THE UNIVERSITY OF MICHIGAN

COLLEGE OF ENGINEERING
Department of Meteorology and Oceanography

THE TRANSIENT PART OF THE ATMOSPHERIC PLANETARY WAVES

JAMES H. S. BRADLEY

AKSEL C. WIIN-NIELSEN
PROJECT DIRECTOR

ORA Project 06372

supported by:

NATIONAL SCIENCE FOUNDATION
GRANT NO. GP-2561
WASHINGTON, D.C.

administered through:

OFFICE OF RESEARCH ADMINISTRATION ANN ARBOR

JUNE 1967

This report was also a dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in The University of Michigan, 1967.

ACKNOWLEDGEMENTS

The author thanks the members of his Doctoral Committee, especially the Chairman, Dr. Aksel Wiin-Nielsen, and the Director of the Computing Center, Dr. R. C. F. Bartels, who also provided computing facilities and courses in numerical analysis. Many of the Computing Center Staff provided advice and assistance with language and hardware problems. IBM Corporation kindly modified the data channels of the 7090 computer to overcome some tape parity problems. Miss Margaret Drake of the National Center for Atmospheric Research provided data, some programs, and tuition in machine language. The major part of the programming was done by Mr. A. Pajas and Mr. H. Akfirat. Dr. A.H. Murphy's extensive personal library considerably eased literature searches. The author's wife, Estella, acted as editorial adviser and proof reader, and provided other encouragements.

Mrs. Francis Kalton, Barbara Koppe, and Kathy Hetmanski typed the drafts and text with skill and patience.

TABLE OF CONTENTS

TABLE OF CONTENTS (concluded)

LIST OF TABLES

LIST OF TABLES (concluded)

LIST OF FIGURES

ABSTRACT

The transient parts of the atmospheric planetary waves are studied observationally, using characteristic patterns and simple time filters to separate free, forced and standing modes.

The fast moving parts of the planetary waves have phase angles and velocities which are independent of meridional wavenumber but a function of zonal and vertical wavenumber, except when a subdominant meridional characteristic pattern is excited. The phase speeds have approximately the Rossby-Haurwitz values corresponding to the dominant meridional characteristic pattern.

Three universal characteristic patterns of pressure coincide with those of Obukhov (1960) and Holmström (1963); modes 1 and 3 are of much smaller divergence than mode 2, which has vertical mean and 500 mb value near zero. Nine analytical and numerical conclusions of Mashkovich (1961) are confirmed; his conclusion 1 on the dependence on meridional wavenumber is rejected. It is suggested that the scale analysis of Deland (1965b) is valid for all modes, when the divergence of mode 2 is taken into account. The vertical structure of all modes is consistent with that observed by Eliasen and Machenauer (1965) and computed by Mashkovich (1961).

Four universal meridional characteristic patterns of the fast-moving modes may be used for diagnostic and prognostic studies: modes 1 and 2 resemble the results of a numerical experiment by Mashkovich (1964b) in shape and typical frequency.

The slow-moving waves have individual meridional structures and typical periods of 21 to 28 days, in agreement with Mashkovich (1961). Beats between the standing, slow-moving and fast-moving planetary waves explain the dependence of phase velocity on meridional wavenumber (Eliasen and Machenauer, 1965; Deland and Lin, 1967) and amplitude (Eliasen, 1958) in unfiltered data.

The observed modes are strongly coupled by the vertical and meridional shear of the horizontal wind, and energy is exchanged at typical periods short compared to the typical periods of fluctuations in phase velocity.

Limits of random, contingent and systematic error of the objective analysis system are estimated, and the feasibility suggested of extracting greater value from existing data.

1. INTRODUCTION

1.1 OUTLINE[1]

The largest scales of motion or planetary waves in the atmosphere are insufficiently understood even for routine forecast requirements, despite many observational and theoretical studies by zonal and surface harmonics and by finite differences. This observational study divides the planetary waves into three classes: the standing or long period time average; the forced transient modes, which are approximately the seasonal variations of the standing part plus some interactions of the free modes; and the free transient modes. (The terms wave and mode are used synonymously.) This study is concerned primarily with the free modes. Tools newly applied to the planetary waves in this study are characteristic patterns (empirical orthogonal functions), and a simple time filtering technique. The established tools of analysis in analytically defined orthogonal functions in 1, 2 and 3 dimensions are also used.

According to present theory, the standing part results from the time average diabatic effects (Blinova, 1943; Smagorinsky, 1953), from interactions of the flow with mountains (Charney and Eliasen, 1949), and from interactions between the transient modes. Holopainen (1966) showed that interactions of the transient modes are important in the energy

[1]Appendix D is a Glossary.

balance of the standing part, but minor in the energy budget of the transient modes.

The forced modes may be separable into two parts, which, however, cannot be distinguished observationally without a much longer series of data than was used in this study. Mashkovich (1964b) predicted a set of slow moving waves generated by the interactions of the fast moving free waves: waves with periods about one month are observed in this study, which is too fast for them to be exclusively seasonal variations. The seasonal forced modes may be defined as those which would not exist if the seasonal cycle of energy sources and sinks were suppressed: they may be regarded as the seasonal variation of the standing part. Because both the slow modes predicted by Mashkovich and the standing part are maintained by the interactions of the free modes, it may be presumed that the seasonally forced modes are similarly maintained.

The transient free modes are truly propagational in the sense that if excited by any means they will exist and move even if the seasonal variation of the energy cycle is suppressed. The phase speeds of the free modes are much higher than the phase speeds of the forced modes: the free modes represent day-to-day variations, and all of them except the most divergent modes move with speeds near the Rossby-Haurwitz values for a meridional wavenumber of 2, 3 or 4.

Although this study is concerned mainly with the free transient waves, Chapter 3 mentions a few properties of the forced modes, which have not previously been studied except in the numerical models of Mashkovich (1961, 1964b). Because of discrepancies between existing theories and observations, and because no model simpler than the primitive Newtonian equations of motion has yet succeeded in forecasting the motion of the divergent free modes, this report uses the primitive Newtonian equations of motion to estimate the interactions between modes represented by various terms in an expansion in orthogonal functions. Because the transient part of the planetary waves is of small amplitude in the tropics and because no data is available in either the tropics or Southern Hemisphere, symmetry about the equator is built into the analysis.

The work described in this report could not be undertaken without extreme vigilance in both the numerical analysis and the calculations, which in many respects use the full facilities of the machine, system and library. The difficulties encountered were commensurate with those of others undertaking work of a similar scope; the high standards of the University of Michigan Computing Center have eased the problems encountered.

The compiler language exponent notation of FORTRAN, ALGOL, MAD, etc. is used throughout this report: e.g. 1E3 means 1000.

1.2 BRIEF REVIEW OF PREVIOUS WORK

The structure and motion of surface harmonic waves were studied observationally by Eliasen and Machenauer (1965), Deland (1965a) and Deland and Lin (1967), who computed the amplitude and east-west phase angle of the 500 mb stream function over a period of time, and applied some simple time filters to them. They found that the transient part of the planetary modes of motion at 500 mb did not have the barotropic Rossby-Haurwitz speed, and that no satisfactory barotropic divergence term could be introduced even with an empirical coefficient. They observed zonal phase speeds which depended on meridional wavenumber, the motion becoming less westerly or more easterly with increasing degree of the surface harmonic.

Eliasen and Machenauer, using only 1000 and 500 mb levels, found all the transient surface harmonics to be strongly baroclinic, the ridge or trough lines tilting to the west at higher levels.

Blinova (1943) first adapted the methods of spherical harmonics from theoretical physics (Gaunt, 1929; Infeld and Hull, 1951) to the numerical solution of the simultaneous non-linear partial differential equations of hydrodynamics.

She dealt with the problems of the standing part of the planetary scales (1943, 1953) and of 10-15 day hydrodynamic forecasts, primarily of the zonal circulation index or angular velocity of solid rotation of the atmosphere (1956, 1957, 1961, 1964, 1965, 1966). She showed that the solid rotation may be forecast by barotropic approximations for periods up to 15 days.

Mashkovich (1957, 1960, 1961, 1963, 1964b) applied spherical harmonics to the analytical and numerical study of the interaction of different scales of motion with the solid rotation and with each other, without friction. Some of Mashkovich's conclusions are translated as Appendix C. He concluded that the vertical, meridional and zonal shears of the wind are of comparable importance in the energetics of the atmosphere.

Scale analyses of the planetary waves have been discussed by Burger (1958) and Murakami (1963) on the assumption of quasi-stationary waves, and by Deland (1965b) on the assumption of speeds of the order of the Rossby-Haurwitz velocity. Burger concluded that the stationary planetary waves represented a balance between the divergence and beta effects; Murakami concluded that both the vorticity equation and the divergence equation become diagnostic; Deland concluded that the vorticity equation remains prognostic for fast-moving waves.

A list of papers at a conference on the general circulation (Izvestiya A.N. SSSR, Seriya Atm. i Okean. Fiz., 1, no. 3, 1965) suggests that B. L. Dzerdzeevskii may have priority in some of the work on characteristic patterns reported here, but the author has not read his paper.

1.3 AIM OF THE STUDY

The aim of the study is to obtain a better description, a comparison with previous theories, and a partial diagnostic understanding of the structure and motion of the fast-moving modes of the planetary waves.

The established tools of zonal and surface harmonic analysis and the new tools of characteristic patterns and a limited time spectrum analysis are the chief descriptive methods of this study. The spectral method is the chief diagnostic tool.

The principal new observational evidence is the availability of data at eight pressure levels, as contrasted to the one or two levels used in previous studies.

In any data study it is of primary importance to investigate the accuracy of the individual data and to estimate the influence of a less than ideal coverage of data over the region of interest.

In order to estimate the limits of error of deductions from the data, the problem of the existence of good data coverage only over land north of 30°N is examined on the basis of Gandin's (1963) method of optimum interpolation (after Kolmogorov (1941) and Wiener (1949), as further examined by Mashkovich (1964a)). Some new results are obtained by a model experiment on the statistical bounds of a generalized aliasing between wave vectors which bothered Gauss (1870); see Chapman and Bartels (1940).

1.4 ORTHOGONAL FUNCTION METHODS

Only a brief sketch is given here, as the use of orthogonal functions and more general methods are extensively treated in the literature (Infeld and Hull, 1951; Blinova, all references; Mashkovich, all references; Baer, 1964; Gavrilin, 1965; Gaunt, 1929; and many others).

In meteorological uses, the variables are written as a product of functions of space and another function of time. The orthogonality relations for the functions of space are then used to isolate the derivative of each function of time as a function of the other coefficients and set of constant interaction integrals.

Meteorological variables are assumed to be continuous with piecewise continuous derivatives in space and time, except for finite steps in the horizontal wind components at the north and south poles because of singularities in the coordinate system. Standard texts show that such functions may be expanded in series of orthogonal functions (Jeffreys and Jeffreys, 1962).

Consider, for instance, the east-west wind component

$$U(\theta,\lambda,p,t) = \sum_{M,N} \sum_{I} Y_N^M(\theta,\lambda) H_I(p) U_{M,N,I}(t) \qquad 1.4.1$$

where Y_N^M is the surface harmonic of colatitude θ and longitude λ with rank (zonal wavenumber) M and degree (zonal plus meridional wavenumber) N, H_I is the Ith pressure mode, and $U_{M,N,I}$ is a function of time only. In general, the summation has to be over a complete set of Y_N^M and H_I: in any application the series is necessarily truncated.

The orthonormality relations are

$$\int_S Y_N^M Y_Q^P \, dS \quad \begin{cases} = 0 & \text{if} \quad N \neq Q \quad \text{or} \quad M \neq P \\ = 1 & \text{if} \quad N = Q \quad \text{and} \quad M = P \end{cases} \qquad 1.4.2$$

where S is the surface of a sphere

$$\int_{p_o}^{p_1} H_I H_J \, dp \quad \begin{cases} = 0 & \text{if} \quad I \neq J \\ = 1 & \text{if} \quad I = J \end{cases} \qquad 1.4.3$$

Consider a non-linear equation of the form

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{A\sin\theta\partial\lambda} + \text{other terms} = 0 \qquad 1.4.4$$

where A is the radius of the earth.

Then the two terms shown may be written

$$\sum_{M,N} \sum_{I} Y_N^M H_I \, dU_{M,N,I}/dt$$
$$+ \sum_{M,N} \sum_{I} Y_N^M H_I U_{M,N,I} \sum_{P,Q} \sum_{J} \frac{\partial Y_Q^P}{A\sin\theta\partial\lambda} H_J U_{P,Q,J} \qquad 1.4.5$$

where the time derivative is an ordinary derivative because the U's are functions only of time.

Multiplying through by some particular $Y_T^R H_K$ and integrating with respect to area and pressure

$$dU_{R,T,K}/dt + \frac{1}{A} \sum_{M,N} \sum_{I} \sum_{P,Q} \sum_{J} U_{M,N,I} \; U_{P,Q,J} \qquad 1.4.6$$

$$\int_{p_0}^{p_1} H_I H_J H_K dp \qquad \int_S \frac{Y_N^M \; Y_T^R \partial \; Y_Q^P}{\sin\theta \partial\lambda} \, ds$$

Thus the non-linear partial differential equation 1.4.4 has been reduced to a set of simultaneous non-linear ordinary differential equations 1.4.6, provided the interaction integrals

$$\int_{p_0}^{p_1} H_I H_J H_K dp \quad \text{and} \quad \int_S Y_N^M \; Y_T^R \frac{1}{\sin\theta} \; \frac{\partial Y_Q^P}{\partial\lambda} \, ds \qquad 1.4.7$$

can be evaluated either analytically or numerically.

See Infeld and Hull (1951) for forms which cannot be handled by this simple technique.

Tables of associated Legendre functions were published by Kheifets (1950) ($1 \leq m \leq 12$, $m \leq n \leq 20$), Belousov (1956) ($0 \leq m \leq 36$, $m \leq n \leq 56$), Haurwitz and Craig (1952) ($0 \leq m \leq 8$, $m \leq n \leq 8$, but these contain errors), and by the Mathematical Tables Project (1945) ($0 \leq m \leq 4$, $m \leq n \leq 10$). All coefficients and functional values used in the present study were obtained by the recurrence formulae and checked against the values given by Belousov (1956), who devoted several man years to studying the propagation of errors in his recurrence formulae. No attempt was made or should be made to punch tables from type because of the risk of errors.

2. DEFINITIONS AND DATA PROBLEMS

2.1 DEFINITION OF STANDING, FORCED AND FREE MODES

For any field of a scalar or vector A resolved into a set of waves A_N, the standing part $\overline{A}_N$ is defined as the time average of the corresponding A_N, and the transient part as

$$A^*_N = A_N - \overline{A_N} \qquad 2.1.1$$

More precisely,

$$\overline{A_N} = \underset{T\to\infty}{\mathrm{Lt}} \frac{1}{T} \int_t^{t+T} A_N(S)\, dS \qquad 2.1.2$$

Given the fact that the available data tapes hold only three months of data, in practice the averaging period was three months. It was verified a posteriori that this approximation had a negligible effect on the free modes, which have typical rotational periods of 5 to 15 days.

The forced modes are defined as a moving time average minus the standing part

$$\tilde{A}_N = \frac{1}{2\tau} \int_{t-\tau}^{t+\tau} A_N(S)\, dS - \overline{A_N} \qquad 2.1.3$$

The selection of τ in relation to the physics of the system is discussed in the next section: in practice the value used was normally

$$2\tau = 5.5 \text{ days} \qquad 2.1.4$$

or in some cases 10.5, 20.5 and 30.5 days. When one is working with equally spaced observations, it is convenient to take the running mean over an odd number of observations in order to evaluate the deviation.

The free modes are defined by

$$A'_N = A_N - \tilde{A}_N - \overline{A_N} \qquad 2.1.5$$

The actual filter used allowed the free modes to be approximately those with rotational periods less than 15 days (Figure 3.2.1).

The forced modes were defined by moving averages of the A and B coefficients (see Glossary) separately.

2.2 SELECTION OF TIME SCALES

The time scale required for an effective separation of the free and forced modes depends on a partial knowledge of the power spectrum of the phenomena being studied. The findings of Mashkovich (1961, 1964b) and Eliasen and Machenauer (1965) suggest the existence of a group of modes with periods in the order of 5 to 15 days, and another group with periods greater than 21 days. This working hypothesis was verified by the results obtained.

A running time average over 5.5 days therefore appeared a reasonable time filter: the result of using some other comparable period would merely be some quantitative change in the amplitudes of the filtered waves with no significant change in the physics of the conclusions. Much more refined time filters, which will be needed in future investigations of the slow-moving forced modes, may be designed by the methods of Blackman and Tukey (1959).

Some other facts about the characteristic times of atmospheric processes are useful in interpreting the observations, especially as it turns out that the amplitude or energy of the free modes fluctuates much more rapidly than their phase velocity. Such information can be obtained from available studies of atmospheric energetics.

The essential part of atmospheric energetics is that the atmosphere contains two energy reservoirs, the available potential and the kinetic energies. Internal transformations of the kinetic energy between different modes occur through the vertical,zonal and meridional shears of the quasi-horizontal wind; these interactions are comparable to the conversions from available potential energy and faster than the frictional dissipation.

The picture is completed by the diabatic generation of available potential energy with a characteristic time about 15 to 20 days, and a frictional dissipation with a characteristic time of 5 days.

To illustrate the orders of magnitude involved, one may use values estimated by various authors (e.g. as tabulated by Oort (1964)) even when these have a substantial margin of uncertainty. Thus from the mean values in Oort's Figure 1, reproduced as Figure 2.2.1 one finds the residence times (see Glossary) shown in Table 2.2.1.

If one follows his notation, letting brackets [ ] represent a zonal average

$$[x] = \frac{1}{2\pi}\int_{0}^{2\pi} x\, d\lambda \qquad 2.2.1$$

an asterisk * a deviation from the zonal average $x^* = x-[x]$

a bar — a time average

$$\overline{x} = \frac{1}{\tau}\int_{t-\tau/2}^{t+\tau/2} x\, dt \qquad 2.2.2$$

and a prime ' a deviation from the time average $x' = x-\overline{x}$, and in all $x = [\overline{x}] + \overline{x}^* + [x]' + x'^*$ (note Oort's notation that a double prime represents a deviation from an area average), then the quantities in Table 2.2.1 are the zonal kinetic energy

$$K_M = \frac{1}{2}\int \overline{([u]^2 + [v]^2)} \quad dM \qquad 2.2.3$$

TABLE 2.2.1

RESIDENCE TIMES FOR ENERGY IN VARIOUS MODES: FROM OORT (1964)

Generation of Zonal Available Potential Energy	40/3 .1E-5 =	12.9E5 secs.	= 14.9 days
Conversion from Zonal to Eddy Available Potential Energy			
In Zonal Box	40/3.0E-5 =	13.3E5	= 14.2
In Eddy Box	15/3.0E-5 =	5.0E5	= 5.8
Generation of Eddy Available Potential Energy	15/0.8E-5 =	18.8E5	= 21.8
Conversion from Zonal Available to Zonal Kinetic Energy			
In Available Box	40/0.1E-5 =	400E5	= 462
In Kinetic Box	8/0.1E-5 =	80E5	= 92.6
Conversion from Eddy Available to Eddy Kinetic Energy			
In Available Box	15/2.2E-5 =	6.8E5	= 7.9
In Kinetic Box	7/2.2E-5 =	3.2E5	= 3.6
Conversion from Eddy to Zonal Kinetic Energy			
In Eddy Box	7/0.4E-5 =	17.5E5	= 20.3
In Zonal Box	8/0.4E-5 =	20E5	= 23.1
Dissipation of Eddy Kinetic Energy	7/1.8E-5 =	3.9E5	= 4.5
Dissipation of Zonal Kinetic Energy	8/0.5E-5 =	16E5	= 18.5

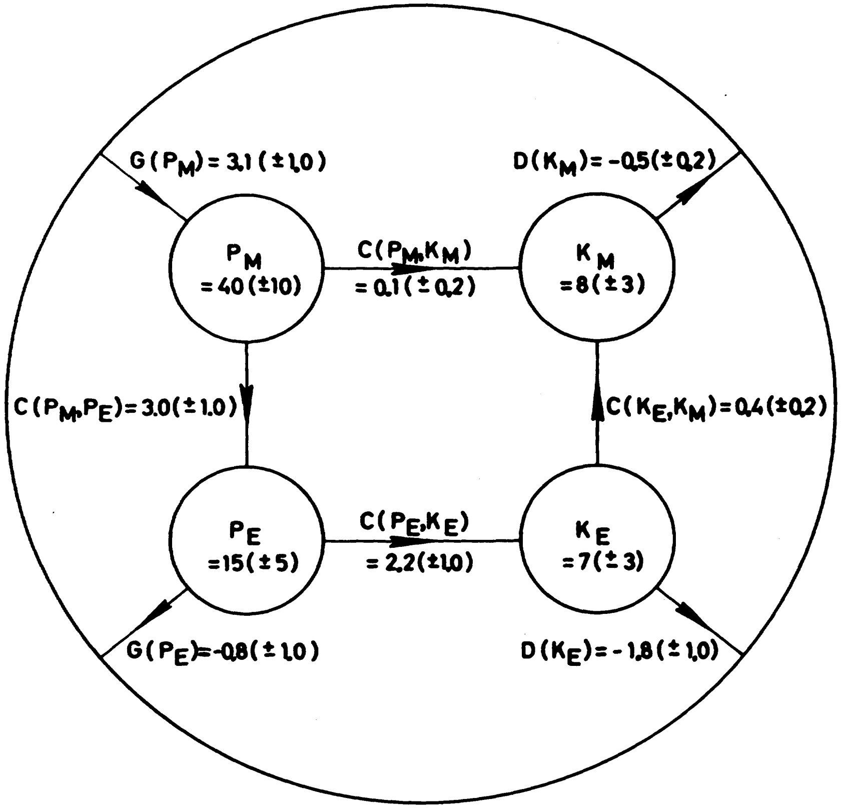


Figure 1.—Tentative flow diagram of the atmospheric energy in the space domain. Values are averages over a year for the Northern Hemisphere. Energy units are in 10^5 joule $m.^{-2}$ ($=10^8$ erg $cm.^{-2}$); energy transformation units are in watt $m.^{-2}$ ($=10^3$ erg $cm.^{-2}$ $sec.^{-1}$).

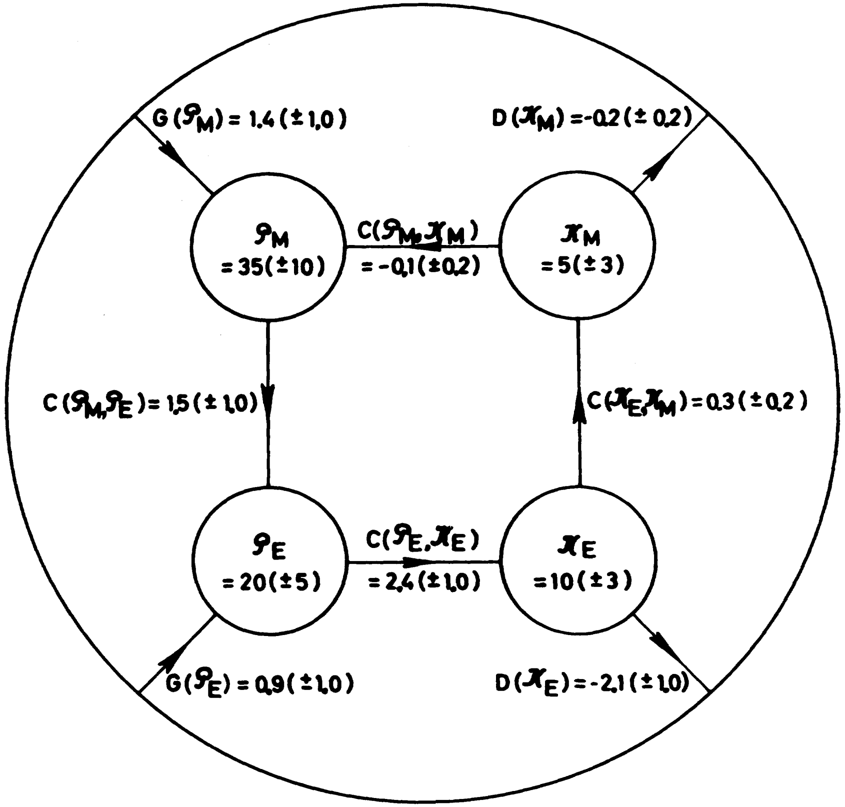


Figure 2.—Tentative flow diagram of the atmospheric energy in the mixed space-time domain. Values are averages over a year for the Northern Hemisphere. Energy units are in 10^5 joule $m.^{-2}$; energy transformation units are in watt m^{-2}

FIGURE 2.2.1.

FLOW DIAGRAM OF THE ATMOSPHERIC ENERGY CYCLE.
After Oort (1964).

eddy kinetic energy

$$K_E = \frac{1}{2}\int \overline{([u^{*2}] + [v^{*2}])} \; dM \qquad 2.2.4$$

zonal available potential energy

$$A_Z = \frac{c_p}{2}\int \overline{\gamma \; [T]''^2} \; dM \qquad 2.2.5$$

and eddy available potential energy

$$A_E = \frac{c_p}{2}\int \overline{\gamma[T^{*2}]} \; dM \qquad 2.2.6$$

as defined in Oort's space domain where T is the temperature. One would like these quantities

$$K_M = \frac{1}{2} \int [\bar{u}^2 + \bar{v}^2] \; dM \qquad 2.2.7$$

$$K_E = \frac{1}{2} \int [\overline{u'^2} + \overline{v'^2}] \; dM \qquad 2.2.8$$

$$P_M = \frac{c_p}{2} \int \gamma[\bar{T}'^{\,\prime 2}] \; dM \qquad 2.2.9$$

and

$$P_E = \frac{c_p}{2}\int \gamma[\overline{T'^2}] \; dM \qquad 2.2.10$$

As the transient eddies have amplitudes comparable to those of the standing eddies, we may use the residence times computed here.

Thus the most rapid processes after the non-linear interchange between different wave vectors are the conversion from zonal to eddy available potential energy (5.8 days), the conversion from eddy available to eddy kinetic energy (3.6 days), and the dissipation of eddy kinetic energy (4.5 days). Kung (1966) gives a depletion time of 2.7 days for the total kinetic energy over N. America. Holopainen (1966) studied the maintenance of the standing eddies; for the winter he found kinetic energy of 101 joules/kg in the transient eddies, and a conversion of 0.16 watts/m^2 between the standing and the transient eddies. As he remarks, this is of little importance to the transient eddies, corresponding to a residence time of 1,600 hours or about 9 weeks: this supports the separation made in this report. His conversion of 0.36 watts/m^2 between the transient eddies and the zonal flow corresponds to a residence time of about 4 weeks. One may adapt Holopainen's figure for the dissipation in the standing eddies to the transient eddies by assuming an equal residence time, i.e. a boundary layer dissipation proportional to the kinetic energy of the disturbance; this time is 6 joules kg^{-1}/0.11 watts m^{-2}, 139 hours or 5.8 days.

Now Holopainen (1966) claimed only an approximate estimate of the boundary layer dissipation, whereas Kung calculates only 31% of the dissipation to be in the boundary layer. It might therefore be necessary to reduce the estimate of 5.8 days for the dissipative residence time to about 2 days. Holopainen

(1963) studied the British Isles, finding a dissipation of 10 watts m^{-2} with 4 watts m^{-2} of this in the friction layer, and with a mean tropospheric kinetic energy of 24E5 joules m^{-2}. The residence time is then 10/3.6 = 2.8 days. Nowhere in the literature has the author found any estimate of the frictional dissipation in the wavenumber regime, and it is difficult to see how observational studies using either station data or grid point analyses could distinguish between friction by cumulus convection or by non-linear interactions which degrade energy to short wave vectors unresolvable in the existing data.

This study assumes as a working hypothesis that the published estimates of the frictional dissipation are too high, because they do not balance the estimated generation of available potential energy. All the published estimates of frictional dissipation evaluate it either as a residual or as an ageostrophic effect.

In contrast to the various zonal models summarized by Derome and Wiin-Nielsen (1966), in which the mechanism of the friction may play a large role in the structure of the atmosphere, the author uses as a working hypothesis the viewpoint of Mashkovich (1964b) who pointed out that the vertical and meridional windshears play comparable roles in atmospheric energetics. In this interpretation, the laws of friction and diabatic heating are not significant to the large scale structure and mechanisms of the atmosphere. This hypothesis is consistent with the difficulty encountered in deducing the frictional dissipation from observations of the system (Kung, 1966; Holopainen, 1963).

2.3 PROBLEMS OF AUTOMATIC ANALYSIS

The field of automatic machine analysis, so-called objective analysis, has always been controversial. Gandin (1963) remarks in his comprehensive review: "Although to date no comparison of the different methods of objective analysis has been made, with each applied to the same data, it is clear that the results of different objective analyses of the same situation may differ about as much as subjective analyses by different forecasters".

This report uses and extends previous methods of assessing the reliability of data analyses both by examining the internal consistency of the results and by extending the definition of aliasing to include the statistical errors of interpolation (see Glossary). The vertical structure of the analyses used in this report appears to be good, because more than 98% of the variance of every wave vector can be explained by three empirical orthogonal functions very similar to those which Obukhov (1960) and Holmström (1963) found for station soundings.

The main problems therefore appear to be in the horizontal plane, due to the sparsity of data south of 30^{o}N and over the oceans. Most analysis schemes use a procedure by which a preliminary field based on a 12 hour forecast from the preceding analysis gradually is modified by the available data. It appears probable that some further conclusions could be validated by a rigorous examination of doubtful features in the various steps in an operational analysis system.

Mashkovich (1964a) used Gandin's (1963) method of optimum interpolation to estimate the relative error of interpolation defined as mean square error of interpolation/mean square deviation of variable (Figure 2.3.1); the R.M.S. error of interpolation calculated by Mashkovich is 20 meters or more in the height field of an isobaric surface over about 80% of the Northern Hemisphere, and considerable contributions clearly exist to all the planetary scales of motion, a conclusion which was quantified by the model experiment described below. Gandin's method is optimum in the sense that it minimizes the mean square error of interpolation under the assumption that the deviations from the local long term averages have a structure function which depends only on

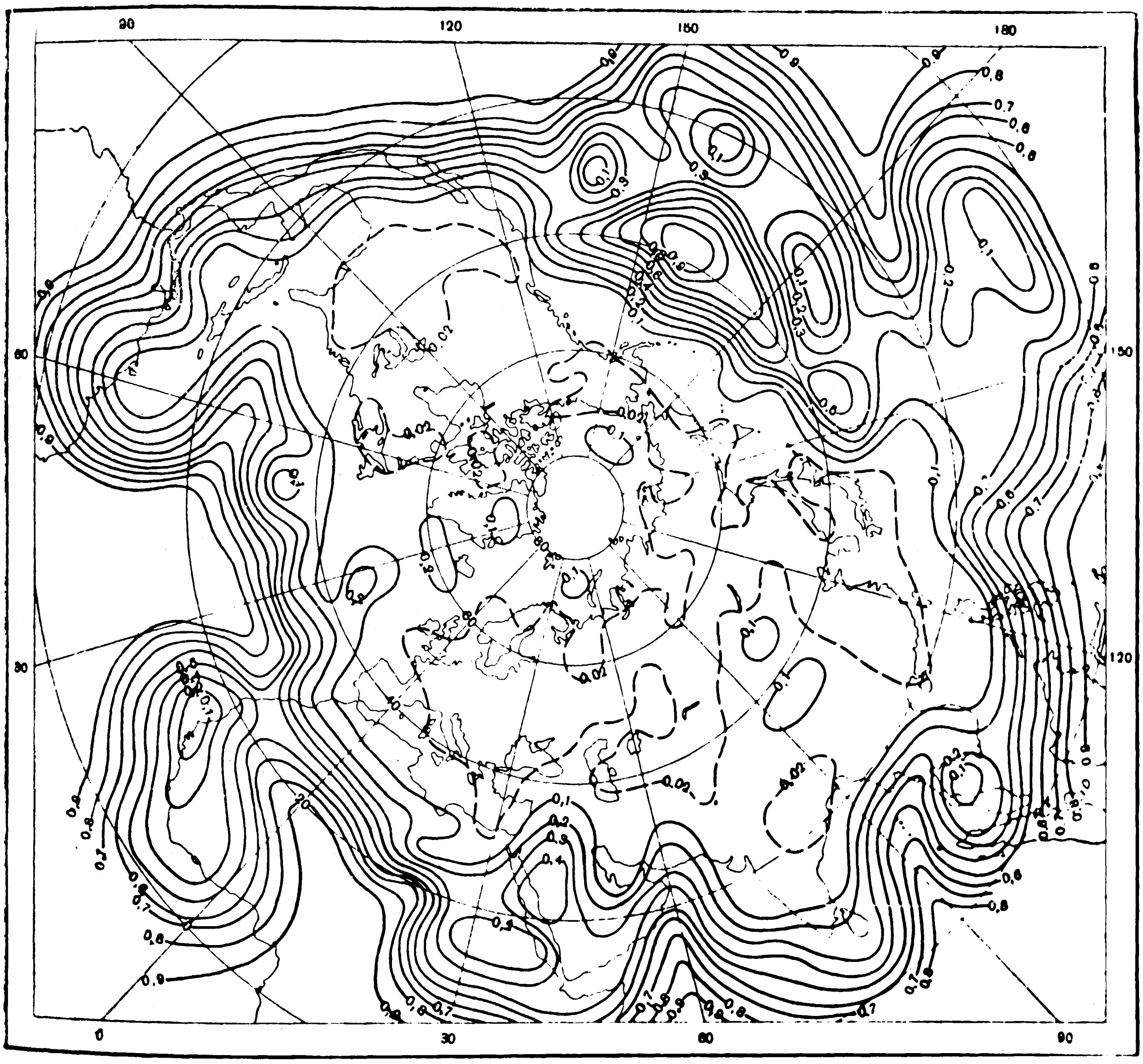


FIGURE 2.3.1.

RELATIVE ERROR OF INTERPOLATION.
After Mashkovich (1964a).

distance (the field of deviations is statistically homogeneous and isotropic). The interpolation in the data used in this study does not use the structure function, makes no allowance for errors of observation, and is quadratic, but the results of a model experiment suggest that the numbers which Gandin gives for these effects are rather small compared to the actual uncertainties, which are in the order of 30% of the square root of the variance of the geopotential.

In order to perform the model experiment, one must give an approximate linear theory of the operational analysis system.

Assume that by some means one had a vector W of the amplitudes of every wave vector in the atmosphere; each vector would be in practice the product of a sine or cosine of pressure, a sine or cosine of longitude, and an associated Legendre polynomial of latitude. Then, as the state of the atmosphere would be exactly defined, one could calculate a non-square matrix A such that the vector S of geopotentials at each station obeys

$$S = AW \qquad 2.3.1$$

The vector S is to be processed as though it were a set of observational data in order to obtain some vector Z which is an estimate of W.

In practice the vector S undergoes a long series of linear operations which normally introduce little rounding error: it is interpolated or extrapolated to a square lattice in some map projection ; smoothed; reinterpolated to a latitude-longitude grid; Fourier analyzed in zonal harmonics; and finally converted by a quadrature to the estimate Z of W, obeying

$$Z = BS \qquad 2.3.2$$

where B is some non-square matrix. In the perfect analysis system

$$AB = I \qquad 2.3.3$$

where I is the identity matrix, and in the practical system one may define an error matrix E by

$$E = AB - I \qquad 2.3.4$$

Neither A nor B can be evaluated explicitly, because they would be dimensioned about the number of stations (400) times the number of grid points (2000); E would be dimensioned about 400 by 400. The model experiment (Appendix A) gives an approximation to E.

It was concluded that any apparent effect exceeding 30% of the apparent variance of the relevant parameter (for example, the amplitude of a wave) was real, and that effects of less than 30% could be substantiated only by detailed scrutiny. In particular there is about 30% noise in the apparent phase velocities of fast-moving waves.

It was further concluded that a large part of the errors of interpolation have a quasi-systematic or contingent character, and that there is strong evidence against any assumption of randomness.

This systematic or contingent nature reflects the fact that the errors of interpolation are a function of the state of the atmosphere, the preceding forecast, and the distribution of data, as well as of approximately random errors such as instrument and transmission errors. A detailed examination of the entire analysis system would therefore allow greater value to be extracted from the existing data, especially for research in which continuity in both directions in time may be applied; some time correlation approach comparable to the structure function methods of Gandin (1963) might be based on the conclusions included in this report.

The results of the experiment suggest that various other effects are small compared to the errors of interpolation: for instance, the effects of constraint by the preceding forecast, satellite observations, interpolated or extrapolated pilot reports, moving ships, drifting icefloes, late or missing reports which make it uncertain what data went into any particular analysis, and errors of observation. Gandin (1963) found that errors of observation have a standard deviation of about 15 meters, while Hodge and Harmantas (1965) give a standard deviation of 1.5 mb (about 15 m.) for U.S. radiosondes; Riehl (1954) gives a similar value for the diurnal pressure cycle in the tropics, independent of height. Comparable figures are in Haurwitz and Cowley (1965) and Chapman and Westfold (1956). Even though missing observations are found preferentially in the strongest winds and in the regions of sparsest data, the aggregate result is that for the planetary waves the results of a linear model experiment are fairly quantitative.

2.4 NEGLECT OF THE TROPICS

Although the author had no choice but to omit the tropics from his calculations for lack of data, one must examine the possible effects of this approximation. Riehl (1954) gives a qualitative description of the stagnation of tropical weather. Newell, Wallace and Mahoney (1966) give data abstracted from Crutcher (1959) for the zonal average of the standard (R.M.S.) deviation of the meridional wind component in the standing and transient eddies, partly reproduced as Tables 2.4.1 and 2.4.2.

TABLE 2.4.1

STANDARD DEVIATION OF MERIDIONAL WIND COMPONENT.
ZONAL AVERAGE FROM CRUTCHER'S DATA.
Units: meters sec^{-1}
Dec./Jan./Feb. Standing eddy component

Lat.	80	70	60	50	40	30	20	10
Level (mb)								
100	2.2	3.9	4.1	3.6	2.7	3.3	3.4	2.1
150	2.3	4.1	5.3	4.9	3.1	3.4	4.7	2.4
200	2.7	3.8	5.6	6.1	4.3	3.3	4.9	2.5
300	2.9	3.5	5.1	5.9	4.6	2.7	3.8	2.0
400	2.9	3.3	4.4	5.2	3.9	2.1	3.0	1.8
500	2.6	2.9	3.5	4.3	3.2	1.8	2.6	1.5
600	2.3	2.4	2.9	3.6	2.7	1.6	2.1	1.3
700	1.7	1.7	2.0	2.9	2.5	1.8	1.8	1.3
800	1.0	1.1	1.8	2.6	2.4	1.7	2.1	1.2

TABLE 2.4.2

STANDARD DEVIATION OF MERIDIONAL WIND COMPONENT.
ZONAL AVERAGE FROM CRUTCHER'S DATA.
Units: meters sec^{-1}
Dec./Jan./Feb. Transient eddy component

Lat.	80	70	60	50	40	30	20	10
Level (mb)								
100	8.4	9.7	10.6	10.9	11.3	10.2	8.1	7.1
150	8.5	10.3	11.8	12.4	13.7	13.2	10.6	8.1
200	9.0	10.8	13.0	13.8	15.2	14.5	11.3	8.2
300	10.4	11.9	14.6	15.8	15.6	13.6	9.8	6.8
400	10.1	11.2	13.4	14.4	13.7	11.3	8.2	5.5
500	9.1	10.0	11.6	12.4	11.6	9.5	6.9	4.5
600	7.8	8.7	10.1	10.7	9.8	8.1	5.9	4.0
700	6.8	7.6	8.7	9.2	8.2	6.9	5.0	3.8
800	5.9	6.6	7.8	8.4	7.4	6.1	4.6	3.7

To supplement these data, the author abstracted some standard deviations of the vector mean wind at 850 and 200 mb from Crutcher (1959). Like Newell et al. he was obliged to extrapolate in regions where Crutcher was not willing to draw an isopleth, but this does not greatly influence the results shown in Table 2.4.3.

TABLE 2.4.3

MEAN SQUARE DEVIATION OF THE VECTOR MEAN WIND IN $(m.\ sec^{-1})^2$

	15^oN	40^oN
850 mb	136	435
200 mb	746	1803

All these figures show that the energy of the transient eddies in the tropics is much smaller than in middle latitudes, and suggest that the energy contained in the belt from the equator to 18.75^oN (0.32144 of the area of the Northern Hemisphere) is in the order of 10% of the total, which is no more than comparable to other unavoidable errors in the analysis. The empirical orthogonal functions of latitude suggest 1% if one assumes symmetry about the equator. One must also remark that during the forecast periods used in this study the most important energy transports are east-west, so that the approximation of neglecting the transient eddies in the tropics is much smaller than other sources of error.

3. OBSERVATIONAL RESULTS

In this chapter, the method of characteristic patterns (Appendix B) is applied in turn to evaluate a) the pressure modes of the transient part, b) the pressure modes of the free and forced transient waves separately, c) the meridional modes of the free and forced waves separately, d) the zonal characteristic patterns of the transient waves.

3.1 CHARACTERISTIC PATTERNS OF PRESSURE

Characteristic patterns of the geopotential as a function of pressure were evaluated for the A and B coefficients of surface harmonics for zonal and meridional wavenumbers 0 to 6 for February, March and June 1963. In all cases three modes explained 98% or more of the variance of the sample: these three modes showed little case-to-case variation, and were very similar to those obtained by Obukhov (1960) and Holmström (1963) from statistical ensembles of station soundings. Indistinguishable results were obtained from monthly ensembles of zonal harmonics 0 to 15 at several latitudes, and from 62 observation ensembles of the deviation from a 5.5 day running mean of zonal wavenumbers 0 to 15 at several latitudes. Typical percentage variances explained are shown in Tables 3.1.1 and 3.1.2.

TABLE 3.1.1

PERCENTAGE VARIANCE EXPLAINED BY EACH MODE, FEBRUARY 1963

M.W.N.	Mode \ Z.W.N.	0 A	1 A	1 B	2 A	2 B	3 A	3 B	4 A	4 B	5 A	5 B	6 A	6 B
0	1	92.8	34.4	73.1	78.5	69.7	71.9	90.5	91.6	79.4	77.5	84.9	93.7	92.0
	2	4.6	57.8	8.9	18.6	19.6	22.6	7.9	5.7	15.6	18.9	11.0	4.2	6.1
	3	1.4	6.8	17.4	2.2	8.8	4.0	1.1	1.8	3.3	0.2	2.6	1.3	1.0
2	1	94.3	88.4	83.8	83.0	81.4	79.1	94.6	89.1	72.6	77.2	80.9	93.0	91.4
	2	4.0	6.4	4.6	15.3	15.1	18.4	4.2	8.0	23.1	18.6	15.0	4.9	7.2
	3	1.1	4.6	11.4	1.3	2.3	1.9	0.9	2.0	2.5	2.8	3.0	1.3	0.8
4	1	65.2	93.3	90.6	94.4	95.7	95.3	89.7	86.1	81.1	88.5	70.6	83.4	78.7
	2	28.5	5.3	8.0	4.0	3.4	3.8	8.4	10.7	14.3	6.8	23.0	12.0	17.4
	3	5.4	1.1	0.9	1.3	0.7	0.6	1.6	2.2	3.0	2.9	5.2	2.8	2.3
6	1	75.7	81.8	92.2	85.9	83.8	92.4	88.5	82.8	86.0	77.7	80.3	82.5	93.7
	2	13.6	15.7	4.9	11.8	8.3	5.4	7.7	11.3	9.1	17.7	15.5	12.2	4.8
	3	10.1	1.8	2.5	1.6	6.7	1.7	3.0	4.0	3.5	2.9	2.2	3.6	0.9

TABLE 3.1.2

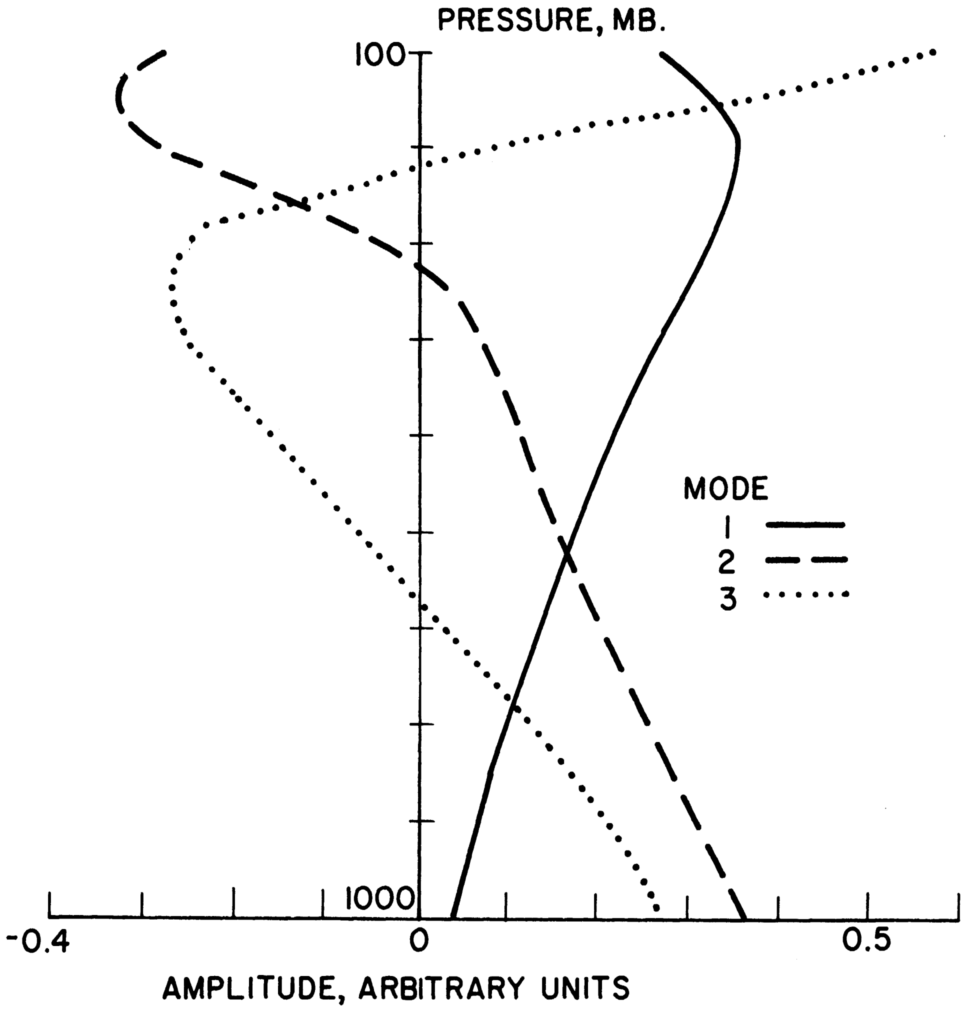
PERCENTAGE VARIANCE EXPLAINED BY EACH MODE, MARCH 1963

Z.W.N.		0	1		2		3		4		5		6	
M.W.N.	Mode	A	A	B	A	B	A	B	A	B	A	B	A	B
0	1	90.9	68.0	86.7	73.6	79.1	88.5	93.6	95.1	94.1	95.9	71.7	93.7	92.8
	2	6.7	29.8	10.0	19.3	15.3	7.7	3.7	4.0	4.9	3.3	24.0	4.8	5.1
	3	1.4	1.4	1.9	5.1	4.3	3.0	1.8	0.6	0.6	0.5	3.2	0.9	1.3
2	1	79.3	79.4	90.3	52.8	81.5	92.9	93.1	94.0	96.1	93.9	72.4	92.6	91.8
	2	17.7	18.2	8.1	34.4	11.9	5.9	4.9	5.2	3.2	5.2	23.0	6.2	6.3
	3	1.9	2.0	1.0	10.6	5.8	0.8	1.3	0.5	0.5	0.7	3.8	0.7	1.3
4	1	82.4	89.2	73.1	87.4	69.5	96.9	73.1	93.3	93.3	91.0	89.4	85.9	86.7
	2	15.7	9.0	23.2	10.3	24.4	2.6	22.9	5.3	5.2	8.0	9.2	10.6	10.1
	3	1.3	1.2	2.7	1.6	5.1	0.3	2.7	0.7	1.0	0.5	1.0	2.1	2.1
6	1	82.7	85.5	74.7	80.6	93.0	86.3	80.2	85.0	83.6	90.3	84.0	89.0	86.4
	2	13.6	10.4	21.0	14.1	5.3	8.9	13.3	11.0	12.4	7.8	13.2	7.9	10.7
	3	2.6	3.2	3.3	4.6	1.4	3.0	4.0	2.6	2.6	1.2	2.1	2.1	1.8

TABLE 3.1.3

NORMALIZED HOLMSTRÖM (1963) MODES USED AS STANDARDS

Pressure (mb)	1000	850	700	500	300	200	150	100
Mode								
1	.0634	.1256	.1959	.3111	.4609	.5116	.4758	.3802
2	.4823	.3834	.2783	.1546	-.0515	-.3813	-.4576	-.4019
3	.2522	.1619	.0290	-.1687	-.2454	.1704	.5283	.7158



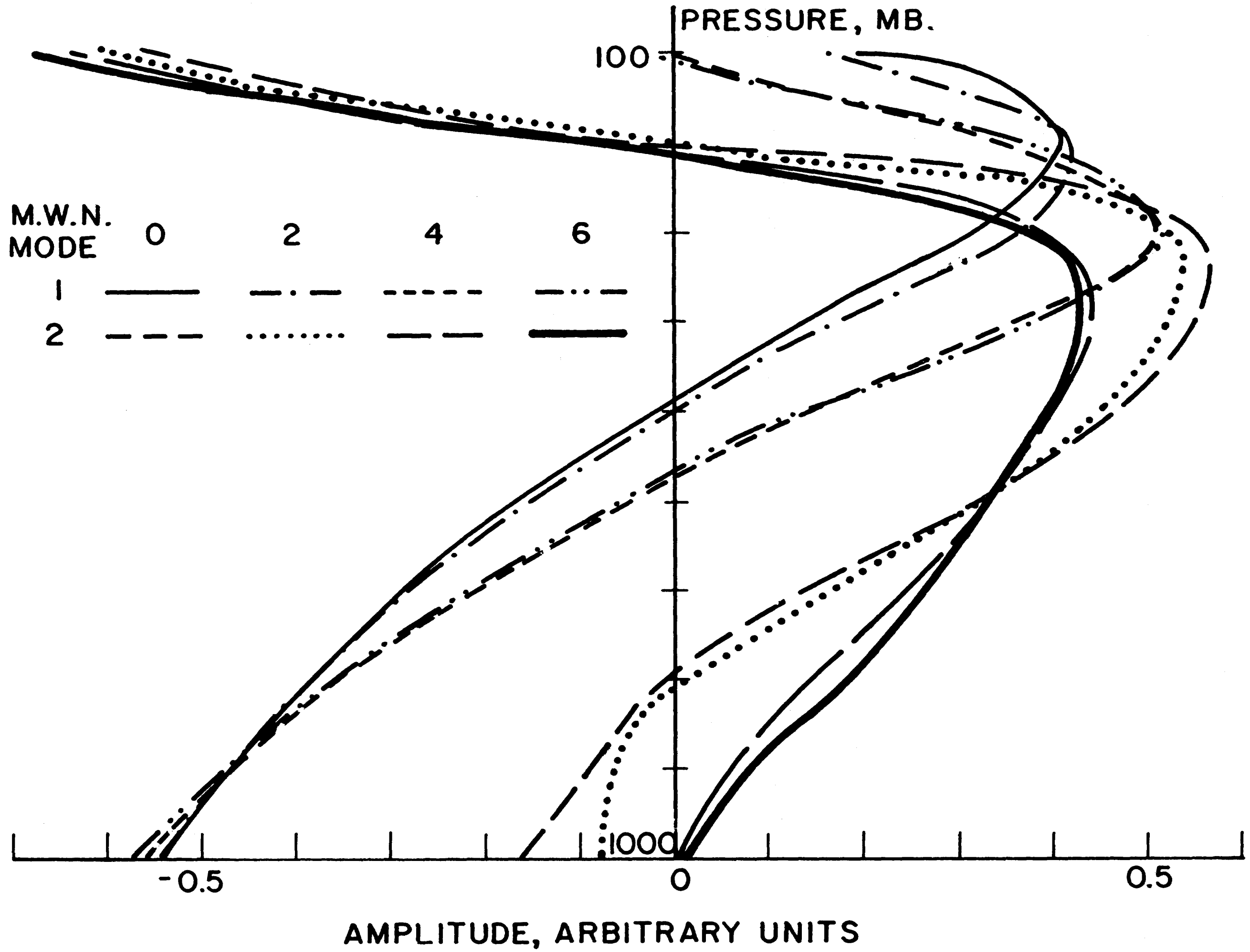
Observed Characteristic Patterns of the Geopotential
After Holmstrom, 1963.

FIGURE 3.1.1.

The modes shown in Table 3.1.3 and Figure 3.1.1 were adopted as standard, from Holmström's Figure 6, orthonormalized. No change in the conclusions of this report would result from adopting some other specimens as the standard modes: the residuals are comparable to those shown by Holmström.

Modes 1 and 3 have much larger vertical mean values than mode 2: it is shown later that their motions are similar to each other and to the motion of the vertical mean or barotropic component of the geopotential field. Mode 2 would be highly divergent in geostrophic flow because of its relatively steep gradient, and is probably so in nature. Although the transports of energy, momentum and matter, and the conversions between available potential and kinetic energy have not been calculated in terms of the vertical modes, it appears likely that a large part of the vertical velocity field is connected with mode 2. The importance of vertical mode 2 in the surface harmonic (1,0) in February 1963 is consistent with the finding of Wiin-Nielsen and Drake (1966) that the divergent energy conversions were unusually large at that time.

It may be deduced that only three properly selected levels with appropriately parameterized interactions are needed for the proper dynamic representation of the tropo-



Observed Characteristic Patterns of the Geopotential with the Vertical Mean Removed. Zonal Wavenumber 4, March 1963.

FIGURE 3.1.2

sphere and lower stratosphere. This is consistent with the small differences which Mashkovich (1961, 1963, 1964b) found between his 4 level and 10 level models.

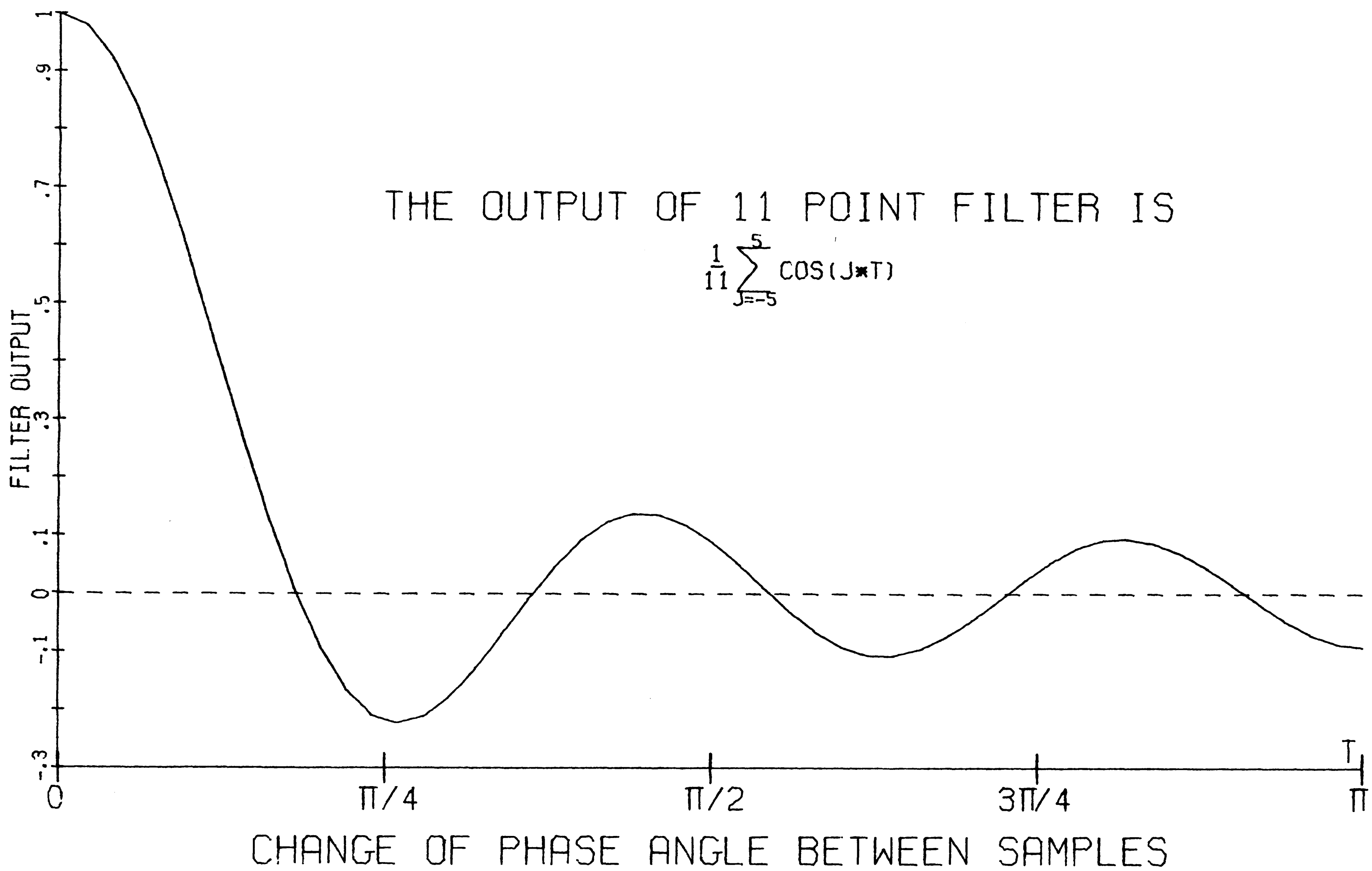
The vertical mean may be removed either by orthonormalization or by a direct evaluation of characteristic patterns from data with the vertical mean removed: the results are similar. Figure 3.1.2 shows the two modes determined for the A coefficient of zonal wavenumber 4, meridional wavenumbers 0, 2, 4, 6, March 1963. The variation of the modes with meridional wavenumber reflects the variation of tropopause height with latitude: when the vertical mean is retained, the variation with meridional wavenumber is smaller, and not necessarily significantly different from zero. The percentage variance explained by each mode is typically 70-80% by the vertical mean, 10-20% by the new mode 1, and 1-10% by the new mode 2, with the total about 98%. The case-to-case variation is several times larger and might restrict the utility of such modes. No further work has been done using modes with the vertical mean removed.

It is concluded that three vertical modes, or the vertical mean and two vertical modes, explain more than 98% of the variance of the geopotential in any statistical ensemble of station soundings, zonal harmonics, or surface harmonics for either free or forced modes. Modes 1 and 3 are approximately non-divergent, but mode 2 is strongly divergent and was abnormally important at a time when the divergent energy conversions were abnormally large.

3.2 MOTION OF FREE AND FORCED PRESSURE MODES

The expansion of data in terms of pressure modes shows at once that the phase velocity of any surface harmonic becomes more uniform than that of the component at any given pressure, that nearly all the time the phase angle is a function of zonal wavenumber and pressure mode but not a function of meridional wavenumber, and that the direction and mean phase velocity of motion depend on the vertical mode number. Typical rotational periods of 21 to 28 days appear, and may tentatively be identified with cyclone families. A diurnal cycle with an amplitude of about 20 meters and in the same phase for all vertical modes is obvious in all vertical modes of zonal wavenumber 1, but it reverses phase on February 15 and has a small average value; statistics are shown in Table 3.2.1, from the mean amplitudes at 0000 and 1200 GMT evaluated separately.

Much clearer results are obtained, however, when a simple time filter is used to separate the free and forced modes predicted by Mashkovich (1964b). A 5.5 day running mean and



THE OUTPUT OF 11 POINT FILTER IS
$\frac{1}{11}\sum_{J=-5}^{5} COS(J*T)$
FILTER OUTPUT
1
.9
.7
.5
.3
.1
0
-.1
-.3
0
Π/4
Π/2
3Π/4
Π
T
CHANGE OF PHASE ANGLE BETWEEN SAMPLES

FIGURE 3.2.1.

the deviation from it were used, with the filter response shown in figure 3.2.1. The phase angles of both free and forced modes are again independent of meridional wavenumber except on rare occasions when a sub-dominant meridional mode is excited, with a small systematic difference corresponding to an average NE-SW tilt of the ridge and trough lines. The phase angles of vertical modes 1 and 3 are normally close together. The phase speeds of the unfiltered transient waves are higher when the amplitudes of the free modes are large, which extends and refines the conclusions of Eliasen (1958).

In the free, fast moving modes, all vertical modes of zonal wavenumbers greater than 3 move east; for zonal wavenumbers 1, 2, and 3 vertical modes 1 and 3 move predominantly west, and mode 2 moves predominantly east.

The average angular velocities of the free modes are shown in Table 3.2.2, and of the forced modes in Table 3.2.3. The tables are constructed by human interpretation of plots of 90 days of data, a task which is too involved to be expedient to program for a machine: phase angles are interpolated wherever the amplitude falls below 1 decameter. A few discontinuous jumps remain even at moderate amplitudes. The mean phase speeds of the forced modes are very approximate, and not necessarily significantly different from zero. As

TABLE 3.2.1.

MEAN AMPLITUDE AND DIURNAL TIDE OF SURFACE HARMONICS OF 3 PRESSURE MODES, FEBRUARY AND MARCH 1963, METERS.

Z.W.N.		0		1		2		3		4		5		6	
MWN.	VERT.	A	T	A	T	A	T	A	T	A	T	A	T	A	T
0	1	44.94	.85	54.35	.88	44.05	.75	41.42	1.05	44.93	.83	34.33	1.22	29.39	.25
	2	24.38	.71	42.54	2.23	31.61	.45	23.28	.55	23.10	.10	18.25	1.01	15.08	.09
	3	27.14	.71	35.51	.20	23.20	.10	22.17	.47	23.24	.32	17.62	.83	15.71	.17
2	1	51.29	.02	99.16	1.20	93.50	.19	98.53	1.00	112.79	.46	81.00	.67	70.30	.41
	2	25.03	.78	54.22	.21	47.98	.23	43.76	.43	47.60	.52	38.90	1.32	33.38	.05
	3	34.60	.47	59.20	1.15	50.84	.71	48.15	.87	52.79	.35	38.73	.83	35.85	.20
4	1	62.84	.67	121.08	1.55	104.25	1.66	98.58	.60	95.53	.24	67.08	.99	44.62	.19
	2	27.93	.29	54.23	.57	51.42	.21	40.64	.01	31.66	.08	28.33	.46	21.11	.03
	3	35.73	.24	64.05	.44	55.99	.78	46.81	.27	40.30	.72	28.48	.20	19.12	.14
6	1	36.65	1.79	101.42	.96	83.60	1.33	55.89	.78	53.02	1.89	45.19	.44	38.15	.98
	2	19.05	.21	40.85	.17	37.20	1.43	23.99	.41	22.80	.92	19.97	.17	17.15	.30
	3	20.53	.89	50.58	.08	37.80	.82	24.58	.41	22.65	1.40	18.34	.75	15.50	.62

TABLE 3.2.2.

AVERAGE JAN.-MARCH 1963 PHASE VELOCITIES IN DEGREES PHASE AND LONGITUDE PER DAY, FREE MODES

ZONAL W.N.		1	2	3	4	5	6
VERT. MODE							
1	PHASE	-32.5	-33.0	-9.0	18.0	43.9	45.8
	LONG.	-32.5	-16.5	-3.0	4.5	8.8	7.6
2	PHASE	16.0	38.3	36.0	45.0	54.8	50.6
	LONG.	16.0	19.1*	12.0	11.3	11.0	8.4

TABLE 3.2.3.

AVERAGE JAN.-MARCH 1963 PHASE VELOCITIES IN DEGREES PHASE AND LONGITUDE PER DAY, FORCED MODES

ZONAL W.N.		1	2	3	4	5	6
VERT. MODE							
1	PHASE	-7.0	-6.8	-1.5	-2.0	-2.5	5.0
	LONG.	-7.0	-3.4	-.5	-.5	-.5	.8
2	PHASE	-7.0	2.3	5.5	1.5	1.5	4.5
	LONG.	-7.0	1.1	1.8	.4	.3	.8

*IN PERIODS WHEN SMALL AMPLITUDES DO NOT REQUIRE INTERPOLATIONS, THIS VALUUE IS 15.0 DEGREES LONGITUDE PER DAY.

TABLE 3.2.4.

ROSSBY-HAURWITZ SPEEDS IN DEGREES LONGITUDE PER DAY, WITH THE INDEX OF SOLID ROTATION $\bar{\alpha}$ =15.58 DEGREES LONG. PER DAY.

$$\omega_{R-H} = \bar{\alpha} - \frac{2(360+\bar{\alpha})}{N(N+1)}$$

WHERE THE DEGREE 'N' IS THE SUM OF THE ZONAL AND MERIDIONAL WAVE NUMBERS.

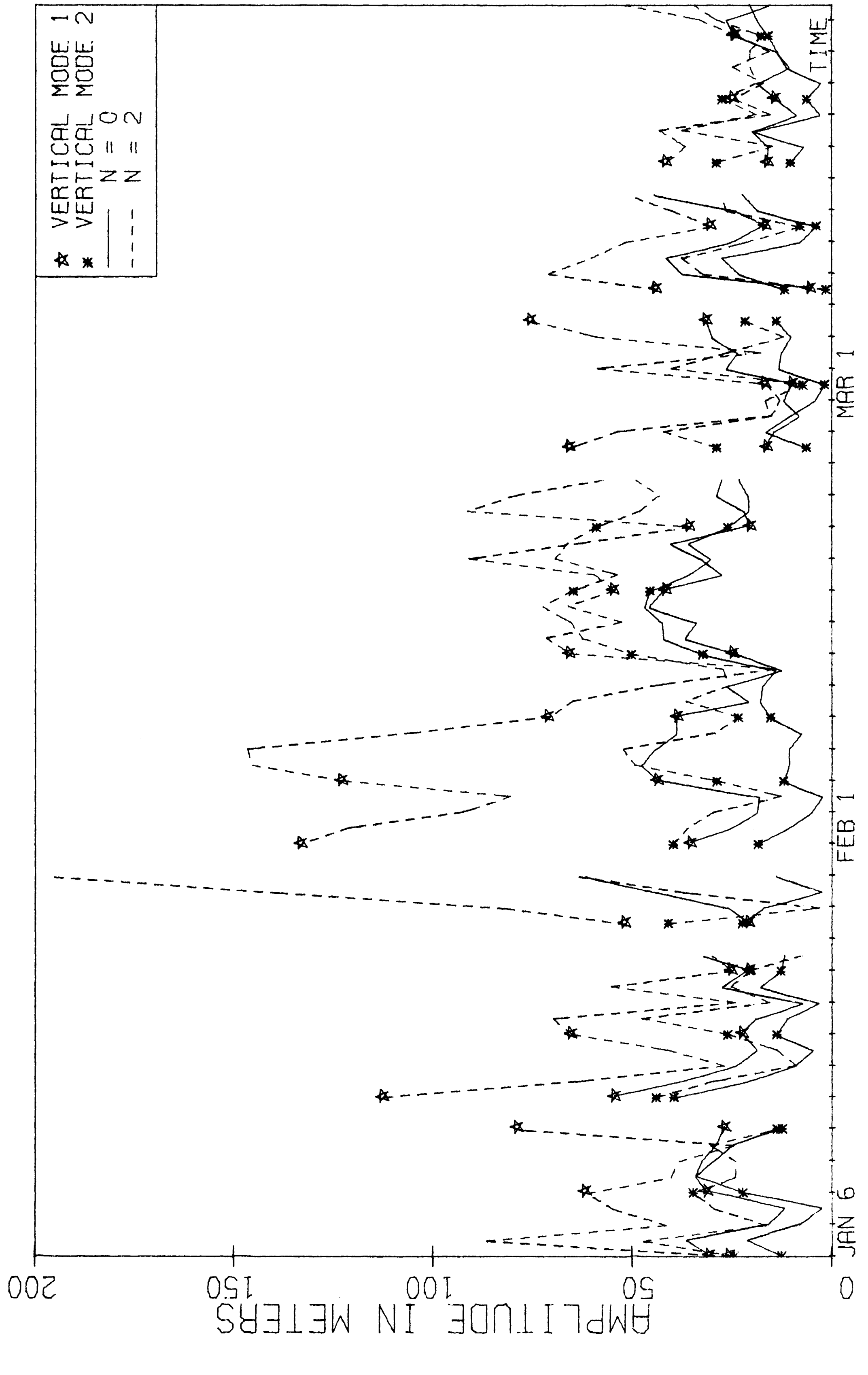
DEGREE	1	2	3	4	5	6	7	8	9	10	11	12
ω_{R-H}	-359.5	-109.5	-42.3	-21.9	-9.4	-2.3	2.2	5.2	7.2	8.8	9.9	10.8

the forced modes were evaluated as the difference between a 5.5 day running mean and a 90 day fixed mean (January 1 to March 31, 1963), their motions may be clarified by more refined time filters. The speed of 19.1 deg. longitude per day for zonal wavenumber 2 is probably an error of interpretation in a period of small amplitude.

The speeds shown in Tables 3.2.2 and 3.2.3 are relative to the earth, with no allowance for the angular velocity of solid rotation of the atmosphere, of which tables were published by Smirnov and Kazakova (1965, 1966). Allowance for the solid rotation would reduce but not remove the vacillation observed in the present graphs of the phase angle of the free and forced modes against time; part of the residue may be due to the excitation of sub-dominant meridional modes, part to imperfections of the time filter, a part to the contingent errors of aliasing, and part may be real. Although it is simple enough to correct the free modes for the solid rotation, the forced modes would require care to separate the part due to the interaction of the free modes from the seasonally forced part for which longitude is the natural frame of reference. Tables 3.2.2 and 3.2.3 may probably be considered as showing the mean phase speeds of the dominant meridional mode: a substantial part of the vacillation appears to be connected with the excitation of sub-dominant meridional modes, but no estimates of their mean phase velocities are yet available. The non-divergent modes 1 and 3 have phase velocities near the Rossby -Haurwitz value assuming a meridional wavenumber of 2, 3, or 4 (Table 3.2.4), which is an approximation to the dominant meridional characteristic pattern (Table 3.4.2), or to the dominant meridional wavenumber (Table 3.4.3).

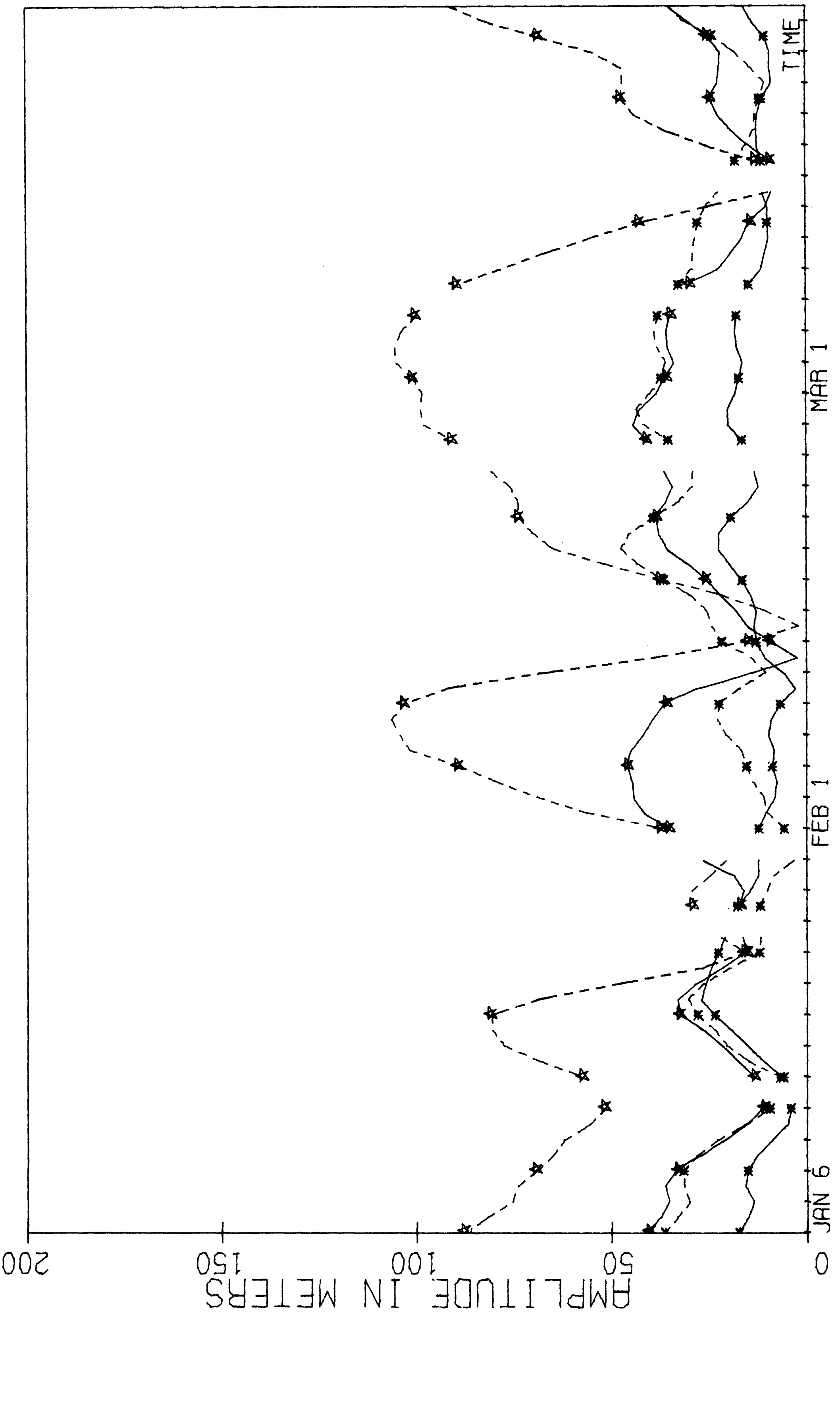
The divergent modes of the longest waves move almost with the solid rotation of the atmosphere when their amplitudes are large enough for reliable estimates. Meaningful statistics on the vacillation could be obtained only with a time filter much more carefully designed than those used in this study, and with careful corrections for aliasing.

Both the free and forced modes have the unexpected property that their amplitude or energy fluctuates with time periods short compared to the periods of fluctuations in their phase velocities. As illustrations of this and other phenomena, Figures 3.2.2 to 3.2.5 plot the amplitudes and phase angles against time for several free and forced modes of zonal wavenumber 3. The phase angles are hand interpolated after examination of detailed listings wherever the amplitude falls below 1 decameter. The phase may be discontinuous



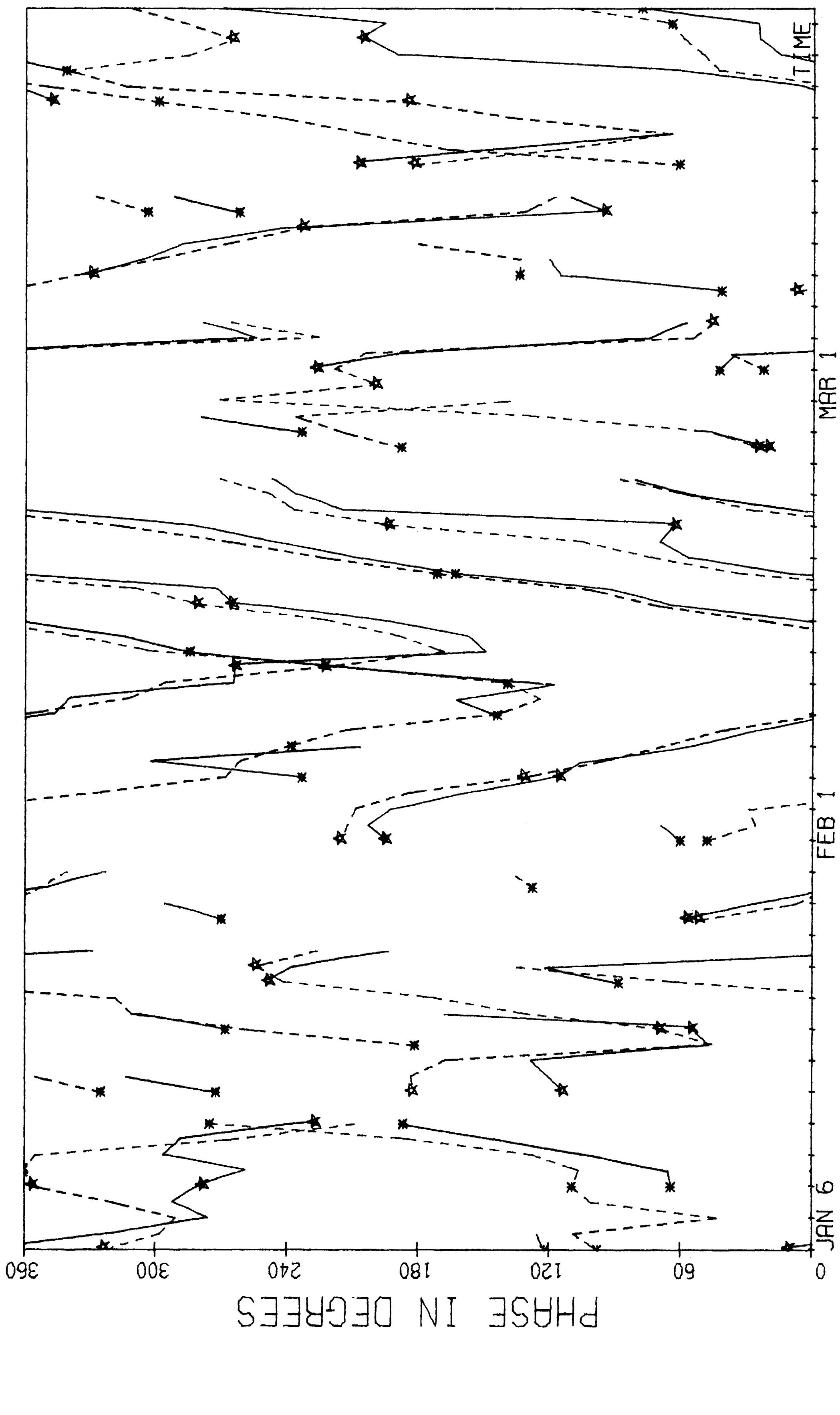
ZONAL WAVE NUMBER 3 – AMPLITUDE OF D, N=0,2

FIGURE 3.2.2



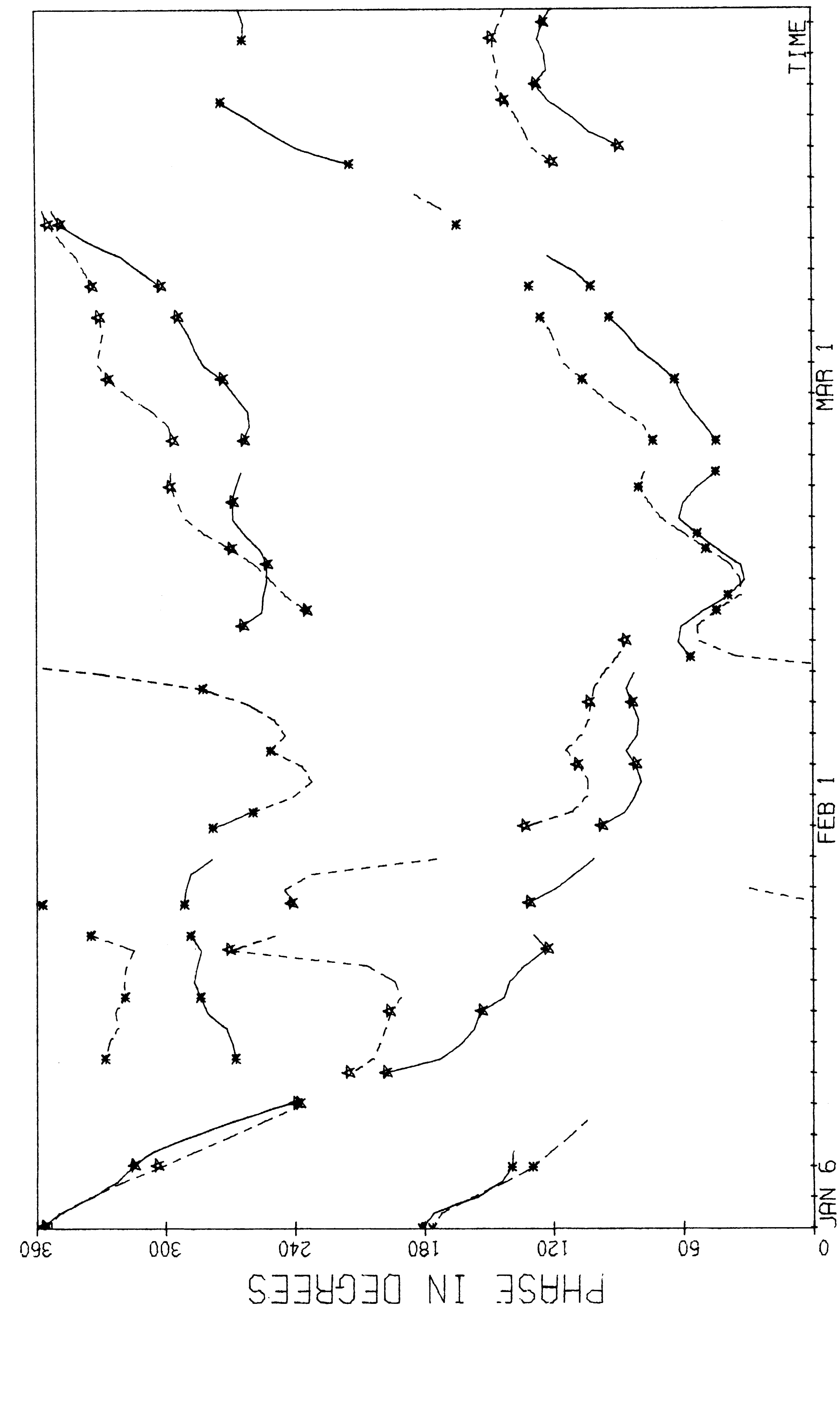
ZONAL WAVE NUMBER 3 – AMPLITUDE OF M, N=0,2

FIGURE 3.2.3



ZONAL WAVE NUMBER 3 - PHASE OF D, N=0,2

FIGURE 3.2.4



ZONAL WAVE NUMBER 3 - PHASE OF M, N=0,2

FIGURE 3.2.5

TABLE 3.2.5

AMPLITUDES OF FREE AND FORCED MODES

3 MONTH MEAN, JAN.-MAR. 1963, METERS

		FREE MODES				FORCED MODES (5.5 DAY MEANS)			
	N	0	2	4	6	0	2	4	6
M	H								
0	1	9.9	20.2	21.2	29.1	38.4	38.1	54.0	26.3
	2	6.7	9.0	13.3	13.1	20.8	18.5	20.9	11.5
	3	5.7	12.1	12.8	14.1	23.6	27.0	30.6	14.9
1	1	35.7	60.6	59.8	52.9	33.3	66.7	99.8	78.2
	2	18.4	25.7	30.1	28.7	32.3	40.1	38.8	28.0
	3	22.3	34.3	31.1	26.6	23.4	41.1	52.1	40.3
2	1	28.6	51.3	70.0	48.5	28.2	72.1	71.7	58.6
	2	18.6	30.2	33.6	25.8	23.4	33.2	36.1	20.7
	3	17.3	29.6	33.5	21.1	12.4	35.4	41.1	26.0
3	1	26.6	59.6	65.5	41.0	27.2	62.3	66.9	29.6
	2	16.9	31.9	30.7	21.3	13.7	24.2	22.9	11.3
	3	16.1	31.2	32.4	18.6	13.0	27.7	31.0	12.9
4	1	31.4	63.2	55.6	42.8	30.1	84.3	63.9	25.5
	2	19.3	36.6	25.0	20.8	13.9	30.9	17.5	9.8
	3	17.6	33.0	25.5	19.0	15.0	39.0	27.2	10.1
5	1	21.9	52.9	55.0	39.2	20.9	49.9	33.9	17.6
	2	14.2	32.5	27.8	18.8	9.7	18.6	11.1	5.9
	3	11.9	26.8	25.0	16.6	10.1	22.1	14.5	6.5
6	1	23.7	60.1	42.0	33.6	15.7	34.2	14.4	10.7
	2	12.4	29.7	20.0	15.2	6.6	12.5	6.8	6.0
	3	12.6	30.1	18.3	13.6	8.0	17.0	6.1	4.9

at such times.

3.3. COMPARISON WITH PRESSURE COSINE MODES

Pressure modes were evaluated in analytic functions only with a view to a possible forecast model; consequently, a reflection was made at 1000 mb, and the Fourier series expanded only in the cosine terms.

The motion of the vertical mean strongly resembles the motions of vertical modes 1 and 3, because they have substantial vertical mean values, which indicates that modes 1 and 3 are approximately non-divergent. The motion of the other vertical cosine modes varies, but is uninformative; such expansions do not appear worth pursuing as either diagnostic or prognostic aids.

3.4 CHARACTERISTIC PATTERNS OF LATITUDE

Characteristic patterns of latitude were first evaluated from zonal harmonics at a given pressure level weighted by the cosine of the latitude: it was noted that the first four of the resultant modes explained about 95% of the variance and were approximately independent of pressure and zonal wavenumber for zonal wavenumbers greater than 6. This is now seen to be because the forced modes are of relatively small amplitude in the higher zonal wavenumbers.

The application of a 5.5 day time filter and an expansion in vertical characteristic patterns yielded the much clearer result that for the free modes four universal meridional characteristic patterns emerge which are independent of pressure mode and zonal wavenumber for zonal wavenumbers greater than 2, and which explain typically 60-70, 10-30, 5-10 and 1-5% of the sample variance. Zonal wavenumbers 1 and 2 exhibit the universal meridional modes when a 10 day filter is used, due to the unsatisfactory discrimination of the 5.5 day filter at their observed rotational periods. Figure 3.4.1 shows examples of the residual case-to-case variation. The dominance order is occasionally interchanged from unknown causes.

Meridional mode 1 is absent from the transient part of vertical modes 1 and 3 of the zonal mean. The meridional C.P.'s of the forced modes are all individual, and some resemblances observed in the higher C.P.'s may be due to imperfections of the time filter.

The modes of the A coefficient, zonal wavenumber 7, vertical mode 1, March 1963, were arbitrarily adopted as standard because they appeared near the average, and are listed in Table 3.4.1. A cosine latitude weight is included. Even though no separation of free and forced modes was made,

TABLE 3.4.1.

STANDARD MERIDIONAL MODES: A COEFFICIENT, ZONAL WAVE NUMBER 7, VERTICAL MODE 1, MARCH 1963.

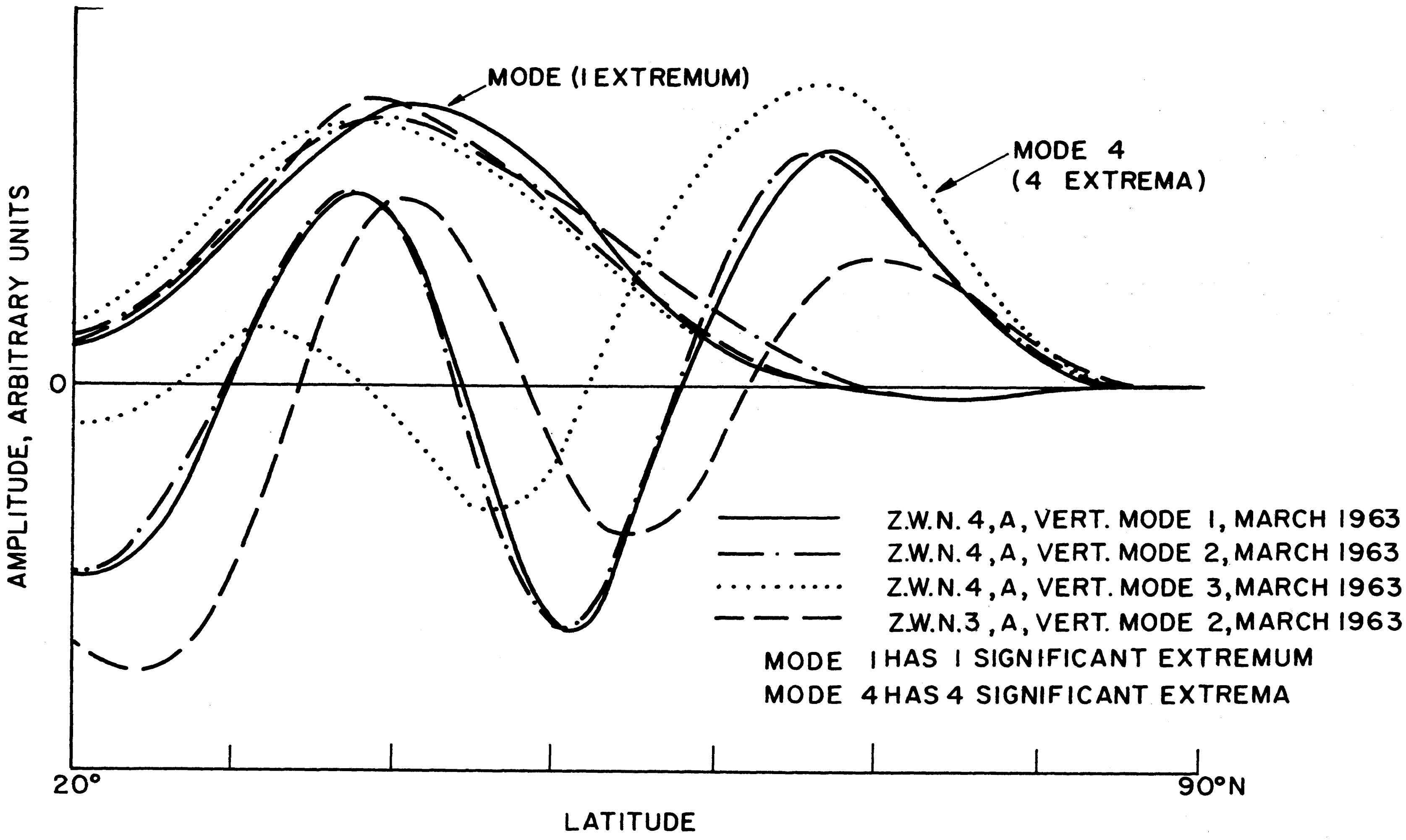
LAT.	20.0	22.5	25.0	27.5	30.0	32.5	35.0	37.5	40.0	42.5	45.0	47.5	50.0	52.5
	55.0	57.5	60.0	62.5	65.0	67.5	70.0	72.5	75.0	77.5	80.0	82.5	85.0	87.5
K														
1	.1154	.1698	.2314	.2947	.3493	.3826	.3886	.3700	.3293	.2710	.2056	.1446	.0986	.0663
	.0401	.0172	.0013	-.0052	-.0044	-.0009	.0006	.0001	-.0005	-.0005	-.0001	.0001	-.0000	-.0000
2	-.1162	-.1550	-.1961	-.2285	-.2327	-.1983	-.1134	.0200	.1731	.3035	.3829	.4043	.3740	.3109
	.2387	.1659	.0980	.0446	.0106	-.0053	-.0093	-.0067	-.0037	-.0010	-.0002	-.0000	.0001	.0000
3	.2611	.3026	.3073	.2525	.1312	-.0208	-.1674	-.2645	-.2836	-.2188	-.0847	.0778	.2166	.2950
	.3095	.2817	.2315	.1801	.1341	.0920	.0520	.0232	.0077	.0022	.0001	.0000	-.0000	.0000
4	-.2567	-.2920	-.2631	-.1498	.0352	.2157	.3045	.2344	.0279	-.1847	-.2839	-.2234	-.0600	.1203
	.2567	.3159	.3036	.2520	.1894	.1256	.0680	.0263	.0046	-.0015	-.0014	-.0002	.0002	.0000
5	.3483	.2431	.0891	-.1100	-.2652	-.2600	-.0910	.1374	.2947	.2672	.0637	-.1778	-.3087	-.2609
	-.0677	.1648	.3046	.3065	.2147	.1074	.0321	-.0010	-.0063	-.0035	-.0006	-.0005	-.0002	.0000
6	.2704	.2120	.0499	-.1742	-.2946	-.1276	.1732	.2817	.1061	-.1497	-.2581	-.1636	.0514	.2253
	.2521	.1154	-.1086	-.2832	-.3481	-.3118	-.2235	-.1266	-.0520	-.0151	-.0030	-.0006	-.0002	-.0001

TABLE 3.4.2.

EXPANSION OF STANDARD MERIDIONAL MODES IN ASSOCIATED LEGENDRE POLYNOMIALS

	M=1						M=2						M=3					
N	C	2	4	6	8	10	0	2	4	6	8	10	0	2	4	6	8	10
K																		
1	.173	.092	-.160	.028	.065	-.044	.159	.154	-.128	-.025	.067	-.025	.143	.192	-.082	-.060	.055	-.009
2	.009	.128	.049	-.171	-.003	.110	-.011	.116	.131	-.129	-.079	.088	-.024	.089	.182	-.059	-.117	.048
3	.074	.040	.049	.063	-.163	-.021	.066	.035	.046	.134	-.111	-.100	.061	.029	.021	.175	-.031	-.139
4	-.017	.075	.065	-.020	.018	-.116	-.029	.052	.083	.009	.069	-.085	-.035	.030	.084	.015	.108	-.014
5	.031	.013	.035	.040	-.033	-.003	.028	.006	.025	.061	-.003	.035	.027	.002	.010	.064	.010	.073
6	.005	-.044	-.046	-.032	-.042	.051	.013	-.027	-.041	-.035	-.079	-.002	.016	-.015	-.031	-.019	-.087	-.041

	M=4						M=5						M=6					
N	0	2	4	6	8	10	0	2	4	6	8	10	0	2	4	6	8	10
K																		
1	.128	.215	-.034	-.077	.038	.000	.113	.227	.011	-.080	.021	.005	.100	.230	.051	-.073	.007	.007
2	-.033	.056	.205	.014	-.121	.008	-.037	.025	.206	.080	-.101	-.022	-.039	-.002	.192	.131	-.067	-.039
3	.058	.027	-.011	.184	.053	-.138	.056	.029	-.042	.166	.123	-.109	.054	.035	-.067	.132	.173	-.065
4	-.038	.012	.079	.006	.120	.068	-.039	-.002	.074	-.009	.107	.139	-.039	-.013	.071	-.022	.074	.186
5	.026	.001	-.005	.057	.006	.091	.026	.003	-.016	.050	-.007	.080	.026	.006	-.023	.045	-.020	.046
6	.018	-.008	-.025	-.000	-.074	-.056	.018	-.003	-.023	.015	-.054	-.053	.019	.001	-.024	.024	-.035	-.042



Typical Case to Case Variations of Meridional C.P.'s of the Free Waves: Modes 1 and 4 are Shown.

FIGURE 3.4.1

these four meridional modes explained from 86 to 99% of the variance in any sample of zonal wavenumbers greater than 1, which is sufficient for diagnostic and some prognostic studies. Modes 1 to 4 have respectively 1 to 4 extrema of significant amplitude between 20°N and the pole; modes 5 and 6 have 5 and 6 significant extrema respectively. The expansion of the standard modes as linear combinations of associated Legendre polynomials is given in Table 3.4.2. The residuals are too large to allow a transformation from surface harmonics to meridional characteristic patterns except for modes 1 and 2. Characteristic patterns which are near zero at the equator cannot be single associated Legendre polynomials of even meridional wavenumber.

From an examination of case-to-case variations it was concluded that meridional mode 4 contains comparable amounts of signal and noise: modes 5 and 6 are not entirely noise, explaining up to 8% of the variance of the forced part of high zonal wavenumbers. Modes 5 and 6 appear to be characteristic of the forced waves.

The internal consistency of any meridional mode, and the contribution of tropical and inter-hemispheric effects to it, may be judged from the ratio of its amplitude at 20°N to its extrema: modes 4, 5 and 6 have ratios near 1, mode 3 has 0.8, and modes 1 and 2 about 0.25. This again indicates that modes 3 and 4 are useful in diagnosis but of restricted prognostic value.

Mashkovich (1964b) shows examples of the meridional profile of the zonal mean as a function of time given by a low order model; resemblances to the characteristic patterns reported here are obvious.

3.5 CHARACTERISTIC PATTERNS OF LONGITUDE

The existence of characteristic patterns of longitude would imply that the values of the A and B coefficients of zonal harmonics were correlated. Because the raw data of this study was already expressed in zonal harmonics, a procedure was adopted which would have given the coefficients of the expansion of zonal characteristic patterns in zonal harmonics. The 31 A and B coefficients of zonal wavenumbers 0 to 15 at any one latitude and any one pressure level for any one month were used to form a covariance matrix in the usual manner (Appendix B).

The resultant characteristic patterns were so unstable from one latitude circle or month to another as to be entirely attributable to the forced modes and aliases. Consequently, zonal harmonic analysis appears the most valid form of longitudinal analysis.

Analyses of data resolved in vertical modes, meridional characteristic patterns, and time filtered showed that the free modes have no zonal C.P.'s.

For the forced modes, many months of data would be required for a significant result.

3.6 UNSTEADY FLUX TERMS

Calculations of unsteady Reynold's stress terms discussed by Bernstein (1966) were made in connection with a forecast model of the planetary waves. These terms arise when a moving time average and deviation from the moving time average are used in the equations of motion, as might be done for instance in calculating the instantaneous phase velocities of the free planetary modes and the forced modes while allowing their interactions.

Consider a moving average of any quantity over a period 2τ such that

$$\bar{x}(t) = \frac{1}{2\tau} \int_{t-\tau}^{t+\tau} x \, ds \qquad 3.6.1$$

and

$$x'(t) = x(t) - \bar{x}(t) \qquad 3.6.2$$

Then the usual Reynold's stress terms are of the form $x'y'$, and the unsteady terms are of the form $x'\bar{y}$, with mean values $\overline{x'y'}$ and $\overline{x'\bar{y}}$, respectively.

In the present study estimates were based only on the geopotential ϕ, with 10.5 day, 20.5 day and 30.5 day averaging periods 2τ. For a 10.5 day moving average the unsteady flux[1] term $\overline{\phi'\bar{\phi}}$ is comparable to the mean flux term $\overline{\phi'^2}$; for a 30.5 day average the unsteady flux term falls to the order of one quarter of the mean flux term, while a 20.5 day average gives intermediate results. These results are consistent with the other conclusions of this study in stressing the significance of the forced modes.

It is concluded that extreme care is necessary in attempting to construct any prognostic modes with a separation of the free and forced planetary modes, as the unsteady flux term appears in the equations for both the time average part $\bar{\phi}$ and the deviation ϕ'.

3.7 TILT OF SURFACE HARMONICS EXPANSIONS

The tilt of the surface harmonics of the geopotential was examined only by the mean differences of the 800 mb, 500 mb, and 200 mb phase angles (1000, 150 and 100 mb data being un-

---

[1] for any surface harmonic, the amplitude is proportional to the geostrophic wind.

available for January 1963), equally weighted over the available observations without regard to amplitude. All differences were brought into the interval ($-\pi,\pi$) before averaging.

Table 3.7.1 shows the monthly values in radians for January, February and March.

The considerable month to month changes have no evident systematic character.

Table 3.7.2 shows the three monthly values for the free and forced modes separately.

The tilts of the free and forced modes are mostly similar, to the west from 850 to 500 mb, and approximately vertical from 500 to 200 mb. The recurvature above 500 mb usually increases with increasing meridional wavenumber and decreasing zonal wavenumber: the variation with meridional wavenumber may merely reflect the variation of tropopause height with latitude. The 500 to 200 mb differences are consistent with the prediction of Mashkovich (1961, 1946) of a tilt to the east in the upper troposphere and lower stratosphere.

No calculations were made in meridional characteristic patterns.

TABLE 3.7.1

TILT OF SURFACE HARMONICS OF THE GEOPOTENTIAL, LESS 3 MONTH MEAN, JANUARY, FEBRUARY, MARCH 1963. DIFFERENCES OF EAST-WEST PHASE ANGLE IN RADIANS OF PHASE, 850-500 MB, 850-200 MB VALUES

MERIDIONAL WAVENUMBER		0		2		4		6		8	
ZONAL WAVE-NUMBER		850-500	850-200	850-500	850-200	850-500	850-200	850-500	850-200	850-500	850-200
1	Jan.	.20	.25	.03	-.16	.37	.69	.15	.25	.20	.33
	Feb.	.29	-.25	.11	.01	-.05	-.03	-.01	-.13	.39	.47
	Mar.	.39	.75	.05	.10	.27	.21	.17	.31	.25	.21
2	Jan.	.53	.29	.03	.06	.25	.23	.51	.57	.37	.51
	Feb.	.45	.60	.41	.48	.32	-.00	.35	-.08	.32	.25
	Mar.	.59	.84	.58	.93	.05	.03	.25	.39	.55	.65
3	Jan.	.46	.57	.20	.32	.28	.54	.43	.66	.32	.40
	Feb.	.41	.41	.69	1.05	.33	.33	.52	.60	.14	.53
	Mar.	.60	.56	.25	.25	.08	-.13	.35	.25	.29	.26
4	Jan.	.51	.64	.17	.32	.20	.27	.25	.39	.11	.08
	Feb.	.20	.28	.34	.38	.21	.29	.16	.21	.16	.11
	Mar.	.19	.40	.27	.52	.24	.31	-.09	.16	.35	.35
5	Jan.	.18	.23	.22	.64	.29	.44	.26	.38	.22	.18
	Feb.	.55	.86	.37	.54	.41	.63	.35	.74	.31	.45
	Mar.	.43	.56	.31	.38	.50	.47	.40	.39	.24	.08
6	Jan.	.36	.36	.27	.26	.49	.48	.17	.26	.44	-.22
	Feb.	.08	.02	.12	.01	.21	.38	.30	.35	.43	.42
	Mar.	.47	.76	.60	.60	.41	.55	.45	.44	.18	.33

TABLE 3.7.2

TILT OF SURFACE HARMONICS OF THE GEOPOTENTIAL: 5.5 DAY MEAN AND DEVIATION, LESS 3 MONTH MEAN JAN. - MARCH 1963. DIFFERENCES OF EAST - WEST PHASE ANGLE IN RADIANS OF PHASE MODULO 2π; 850 MB - 500 MB, 850 MB, - 200 MB VALUES

MERIDIONAL WAVENUMBER		0		2		4		6		8	
ZONAL WAVE-NUMBER		850-500	850-200	850-500	850-200	850-500	850-200	850-500	850-200	850-500	850-200
1	Free	1.43	1.37	1.41	1.22	1.32	1.19	1.05	1.06	1.03	1.12
	Forced	0.96	1.31	1.25	1.25	0.76	0.92	1.64	1.44	0.78	0.91
2	Free	1.43	1.22	1.12	1.05	1.10	1.29	0.89	1.00	1.16	1.04
	Forced	0.57	0.77	0.84	0.70	1.13	1.07	0.43	0.62	0.76	0.77
3	Free	0.96	1.07	0.65	0.93	1.34	1.25	0.94	1.01	0.94	1.07
	Forced	0.25	0.77	0.28	0.17	1.10	0.99	0.68	0.76	1.41	1.48
4	Free	0.97	0.97	1.04	1.00	0.95	0.96	0.77	0.91	0.96	0.97
	Forced	0.68	0.78	0.54	0.66	0.86	0.79	1.58	1.44	1.19	1.27
5	Free	0.76	0.92	0.95	0.93	1.07	1.16	1.04	0.98	0.98	0.92
	Forced	1.23	0.93	0.68	0.90	0.94	0.98	1.27	1.39	1.23	1.49
6	Free	0.73	0.85	0.84	1.03	1.07	1.09	0.74	0.74	0.94	0.94
	Forced	1.21	1.18	0.97	1.01	1.23	1.17	1.13	0.96	1.21	1.50

4. COMPARISON WITH OTHER PUBLICATIONS

4.1 ELIASEN AND MACHENAUER

The main observational studies of the transient part of the planetary waves are those of Eliasen and Machenauer (1965) and Deland and Lin (1967). Both these studies were mainly of the stream function at 500 mb, where the value of vertical mode 2 is near zero, although Eliasen and Machenauer also studied the 1000 mb field. No previous study has used three or more levels.

Eliasen and Machenauer also applied a simple time filter with an averaging period which they made a function of meridional wavenumber because of their finding that phase speeds depend on meridional wavenumber. Their observed phase velocities are generally similar to those found for vertical modes 1 and 3 in the present study; their finding that the phase angles at 500 mb and 1000 mb are usually little different was also observed in this study, but both this observation and occasional departures from it are due to the ratio of the percentage variances explained by vertical modes 1 and 2: cases were noted when the 200 mb and 850 mb phase velocities were large and of opposite signs.

The dependence of phase velocity on meridional wavenumber is due mainly to a time filtering technique which retained the forced modes: if Eliasen and Machenauer had worked at levels far from 500 mb, then vertical mode 2 would have been an additional complication, but no qualitative change would have been necessary in their conclusions. The use of cosine or other vertical modes also leads to a dependence of phase velocity on meridional wavenumber over short time periods.

Their other conclusions are consistent with everything observed in the present study, although their opinion of the nature of slow-moving waves conflicts with the results of Mashkovich (1964b).

4.2 DELAND AND LIN

The main result of Deland and Lin (1967) on which the present study casts new light, and which is not comparable to any result of Eliasen and Machenauer (1965), is that the daily changes of phase angle of surface harmonics are correlated. The existence of characteristic patterns leads one to expect such correlations, though substantial calculations would be required for quantitative comparisons.

It must also be pointed out that many of the correlation coefficients reported by Deland and Lin are comparable in magnitude to the space aliases found in the model experiment of this report: because the simple model experiment says nothing about the sign of contingent errors, a further investigation is needed before anything more can be said.

Although no regression of one day's phase velocity on another has been made in this study, it appears likely from the available graphs that the separation of modes by time filtering and characteristic patterns would improve on the maximum correlation coefficient of 0.69 reported in Deland and Lin's Table 4.

The difficulty which Deland and Lin found in choosing a satisfactory empirical coefficient for the divergence is due to the facts that they did not use a time filter, and that the main part of the divergence is in vertical mode 2, which is by definition statistically uncorrelated with the dominant vertical model. Operational analysis and forecast models also fail to separate the standing, forced and free modes.

5. COMPARISON WITH THEORY

5.1 ESTIMATES OF INTERACTIONS BETWEEN MODES

The estimates derived here are of the instantaneous non-linear interactions between different modes of the geopotential by the transports and shears of the quasi-horizontal wind. No estimates are made of the frictional dissipation nor the conversion between available potential and kinetic energies.

The orthogonal function method is used to examine the effects of vertical and meridional shear of the quasi-horizontal wind. For the full formulation of a forecast model see Gavrilin (1965).

The equations of motion and the continuity equation with the hydrostatic approximation may be written in meteorological coordinates as

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \omega\frac{\partial u}{\partial p} + \frac{\partial \phi}{\partial x} - fv = 0 \qquad 5.1.1$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \omega\frac{\partial v}{\partial p} + \frac{\partial \phi}{\partial y} + fu = 0 \qquad 5.1.2$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0 \qquad 5.1.3$$

where

u is the eastward windspeed

$\omega = dp/dt$ is the pressure vertical velocity

$f = 2\Omega$ sine latitude is the Coriolis parameter

Ω = is the angular velocity of the earth, 2π radians/day.

ϕ is the geopotential

x is the eastward coordinate, in a local tangent plane

y is the northward coordinate, in a local tangent plane

Derivations are given in standard textbooks, e.g. Kibel (1957).

The horizontal wind components are observed to be nearly geostrophic, so that

$$\frac{\partial \phi}{\partial x} \approx fv \qquad 5.1.4$$

$$\frac{\partial \phi}{\partial y} \approx -fu \qquad 5.1.5$$

and therefore the vertical structure of the horizontal wind components may be expanded in the same characteristic patterns as the geopotential: the same applies to the simplified balance stream function ψ given by

$$f\nabla^2 \psi = \nabla^2\phi - \nabla f \nabla \phi \qquad 5.1.6$$

Let

$$u(x,y,p,t) = u_1H_1 + u_2H_2 + u_3H_3 \qquad 5.1.7$$

$$v(x,y,p,t) = v_1H_1 + v_2H_2 + v_3H_3 \qquad 5.1.8$$

$$\phi(x,y,p,t) = \phi_1H_1 + \phi_2H_2 + \phi_3H_3 \qquad 5.1.9$$

H_1, H_2, H_3 (p) are the observed vertical characteristic patterns of the geopotential, and u_1, u_2, u_3, v_1, v_2, v_3, ϕ_1, ϕ_2, ϕ_3 are functions of x, y, and t only.

Substituting in the continuity equation 5.1.3 and integrating with respect to pressure

$$\omega-\omega(p_o) = \sum_I \frac{\partial u_I}{\partial x} \int_{p_o}^{p} H_I \, dp + \sum_I \frac{\partial v_I}{\partial y} \int_{p_o}^{p} H_I \, dp \qquad 5.1.10$$

One may select a level p_o (e.g. 1000mb) such that $\omega(p_o)$ is sufficiently near zero for the purpose of the present estimates.

Then

$$\omega = \sum_I \omega_I \int_{p_o}^{p} H_I \, dp \qquad 5.1.11$$

substituting into 5.1.1 from 5.1.6 to 5.1.11

$$\sum_I H_I \frac{\partial u_I}{\partial t} + \sum_I H_I u_I \sum_J H_J \frac{\partial u_J}{\partial x} + \sum_I H_I u_I \sum_J H_J \frac{\partial u_J}{\partial y}$$

$$+ \sum_I \omega_I \int_{p_o}^{p} H_I \, dp \sum_J u_J \frac{\partial H_J}{\partial p} + \sum_I H_I \frac{\partial \phi_I}{\partial x} - f \sum_I v_I H_I = 0 \qquad 5.1.12$$

Multiplying through by H_K and integrating over the domain of orthonormality

$$\frac{\partial u_K}{\partial t} + \sum_{I,J} u_I \frac{\partial u_J}{\partial x} \int_{p_o}^{p_1} H_I H_J H_K \, dp + \sum_{I,J} v_I \frac{\partial u_J}{\partial y}$$

$$\int_{p_o}^{p_1} H_I H_J H_K \, dp + \sum_{I,J} \omega_I u_J \int_{p_o}^{p_1} H_K \frac{\partial H_J}{\partial p} \int_{p_o}^{p} H_I \, dp \, dp + \frac{\partial \phi}{\partial x} - f v_K = 0 \qquad 5.1.13$$

In this equation, and in the similar equation for v, the vertical interaction integrals are of the form

$$\int_{p_o}^{p_1} H_I \; H_J \; H_k \; dp \qquad 5.1.14$$

$$\int_{p_o}^{p_1} H_K \; \frac{\partial H_J}{\partial p} \int_{p_o}^{p} H_I \; dp \; dp \qquad 5.1.15$$

Values of these integrals are given in Table 5.1.1, estimated from eigenvectors at 50 mb intervals. The absolutely largest elements run around 0.1 to 0.2.

One may examine the meridional shear of the quasi-horizontal wind in the same way by writing

$$u(x,y,p,t) = \sum_I u_I \; L_I \qquad 5.1.16$$

etc., where L_I are meridional characteristic patterns and the u_I are now functions of x, p and t only.

Then algebraic manipulation gives

$$\frac{\partial u_K}{\partial t} + \sum_{I,J} u_I \frac{\partial u_J}{\partial x} \int L_I L_J L_K \, dy + \sum_{I,J} v_I \; u_J \int L_I \frac{\partial L_J}{\partial y} L_K \, dy$$

$$-\sum_{I,J} \frac{\partial u_I}{\partial p} \int_{p_o}^{p} \frac{\partial u_J}{\partial x} \, dp \int L_I L_J L_K \, dy - \sum_{I,J} \frac{\partial u_I}{\partial p} \int_{p_o}^{p} v_J \, dp$$

$$\int L_I \frac{\partial L_J}{\partial y} L_K \, dy + \frac{\partial \phi_K}{\partial x} - \sum_I v_I \int f \, L_I L_K \, dy = 0 \qquad 5.1.17$$

TABLE 5.1.1

INTERACTION COEFFICIENTS OF NORMALIZED HOLMSTRÖM MODES AT 50 MB INTERVALS

VALUES OF $\int_{p_0}^{p_1} H_I H_J H_K \, dp$

MODES	1,1	1,2	1,3	2,2	2,3	3,3
1	.2898	-.0616	-.0124	.1636	-.0786	.2392
2	-.0616	.1636	-.0786	.1207	.1925	-.0499
3	-.0124	-.0786	.2392	.1925	-.0499	.2216

VALUES OF $\int_{p_0}^{p_1} H_I \int_{p_0}^{p} H_J ds \frac{\partial H_K}{\partial p} dp$

I \ J,K	1,1	1,2	1,3	2,1	2,2	2,3	3,1	3,2	3,3
1	.0073	.4152	-1.0040	-.0991	.4434	-.5502	-.0215	.0261	.0379
2	-.1360	-.1145	.9365	-.1391	-.0133	.7292	-.0563	.0617	.1508
3	.2317	-.1225	-.8651	.1644	-.1533	-.4306	.0236	.0163	-.1243

The interaction coefficients are too numerous to tabulate, but the absolutely largest values of

$$\int L_I L_J L_K \, dy \qquad \text{and} \qquad \int L_I \frac{\partial L_J}{\partial y} L_K \, dy$$

run around 4E-7.

If one takes characteristic windspeeds of 10m/sec, then the characteristic times for a 100% change due to the zonal and meridional shears of the quasi-horizontal wind are of the order of 15 days due to the interaction between only two wave vectors. Thus the characteristic periods of fluctuations involving many wave vectors may be in the order of 5 days, which is comparable to the observed periods and in general agreement with the conclusions of Mashkovich (1961, 1963, 1964b) and Blinova (1956, 1957, 1961, 1964, 1965).

The interaction terms involving the vertical velocity are more difficult to handle, as the horizontal divergence and vertical velocity are largely ageostrophic effects. Even the quasi-geostrophic omega equation (Wiin-Nielsen, 1959; Thompson, 1961) is a second order partial differential equation in which the vertical velocity is multiplied by the static stability. Although the solution of the omega equation is not difficult numerically, a simpler estimate may be used here by simply taking a typical value of the vertical velocity as 1 cm/sec. or .001 mb/sec. (Kibel, 1957) and of the vertical shear of the horizontal wind as .02 m/sec/mb. The vertical interaction terms in Table 5.1.1 attain magnitudes about unity. Thus a characteristic time for the vertical transports may be of the order of 5 days, which is comparable both to the periods of the observed fluctuations and to the characteristic time of the conversions between eddy available and eddy kinetic energy.

5.2 CHARNEY

Some comparisons may be made with Charney (1947), reprinted in Saltzmann (1962). In particular, Charney recognized the distinction between the effects of the vertical shear of the horizontal wind, which results from the horizontal temperature gradient in the atmosphere, and the conversion between available potential and kinetic energies, which involves the vertical velocity and vertical temperature gradient.

Charney's generalization of the Rossby-Haurwitz formula (his equation 56') is

$$\int_{p_o}^{o} (\overline{u}-c-u_c)\ V\, d\overline{p} = - \frac{f^2 c}{\mu^2} \overline{\rho_o}\ V_o \qquad 5.2.1$$

where quantities with a bar refer to the basic state and other quantities to the small perturbation, and where

$\bar{u}(\bar{p})$ is the zonal windspeed

c is the perturbation phase velocity

$u_c = 2\,\Omega\, A \cos^3 \phi/N$

N is the zonal wavenumber

Ω is the angular velocity of the earth, 2π radians/day

ϕ is the latitude

V is an amplitude function of the perturbation, which we shall identify with a characteristic pattern

$\bar{p}_o$ is the pressure of the basic state at the ground

$f = 2\,\Omega \sin\phi$ is the coriolis parameter

$\bar{\rho}_o$ is the density of air in the basic state at the ground

V_o is the magnitude of V at the ground

$\mu = 2\pi/L = N/R\cos\phi$

L is the zonal wavelength

Charney suggests that the variation of $\bar{u}$ and V with pressure may be taken from observations: but one may as well express them in vertical characteristic patterns and employ the orthonormality relations to approximate

$$\bar{u}_I - (c+u_c)\int_{p_o}^{p_1} V_I \, dp = -\frac{f_c^2}{\mu^2}\,\bar{\rho}_o\, V_{I,o} \qquad 5.2.2$$

where I is the vertical mode number.

For vertical mode 2, the integral is small compared to other terms, whereas for vertical mode 1 the right hand side is small compared to the integral.

Thus Charney's theory predicts that for vertical mode 1 the phase speed of the perturbation relative to the basic flow should be a constant times the Rossby value; for vertical mode 2, the phase speed should vary as the square of the zonal wavenumber. Neither prediction is observed, and the difficulty may lie partly in the assumptions about the shape of the vertical profile of the zonal wind partly in various assumptions about the magnitudes of wave speed, but mainly in the neglect of the meridional shear of the zonal wind (Mashkovich, 1964b).

5.3 MASHKOVICH[1]

Many of the results of this study were predicted by Mashkovich (1957, 1960, 1961, 1963, 1964b), in a wide range of studies using grid point and spectral models, using instantaneous tendencies of surface harmonics in a time-invariant basic flow, and using low order models based on those of Lorenz (1962, 1963a,b, 1965).

Appendix C is a translation of the conclusions of Mashkovich (1961) of which points 4 and 7 to 14 are supported and point 1 appears to be contradicted by this study. In

[1] Note added in proof: the paper of T. Nitta, M.W.R., 95, 319-339 is also relevant.

particular, the motion and development of disturbances depends on their vertical structure and the vertical structure normally changes rapidly with time.

The more rapid amplification of disturbances in the upper troposphere predicted by Mashkovich agrees with the strong vertical shears of the horizontal wind shown in the observed characteristic patterns.

The predicted inclinations of the trough and ridge lines to the west in the lower troposphere and to the east in the upper troposphere are observed for all waves expressed in surface harmonics.

Wave motions of unbounded amplitude are not observed in the atmosphere, nor in low order models (Mashkovich, 1963, 1964b) but only in models with a time invariant basic flow (Mashkovich, 1961), i.e. with an infinite source of energy for the disturbances. No explanation is offered for the apparent breakdown of Mashkovich's (1961) prediction that the region of baroclinic instability is a function of the degree of a surface harmonic; the higher meridional wavenumbers were found to be of comparatively small amplitude, in agreement with the observations of Eliasen and Machenauer (1965).

Mashkovich stresses the importance of the meridional shear of the quasi-horizontal wind, which is consistent with the results of this study.

5.4 SCALE ANALYSES

Scale analyses of the planetary waves have been published by Burger (1958), Murakami (1963), and Deland (1965b).

Burger and Murakami show that in slow-moving planetary waves the vorticity equation and the divergence equation both become diagnostic. Vertical mode 2 of the observed free modes may meet their assumptions; vertical mode 2 of zonal wavenumbers 1 and 2 have mean phase speeds very close to the angular velocity of solid rotation of the atmosphere, although periods of considerable vacillation are observed.

The basic difference between the scale analysis of Deland (1965b) and those of Burger and Murakami is that Deland assumes wave speeds in the order of the Rossby-Haurwitz velocity. The horizontal advection of vorticity is not then negligible, and Deland concludes that there may exist modes for which the vorticity equation has prognostic value and which move with speeds in the order of the Rossby-Haurwitz values. This study has shown the existence of vertical modes 1 and 3 of the free waves, which meet Deland's assumptions. Zonal wavenumbers 5 and 6 of vertical mode 2 of the free waves also approach Deland's assumptions and conclusions.

Zonal wavenumbers 3 and 4 of vertical mode 2 of the free waves have properties intermediate between those assumed by

Deland and Murakami.

It remains to be clarified whether any scale analysis is possible for the forced modes.

6. CONCLUSIONS

1. Any planetary wave or station sounding of the geopotential may be represented by three universal vertical characteristic patterns, or by a vertical mean and two characteristic patterns. Typical percentage variances explained by each mode are 70-90, 10-20 and 1-10%, with the total greater than 98%. Vertical modes 1 and 3 have non-zero vertical means, and their motions are very similar in all cases studied. Mode 2 is of opposite signs but comparable magnitudes in the upper and lower troposphere, with a vertical mean near zero, and is therefore thought to explain most of the horizontal divergence and vertical velocity of the wind. These three vertical modes are indistinguishable from those found by Obukhov (1960) and Holmström (1963).

The latitudinal structure of the free modes of all zonal wavenumbers may be represented by four universal characteristic patterns, with typical percentage variances explained 60-70, 10-30, 5-10 and 1-5%, with the total about 95%. Modes 1, 2, 3 and 4 have respectively 1, 2, 3 and 4 extrema of significant amplitude between 20°N and the North Pole, and are close to zero from 70°N to the pole. The fourth meridional mode is about equally signal and noise: it and higher modes have large values at 20°N, and are thus not correctly represented in the present study or the available data, and may be substantially influenced by inter-hemispheric exchanges.

2. The characteristic patterns of pressure, and the C.P.'s of latitude of fast-moving waves are sufficiently universal that the conclusions of this report are not significantly influenced by their case-to-case variations, and sufficiently universal that diagnostic or prognostic spectral dynamic models may be formulated in terms of them.

3. Slower moving waves exhibit individual meridional characteristic patterns; the meridional C.P.'s of zonal wavenumbers greater than 6 in winter and greater than 3 in June approximate the universal functions, because the slow-moving parts are small relative to the fast-moving parts.

4. The divergent vertical mode 2 was relatively important in February 1963; no results are available for January 1963. This is consistent with the results of Wiin-Nielsen and Drake (1966).

5. The vertical C.P.'s obtained when the vertical mean is removed are approximately those found by orthonormalizing the usual vertical modes to a constant. Changes in the pressure levels of the extrema with meridional wavenumber correspond to the variation of tropopause height with latitude.

6. No zonal characteristic patterns were identified by this study.

7. The zonal phase angle and speed of any given wave are usually approximately independent of meridional wavenumber for both fast- and slow-moving waves. This observation, and occasional departures from it, are explained by the usual dominance of one meridional characteristic pattern.

8. The transient planetary waves (less the 3 month mean) exhibit rotational periods about 21 to 28 days.

9. Vertical modes 1 and 3 of any given zonal wavenumber are normally close together (modulo π) in zonal phase angle, for both fast- and slow-moving waves.

10. Vertical mode 2 of any given zonal wavenumber moves predominantly eastward unless a time filter is applied: modes 1 and 3 require more detailed descriptions.

11. Vertical modes 1 and 3 of zonal wavenumbers 1 through 5 move predominantly west with much vacillation, unless a time filter is applied: vertical modes 1 and 3 of zonal wavenumber 6 move predominantly rapidly eastward.

12. The vertical mean wave has the same characteristics as vertical modes 1 and 3, but the meridional wavenumbers are less closely coupled. Expansions in vertical cosine modes

are uninformative, and expansions have not been made in characteristic patterns with the vertical mean removed.

13. The 5.5 day mean of the waves shows no mean motion significantly different from zero relative to the earth.

14. The deviation from a 5.5 day mean has characteristics that all vertical modes of zonal wavenumbers greater than 3 move east. For zonal wavenumbers 1, 2 and 3, vertical modes 1 and 3 move west with much vacillation, and vertical mode 2 moves east with much vacillation.

These results are independent of meridional wavenumber except rarely when a sub-dominant meridional mode is excited.

15. The zonal phase velocities of the total transient, 5.5 day average, and fast-moving modes are all independent of the amplitude of the corresponding wave; large energy exchanges occur on a time scale short compared to changes in the zonal phase velocity.

16. The vertical modes are closely coupled by the vertical shear of the quasi-horizontal wind; the typical times for 100% energy changes by this mechanism are comparable to the periods of the free modes.

17. Vertical modes 1 and 3 have phase speeds close to non-divergent Rossby-Haurwitz values assuming a meridional wavenumber of 2, 3, or 4, which is an approximation to the dominant meridional mode.

18. The divergent modes have zonal phase speeds absolutely smaller than the relevant Rossby-Haurwitz values, and independent of meridional wavenumber; an anomaly in zonal wavenumber 2 may be an error of interpretation.

19. The predominant zonal phase relation of the several meridional wavenumbers of any given wave indicates a NE-SW tilt of the ridge or trough lines: this result is explained by the fact that the dominant meridional mode has a maximum in middle latitudes.

20. The vertical structure of surface harmonic expansions was investigated only by differences of the 850, 500 and 200 mb zonal phase angles. The tilt of all waves is westward from 850 to 500 mb, and vertical or slightly eastward from 500 to 200 mb, independently of time averaging period.

21. Little attention was given to the time spectrum of energy in this study, but the broad indications are of bands around 5-10 days (free modes) and 28 days (forced modes). These groups of modes have different meridional structures; the clearest results are obtained if the expansions are performed in the order zonal, pressure or time, and lastly meridional.

22. No theory known to the author adequately describes the observations; the treatment of Murakami (1963, p. 136)

is consistent with the observations for the 5.5 day mean of vertical modes 1 and 3 of the vertical mean of zonal wavenumbers less than 6. The scale analysis of Deland (1965b) applies to the weakly divergent free modes of all wavenumbers. Conclusions 4 and 7 through 14 of Mashkovich (1961) are confirmed. His conclusion 1 appears to be contradicted, and no light is cast on his other points.

23. The observations of previous authors of the apparent variation of the zonal phase speed of zonal harmonics with amplitude (Eliasen, 1958), and the vacillation and dependence on meridional wavenumber of the zonal phase speed of surface harmonics (Eliasen and Machenauer, 1965; Deland and Lin, 1967) appear to be due to the omission of the vertical structure and the non-separation of free and forced modes in previous studies.

24. The main errors of analysis are contingent, i.e. a function of the state of the atmosphere. They cannot, therefore, be eliminated by any purely statistical technique. In the author's opinion, a carefully designed study of the actual analysis system would enable greater value to be extracted from the available data. Even if critical points did not exist, the amplitudes of waves often fall within the quasi-random noise level.

7. SUGGESTIONS FOR FUTURE WORK

7.1 DIRECT EXTENSIONS

Direct extensions of the present work are by the study of data in the tropics, Southern Hemisphere, and above 100 mb. Satellite observations of the vertical and horizontal structure of the atmosphere may conveniently employ characteristic patterns as an efficient noise filter.

Existing observational studies of the transports and conversions of available potential energy, kinetic energy, momentum, angular momentum and matter (air, water, ozone, dust) may be formulated in characteristic patterns. In particular, the causes and roles of the characteristic patterns are not understood.

The budgets of all the above parameters may be evaluated for the standing, forced and free modes of the atmosphere: a few illustrative references are Oort (1964), Wiin-Nielsen and Drake (1966), Gavrilin (1965), Holopainen (1966).

The forced modes need further description, but little can be added to the description of the free modes without a detailed knowledge of the contingent errors of the analysis system.

7.2 COGNATE APPLICATIONS

Characteristic patterns may be applied to scale analyses (Murakami, 1963; Deland, 1965b), to prognostic models (Mashkovich, 1964b; Baer, 1964) and non-linear diagnostic calculations (Chapter 5). They are more economical than grid points, surface harmonics or solid harmonics for diagnostic calculations, and are more economical for prognostic models provided the boundary conditions can be formulated in characteristic patterns, e.g. friction, topography and diabatic effects.

The role and mechanism of geostrophic balance are questions which can probably be treated more economically and meaningfully in characteristic patterns than in any other formulation on a spherical earth without friction, topography or diabatic effects.

The vertical and meridional characteristic patterns of the geopotential observed in this study could be used in the evaluation of satellite cloud and temperature data.

APPENDIX A

VALUES OF THE SPACE ALIAS FOR DIFFERENT WAVE VECTORS

The aim of the experiment was to estimate the matrix E of errors of interpolation in each wave vector.

In order to make the problem tractable, the explicit form of a solid harmonic of geopotential was used:

$$\phi(P,M,N) = A(P,M,N) \cos \left(\frac{P(p-p_o)\pi}{p_1 - p_o} \right) \begin{matrix}\cos \\ \sin\end{matrix} (M\lambda)\ P_N^M (\mu) \qquad A.1$$

The form A.1 shows that the desired effect, called an alias by extension of the established meaning (see Glossary), is the product of an alias which is a function of pressure only, and space alias which is a function of latitude and longitude. The space alias may be derived from a zonal alias which is a function only of zonal wavenumber M and latitude.

A latitude-longitude grid with 2.5^o increments was defined from 20^oN to the pole. The R.M.S. error of interpolation was assumed zero over continents, unity over oceans except near a station, and to be

$$\epsilon = 1.0 - \frac{N^2-h^2}{N^2+h^2} \qquad A.2$$

near a station, where N was defined as 3.0 degrees of latitude, and where the distance h was defined by

$$h^2 = (\theta_s - \theta_g)^2 + (\lambda_s - \lambda_g)^2 \cos^2 (\theta_s) \qquad \text{A.3}$$

where θ_s and θ_g are the latitude of the station and gridpoint respectively, and λ_s and λ_g are the longitude of the station and gridpoint respectively.

Continents were defined to occupy the inclusive areas 10°W to 145°E, 70°N to 30°N; 145°E to 125°W, 70°N to 50°N; 125°W to 60°W, 80°N to 30°N. Stations were defined by table A.1 in approximately the positions of real radiosonde stations.

TABLE A.1

ASSUMED POSITIONS OF 'STATIONS'
APPROXIMATING REAL STATIONS

	°N	°LONG.		°N	°LONG.
4YA	62	33W	04-340	70.5	21.5W
4YB	56.5	51W	04-320	77	18W
4YC	53	36W	04-310	82	15W
4YD	44	41W	01-001	71	8W
4YE	35	48W	01-028	74	18E
4YI	59	19W	01-005	78	10E
4YJ	53	20W	20-047	80	57E
4YK	45	16W	20-274	77.5	82E
4YN	30	140W	20-069	79	105E
4YV	34	166W	91-165	21.9	159.3W
08-512	37.7	25.5W	91-066	28.2	177.3W
72-807	47.2	54W	91-245	19.2	166.6E
04-270	61	45W	91-115	24.7	154E
04-220	68.7	52.5W			

The field of R.M.S. relative error of interpolation [square root of (mean square error of interpolation/variance of geopotential)] was analyzed into zonal aliases at 28 latitudes, 20° by 2.5° to 87.5°N. For zonal wavenumbers M and MP,

$$\text{VSS(LAT,M,MP)} = \frac{1}{72} \sum_{I=1}^{144} \text{FIELD(LAT,I)*SIN.(M*I*}\pi\text{/72)} \text{*SIN.(MP*I*}\pi\text{/72)} \quad \text{A.4}$$

$$\text{VSC(LAT,M,MP)} = \frac{1}{72} \sum_{I=1}^{144} \text{FIELD(LAT,I)*SIN.(M*I*}\pi\text{/72)} \text{*COS.(MP*I*}\pi\text{/72)} \quad \text{A.5}$$

$$\text{VCS(LAT,M,MP)} = \text{VSC(LAT,MP,M)} \quad \text{A.6}$$

$$\text{VCC(LAT,M,MP)} = \frac{1}{72} \sum_{I=1}^{144} \text{FIELD(LAT,I)*COS.(M*I*}\pi\text{/72)} \text{*COS.(MP*I*}\pi\text{/72)} \quad \text{A.7}$$

where I is the longitude counter, and where the above expressions were halved for MP = 0.

Thus the R.M.S. relative contribution of the zonal mean (M = 0) to a wave (MP) is half the R.M.S. relative contribution of the wave to the zonal mean.

The meridional analysis was performed using the same quadrature as in the data analysis:

$$\text{ALIASS(M,MP,N,NP)} = \frac{\pi}{36.}\sum_{J=1}^{28} \text{VSS(J,M,MP)*PMN(M,N,J)} \tag{A.8}$$

$$\text{*PMN(MP,NP,J)*COS.((J+7)*}\pi\text{/72)}$$

where J is the latitude counter (J = 1 at 20°N) and PMN the associated Legendre function, is the alias from the zonal wavenumber M sine to the zonal wavenumber MP sine, and thence from the meridional wavenumber N to the meridional wavenumber NP. The normalizing factor $\pi/36.$ is accurate to 1/2 % for all meridional and zonal wavenumbers 0 through 15.

Similarly, one obtains ALIASC from VSC, ALIACS from VCS, and ALIACC from VCC. Only the alias between the zonal mean (M = 0) and some other zonal wavenumber (MP) can have an east-west phase, which further is not a function of the associated meridional wavenumbers (N,NP). The alias between any two waves consists of four components unless one of the zonal wavenumbers is zero.

Illustrative values are given in Tables A.2 et seq.

It is seen that the R.M.S. relative error in zonal wavenumbers is high when the wavenumbers are close together, and tails off to the order of 5% for zonal wavenumbers 5 or 6 apart. The cross-aliases between sine and cosine terms (VSC and VCS) tend to be somewhat smaller than the direct sine-sine and cosine-cosine aliases (VSS and VCC), except for well separated zonal wavenumbers. The values at high and low latitudes are worse: the influence of Alaska, Siberia, Greenland and Iceland is clearly seen in the superiority of the belt 50° to 70°N. The automatic rounding error estimate averages less than one decimal digit and never exceeds three (in a 27 bit mantissa or 8 decimal digits) except in numbers which print as zero. The automatic estimate is not meant to be precise in cases of small loss: for exact treatments see Wilkinson (1963).

The hemispheric aliases show 32% in the hemispheric mean, 32% from the hemispheric mean to MP = 0, NP = 1, 4% from the hemispheric mean to MP = 0, NP = 2 or the fundamental of the meridional profile of the zonal average, -7% to M = 0, N = 4 and 25% to M = 0, N = 6. These figures are disturbing, and a substantial restriction on the classes of energetic study which can be undertaken with the present data. They clearly show the advantage of using seasonally varying climatological means in optimum interpolation, to allow for the quasi-stationary responses to orography and heating. Separation of meridional wavenumbers is seen to cause a reduction of aliasing, down to the order of 1% for (0,6) and (6,0) (remember the meridional wavenumbers are reduced; the difference of the usual upper and lower parameters of the Legendre polynomials). Again the cross-aliases ALIASC and ALIACS tend to be smaller than the direct aliases ALIASS and ALIACC. The rounding errors in the meridional quadrature are everywhere negligible, and the uncertainties in the printed hemispheric aliases are considered dominated by the design of the experiment.

What we may call the auto-aliases, from (M,N) to (M,N), run from 27% for (0, 1) to values fluctuating from 15 to 50% for (6,6) in the direct aliases, while the indirect auto-aliases run two orders of magnitude smaller. This may indicate that one can place a fair reliance on the east-west phase of a given wave vector.

The automatic rounding error estimate was developed for use with series in which the partial sums may be absolutely much larger than the total sum, and in which the majority of the rounding error arises as the small difference of two large terms: many of the series arising in connection with Legendre polynomials have individual terms several orders of magnitude larger than the sum of the series. The automatic error estimate is therefore (Wilkinson, 1963, p.7 et seq.)

$$\log_{10} \left(\left| \max |S_I| \right| / |S| \right) \qquad \text{A.9}$$

where S is the total sum of the series, S_I any partial sum,

and max $|S_I|$ is the absolutely largest of the S_I.
Rigorous treatments are given by Wilkinson (1963).

TABLE A.2.

M= 0 MP= 0

20, 55												52.5, 87.5	
VSS - ZONAL SINE-SINE ALIAS													
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
VSC - ZONAL SINE-COSINE ALIAS													
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
VCS - ZONAL COSINE-SINE ALIAS													
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
VCC - ZONAL COSINE-COSINE ALIAS													
.987	.986	.992	.989	.360	.366	.354	.362	.373	.366	.349	.360	.132	.1C2
.112	.108	.096	.102	.132	.114	.085	.769	.768	.654	.679	.934	1.000	1.000

ALIAS FROM WAVE VECTOR (M,N) TO (MP,NP) , TIMES TEN.

N/NP	0	1	2	3	4	5	6	0	1	2	3	4	5	6
	ALIASS							ALIASC						
0	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
1	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
2	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
3	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
4	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
5	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
6	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.00C	.000
	ALIACS							ALIACC						
0	.000	.000	.000	.000	.000	.000	.000	3.182	3.150	.401	-1.538	-.727	1.524	2.509
1	.000	.000	.000	.000	.000	.000	.000	3.150	3.541	1.467	-.282	-.016	1.547	2.447
2	.000	.000	.000	.000	.000	.000	.000	.401	1.467	2.815	3.145	2.043	.658	.415
3	.000	.000	.000	.000	.000	.000	.000	-1.538	-.282	3.145	5.133	3.794	.860	-.494
4	.000	.000	.000	.000	.000	.000	.000	-.727	-.016	2.043	3.794	3.934	2.629	1.168
5	.000	.000	.000	.000	.000	.000	.000	1.524	1.547	.658	.860	2.629	4.238	3.758
6	.000	.000	.000	.000	.000	.000	.000	2.509	2.447	.415	-.494	1.168	3.758	4.496

TABLE A.3.

M= 1 MP= 1

20, 55													52.5, 87.5
VSS - ZONAL SINE-SINE ALIAS													
.998	.996	.997	.999	.171	.178	.168	.176	.181	.176	.168	.164	.082	.071
.068	.065	.063	.063	.082	.062	.057	.658	.647	.511	.502	.982	1.000	1.000
VSC - ZONAL SINE-COSINE ALIAS													
-.001	-.001	.006	-.003	-.051	-.043	-.032	-.029	-.038	-.032	-.020	-.025	-.106	-.084
-.090	-.087	-.078	-.079	-.106	-.089	-.073	-.004	-.012	.004	-.023	.021	-.000	-.000
VCS - ZONAL COSINE-SINE ALIAS													
-.001	-.001	.006	-.003	-.051	-.043	-.032	-.029	-.038	-.032	-.020	-.025	-.106	-.084
-.090	-.087	-.078	-.079	-.106	-.089	-.073	-.004	-.012	.004	-.023	.021	-.000	-.000
VCC - ZONAL COSINE-COSINE ALIAS													
.976	.977	.986	.978	.550	.555	.540	.547	.565	.557	.529	.555	.181	.134
.157	.151	.129	.142	.182	.167	.114	.879	.888	.797	.856	.885	1.000	1.000

ALIAS FROM WAVE VECTOR (M,N) TO (MP,NP) , TIMES TEN.

N/NP	0	1	2	3	4	5	6	0	1	2	3	4	5	6
	ALIASS							ALIASC						
0	2.512	2.528	.203	-1.889	-1.496	.700	2.025	-.161	-.251	-.229	-.121	-.005	.053	.050
1	2.528	2.729	.642	-1.426	-1.305	.592	1.924	-.251	-.405	-.400	-.255	-.074	.050	.089
2	.203	.642	1.084	1.079	.543	-.008	.017	-.229	-.400	-.452	-.371	-.207	-.033	.092
3	-1.889	-1.426	1.079	3.030	2.358	-.012	-1.281	-.121	-.255	-.371	-.410	-.336	-.164	.034
4	-1.496	-1.305	.543	2.358	2.479	1.087	-.110	-.005	-.074	-.207	-.336	-.371	-.270	-.080
5	.700	.592	-.008	-.012	1.087	2.381	2.513	.053	.050	-.033	-.164	-.270	-.287	-.206
6	2.025	1.924	.017	-1.281	-.110	2.513	3.811	.050	.089	.092	.034	-.080	-.206	-.277
	ALIACS							ALIACC						
0	-.161	-.251	-.229	-.121	-.005	.053	.050	3.492	3.912	1.150	-1.911	-2.299	-.147	1.810
1	-.251	-.405	-.400	-.255	-.074	.050	.089	3.912	4.722	2.096	-1.276	-2.267	-.534	1.562
2	-.229	-.400	-.452	-.371	-.207	-.033	.092	1.150	2.096	2.344	1.593	.283	-.587	-.306
3	-.121	-.255	-.371	-.410	-.336	-.164	.034	-1.911	-1.276	1.593	3.880	3.233	.499	-1.282
4	-.005	-.074	-.207	-.336	-.371	-.270	-.080	-2.299	-2.267	.283	3.233	4.100	2.538	.447
5	.053	.050	-.033	-.164	-.270	-.287	-.206	-.147	-.534	-.587	.499	2.538	4.052	3.634
6	.050	.089	.092	.034	-.080	-.206	-.277	1.810	1.562	-.306	-1.282	.447	3.634	5.178

TABLE A.4.

M= 1 MP= 6

20, 55													52.5, 87.5
VSS - ZONAL SINE-SINE ALIAS													
.006	.007	.004	-.000	-.043	-.026	-.008	-.034	-.030	-.023	-.022	-.014	-.046	-.048
-.034	-.033	-.034	-.031	-.046	-.024	-.041	-.017	-.025	-.032	.039	-.020	-.000	-.000
VSC - ZONAL SINE-COSINE ALIAS													
-.008	.003	-.009	.009	.051	.072	.064	.076	.066	.058	.074	.054	-.009	.009
-.014	-.006	-.004	-.025	-.009	-.028	.022	.054	.035	.029	.049	.080	-.000	-.000
VCS - ZONAL COSINE-SINE ALIAS													
-.004	-.001	.005	.001	.023	.025	.026	.019	.025	.021	.022	.040	.025	.007
.024	.031	.012	.003	.025	.038	.030	.032	.043	.061	-.053	-.017	.000	.000
VCC - ZONAL COSINE-COSINE ALIAS													
-.001	-.019	-.012	.016	-.039	-.043	-.043	-.024	-.038	-.034	-.038	-.052	-.061	-.031
-.055	-.064	-.040	-.025	-.061	-.071	-.055	-.001	.026	-.010	-.047	-.022	-.000	-.000

ALIAS FROM WAVE VECTOR (M,N) TO (MP,NP) , TIMES TEN.

N/NP	0	1	2	3	4	5	6	0	1	2	3	4	5	6
	ALIASS							ALIASC						
0	-.027	-.067	-.108	-.124	-.103	-.057	-.018	.069	.159	.231	.231	.139	-.004	-.116
1	-.038	-.095	-.153	-.182	-.165	-.115	-.068	.092	.211	.311	.318	.204	.018	-.141
2	-.026	-.065	-.108	-.141	-.157	-.153	-.137	.049	.116	.178	.197	.152	.054	-.055
3	.004	.006	-.002	-.032	-.086	-.144	-.176	-.028	-.060	-.075	-.051	.008	.064	.071
4	.029	.065	.086	.069	.006	-.076	-.137	-.081	-.180	-.255	-.245	-.137	.016	.123
5	.030	.067	.097	.102	.077	.029	-.025	-.066	-.153	-.231	-.253	-.193	-.072	.054
6	.006	.016	.036	.066	.101	.119	.099	.001	-.009	-.038	-.086	-.129	-.132	-.073
	ALIACS							ALIACC						
0	.029	.067	.099	.105	.078	.034	.001	-.056	-.121	-.167	-.169	-.129	-.076	-.041
1	.038	.090	.137	.153	.126	.074	.028	-.070	-.156	-.230	-.255	-.222	-.161	-.108
2	.022	.054	.089	.115	.120	.104	.077	-.032	-.084	-.151	-.208	-.234	-.222	-.185
3	-.009	-.016	-.010	.017	.060	.099	.112	.024	.034	.009	-.058	-.144	-.208	-.225
4	-.030	-.065	-.086	-.073	-.021	.046	.094	.049	.102	.128	.096	.007	-.100	-.177
5	-.025	-.057	-.088	-.099	-.079	-.034	.020	.030	.078	.132	.160	.136	.059	-.040
6	.000	-.007	-.027	-.058	-.088	-.095	-.066	-.009	.002	.045	.113	.172	.180	.124

TABLE A.5.

M= 6 MP= 6

20, 55 — VSS - ZONAL SINE-SINE ALIAS — 52.5, 87.5

.981	.987	.994	.991	.381	.387	.363	.390	.402	.392	.363	.384	.154	.124
.128	.113	.100	.133	.154	.129	.086	.783	.772	.638	.688	.929	1.000	1.000

VSC - ZONAL SINE-COSINE ALIAS

.010	.002	-.008	-.010	.008	.018	.017	.005	.014	.023	.027	.004	.013	.015
.008	.002	.016	.019	.013	.001	-.011	.018	.019	.014	.012	.006	.000	.000

VCS - ZONAL COSINE-SINE ALIAS

.010	.002	-.008	-.010	.008	.018	.017	.005	.014	.023	.027	.004	.013	.015
.008	.002	.016	.019	.013	.001	-.011	.018	.019	.014	.012	.006	.000	.000

VCC - ZONAL COSINE-COSINE ALIAS

.992	.985	.990	.986	.340	.346	.345	.333	.344	.341	.334	.335	.109	.081
.097	.103	.091	.072	.109	.099	.085	.755	.763	.670	.669	.938	1.000	1.000

ALIAS FROM WAVE VECTOR (M,N) TO (MP,NP) , TIMES TEN.

N/NP	0	1	2	3	4	5	6	0	1	2	3	4	5	6
	ALIASS							ALIASC						
0	1.855	2.950	2.323	.215	-1.578	-1.486	.130	.009	.017	.020	.017	.010	.004	-.003
1	2.950	4.836	4.097	.910	-2.082	-2.346	-.129	.017	.035	.046	.045	.032	.012	-.009
2	2.323	4.097	4.051	1.931	-.640	-1.646	-.682	.020	.046	.070	.077	.061	.026	-.012
3	.215	.910	1.931	2.557	2.070	.572	-.900	.017	.045	.077	.095	.083	.044	-.003
4	-1.578	-2.082	-.640	2.070	3.752	2.728	-.163	.010	.032	.061	.083	.084	.060	.025
5	-1.486	-2.346	-1.646	.572	2.728	3.038	1.268	.004	.012	.026	.044	.060	.068	.060
6	.130	-.129	-.682	-.900	-.163	1.268	2.201	-.003	-.009	-.012	-.003	.025	.060	.081
	ALIACS							ALIACC						
0	.009	.017	.020	.017	.010	.004	-.003	1.834	2.900	2.250	.144	-1.615	-1.477	.168
1	.017	.035	.046	.045	.032	.012	-.009	2.900	4.722	3.931	.748	-2.175	-2.341	-.059
2	.020	.046	.070	.077	.061	.026	-.012	2.250	3.931	3.811	1.686	-.802	-1.681	-.616
3	.017	.045	.077	.095	.083	.044	-.003	.144	.748	1.686	2.287	1.852	.462	-.897
4	.010	.032	.061	.083	.084	.060	.025	-1.615	-2.175	-.802	1.852	3.518	2.535	-.264
5	.004	.012	.026	.044	.060	.068	.060	-1.477	-2.341	-1.681	.462	2.535	2.804	1.072
6	-.003	-.009	-.012	-.003	.025	.060	.081	.168	-.059	-.616	-.897	-.264	1.072	1.971

APPENDIX B

FORMALISM OF CHARACTERISTIC PATTERNS

Characteristic patterns or C.P.'s are also known as empirical orthogonal functions and principal factors. More complete treatments and proofs of results quoted here are given by Mateer (1965), Lawley and Maxwell (1963), and Lorenz (1956).

Consider any scalar which is piecewise continuous in space and time: the amount of information about the variations of the scalar is not proportional to the density of observations of it in space or time, because of its continuity. Suppose the values of the scalar are represented by a matrix $Y = \{y_{ij}\}$, where $1 \leq i \leq M$ is the space subscript and $1 \leq j \leq N$ is the time subscript. Let the mean state be removed by requiring that

$$\sum_{j=1}^{N} y_{ij} = 0 \qquad \text{for } 1 \leq i \leq M \qquad \text{B.1}$$

Form the covariance matrix

$$L = YY' \qquad \text{B.2}$$

where Y' is the transpose of Y. L is a square M by M, symmetric matrix with real positive eigenvalues λ_i, where $1 \leq i \leq M$.

Further

$$\sum_{i=1}^{M} \lambda_i = \sum_{i=1}^{M} \sum_{j=1}^{N} Y_{ij}^2 \qquad \text{B.3}$$

which is the total variance of the matrix Y.

Each eigenvector

$$V_k = \{V_{ik}\} , \qquad 1 \leq i \leq M, \qquad 1 \leq k \leq M \qquad \text{B.4}$$

of L explains a fraction of the total variance of Y

$$\Lambda_i = \lambda_i / \sum_{j=1}^{M} \lambda_j \qquad \text{B.5}$$

The algorithm therefore proceeds by

1. establishing the matrix Y (e.g. the amplitude of a given surface harmonic at eight pressure levels twice a day for a month)
2. Subtracting out the row mean of Y according to equation B.1.
3. Forming the covariance matrix L by equation B.2.
4. Evaluating the eigenvalues and eigenvectors of L in sequence by any standard method.
5. Normalizing the eigenvectors, and dividing the eigenvalues by the sum of squares of Y, according to equations B.3 and B.5.

Both the Jacobi method and the improved power method were used in this study (Fox, 1964, p. 223; Fadeev and Fadeeva, 1960, Sect. 54). The improved power method is well suited to characteristic pattern calculations because the eigenvalues are real and well separated, because good guesses at the corresponding eigenvectors are usually available from previous work, and because the sequence of eigenvalues (dominance order) is usually known from previous work.

Working in single precision floating point on a computer with a 27 bit mantissa, the following precautions were found sufficient when only three or four eigenvalues and vectors were needed, with the ratio of the largest to the smallest desired eigenvalue not greater then 20, and with three significant digits sufficient.

1. The dominant eigenvector V_1, corresponding to the largest eigenvalue λ_1, should be given ten iterations.

2. The initial guess at each subdominant eigenvector V_k should be orthogonalized to each more dominant eigenvector V_j $(k>j\geq 1)$ every five iterations, always in the order $j = k-1$ to $j = 1$ to prevent the orthogonalization of V_k to the available estimate of V_j from leaving any residual contribution from V_L, where $j > L > 1$.

3. If the dominance ratio λ_1/λ_k is estimated at more than 20, one must either work in double precision or reorthogonalize

sufficiently often to prevent the growth of the dominant eigenvector V_1 in the estimate of V_k.

4. The results must be examined for interchanges of the usual dominance order, which mean that the initial guesses at two or more eigenvectors are very poor instead of very good. This may be done by testing the estimated eigenvectors for their stability from additional iterations.

5. If for any k in the desired range

$$\lambda_{k-1}/\lambda_k \lesssim 1.5 \qquad \text{B.6}$$

then the eigenvector V_{k-1} must be iterated many times to ensure valid results for V_{k-1} and V_k.

APPENDIX C.

CONCLUSIONS OF MASHKOVICH (1961).

Translated by J. H. Bradley

1. Wave disturbances, the amplitude of which grows unboundedly with time, can arise in a baroclinic atmosphere. Instability occurs for waves of degree N greater than 7. The long waves are stable.

2. The instability of the motion is greater the faster the speed of the basic (zonal) motion increases with height.

In the case when the speed of the basic current is independent of height (a quasi-barotropic basic state) all waves are stable.

3. The difference in the distribution in the vertical of the speed of the basic current in the summer and winter months causes a greater stability of the motion in summer than in winter.

4. A lowering of the tropopause level raises the stability of the motion.

5. Temperature advection is one factor causing instability. Vertical motions partly compensate the effect of temperature advection and stabilize the motion. In the case of a neutral stratification the instability is substantially stronger than for ordinary values of γ (the lapse rate--Trans.).

6. The translational speed of the waves is greater (less in absolute value) in winter than in summer.

7. The nature of the development of a disturbance depends essentially on its initial vertical structure.

8. Disturbances of which the initial amplitude and phase are the same at all levels are unstable and change shape substantially in a short time. The amplitude of the disturbance in the upper and lower layers of the troposphere increases rapidly, while a slight decrease in amplitude occurs in the middle troposphere. The speed of the waves depends on height. The axis of an initially vertical disturbance acquires a tilt to the west in the upper troposphere and to the east in the lower troposphere. The signs of the vertical gradient of the pressure disturbance become opposite in the upper and lower layers.

9. The computational results indicate definite differences in the development of disturbances initially localized in different layers of the atmosphere.

10. The amplification of the disturbance is more intense in the upper than the lower layers.

11. The speed of motion of waves formed below the level at which the initial disturbance was localized (the "initial" level) is close to the speed of the disturbance at the initial level. Waves formed above the initial level move eastward substantially faster.

12. A pressure disturbance initially localized in the lower layer of the atmosphere rapidly propagates upward, while the disturbance at the ground weakens slightly. The amplitude of the disturbance in the lower half of the atmosphere soon evens out. The disturbance at the ground is quasi-stationary.

13. Disturbances initially located in the upper troposphere and lower stratosphere are unstable; their amplitude grows rapidly with time, while their speed practically does not change. The disturbance propagates into the lower layers, but the amplitude of the wave below is several times smaller

than the amplitude at the initial level. The greatest kinetic energy is generated in this case as compared to the other variants mentioned.

14. Pressure waves arising as a result of the development of disturbances in the lower layer have an inclination of the axis to the west in the lower layers of the troposphere, and to the east in the upper layers. The opposite result is obtained for disturbances arising in the upper and middle troposphere.

15. A conversion of the potential energy of a disturbance to its kinetic energy occurs during the motion of a single wave in a baroclinic atmosphere. This energy conversion results from the vertical redistribution of air masses of non-uniform temperature by rising and sinking currents. The potential energy of the disturbance also changes by temperature advection.

APPENDIX D

GLOSSARY

1. NOTATION

Where compiler language (ALGOL, FORTRAN, MAD, etc.) notation is used the following rules apply.

1. E in a number denotes a power of 10: e.g. 1E3 means 1000; 1.5E0 means 1.5; 27.182818285E-1 means 2.7182818285; etc.
2. Multiplication is explicitly denoted by the operator *; implicit multiplication is forbidden in mixed compiler-algebraic expressions except where parentheses make it clear, but allowed in purely algebraic expressions.
3. Variable names consist of 1 to 6 letters and numbers, of which the first is a letter.
4. Function names end with a period as in MAD: e.g. SIN., SORT.
5. One or more subscripts or function arguments may follow the name in brackets, separated by commas: e.g. A(1); B(2,7); C(I,J+1); SIN.(PI); COS.(MP*I*PI/72.0).

2. JARGON

A coefficient: a coefficient corresponding to a zonal harmonic cos ($m\lambda$), with the origin at the Greenwich meridian. A(o) is the zonal mean.

Alias: by extension of the usual meaning (Blackman and Tukey, 1959), a contingent error by which the analysis system does not correctly represent each wave.

Available potential energy: the potential energy releasable by adiabatic motions to a barotropic state, in which temperature is a function of pressure only.

Baroclinic: a state in which temperature is a function of space and pressure (and possible time).

Barotropic: a state in which temperature is a function of pressure (and possibly time) but not space.

B coefficient: a coefficient corresponding to a zonal harmonic sine ($m\lambda$), with the origin at the Greenwich meridian. B(o) is zero.

Characteristic Pattern: empirical orthogonal function, principal factor.

C.P.: characteristic pattern.

Characteristic Time: a time constant, residence time, or other typical time which may be used for order-of-magnitude estimate, scale analyses or other semi-quantitative examination of the main properties of a system.

Degree: the lower parameter, or meridional plus zonal wavenumber of a surface harmonic.

Deviation: the instantaneous amplitude minus the running time average of a wave.

Diabatic: influx or efflux of heat.

Diagnostic equation: an equation not containing time derivatives; a windspeed is not counted as a time derivative.

Dominance order: the sequence of eigenvalues of a matrix arranged in order of absolute value.

Dominance ratio: the ratio of any two eigenvalues of a matrix, in connection with an iterative process.

Forced mode: a slow-moving wave generated by the seasonal cycle of heating or by the interactions of fast-moving waves.

Instability: classically the instantaneous tendency of the small amplitude of a wave in a time-independent basic flow.

Interaction integral: the contribution of any two wave vectors in a non-linear term to some third wave vector.

Meridional: north-south, latitudinal. Wavenumber denoted by N.

Mode: a waveform defined either analytically or by a characteristic pattern.

Mode number: a sequential numbering of the usual dominance order of characteristic patterns. The mode number cannot conveniently be defined by the number of zeros or extrema of a characteristic pattern unless one gives involved criteria to exclude minor statistical fluctuations.

Orography: topographic; relief; mountain.

Primitive Equations: the Newtonian equations of motion for a compressible fluid, usually with the hydrostatic approximation applied. Sometimes found with other assumptions, e.g. restrictions on the vertical velocity.

Prognostic equation: an equation containing one or more time derivatives.

Rank: the upper parameter, or zonal wavenumber of a surface harmonic.

Residence time: the energy of a particular mode (kinetic or available potential; zonal or eddy; standing or transient) divided by the sum of one or more energy conversions.

Solid rotation: the mean over the mass of the atmosphere of the angular velocity about the earth's axis of the zonal wind U.

Spectral: A Fourier expansion in orthogonal eigenfunctions.

Spherical harmonic: properly the product of a surface harmonic by a function of radius orthogonal in the range zero to infinity. Often loosely used as a synonym for surface harmonic.

Standing: long period time average.

Surface harmonic: an eigenfunction of Laplace's equation, orthogonal over the surface of a sphere. Explicitly $Y_N^M = {}^{\sin}_{\cos}(m\lambda) P_N^M(\cos\theta)$, where P_N^M is the associated Legendre polynomial of rank M and degree N.

Transient: moving, or instantaneous minus time average.

Wavenumber: the number of cycles of a wave in a given space. Meridional wavenumbers are the number of zeros in the open interval between the poles; zonal wavenumbers are in 2π radians of longitude

Zonal: east-west, longitudinal. Wavenumber denoted by M.

BIBLIOGRAPHY

NOTE: 'Izvestiya' and some 'Dokladi' are translated. No 'Trudi' are translated to the author's knowledge. Translations of books are shown. Good, non-literal title translations are provided.

Baer, F., 1964: Integration with the Spectral Vorticity Equation. Journal of the Atmospheric Sciences, 21, no.3, 260-276.

Belousov, S.L., 1956: Tablitsi Normirovanikh Prisoedinennikh Polinomov Lezhandra. Akademiya Nauk SSSR. Reprinted as: Normalized Associated Legendre Polynomials; Mathematical Tables Series vol. 18. Pergamon, 1962.

Bernstein, A.B., 1966: Examination of Certain Terms Appearing in Reynolds' Equations under Unsteady Conditions and their Implications for Micrometeorology. Quart.J.Roy. Met.Soc., 92, no.394, 533-542.

Blackman, R.B. and J.W.Tukey, 1959: The Measurement of Power Spectra. Dover, 190pp.

Blinova, E.N., 1943: Gidrodinamicheskaya Teoriya Voln Davleniya, Temperaturnikh Voln i Tsentrov Deistviya Atmosferi. Dokladi Akademii Nauk SSSR, 39, no.7, 284-287. (A Hydrodynamic Theory of Pressure and Temperature Waves and the Centers of Action of the Atmosphere.)

______, 1953: K Voprosu ob Opredelenii Davleniya na Urovne Morya. Dokladi Akademii Nauk SSSR, 92, no.3, 557-560. (On the Question of Determining the Sea Level Pressure.)

______, 1956: Metod Resheniya Nelineinoi Zadachi ob Atmosfernikh Dvizheniyakh Planetarnogo Masshtaba. Dokladi Akademii Nauk SSSR, 110, no.6, 975-977. (A Method of Solving the Non-Linear Problem of Atmospheric Motions of Planetary Scale.)

______, 1957: Hydrodynamic Methods of Short and Long Range Forecasting. Tellus, 9, no.4.

______, 1961: Gidrodinamicheskoe Issledovanie Nestatsionarnikh Atmosfernikh Protsessov Planetarnogo Masshtaba. Dokladi Akademii Nauk SSSR, 140, no.2, 354-357. (A Hydrodynamic Study of Non-Stationary Processes of Planetary Scale.)

BIBLIOGRAPHY (continued)

______, 1964: Long Range Hydrodynamic Weather Forecasting. Israel Program for Scientific Translations, 1965, 123pp.

______, 1965: Dinamika Atmosfernikh Dvizhenii Planetarnogo Masshtaba i Gidrodinamicheskoi Dolgosrochnii Prognoz Pogodi. Trudi Mirovogo Meteorologicheskogo Tsentra, vip. 5, 84pp.
(The Dynamics of Atmospheric Planetary Scale Motions and Hydrodynamic Long Period Weather Forecasting.)

______, 1966: Ob Obespechenii Danimi Nabliudenii Operativnoi i Issledovatelskoi Raboti po Gidrodinamicheskomu Dolgosrochnomu Prognozu Pogodi. Trudi Mirovogo Meteorologicheskogo Tsentra, vip. 12, 3-20.
(On the Provision of Observational Data for Operational and Research Work on Hydrodynamic Long Period Weather Forecasting.)

Burger, A.P., 1958: Scale Considerations of Planetary Motions of the Atmosphere. Tellus, 11, 195-205.

Chapman, S. and J.Bartels, 1940: Geomagnetism. Oxford University Press, 2 vols.

______, and K.C.Westfold, 1956: A Comparison of the Annual Mean Solar and Lunar Atmospheric Tides in Barometric Pressure, as regards their Worldwide Distribution of Amplitude and Phase. Journal of Atmospheric and Terrestrial Physics, 8, no.1/2, 1-23.

Charney, J.G., 1947: The Dynamics of Long Waves in a Baroclinic Westerly Current. Journal of Meteorology, 4, no.5.

______, and A.Eliassen, 1949: A Numerical Method for Predicting the Perturbations in the Middle-Latitude Westerlies. Tellus, 1, no.2, 38-54.

Crutcher, H., 1959: Upper Wind Statistics Charts of the Northern Hemisphere. Office of the Chief of U.S. Naval Operations. NAVAER 50-1c-535.

BIBLIOGRAPHY (continued)

Deland, R.J., 1965a: Some Observations of the Behavior of Spherical Harmonic Waves. Monthly Weather Review, 93, no.5, 307-312.

______, 1965b: On the Scale Analysis of Travelling Planetary Waves. Tellus, 17, no.4, 527-528.

______, and Yeong-jer Lin, 1967: On the Movement and Prediction of Traveling Planetary-Scale Waves. Monthly Weather Review, 95, no.1, 21-31.

Derome, J.F. and A.Wiin-Nielsen, 1966: On the Baroclinic Stability of Zonal Flow in Simple Model Atmospheres. University of Michigan, Project Report 06372-2-T.

Dobrishman, E.M., S.A.Mashkovich and A.L.Chubukova, 1963: Vremenie Spektralnie Funktsii Indeksov Tsirkulatsii na Raznikh Urovyakh. Trudi Vsesoyuznogo Nauchnogo Meteorologicheskogo Soveshchaniya. Dinamicheskaya Meteorologiya, 2.
(Time Spectral Functions of Circulation Indices at Different Levels.)

Eliasen, E., 1958: A Study of the Long Atmospheric Waves on the Basis of Zonal Harmonic Analysis. Tellus, 10, 206-215.

______, and B.Machenauer, 1965: A Study of the Fluctuations of the Atmospheric Planetary Patterns Represented by Spherical Harmonics. Tellus, 17, no.2, 220-238.

Fadeev, D.K. and V.N. Fadeeva, 1960: Computational Methods of Linear Algebra. W.H. Freeman, 1963, 621pp.

Fox, L., 1962: Numerical Solution of Ordinary and Partial Differential Equations. Pergamon, 507pp.

______, 1964: An Introduction to Numerical Linear Algebra. Oxford University Press, 328pp.

Gandin, L.S., 1963: Obektivnii Analiz Meteorologicheskikh Polei. Gidrometeoizdat. Translated as: Objective Analysis of Meteorological Fields. Israel Program for Scientific Translations, 1965.

BIBLIOGRAPHY (continued)

Gaunt, J.A.,1929: On the Triplets of Helium. Phil. Trans., A228, 192-196.

Gauss, K.F.,1870: Werke, Band 5, S.121-193. Paris.

Gavrilin, B.L.,1965: On the Description of the Vertical Structure of Synoptic Processes. Izvestiya Akademii Nauk SSSR, Seriya Atm. i Okean. Fiziki, No. 1, 8-17.

Haurwitz, B. and R.A. Craig,1952: Atmospheric Flow Patterns and Their Representation by Spherical-Surface Harmonics. Geophysical Research Papers, No. 14.

________ and A.D. Cowley,1965: The Lunar and Solar Air Tides at Six Stations in North and Central America. Monthly Weather Review, 93, No. 8, 505-509.

Hodge, M.W. and C. Harmantas,1965: Compatability of United States Radiosondes. Monthly Weather Review, 93, No. 4, 253-266.

Holmström, I.,1963: On a Method for Parametric Representation of the State of the Atmosphere. Tellus, 15, No. 2, 127-149.

Holopainen, E.O.,1963: On the Dissipation of Kinetic Energy in the Atmosphere. Tellus, 15, No. 1, 26-32.

______, 1966: A Diagnostic Study of the Maintenance of Stationary Disturbances in the Atmosphere. University of Michigan, Office of Research Administration, Report 06372-3-T.

Infeld, L. and T.E. Hull,1951: The Factorization Method. Rev. Mod. Phys., 23, 21.

Jeffreys, H. and B. Jeffreys,1962: Methods of Mathematical Physics. Cambridge University Press.

Kheifets, Ya.M.,1950: Tablitsi Normirovanikh Prisoedinennikh Polinomov Lezhandra. Gidrometeoizdat. (Tables of Normalized Associated Legendre Polynomials.)

Kibel, I.A.,1957: Vvedenie v Gidrodinamicheskie Metodi Kratkosrochnogo Prognoza Pogodi. Gostekhizdat. Translated as: Hydrodynamic Methods of Short Period Weather Forecasting. Pergamon, 1963, 383pp.

BIBLIOGRAPHY (continued)

Kolmogorov, A.N., 1941: Interpolirovanie i Ekstrapolirovanie Statsionarnikh Sluchainikh Posledovatelnostei. Izvestiya Akademii Nauk SSSR, seriya matematicheskaya. 5, No. 1.
(Interpolation and Extrapolation of Stationary Random Time Series.)

Kung, E.C., 1966: Large Scale Balance of Kinetic Energy in the Atmosphere. Monthly Weather Review, 94, No. 11, 627-660.

Lawley, D.N. and A.E. Maxwell, 1963: Factor Analysis as a Statistical Method. London, Butterworths, 117pp.

Lorenz, E.N., 1956: Empirical Orthogonal Functions and Statistical Weather Prediction. Sci. Rep. No. 1, Statistical Forecasting Project, Dept. of Met., Mass. Inst. Tech.

______, 1962: The Statistical Prediction of Solutions of Dynamic Equations. Proceedings of the International Symposium on Numerical Weather Prediction in Tokyo, November 7-13, 1960.

______, 1963a: The Predictability of Hydrodynamic Flow. Trans. New York Academy of Sciences, Ser. II, vol. 25, No. 4, 409-432.

______, 1963b: The Mechanics of Vacillation. Journal of the Atmospheric Sciences, 20, No. 5, 448-464.

______, 1965: A study of the Predictability of a 28-variable Atmospheric Model. Tellus, 17, No. 3, 321-333.

Mashkovich, S.A., 1957: K Uchetu Nelineinikh Chlenov Uravnenii pri Reshenii Zadachi o Prognoze Davleniya i Temperaturi Maloi Zablagovremennosti. Trudi Tsentralnogo Instituta Prognozov, Vipusk 60.
(On the Calculation of Non-Linear Terms in the Equations in Short Period Pressure and Temperature Forecasts.)

BIBLIOGRAPHY (continued)

______ and Ya. M. Kheifets,1960: K Teorii Dolgosrochnogo Prognoza s Uchetom Vertikalnoi Stratifikatsii Atmosferi i Turbulentnogo Peremeshivaniya. Trudi Tsentralnogo Instituta Prognozov, Vipusk 93, 3-15. (On the Theory of Long Range Forecasting, Taking Account of the Vertical Stratification of the Atmosphere and Turbulent Exchange.)

______, 1961: K Teorii Voln Davleniya v Baroklinoi Atmosfere. Trudi Tsentralnogo Instituta Prognozov, Vipusk 111, 13-28. (On the Theory of Pressure Waves in a Baroclinic Atmosphere.)

______, 1963: O Svyazi Kolebanii Zonalnoi Tsirkulatsii Atmosferi s Razvitiem Dlinnikh Voln. Sbornik "Obektivnii Analiz i Prognoz Meteorologicheskikh Elementov". Izdatelstvo Akademii Nauk SSSR. (On the Connection of Fluctuations of the Zonal Atmospheric Circulation with the Development of Long Waves. Collection 'Objective Analysis and Prognosis of Meteorological Elements'.)

______, 1964a: Obektivnii Analiz Aerologicheskikh Nabliudenii i Trebovanie k Razmeshcheniya Seti Stantsii. Izvestiya Akademii Nauk SSSR, Seriya Geofizicheskaya, No. 2, 285-292. (Objective Analysis of Aerological Observations and the Need for the Placement of a Station Network.)

______, 1964b: O Vzaimodeistvii Zonalnoi Tzirkulyatsii i Nalozhennikh Na Nee Nezonalnikh Vozmushchenii. Izvestiya Akademii Nauk SSSR, No. 6, 930-946. (On the Interaction of the Zonal Circulation and Superposed Non-Zonal Disturbances.)

Mateer, C.L.,1965: On the Information Content of Umkehr Observations. Journal of the Atmospheric Sciences, 22, no. 4, 370-381.

Mathematical Tables Project, 1945: Tables of Associated Legendre Functions. Columbia University Press, New York.

Murakami,T.,1963: Analysis of Various Large Scale Disturbances and the Associated Zonal Mean Motions in the Atmosphere. Geofisica Pura e Applicata, 54, 119-165.

BIBLIOGRAPHY (continued)

National Academy of Sciences - National Research Council, 1966: The Feasibility of a Global Observation and Analysis Experiment. Publication 1290.

Newell, R.E., J.M. Wallace and J.R. Mahoney, 1966: The General Circulation of the Atmosphere and Its Effects on the Movement of Trace Substances. Part 2. Tellus, 18, No. 2, 363-380.

Obukhov, A.M., 1960: O Statisticheski Ortogonalnikh Razlozheniyakh Empiricheskikh Funktsii. Izvestiya Akademii Nauk SSSR, Seriya Geofizicheskaya, No. 3, 432-439. (On Statistically Orthogonal Expansions in Empirical Functions.)

Oort, A.H., 1964: On Estimates of the Atmospheric Energy Cycle. Monthly Weather Review, 92, No. 11, 483-493.

Riehl, H., 1954: Tropical Meteorology. McGraw Hill, 392pp.

Saltzmann, B., 1962: Theory of Thermal Convection. Dover, 461pp.

Smagorinsky, J., 1953: The Dynamical Influence of Large-Scale Heat Sources and Sinks on the Quasi-Stationary Mean Motions of the Atmosphere. Quarterly Journal of the Royal Meteorological Society, 79, 342-366.

Smirnov, I.P., and L.L. Kazakova, 1965: Kharakteristiki Zonalnoi Tsirkulyatsii Atmosferi za Period 1960 - 1963 gg. Trudi Mirovogo Meteorologicheskogo Tsentra, vipusk 5. (Characteristics of the Zonal Circulation of the Atmosphere for the Period 1960 - 1963.)

______, and L.L. Kazakova, 1966: Kharakteristiki Zonalnoi Tsirkulyatsii Atmosferi v Severnom i Yuzhnom Polushariyakh. Trudi Mirovogo Meteorologicheskogo Tsentra, vipusk 12, 43-63. (Characteristics of the Zonal Circulation of the Atmosphere in the Northern and Southern Hemispheres.)

Thompson, P.D. 1961: Numerical Weather Analysis and Prediction. Macmillan, 170pp.

BIBLIOGRAPHY (concluded)

Wiener, N., 1949: Extrapolation, Interpolation, and Smoothing of Stationary Time Series. New York.

Wiin-Nielsen, A., 1959: On a Graphical Method for an Approximate Determination of the Vertical Velocity in the Mid-Troposphere. Tellus, 11, 432-440.

______, 1963: On Baroclinic Instability in Filtered and Nonfiltered Numerical Prediction Models. Tellus, 15, 1-19.

______, and M. Drake, 1966: The Contribution of Divergent Wind Components to the Energy Exchange between the Baroclinic and Barotropic Components. Monthly Weather Review, 94, No. 1, 1-8.

Wilkinson, J.H., 1963: Rounding Errors in Algebraic Processes. National Physical Laboratory Notes on Applied Science No. 32. London, Her Majesty's Stationery Office, Code 48-120-32, 161pp.

Yang, C.H., 1967: Private communication. To form part of Ph.D. thesis, Univ. of Mich., to be published.