}, \mathrm{C}^{\beta} \mathrm{C}^{\alpha} \mathrm{H}\right) g$ | $0.016^{\mathrm{d}}$ |
| 54. $f(\mathrm{~N}-\mathrm{H} \cdots \mathrm{O}) \mathrm{opb}$ | $0.030^{\mathrm{f}}$ |
| 55. $f(\mathrm{C}=\mathrm{O} \cdots \mathrm{H}) \mathrm{opb}$ | $0.010^{\mathrm{f}}$ |

[^0]used in the least squares refinement procedure. It should be noted that the value of the $\mathrm{N}-\mathrm{H}$ stretching force constant required to give agreement with the unperturbed $\mathrm{N}-\mathrm{H}$ stretching frequency is lower than that required for (Gly I) ${ }_{n}$. This is consistent with the stronger hydrogen bond in the pleated as compared to the rippled-sheet structure. As in the case of (Gly $\mathrm{I})_{n}$, the interchain hydrogen bonds are not colinear with respect to the $\mathrm{C}=\mathrm{O}$ and $\mathrm{N}-\mathrm{H}$ bonds of the peptide groups which they connect.

A criticism of our previous force field ${ }^{11}$ was that it might lack success in predicting the low-frequency modes because no account was taken of interchain interactions via $\mathrm{H}^{\alpha} \ldots \mathrm{H}^{\alpha}$ interactions. This argument is seemingly valid since the $\mathrm{H}^{\omega} \ldots \mathrm{H}^{\alpha}$ distance in the structure of (Gly I) ${ }_{n}$ is only $2.6 \AA$, compared to a close contact of $\sim 2.5 \AA$ in crystalline polyethylene, where such interactions are known to be important. ${ }^{19}$ The $\mathrm{H}^{\alpha} \ldots \mathrm{H}^{\alpha}$ distances in (Ala) ${ }_{n}$ and (Ala-Gly) ${ }_{n}$ are shorter, viz., $2.37 \AA$ and $2.19 \AA$, respectively, and we have therefore included such interactions in our calculations through a Williams potential. ${ }^{20}$

The calculated frequencies are compared with the observed ir and Raman bands in Tables VIII and IX for (Ala) ${ }_{n}$ and (Ala-Gly) ${ }_{n}$, respectively.

## Assignments

Using the observed dichroic behavior, $\mathrm{CH}_{3}$ bending modes in (Ala) $n_{n}$ above $1410 \mathrm{~cm}^{-1}$ are readily assigned. In (Ala-Gly) ${ }_{n}$ both $\mathrm{CH}_{3}$ and $\mathrm{CH}_{2}$ bending modes have characteristic bands in this region. The $1447 \mathrm{~cm}^{-1}$ band in the Raman and ir spectra of (Ala-Gly) $)_{n}$ is most probably associated with the $\mathrm{CH}_{3}$ and $\mathrm{CH}_{2}$ deformations that are calculated at 1453 and 1440 $\mathrm{cm}^{-1}$, respectively. In the spectra of all $\beta$ polypeptides, there is a band at $\sim 1400 \mathrm{~cm}^{-1}$, i.e., $1410 \mathrm{~cm}^{-1}$ in (Gly I) ${ }_{n}, 1400 \mathrm{~cm}^{-1}$ in (Ala) ${ }_{n}$, and $1405 \mathrm{~cm}^{-1}$ in (Ala-Gly) ${ }_{n}$. Because in (Ala) $n$ this band is highly dichroic, we investigated the possibility of its being associated with the crystal lattice. In polypeptides with a beta carbon, the wagging of $\mathrm{H}^{\alpha}$ is invariably the dominant contributor to the potential energy of the $1400 \mathrm{~cm}^{-1}$ band. However,

TABLE VIII
Observed and Calculated Frequencies (in $\mathrm{cm}^{-1}$ ) of Poly(L-alanine)

(continued)

TABLE VIII (continued)

a Only contributions $10 \%$ or greater are included.
b Unperturbed frequency.

TABLE IX
Observed and Calculated Frequencies (in $\mathrm{cm}^{-1}$ ) of Poly(L-alanylglycine)

| Observed <br> Frequencies <br> Infrared | Calculated <br> Frequencies |  |
| :---: | :---: | :---: |

TABLE IX (continued)

| Observed <br> Frequencies <br> Infrared | Calculated <br> Frequencies |  | $A^{\prime}$ |
| :---: | :---: | :---: | :---: |

a Only contributions $10 \%$ or greater are included.
b Unperturbed frequency.
in the absence of the beta carbon, as in (Gly I) $)_{n}, \mathrm{CH}_{2}$ wagging in conjunction with $\mathrm{N}-\mathrm{H}$ in-plane bending was found to give rise to this band. In (AlaGly) $)_{n}$, as would be expected, a combination of $\mathrm{H}^{\alpha}$ wag and $\mathrm{CH}_{2}$ wag contribute to a band at approximately the same frequency $\left(1405 \mathrm{~cm}^{-1}\right)$. This absorption is also present in the $\alpha$-helical form of some polypeptides, e.g., poly(L-leucine), poly(L-norleucine), and (Ala) $n$. It is plausible that this particular band may be indigenous to polypeptides with alphatic side chains and not conformationally dependent. The inclusion of the $\mathrm{H}^{\alpha} \ldots \mathrm{H}^{\alpha}$ intermolecular interaction causes this frequency to shift by only small amounts, viz., about $4 \mathrm{~cm}^{-1}$. Because the $\mathrm{C}^{\alpha}-\mathrm{H}^{\alpha} \ldots \mathrm{H}^{\alpha}-\mathrm{C}^{\alpha}$ geometry has such a small effect on the frequency at $\sim 1400 \mathrm{~cm}^{-1}$, and also on all frequencies above $100 \mathrm{~cm}^{-1}$, the transferability of the portion of this force field associated with
$\mathrm{H}^{\alpha}$ and the side-chain residue to polypeptides in the $\alpha$-helical form should be valid.

Elliott ${ }^{4}$ speculated that the observed ir band at $1308 \mathrm{~cm}^{-1}$ in $\beta$-(Ala) $)_{n}$ was probably due to the presence of some $\alpha$-helical conformation in the predominantly $\beta$ sample. Actually in the ir spectrum of (Ala) ${ }_{n}$ there are two bands in this region, at 1332 and $1308 \mathrm{~cm}^{-1.4}$ From the spectrum it is not clear if these absorptions are indeed spectral contributions from $\beta$ or residual $\alpha$-helical (Ala) ${ }_{n}$. However, in the Raman spectrum of $\beta$-(Ala) ${ }_{n}$ the bands at $1335 \mathrm{~cm}^{-1}$ and $1311 \mathrm{~cm}^{-1}$ have intensities which are too large relative to the intensities of corresponding bands in the $\alpha$ form of the polymer to be assigned to residual $\alpha$ content. Since bands near these frequencies are predicted by our calculations, we assign the observed bands to the $\beta$ form, acknowledging that there may be some contribution from the $\alpha$ form. In (Ala-Gly) ${ }_{n}$, which is almost entirely in the antiparallel chain-pleated sheet structure, there are bands at $1334 \mathrm{~cm}^{-1}\left(1327 \mathrm{~cm}^{-1}\right.$ in Raman) and $1310 \mathrm{~cm}^{-1}$. In both polypeptides, and in both the $\alpha$ and $\beta$ forms, these bands are associated with the $\mathrm{CH}_{3}$ symmetric deformation and a bending motion of $\mathrm{H}^{\alpha}$. Relatively large contributions to the potential energy from motions of the residue are consistent with the apparent conformational independence of these two bands, although some $\mathrm{C}^{\alpha}-\mathrm{C}$ stretching is involved in the absorption around $1311 \mathrm{~cm}^{-1}$.

In (Ala) ${ }_{n}$ the amide III vibrations, which involve $\mathrm{N}-\mathrm{H}$ in-plane bending and $\mathrm{C}-\mathrm{N}$ stretching, account for all observed ir and Raman bands between $1200-1300 \mathrm{~cm}^{-1}$. The strong and highly parallel dichroic ir absorption at $1221 \mathrm{~cm}^{-1}$ is assigned to $\nu(0, \pi)$, i.e., the $B_{1}$ amide III vibration. Similarly, the $1241 \mathrm{~cm}^{-1}$ ir band ( $1243 \mathrm{~cm}^{-1}$ in the Raman), which has perpendicular dichroism, is assigned to the $\nu(\pi, 0)\left(B_{2}\right)$ amide III mode, while the $1223 \mathrm{~cm}^{-1}$ Raman band is assigned to the corresponding $\nu(0,0)$ (A) mode. As can be seen, $\mathrm{H}^{\alpha}$ bend makes significant contributions to these modes.

It has been suggested ${ }^{9}$ that the Raman band at $1243 \mathrm{~cm}^{-1}$ in (Ala) $)_{n}$ is due to the presence of the polymer in the disordered state. The normal vibration analyses of (Ala) $n$ and (Ala-Gly) ${ }_{n}$, however, strongly indicate that what is observed is the splitting of amide III in the antiparallel chainpleated sheet. The results of our subsequent calculations on poly(L-valine) also support these assignments for amide III. It is possible, of course, that the disordered polymer also has a band at $1243 \mathrm{~cm}^{-1}$.

In the amide III region of (Ala-Gly) ${ }_{n}$ ir bands are observed at $1265 \mathrm{~cm}^{-1}$ and $1230 \mathrm{~cm}^{-1}$, and correspond to calculated frequencies at $1269 \mathrm{~cm}^{-1}$ and $1225 \mathrm{~cm}^{-1}$, respectively. Although the $1265 \mathrm{~cm}^{-1}$ mode is predominantly $\mathrm{N}-\mathrm{H}$ in-plane bend (one of the components of amide III), the $1225 \mathrm{~cm}^{-1}$ band contains more of the $\mathrm{CH}_{2}$ twisting vibration associated with the glycyl residue (which predominates at $1248 \mathrm{~cm}^{-1}$ ). Thus, this region contains significant contributions from modes other than those of the peptide group, and it therefore may not be as useful a diagnostic of the conformation of the backbone as might be expected. This conclusion is not modified by
taking interactions into account, since the calculated 1225, 1248, and 1269 $\mathrm{cm}^{-1}$ frequencies are insensitive to the $\mathrm{H}^{\alpha} \ldots \mathrm{H}^{\alpha}$ interactions.

The strong ir band at $1167 \mathrm{~cm}^{-1}$ in (Ala) $)_{n}$ has parallel dichroism and appears in the spectra of both $\alpha$ - and $\beta$-(Ala $)_{n}$. Since the potential energy in this mode contains $30 \% \mathrm{~N}-\mathrm{C}^{\alpha}$ stretch and $27 \% \mathrm{H}^{\alpha}$ bend, it might seem surprising that this frequency is not more sensitive to conformation in view of the possible expectation that modes involving skeletal motions should be conformation sensitive. In fact, an examination of the eigenvectors of this vibration shows that the contribution of $15 \% \mathrm{CH}_{3}$ symmetric rock results in displacements in the $\mathrm{C}^{\alpha} \_\mathrm{C}^{3} \mathrm{H}_{3}$ group that predominate over those in the $\mathrm{N}-\mathrm{C}^{\alpha}$ bond, making this mode in a sense a side-chain mode. The situation is further complicated by the observation that in (Gly I) $n_{n}$ (which of course has no side chain) a band observed at $1162 \mathrm{~cm}^{-1}$ (Raman) is assignable to a mode comprised of $62 \% \mathrm{~N}-\mathrm{C}^{\alpha}$ stretch and $14 \% \mathrm{C}^{\alpha}-\mathrm{C}$ stretch. The important point that emerges is that caution is required in assigning bands of similar frequency in different systems to similar vibrational modes.

Bands at $1096 \mathrm{~cm}^{-1}, 1050 \mathrm{~cm}^{-1}$, and $909 \mathrm{~cm}^{-1}$ also appear in the spectra of both $\alpha$ - and $\beta$-(Ala) $n_{n}$. Corresponding bands appear in (Ala-Gly) ${ }_{n}$ spectra at $\sim 1100 \mathrm{~cm}^{-1}, 1050 \mathrm{~cm}^{-1}$, and $920 \mathrm{~cm}^{-1}$. Since these bands are absent in the spectra of $(\mathrm{Gly} \mathrm{I})_{n}$, it is rewarding to find that all of these bands are predicted to be predominantly associated with the motion of the alanyl residue.

The dichroic character of the $771 \mathrm{~cm}^{-1}$ band in (Ala) $)_{n}$ is not clear. In fact from the ir spectra it is not even clear whether this band is to be identified with the $\beta$ or the $\alpha$ form. Because in the Raman spectra of Frushour and Koenig ${ }^{9}$ a $775 \mathrm{~cm}^{-1}$ band is only observed for the rolled (Ala) ${ }_{n}$ fiber, which is known to be in the $\beta$ form, we have assigned it to a combined $\mathrm{C}=\mathrm{O}$ in-plane bending and $\mathrm{C}^{\alpha}-\mathrm{C}$ stretching mode of the $\beta$ form. Nonetheless some ir absorption at $\sim 776 \mathrm{~cm}^{-1}$ could also be present in the $\alpha$ conformation, although there does not even seem to be agreement on this point. ${ }^{21,22}$

In the adjustment of force constants and the assignment of bands between $695 \mathrm{~cm}^{-1}$ and $776 \mathrm{~cm}^{-1}$ for the polypeptides in the $\beta$ form we again confirm the suggestion that amide V , which is due predominantly to $\mathrm{N}-\mathrm{H}$ out-of-plane bending and $\mathrm{C}-\mathrm{N}$ torsion, is confined to a small $\left(\sim 5 \mathrm{~cm}^{-1}\right)$ region around $700 \mathrm{~cm}^{-1} .{ }^{23}$ Although the amide $V$ vibration near $700 \mathrm{~cm}^{-1}$ has been said to possess perpendicular dichroism, ${ }^{23}$ it does not appear to be highly dichroic. We assume that the reason for this is that the amide V modes of the $B_{1}, B_{2}$, and $B_{3}$ symmetry species all contribute to the absorption near $700 \mathrm{~cm}^{-1}$.

In the region from $690 \mathrm{~cm}^{-1}$ to $250 \mathrm{~cm}^{-1}$ agreement between observed and calculated frequencies is good. Two bands in $\beta$-(Ala) $)_{n}$ at $528 \mathrm{~cm}^{-1}$ and $372 \mathrm{~cm}^{-1}$ are due to the presence of some $\alpha$-(Ala) $)_{n}$ in the samples. In (Ala-Gly) ${ }_{n}$ a band at $370 \mathrm{~cm}^{-1}$ is also assigned to the $\alpha$-helical structure.

A recent study by Itoh and Katabuchi ${ }^{8}$ supports this explanation. They found that (Ala-Gly) ${ }_{n}$ after treatment with dichloroacetic acid assumes the $\alpha$-helical conformation, as indicated by the large intensity increase in the $375-371 \mathrm{~cm}^{-1}$ band in conjunction with enhanced absorption at $527-523$ $\mathrm{cm}^{-1}$.

The bands observed near $240 \mathrm{~cm}^{-1}$ have previously been assigned to methyl torsion. ${ }^{24}$ Using the value of the force constant associated with this assignment ( $f_{T}=0.11 \mathrm{mdyn}-\AA$ ), ${ }^{24}$ the methyl torsions are well predicted at $240 \mathrm{~cm}^{-1}$ and $256 \mathrm{~cm}^{-1}$ in (Ala) ${ }_{n}$ and (Ala-Gly) ${ }_{n}$, respectively. Although these modes at $\sim 250 \mathrm{~cm}^{-1}$ have been most often assigned to $\mathrm{C}-\mathrm{N}$ torsion and referred to as amide VII, our calculations definitely indicate that this is not correct. In all molecules that contain the methyl group some absorption in the $200-260 \mathrm{~cm}^{-1}$ region is observed. Certainly the internal rotation around $\mathrm{C}^{\alpha}-\mathrm{C}^{\beta}$ in polypeptides invites a comparison with similar rotations in $n$-paraffins at $\sim 250 \mathrm{~cm}^{-1}$, where a torsional force constant of $\sim 0.1$ mdyn $-\AA$ is required. ${ }^{25}$ Neutron scattering experiments also support this assignment. ${ }^{26}$ Other aspects of the low-frequency region of the spectrum of (Ala) ${ }_{n}$ have been discussed in detail in the preceding paper. ${ }^{12}$

## CONCLUSIONS

It is seen that the force field transferred from (Gly I) ${ }_{n},{ }^{12}$ to which constants associated with the methyl group have been added, provides a very good prediction of the vibrational spectra of (Ala) ${ }_{n}$ and (Ala-Gly) $)_{n}$ in the antiparallel chain-pleated sheet structure. Since (Gly I) ${ }_{n}$ has a rippledsheet structure, ${ }^{12}$ this force field exhibits a high degree of transferability for $\beta$ polypeptides.

As for (Gly I) ${ }_{n},{ }^{12}$ transition dipole coupling is important in accounting for the splittings in the amide I and amide II modes. The incorporation of this interaction mechanism also permits the determination of the axial shift, $\Delta b$, between chains in a sheet, probably more sensitively than is possible at present by X-ray or electron diffraction techniques on such poorly ordered systems. It is interesting to note that $\Delta b$ is in the range of -0.25 to $-0.45 \AA$ for the pleated sheet (based also on our calculations of other $\beta$ polypeptides), whereas for the rippled sheet of (Gly I) ${ }_{n}$ we find that $\Delta b \cong 0 .{ }^{12}$ Despite these structural differences, the unperturbed peptide group amide I frequency is essentially the same in all $\beta$ structures, $\sim 1673$ $\mathrm{cm}^{-1}$. The unperturbed amide II frequency is not nearly so constant, reflecting the less localized character of this mode. ${ }^{12}$

While the amide I and amide II modes are very characteristic of the main-chain conformation, we see that this is not as true of bands in the amide III region. The reason for this is that modes in this region (1200$1300 \mathrm{~cm}^{-1}$ ) contain significant side-chain contributions. Caution is therefore called for in using this region for diagnostic purposes.

Subsequent papers in this series will consider the analysis of the vibrational spectra of other $\beta$ polypeptides as well as polypeptides in other conformations. The analysis of small peptides as well as proteins will also be discussed.

This work was supported by National Science Foundation Grants BMS74-21163 and MPS75-05239.

## References

1. Miyazawa, T., Shimanouchi, T. \& Mizushima, S. (1958) J. Chem. Phys. 29, 611-616.
2. Itoh, K. \& Shimanouchi, T. (1967) Biopolymers 5, 921-930.
3. Jakeš, J. \& Krimm, S. (1971) Spectrochim. Acta 27A, 19-34.
4. Elliott, A. (1954) Proc. Roy. Soc., Ser. A 226, 408-421.
5. Fraser, R. D. B., MacRae, T. P., Stewart, F. H. C. \& Suzuki, E. (1965) J. Mol. Biol. 11, 706-712.
6. Itoh, K., Nakahara, T., Shimanouchi, T., Oya, M., Uno, K. \& Iwakura, Y. (1968) Biopolymers 6, 1759-1766.
7. Itoh, K., Shimanouchi, T. \& Oya, M. (1969) Biopolymers 7, 649-658.
8. Itoh, K. \& Katabuchi, H. (1972) Biopolymers 11, 1593-1605.
9. Frushour, B. G. \& Koenig, J. L. (1972) Biopolymers 13, 455-474.
10. Fanconi, B. (1973) Biopolymers 12, 2759-2776.
11. Abe, Y. \& Krimm, S. (1972) Biopolymers 11, 1817-1839.
12. Moore, W. H. \& Krimm, S. (1976) Biopolymers 15, 2439-2464.
13. Schachtschneider, J. H. \& Snyder, R. G. (1964) J. Polym. Sci., Part C 7, 99-124.
14. Hsu, S. L., Moore, W. H. \& Krimm, S. (1975) J. Appl. Phys. 46, 4185-4193.
15. Frushour, B. G. \& Koenig, J. L. (1975) Biopolymers 14, 2115-2135.
16. Arnott, S., Dover, S. D. \& Elliott, A. (1967) J. Mol. Biol. 30, 201-208.
17. Colonna-Cesari, F., Premilat, S. \& Lotz, B. (1975) J. Mol. Biol. 95, 71-82.
18. Fukushima, K. \& Miyazawa, T. (1965) J. Mol. Spectrosc. 15, 308-318.
19. Tasumi, M. \& Krimm, S. (1967) J. Chem. Phys. 46, 755-766.
20. Williams, D. E. (1967) J. Chem. Phys. 47, 4680-4684.
21. Itoh, K. \& Shimanouchi, T. (1970) Biopolymers 9, 383-399.
22. Fanconi, B., Small, E. \& Peticolas, W. (1971) Biopolymers 10, 1277-1298.
23. Masuda, Y., Fukushima, K., Fujii, T. \& Miyazawa, T. (1969) Biopolymers 8, 91-99.
24. Moore, W. H., Ching, J. H. C., Warrier, A. V. R. \& Krimm, S. (1973) Spectrochim. Acta 29A, 1847-1858.
25. Moore, W. H. \& Krimm, S. (1973) Spectrochim. Acta 29A, 2025-2042.
26. Drexel, W. \& Peticolas, W. L. (1975) Biopolymers 14, 715-721.

Received May 10, 1976
Accepted July 12, 1976


[^0]:    a Superscript $t$, $g$ refer to trans and gauche (Ref. 13);
    ${ }^{b}$ Transferred from Ref. 13 without refinement.
    c Transferred from Ref. 13 and adjusted by trial and error for agreement between observed and calculated frequencies of (Ala $)_{n}(\alpha \text { and } \beta \text { ) and (Ala-Gly) })_{n}$.
    ${ }^{d}$ Transferred from Ref. 13 and then refined.
    e Transferred from Ref. 25 without refinement.
    ${ }^{f}$ Transferred from Ref. 11 without refinement.

