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CALCULATIONS FOR A THERMAL ANTI-ICER

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FOREWORD

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CALCULATIONS FOR A THERMAL ANTI-ICER

1. Object of the Calculations

In a communication concerning thermal anti-icing made to the same Congress by one of the authors in collaboration with Messrs. Caron and Petit, there were distinguished two possible methods of thermal anti-icing of an airfoil. For complete evaporation, only the area of impingement by the water-drops need be heated, but it is essential that this heating be high enough so that the waterdrops which impinge on the airfoil are individually evaporated, without having the time to agglomerate on each other.

For a "running wet" surface, the impingement area is not heated enough to evaporate all the collected water impinging on the surface. Accordingly, a thin film of water covers the surface and extends beyond the impingement area; to prevent icing it is necessary to heat a larger area of the airfoil.

We shall compare numerically the amount of heat required for two methods of anti-icing, for the case of a Göttingenn 430 airfoil (see Figure 1). The wing considered has a length of 3 meters (measured along the chord) and flies with a theoretical angle of attack of 6°33' at a speed of 91 meters per second ($C_Z = 0.8$) in a cloud composed mainly of waterdrops 30 microns in diameter.

The choice of this profile allows a rapid calculation of the aerodynamic flow fields in the vicinity of the airfoil by a conformal transformation (the work having been accomplished by Mr. Max Plan). At the present time such calculations were taken up again on two sections of the Laté 631 airfoil, for which the aerodynamic flow fields have been determined experimentally in the laboratory of Mr. Malavard, by means of an electrolytic tank.

2. Water Catch on the Airfoil

By a graphical method described in a recent report,[1] we have traced the water droplet trajectories in the vicinity of the airfoil (Figure 2) for

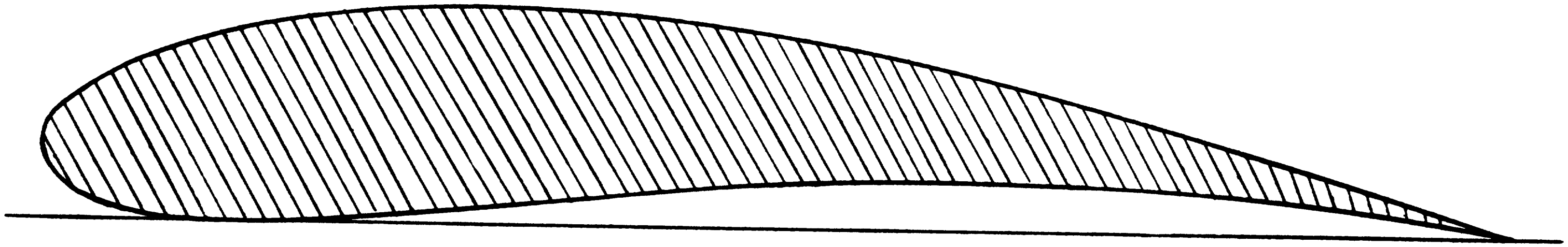


Figure 1. Profile studied. (Gottingen 430)

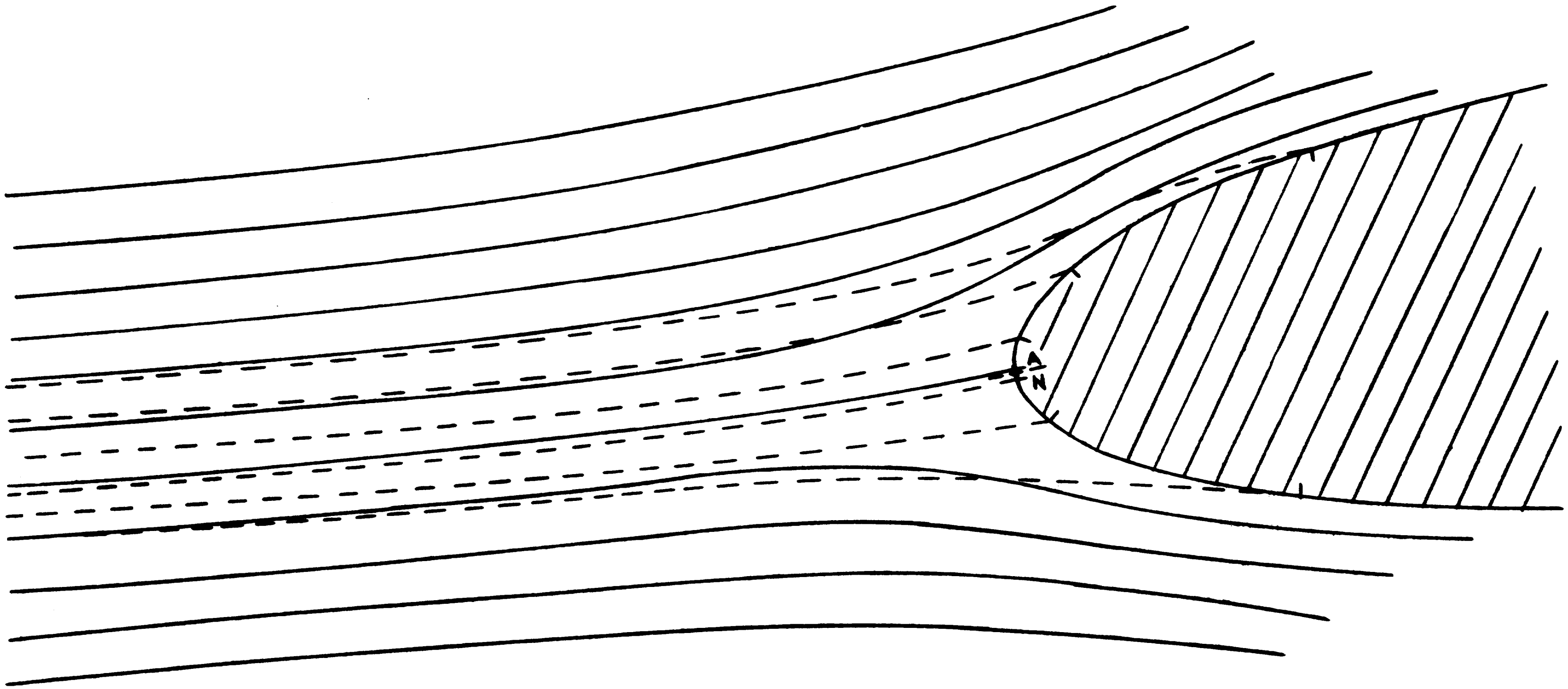


Figure 2. Water droplet trajectories near the airfoil.

the conditions fixed as follows: the tangent trajectories define the efficiency of catch, that is, the relation of the mass of water which, during a unit of time, impinges on a unit of length of the airfoil, to the mass of water which, during the same time, will pass through a unit of length of the maximum thickness in the absence of the airfoil. This efficiency is equal to 0.32; this relatively high value is due to the fact that the waterdrops are rather large; meanwhile, the impingement zone extends only to about 9% chord.

The trace of four intermediate trajectories allows the division of the impingement area into five divisions (see Figure 2) in which each of the efficiencies of average catch is known. By taking the abscissa x along the chord of the airfoil, beginning with the stagnation point, the following table is calculated:

Abscissa, cm	-31		-6.7		-1.6		+2.5		+10.5		+35.5
Efficiency of Catch		0.101		0.471		0.770		0.371		0.121	

It is then possible to trace approximately the graph of the local water-catch efficiency as a function of chordwise distance along the surface from the stagnation point (Figure 3). At the point N (Figure 2) where the waterdrop trajectories are normal to the airfoil, the efficiency will be equal to 1, if the point N coincides with the stagnation point A; in an actual case, the local water-catch efficiency at the point N is a little less than 1.

3. Convection of Heat along the Airfoil

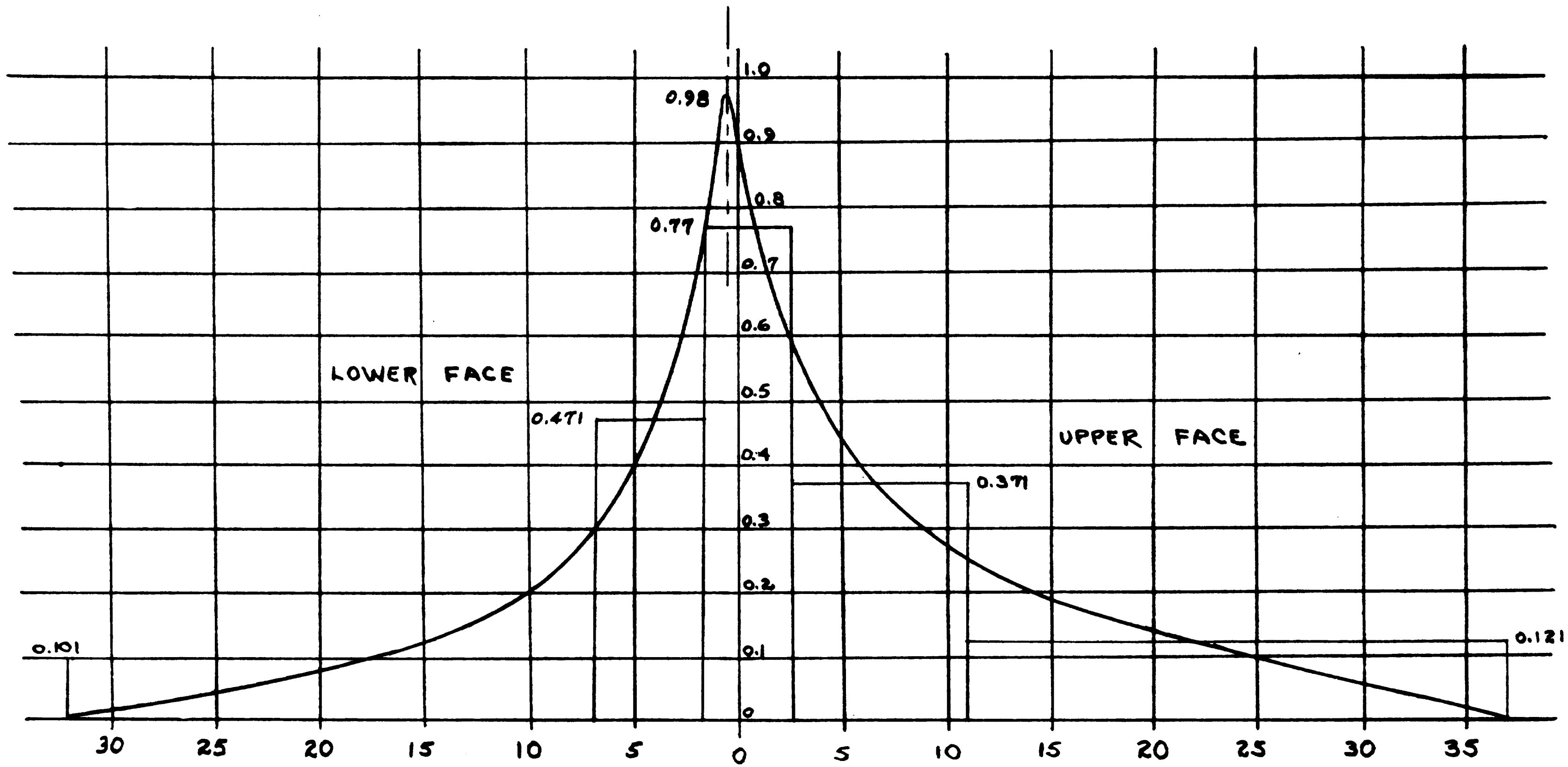
A recent report[2] illustrates the method of calculation of a local convection coefficient along an airfoil, for the case of laminar flow. We shall recall here the elements which permit this calculation.

The following symbols are defined:

x = the abscissa of a point along the surface measured chordwise from the stagnation point.

y = the coordinate of a point normal to the surface measured from the surface.

U = the local velocity at the edge of the boundary layer, at a point on the airfoil defined by the abscissa x.



Distance along the surface from the stagnation point, cm.

Figure 3. Local water-catch efficiency around the airfoil leading edge.

U_o = the free-stream velocity.

ν = the kinematic viscosity of the air.

f_{cx} = the local convection coefficient at a point defined by the abscissa x.*

k = the thermal conductivity of air.**

θ = the difference between the temperature of a point M along the surface and the temperature of a point situated a distance y from the point M on the normal to the surface at this point.

θ_o = the difference between the temperature of a point M on the surface and the temperature of the ambient free stream air.

Consider a wedge*** of included angle β**** placed in an airstream. The local coefficient of convection f_{cx} at a point M on the surface located at a distance x from the stagnation point is given by the relation

$$f_{cx} = \frac{k}{\theta_o}\left(\frac{\partial\theta}{\partial y}\right)_{y=0} . \tag{1}$$

Replacing the coordinate y by a dimensionless number

$$z = \frac{y}{\sqrt{2 - \frac{\beta}{\pi}}}\sqrt{\frac{U}{\nu x}} \tag{2}$$

which, for the given abscissa x, is proportional to it. The coefficient f_{cx} may be written as

$$f_{cx} = \frac{k}{\sqrt{2 - \frac{\beta}{\pi}}}\sqrt{\frac{U}{\nu x}}\left[\frac{\partial\left(\frac{\theta}{\theta_o}\right)}{\partial z}\right]_{z=0} . \tag{3}$$

The calculation shows that the temperature gradient at the surface, put in the form

*In the original French report, α was used for the convection coefficient; this was changed in the translation to f_{cx} to conform to current usage.

**Similarly for k which, although not defined in the original French report, was denoted by λ in the Prandtl number in the same manner as k, the thermal conductivity, is currently used.

***Original report shows "dièdre", which is literally a dihedron.

****α was used in the original French report.

$$\left[\frac{\partial\left(\frac{\Theta}{\Theta_o}\right)}{\partial z}\right]_{z=0} ,$$

is the same at every point. It depends only on the Prandtl number, $Pr = \mu C_p/k$ which characterizes the fluid, and the included angle β of the wedge. Eckert has pointed out that within 2% (and as long as β is not zero) one may write the following:

$$\left[\frac{\partial\left(\frac{\Theta}{\Theta_o}\right)}{\partial z}\right]_{z=0} = A = 0.56\left(\frac{\beta}{\pi} + 0.2\right)^{0.11} Pr^{0.35 + (0.02\beta/\pi)} . \tag{4}$$

We will assume that, in what follows, Pr equals 0.7, which is practically the case for air. Consequently A depends only on β. Taking account of equation (4), the local coefficient of convection may be written

$$f_{cx} = \frac{kA}{\sqrt{2 - \frac{\beta}{\pi}}} \sqrt{\frac{U}{\nu x}} . \tag{5}$$

Designating by Δy^* the discharge thickness of the thermal boundary layer, Δy^* is defined by the following equation:

$$\Delta y^* = \int_0^\infty \left(1 - \frac{\Theta}{\Theta_o}\right) dy . \tag{6}$$

For the dimensionless thickness Δz^* the following equation is written:

$$\Delta z^* = \frac{\Delta y^*}{\sqrt{2 - \frac{\beta}{\pi}}} \sqrt{\frac{U}{\nu x}} . \tag{7}$$

In taking account of equation (7), equation (5) containing the local convection coefficient may be rewritten as

$$f_{cx} = \frac{kA\Delta z^*}{\Delta y^*} . \tag{8}$$

As was the case for A, Δz^* depends only on the Prandtl number and the angle β. For $Pr = 0.7$, the following table is given:

β	$8/5\ \pi$	π	$\pi/2$	$\pi/5$	0	-0.14π
Δ	0.514	0.496	0.470	0.444	0.414	0.370
Δy^*	1.15	1.19	1.24	1.30	1.38	1.53
$A\Delta z^*$	0.591	0.590	0.583	0.577	0.570	0.566

The table shows that the product $A\Delta z^*$ changes only slightly when the angle β varies. It may, with small error, be considered constant and equal to 0.58. The local coefficient of convection f_{cx} varies then according to equation (8), on account of the inverse relationship of the discharge thickness Δy^*.

In the method of approximation which permits the determination of the local coefficients of convection for the airfoil, one may identify each point M of the airfoil with a point P on the wedge (according to Eckert). The angle β of the wedge and the value Δy^* at the point P are defined by the two equations

$$\frac{\beta}{\pi} \Delta z^{*2} = \frac{\Delta y^{*2}}{\nu} \frac{dU}{dx} \tag{9}$$

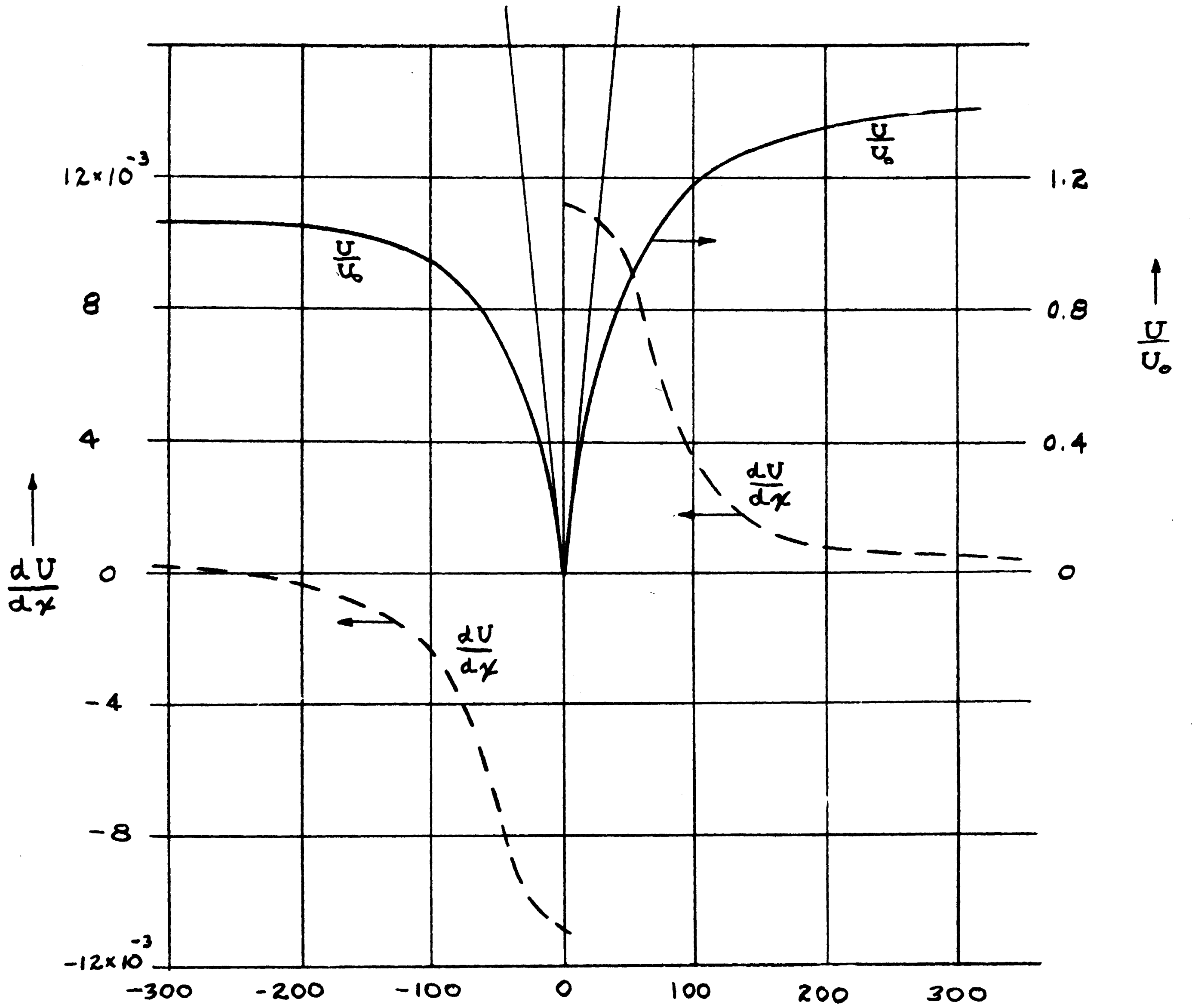
$$\frac{d\Delta y^*}{dx} = \left(1 - \frac{\beta}{\pi}\right) \Delta z^{*2} \left(\frac{\nu}{U\Delta y^*}\right) \tag{10}$$

in which U and dU/dx are known, after differentiation of the velocity, at the outer edge of the boundary layer of the airfoil. For the airfoil considered, this differentiation is easily obtained by a suitable transformation (Figure 4).

We shall commence by representing the numerical correspondence for Pr = 0.7, among the values of β/π, Δz^{*2} and $\beta/\pi\ \Delta z^{*2}$ on the graph of Figure 5.* To find the value of Δy^* as a function of x we shall construct, in a system of x, Δy^* axes, the field of tangents $d\Delta y^*/dx$. For this, we shall arbitrarily fix a pair of values x, Δy^*; equation (9) then gives the values of β/π and Δz^{*2}, and consequently the values of β and Δz^{*2}; next, equation (10) will provide the value of $d\Delta y^*/dx$.

In this way, for example, at a point on the top surface for the abscissa x = 2.4 cm, Figure 4 gives U = 6440 cm/sec, dU/dx = 653 cgs. In

*In the original French report, Figure 5 shows Δz^* and $\beta/\pi\ \Delta z^*$, which is obviously in error.



Distance along the surface
from the stagnation point, mm.

Figure 4. Distribution of velocity around the airfoil leading edge.

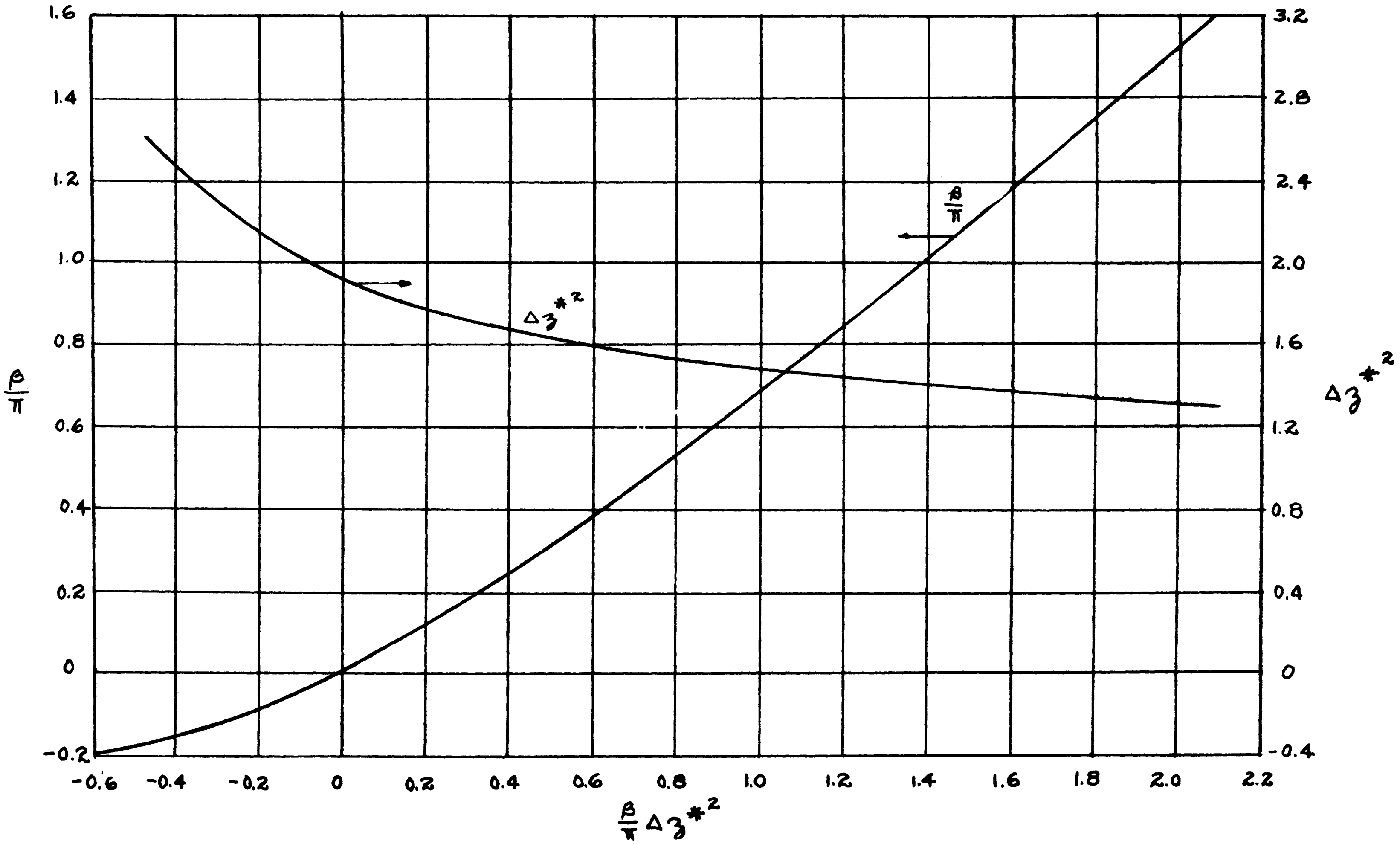


Figure 5. Curves serving in the calculation of the local coefficient of convection.

arbitrarily fixing $\Delta y^* = 0.0153$ cm, equation (9) and the curves of Figure 5 give $\beta/\pi\ \Delta z^{*2} = 1.148$, $\beta/\pi = 0.875$, and $\Delta z^{*2} = 1.46$.

All the values are now known to calculate the gradient $d\Delta y^*/dx$, which is equal to 4.06×10^{-4}. At the same point defined by $x = 2.4$ cm, Δy^* is given successively the values 1.208, 1.26, 1.38, etc; the corresponding values of $d\Delta y^*/dx$ are calculated.

Next, another point on the profile defined by a new abscissa is chosen, and so on. When a sufficient number of tangents have been determined on the plot of x, Δy^* it is easy to trace a network of curves. Only one of these curves is suitable; it is determined by the known quantities at the stagnation point. One may say, in effect, that at the point A, defined by $x = 0$, the tangent is parallel at the x axis. In addition, according to Equation (7), $\Delta y_A{}^* = 1.19\sqrt{\nu x/U}$. The relation x/U is given at the stagnation point in the form 0/0; the value dx/dU replaces it. The value of $(dx/dU)_A = 1/754$ cgs, whereby $\Delta y_A{}^* = 0.0158$ cm. Finally, the graph showing Δy^* as a function of x is represented as shown in Figure 6.

Equation (8) then allows the calculation of f_{cx} as a function of x in which, for each value of x, there corresponds a value of β, $A\Delta z^*$, and Δy^*. In making $k = 5.65\ (10^{-5})$ cal/(cm)(sec)(°C), (air at 0°C), the curve of Figure 7 is thus traced, which gives the local coefficient of convection along the surface from the stagnation point.

4. Evaporation of Water along the Profile

It is convenient to consider a local coefficient of evaporation ψ defined by

$$\psi = \frac{mP_B}{(p_s - p_\infty)\ d} , \tag{11}$$

where m is the mass of water evaporated per unit of surface per unit of time, p_s is the pressure of water vapor* in the immediate vicinity of the surface (the pressure of the saturated water vapor at the temperature beyond the airfoil); p_∞, the pressure of the water vapor at infinity; P_B, the ambient pressure;* and d, the water-vapor density with respect to the air (molecular weight of water with respect to air, = 18/29 = 0.62).

A study of the relation between convection and diffusion phenomena, in a laminar region, gives the relation

$$\psi = \frac{f_c}{C_p} L^{2/3} , \tag{12}$$

*In the original French report, p_p and P were used, respectively.

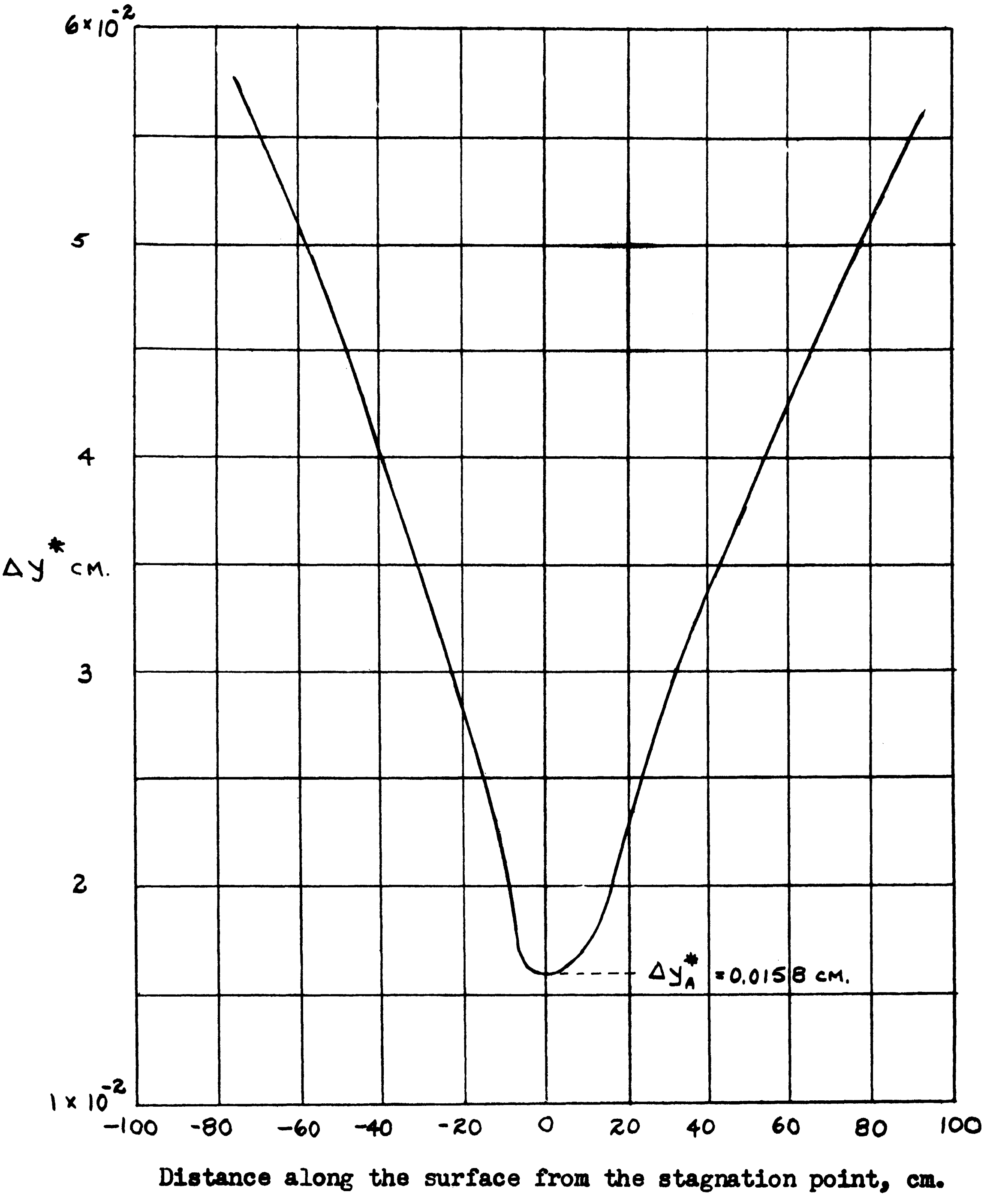


Figure 6. The thermal boundary-layer discharge thickness Δy^* around the airfoil leading edge.

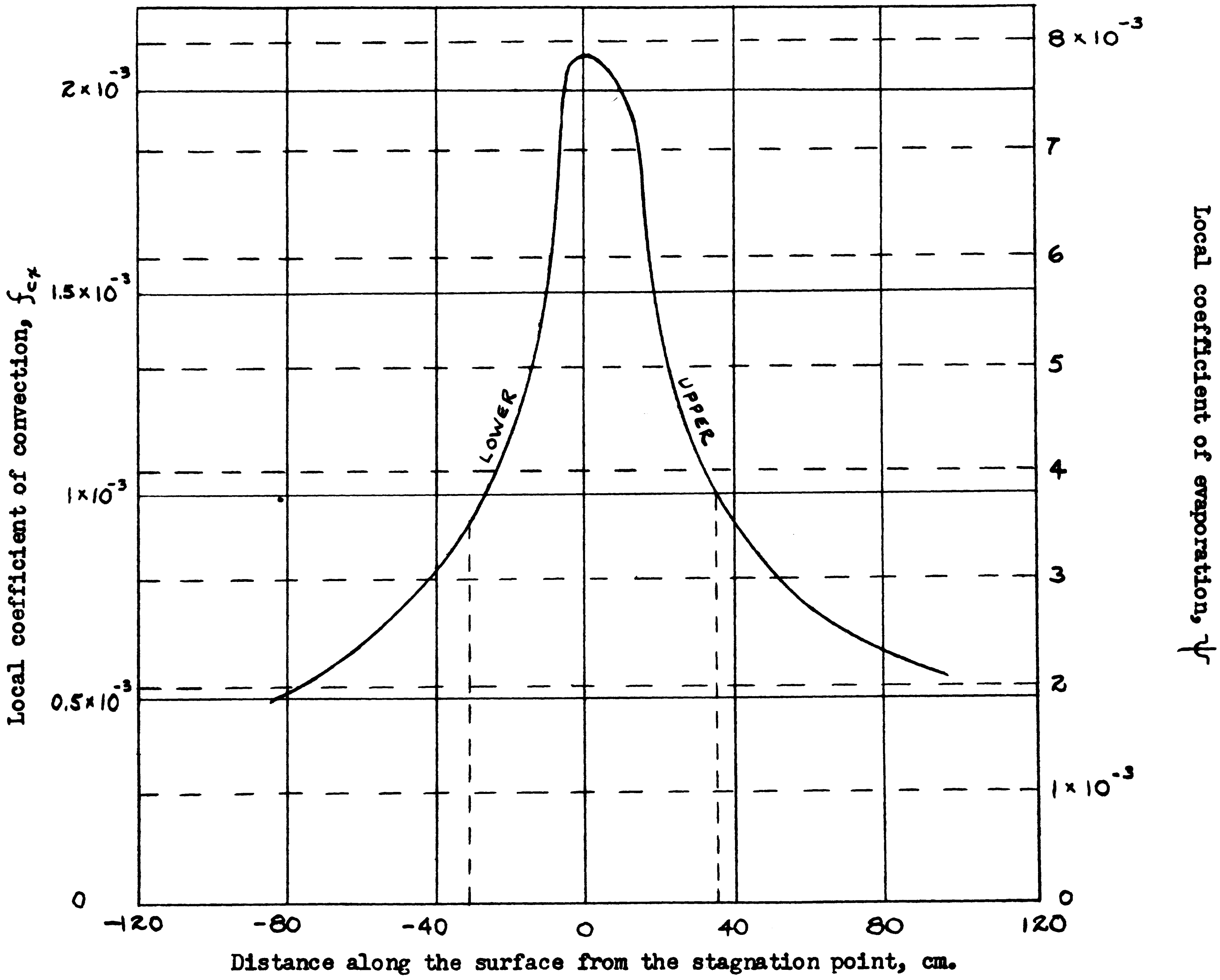


Figure 7. The local coefficients of convection and evaporation around the airfoil leading edge.

where C_p is the specific heat of air which is equal to 0.24 cal/(gm)(°C), and L the Lewis number of the water vapor diffusing into the air (the relation of the coefficient of diffusion of the water vapor in the air at the kinematic viscosity of the air) which is equal to 0.866. Equation (12) then permits one to express numerically an experimental f_c in cal/(cm)^2(sec)(°C) and ψ in gm/(cm)^2(sec) by

$$\psi = 3.78 \, fc \ . \qquad (12a)$$

In this way the graph of Figure 7 represents, with different ordinates, the local coefficient of evaporation just as well as the local coefficient of convection.

5. Anti-Icing with a Running Wet Surface

We shall first be concerned with the area of impact. We shall suppose that, to prevent icing of this area, it will be necessary to maintain a temperature 6° higher than the temperature that the surface would have without heating. Furthermore, in taking account of the effects of frictional heating, anti-icing would still be assured for temperatures in the neighborhood of -10°C.

The heat that is required is evaluated by taking account of the heat removed by convection, the heat removed by evaporation, and the heat which serves to elevate the temperature of all the water caught (sensible heating).

Heat Removed by Convection. The area of impact is contained between the points -30.9 cm and +35.4 cm along the surface. The estimate, on Figure 7, of the area contained on the graph between points -30.9 and +35.4 on the abscissa allows an estimate of an average coefficient of convection in the impact area; it is about 1.55×10^{-3} cal/(cm)^2(sec)(°C). The supply of heat for a temperature differential of 6°C is therefore 9.3×10^{-3} cal/(cm)^2(sec), or 335 kcal/(m)^2(hr).

Heat Removed by Evaporation. The mean coefficient of evaporation in the area of impact is, from Equation (12a), equal to 5.84×10^{-3} gm/(cm)^2(sec). In presupposing that the whole area of impact remains constantly wetted, we shall determine the amount of water evaporated per unit of surface per unit of time by means of Equation (11). We shall suppose that the saturated atmosphere is at -5°C (p_∞ = 0.317 cm of mercury) and the atmospheric pressure is normal (P_B = 76 cm of mercury); the surface is then at a temperature of about +5°C (p_s = 0.651 cm of mercury).

$$m = \frac{(5.84)(10^{-3})(0.62)(0.651 - 0.317)}{76} = 1.59 \times 10^{-5} \text{ gm/(cm)}^2\text{(sec)} \quad .$$

Accomplishing the evaporation at 5°C carries away an amount of heat equal to 603 cal/gm. Consequently, the loss of heat by evaporation is 9.6 x

10^{-3} cal/(cm)^2(sec), corresponding to 346 kcal/(m)^2(hr). This amount of heat is practically equal to that removed by convection.

Heat Absorbed by Elevation of the Water Temperature. The maximum thickness of the airfoil is 42.75 cm. With the efficiency of catch 0.32 and the velocity about 91 m/sec, the amount of water which impinges per meter of span per second in the impingement area is contained within a volume of air equal to

$$V = (0.32)(0.4275)(91) = 12.34 \text{ (m)}^3 \ .$$

This amount of water distributes itself over a surface area of 0.663 m^2 so that the average amount of water caught per second per square meter over the impact area is equal to 18.6 m^3.

If the cloud contains a liquid-water content of 1 gm/m^3, the amount of heat required to raise the water 10° is 0.0186 cal/(cm)^2(sec), corresponding to 670 kcal/(m)^2(hr). In the case of a cloud whose liquid-water content is much less than 0.1 gm/m^3, the amount of heat necessary to raise the water temperature is much less than that necessary for convection or evaporation, 67 kcal/(m)^2(hr). In the case of a high liquid-water content, however, it exceeds the heat losses by convection and evaporation.

Summarizing, in the case just examined, the heat required over the impingement area to prevent the formation of ice is 1350 kilocalories per hour per square meter (335 + 346 + 670). Of course, it is not necessary that the distribution of heat be uniform over the leading edge, and it is easy, by means of the curves presented, to calculate a better distribution. A careful calculation is unnecessary because the distribution law depends essentially on the volume of water in the cloud.

For a cloud of low liquid-water content, for which the heat supplied for heating of the water (sensible heating) is relatively small, the heat supplied is twice as much at the stagnation point as at the tangent impingement point. For a cloud of high liquid-water content, wherein a heavy water-catch is present at the stagnation point (the fifth part of the impingement zone receives half of the water), it is necessary, for clearing the leading edge of ice, to provide an amount of heat much larger at the stagnation point (for example, four times as large as at the tangent impingement point).

The water may run back along the length of the airfoil beyond the zone of impingement; it is therefore necessary to heat a much larger area of the airfoil. However, the heating of this part can be much less for two reasons: first, the water temperature having been raised above 0°C at the impingement zone, no more heat is required to raise the temperature of the water; secondly, the local coefficients of convection and evaporation are quite small, at least as long as the flow is laminar.

Suppose, to take up again the preceding example, that it is desired to maintain the airfoil at +5°C as far aft as 40% chord: this amounts to heating an area about three times larger than that of the impingement area. The average water-catch efficiencies are, on the portion of the airfoil that is being considered, 0.45 times smaller than in the impingement zone. The heat to be supplied over this area is (0.45)(335 + 346) = 300 kcal/(m)^2(hr). On the whole, for this large area, the total expenditure of energy is about that which was necessary over the impingement area.

6. Anti-Icing for Complete Evaporation

The case is first treated of a cloud of low liquid-water content, corresponding to 0.1 gm/(m)^3. An average of 1.86 x 10^{-4} gm/(cm)^2(sec) of water will be deposited. For evaporation to be complete over this area, it is necessary, according to equation (11), that the water-vapor pressure at the surface be

$$p_s = \frac{mP_B}{d\psi} + p_\infty = \frac{(1.86)(10^{-4})(76)}{(0.62)(5.84)(10^{-3})} + 0.317 = 4.22 \text{ cm of mercury} .$$

Consequently, it is necessary that the temperature of the surface at the impingement area be about 35°C.

The heat loss by convection is then six times larger than that calculated for a running wet surface, being about 2000 kcal/(m)^2(hr). To raise 1 gram of water from -5°C to 35°C and to evaporate it at this temperature, 622 calories are necessary. When 6700 gm/(m)^2(hr) of water are deposited, the heat required to evaporate the water is 4170 kcal/(m)^2(hr). Finally, the total heat required for the impingement area is 6170 kcal/(m)^2(hr).

Of course it is enough to heat this impingement area, but, as is the case for a cloud of much lower liquid-water content, the total required heat is much greater than for a running wet surface.

The calculations will be taken up again for a cloud of higher liquid-water content, corresponding to 1 gm/m^3. The water-vapor pressure at the surface is 39.3 cm of mercury, corresponding to a surface temperature of 82°C. With this value, the heat loss by convection is 4600 kcal/(m)^2(hr). The heat required to raise the water from -5°C to 82°C and to evaporate the water at this temperature is 42,600 kcal/(m)^2(hr).

In all, it is necessary to supply, over the impingement area, 47,200 kcal/(m)^2(hr), which is rather high. Complete evaporation can be effective only in the case of a very light icing condition for the airfoil considered here.

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