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ELECTROMAGNETIC SCATTERING BY HIGH DENSITY METEOR TRAILS

by

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ABSTRACT

The discussion first concerns itself with establishing limits on the validity of the low density approximation. These depend not only on the electron line density (as has sometimes been loosely stated) but also on the wavelength of observation and on the altitude of the trail. Next, a model is developed for scattering by a super-critical density distribution of electrons, based on the idea that the process can still be viewed as a superposition of individual Compton effects, but with the wave incident on the electron attenuated because of refraction (in analogy to the skin effect). Results of this model are compared with those obtained by the usual approach of replacing the electron distribution by a metallic scatterer whose surface is the critical density contour. Some calculations with non-Gaussian electron distributions help to clarify the physical interpretation.

1. Phase Error in Individual Scatterer Model.

Scattering by a meteor trail of low electron density is usually treated by considering each electron to be subject to the incident field and to scatter independently (according to the Thomson cross section), the contributions from the electrons being added up coherently.[1]

For a moderately higher density (still below the critical density everywhere), with an electron collision frequency well below the frequency of the radiation, the model is disturbed by the deviation from unity of the index of refraction of the inner region (which alters the phase relations between electrons). As an indication of when this effect can become significant, the phase error made in neglecting the variation of the index of refraction is computed for a ray coming in from infinity to the axis of a cylindrical Gaussian distribution. When this phase error is small, the individual scatterer model is good; as it becomes large, the model breaks down.

The index of refraction, n, is given by

$$n^2 = 1 - (4\pi N r_o / k^2), \tag{1}$$

where

N = electron density

$r_o = e^2/mc^2$ = classical electron radius

$k = 2\pi$/wavelength.

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1. H. Brysk, "Electromagnetic Scattering by Low Density Meteor Trails", J. Geophys. Res., Vol. 63, 693-716(1958).

The electron distribution in a meteor trail is approximated locally by a cylindrical Gaussian

$$N(r) = (\alpha/\pi a^2)\ \exp(-r^2/a^2). \tag{2}$$

The phase error, ϕ is then

$$\phi = k \csc\theta \int_0^\infty (1-n)\, dr \tag{3}$$

where θ is the angle between the trail axis and the ray (under the stipulated conditions, the bending of the ray can be neglected). From equations (1) and (2)

$$n^2(r) = 1-(4\alpha r_o/k^2 a^2)\ \exp(-r^2/a^2) \tag{4}$$

and the minimum value of n is that at $r = 0$:

$$n^2(0) = 1 -(4\alpha r_o/k^2a^2). \tag{5}$$

The expression in equation (3) can be reasonably well bounded by noting that

$$1 - n = (1-n^2)/(1+n). \tag{6}$$

Since

$$1 \geq n \geq n(0), \tag{7}$$

$$2 \geq 1+n \geq 1+n(0) \tag{8}$$

and

$$\left[1 + n(0)\right]^{-1} k \csc\theta \int_0^\infty (1-n^2)dr \geq \phi \geq 2^{-1} k \csc\theta \int_0^\infty (1-n^2)dr. \tag{9}$$

The integral is simple:

$$\int_0^\infty (1-n^2)dr = (4\alpha r_o/k^2a^2)\int_0^\infty dr\ \exp(-r^2/a^2) = 2\,\pi^{1/2}\alpha r_o/k^2a \tag{10}$$

so that

$$2\,\pi^{1/2}\alpha r_o \csc\theta/ka \left\{1 + \left[1-(4\alpha r_o/k^2a^2)\right]^{1/2}\right\} \geq \phi \geq \pi^{1/2}\alpha r_o \csc\theta/ka. \tag{11}$$

It is convenient to refer to the characteristic dimensionless constants

$$B = 2(\alpha r_o)^{1/2} \tag{12}$$

$$K = ka \;. \tag{13}$$

In terms of these, the last equation reads

$$2^{-1}\pi^{1/2}(B^2/K)\csc\theta \left\{1 + \left[1-(B/K)^2\right]^{1/2}\right\}^{-1} \geq \phi \geq 4^{-1}\pi^{1/2}(B^2/K)\csc\theta \;. \tag{14}$$

For $B \ll K$, the phase error, ϕ, is exactly determined. At worst, for $B = K$ (critical density reached on axis), there is a factor of 2 uncertainty.

The definition of what constitutes a small phase error is somewhat arbitrary. The simplest reasonable choice from equation (14) is

$$(B^2/K) \leq 1 \tag{15}$$

(For $\theta = 0$, this leads to $\phi \leq 25^\circ$ from the right-hand estimate). At the same time, if the trail is to be of subcritical density everywhere

$$B \leq K \tag{16}$$

Hence, the rough criterion of applicability of the underdense model is that

$$B \leq K \;, \qquad B < 1 \tag{17}$$

$$B^2 \leq K \;, \qquad B > 1 \tag{18}$$

Figure 1 exhibits the limiting value of the electron line density as a function of the wavelength, using for "a" the initial width computed by Öpik[2] with densities at four altitudes obtained from Watanabe[3], for a meteor velocity of 40 km/sec. This set of curves is to be contrasted with the frequently made unqualified statement[4] that the transition from the underdense to the overdense case occurs at $\alpha = 10^{12}$ electrons/cm.

2. Attenuation Factor Due to Phase Variation on Critical Density Contour

Consider a plane perpendicular to the trail axis. If there is a critical density region, the intersection of this plane with the critical density contour will be a circle. The phase variation of rays reaching this circle will next be investigated. For simplicity, the rays will be assumed parallel. (This is fully justified, comparing the radius of the critical density region, $\bar{R}$, with the source size and range, although it would be incorrect over the length of the trail.)

The phase of a ray coming in from infinity and reaching the critical density contour is in general

$$\phi = k \csc \theta \int_{\bar{R}\cos\beta}^{\infty} n(\beta)\, dz, \tag{19}$$

where β is the azimuthal angle.

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2 . E. J. Öpik, Physics of Meteor Flight in the Atmosphere", Interscience Publishers, Inc., New York (1958).

3. K. Watanabe, "Ultraviolet Absorption Processes in the Upper Atmosphere", Advances in Geophys., Vol. 5, 153-221(1958).

4. L. A. Manning and V. R. Eshleman, "Meteors in the Ionosphere", Proc. IRE, Vol. 47, 186-199(1959).

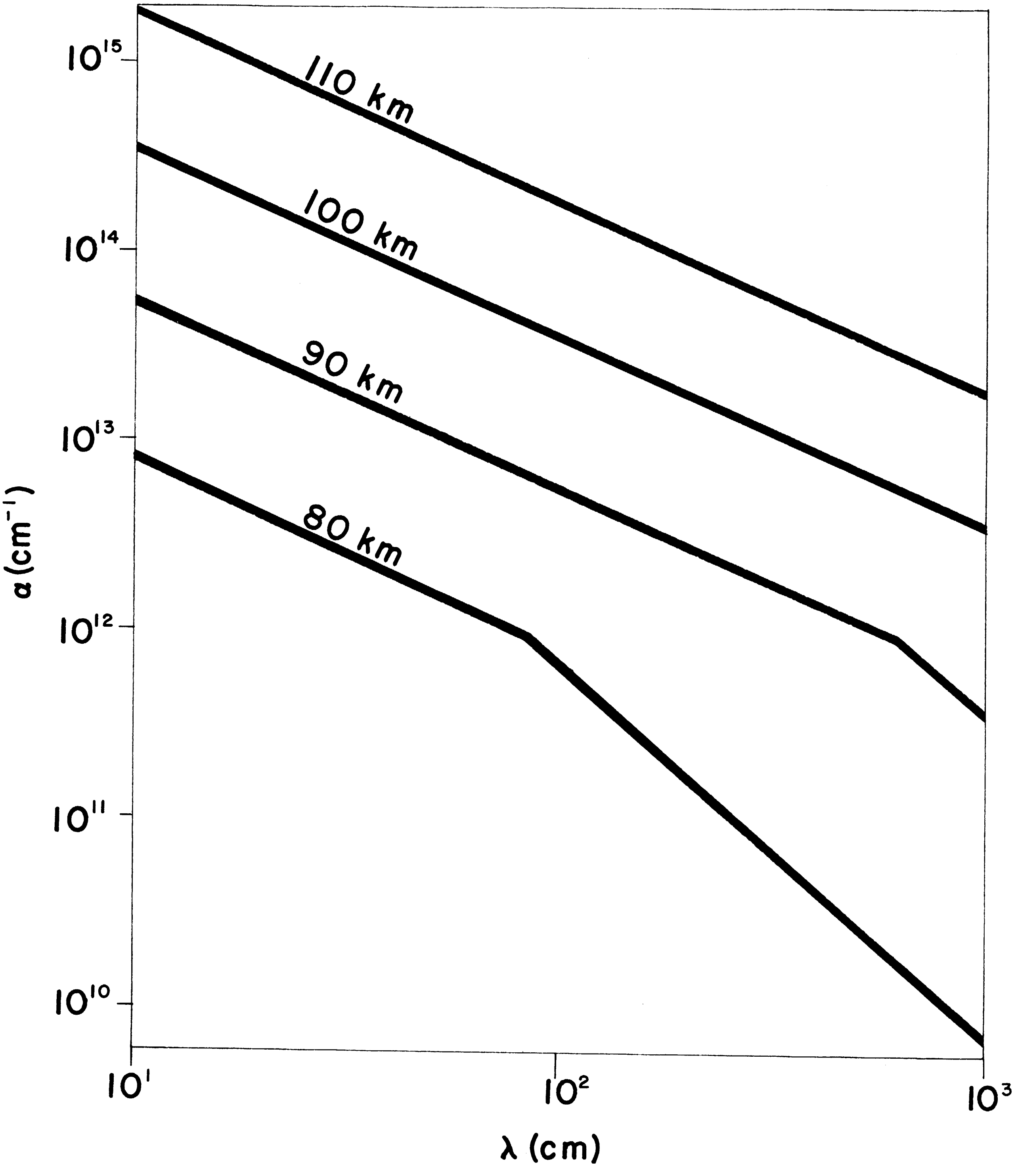

FIG 1: MAXIMUM ELECTRON LINE DENSITY FOR WHICH INDEPENDENT SCATTERER MODEL ASSUMPTIONS ARE VALID

For a ray whose extension would hit the axis of the trail,

$$\phi_o = k \csc \theta \int_{\bar{R}}^{\infty} n_o \, dz \quad . \tag{20}$$

The phase difference between the rays is

$$\phi - \phi_o = k \csc \theta \left\{ \int_{\bar{R}\cos\beta}^{\bar{R}} n \, dz - \int_{\bar{R}}^{\infty} \left[n_o - n\right] dz \right\} \tag{21}$$

The index of refraction is given by

$$n^2 = 1-(B/K)^2 \exp\left[-r^2/a^2\right]$$

$$= 1-(B/K)^2 \exp\left[-(z^2+\bar{R}^2 \sin^2\beta)/a^2\right]. \tag{22}$$

Since the index of refraction is zero at the critical density contour, $(B/K)^2$ and $\bar{R}$ are related, so that equation (22) can be rewritten more concisely as

$$n^2 = 1 - \exp\left[(\bar{R}^2 \cos^2\beta - z^2)/a^2\right] \quad . \tag{23}$$

To evaluate the integrals approximately, an inequality like equation (7) is again resorted to, after expressing

$$n = 1-(1-n^2)/(1+n) \tag{24}$$

$$n_o - n = (n_o^2 - n^2)/(n_o+n), \tag{25}$$

by replacing the denominators by estimated mean values. Thus

$$\phi-\phi_o = k \csc \theta \Big\{ \bar{R}(1-\cos\beta) - \overline{(1+n)^{-1}} \exp(\bar{R}^2\cos^2\beta/a^2) 2^{-1} \pi^{1/2} a\left[\Phi(\bar{R}/a) - \Phi(\bar{R}\cos\beta/a)\right]$$

$$+ \overline{(n_o+n)^{-1}} \left[\exp(\bar{R}^2/a^2) - \exp(\bar{R}^2\cos^2\beta/a^2)\right] 2^{-1} \pi^{1/2} a \left[1 - \Phi(\bar{R}/a)\right] \Big\} \tag{26}$$

In the asymptotic limit for $\bar{R} \cos\beta >> a$, the last two terms in equation (26) are small to order $(a/\bar{R})^2$ compared with the first, so that equation (26) can be reduced approximately to

$$\phi - \phi_o = k\bar{R} \csc\theta\,(1-\cos\beta) \quad . \tag{27}$$

As $(a/\bar{R})$ increases, a less tidy situation ensues. For a reasonable indication of the effect of phase variation (accepting a possible error in phase of as much as a factor of 2), equation (27) will be used.

Within the critical density contour the wave suffers no further phase change. Hence, if the electron distribution has cylindrical symmetry, contributions for all values of β are equiprobable. Due to these phase differences, there is an attenuation factor, f_p, at a given point along the axis, given for a two-way traversal by

$$f_p = (2\pi)^{-1} \int_{-\pi}^{\pi} d\beta \exp\left[-2ik\bar{R} \csc\theta \cos\beta\right] = J_o(2k\bar{R}\csc\theta) \tag{28}$$

For large argument, the asymptotic expression yields

$$f_p = (\pi k\bar{R} \csc\theta)^{-1/2} \cos\left[2k\bar{R} \csc\theta - (\pi/4)\right] \tag{29}$$

Averaging out the oscillatory factor (i.e., averaging over time during the expansion of the trail, or simply recognizing that the critical density contour is less than perfectly sharp), equation (29) reduces to

$$f_p = (2\pi k\bar{R} \csc\theta)^{-1/2} \tag{30}$$

3. Attenuation in Super-Critical Region

Scattering by an overdense region is usually handled by replacing the region by a perfect conductor whose surface is defined by the critical density contour. Aside from problems of computation of the scattering by the conductor, there are intrinsic objections to this model. An ionized region can maintain a transverse current, while a conductor cannot. Within a metal, the real and imaginary parts of the index of refraction are equal in magnitude, whereas within the overdense ionized region the real part is zero (so that phase properties are entirely different). Furthermore, the critical density contour is not a perfect reflector. There is penetration into the overdense region with an attenuation factor, analogous to the skin effect for a metal. The electron density usually increases toward the center of the overdense region, so that there are the counteracting trends of the skin effect tending to reduce the scattering from deep inside per electron while the density distribution indicates an increase in the number of scattering centers.

The approach attempted here is a mongrel model. The scattering is considered to consist of three distinct regimes: the incoming wave, the Compton process itself, and the outgoing wave. The Compton process is treated as an individual particle effect, just as for the underdense case. The waves, on the other hand, are handled from a ray-tracing viewpoint. The amplitude of the wave being scattered by the electron is taken to be the incident amplitude reduced by the skin-effect attenuation, the latter being computed along the shortest optical path from the critical density contour to the electron. The amplitude of the scattered wave is reduced by the same factor in coming out of the overdense region. It should be noted that the skin effect represents a reduction of amplitude due to refraction away from a region rather than absorption in the region. The discussion assumes that the frequency of the electromagnetic radiation is sufficiently higher than the electron collision frequency that absorption can be neglected.

The results will be expressed as the ratio of the scattering amplitude obtained from the overdense region to that which would be obtained from the same number of electrons treated as individual scatterers without phase differences, i.e., in effect, the reduction in effectiveness of scattering due to the denseness.

In particular, for a cylindrically symmetrical distribution of electrons the attenuation factor, f', will be given by

$$f' = \left\{ \int_0^{\bar{R}} r\, dr\, N(r) \exp\left[-2k \int_0^{\bar{R}} \bar{n}(r')\, dr' \right] \right\} \Bigg/ \int_0^{\bar{R}} r\, dr\, N(r) \tag{31}$$

where $\bar{n}$ denotes the absolute value of n.

For the Gaussian distribution, with the further notation

$$x = r/a, \qquad X = \bar{R}/a, \tag{32}$$

the exponent in equation (31) becomes

$$2k \int_r^{\bar{R}} \bar{n}(r')\, dr' = 2K \int_x^X \left[(B/K)^2 \exp(-t^2) - 1 \right]^{1/2} dt$$
$$= 2B \int_x^X \left[\exp(-t^2) - \exp(-X^2) \right]^{1/2} dt \tag{33}$$

The denominator is simply

$$\int_0^{\bar{R}} r\,dr\,N = (\alpha/\pi)\int_x^X x\,dx\,\exp(-x^2) = (\alpha/2\pi)\left[1-\exp(-X^2)\right]. \tag{34}$$

Thus, the attenuation factor is

$$f' = 2\left[1-\exp(-X^2)\right]^{-1}\int_0^X x\,dx\,\exp(-x^2)\exp\left\{-2B\int_x^X\left[\exp(-t^2)-\exp(-X^2)\right]^{1/2}dt\right\} \tag{35}$$

The indicated integrations cannot be done analytically. Two special cases will be studied below.

a. Very High Density

If the electron line density parameter, B, is much larger than the critical-density-region size parameter, K, equation (35) simplifies a bit on going to the limit $X \rightarrow \infty$. (This limit can be used provided X is greater than about 3.) The integral in the exponent becomes

$$\int_x^X\left[\exp(-t^2)-\exp(-X^2)\right]^{1/2}dt \rightarrow \int_x^\infty \exp(-t^2/2)dt = (\pi/2)^{1/2}\mathrm{erfc}(x/2^{1/2}) \tag{36}$$

so that equation (35) reduces to

$$f' = 2\int_0^\infty x\,dx\,\exp(-x^2)\exp\left[-(2\pi)^{1/2}B\,\mathrm{erfc}(x/2^{1/2})\right] \tag{37}$$

In this last form, f' depends only on B (i.e., only on α). Thus for a sufficiently dense region the attenuation depends only on the line density of electrons, and not on the wavelength of the radiation or the width of the distribution. These quantities do enter (in the combination ka) in the determination of what constitutes a "sufficiently dense" region. Figure 2 exhibits f', from equation (37), as a function of α; on a log-log plot, the curve is very nearly a straight line.

The scattering amplitude of the meteor trail is proportional to $\alpha f'$ and hence to B^2f'. In Figure 3, B^2f' has been plotted against B. The curve is fitted by

$$B^2 f' = 0.91 \ln B \tag{38}$$

apart from the small-B end (where equation (37) is not a good approximation anyhow).

For an intuitive grasp of the characteristics of the scattering, it is instructive to compare equation (38) with the scattering amplitude obtained for two simpler electron distributions - a uniform cylinder and two coaxial uniform cylinders - with the same line density and in the high density limit.

For the uniform cylinder, N is a constant, hence $\bar{n}$ is also a constant, and equation (31) reduces to

$$\begin{aligned} f' &= \left\{ N \int_0^{\bar{R}} r\, dr \exp\left[-2k\bar{n} \int_r^{\bar{R}} dr'\right]\right\} \Big/ N \int_0^{\bar{R}} r\, dr \\ &= (2/\bar{R}^2) \left\{ \exp\left[-2k\bar{n}\bar{R}\right] \right\} \int_0^{\bar{R}} r\, dr \exp\left[2k\bar{n}\, r\right] \\ &= (k\bar{n}\bar{R})^{-1} \left\{ 1-(2k\bar{n}\bar{R})^{-1}\left[1-\exp(-2k\bar{n}\bar{R})\right]\right\} . \end{aligned} \tag{39}$$

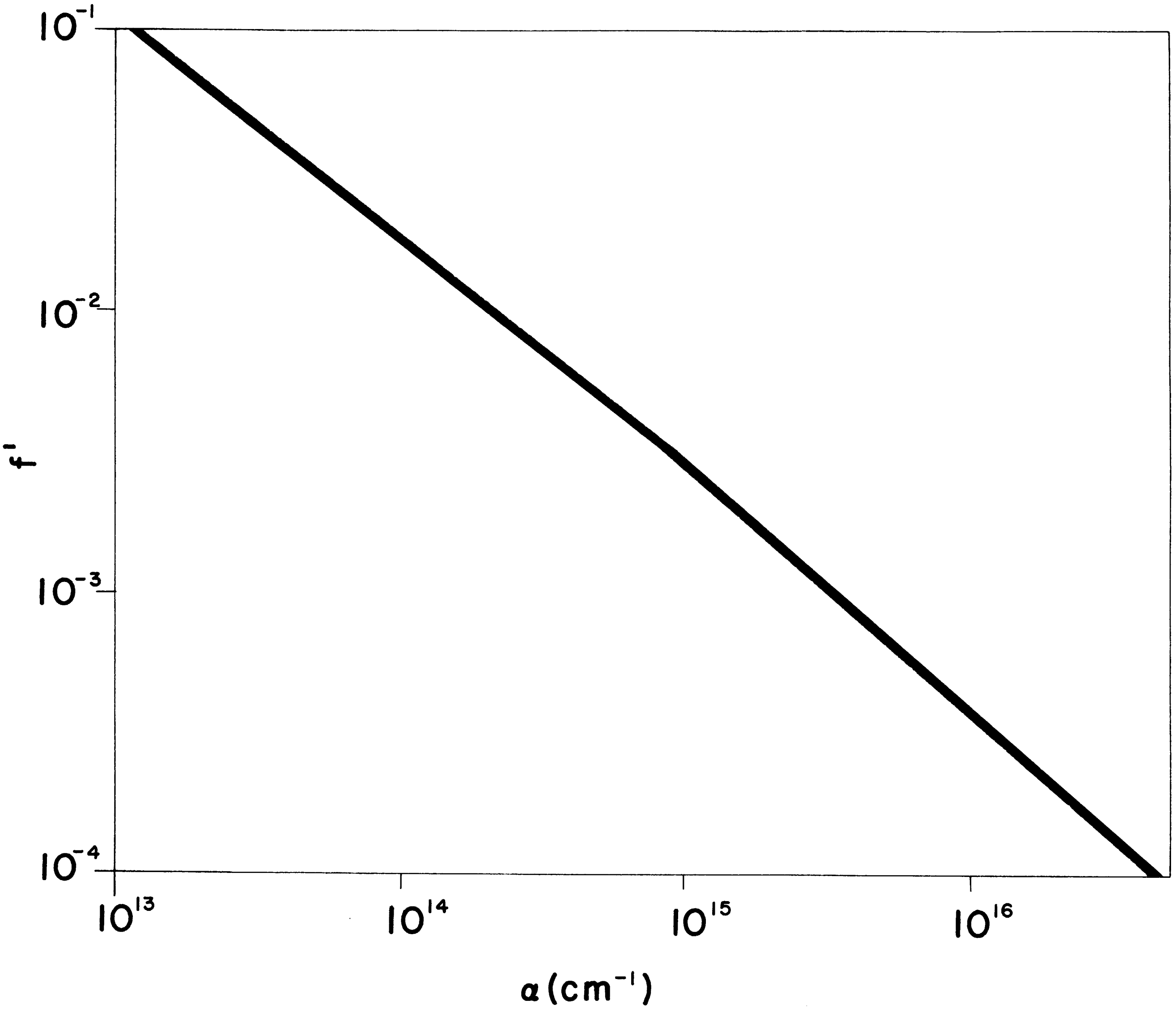

FIG 2: ATTENUATION FACTOR FOR VERY HIGH DENSITY

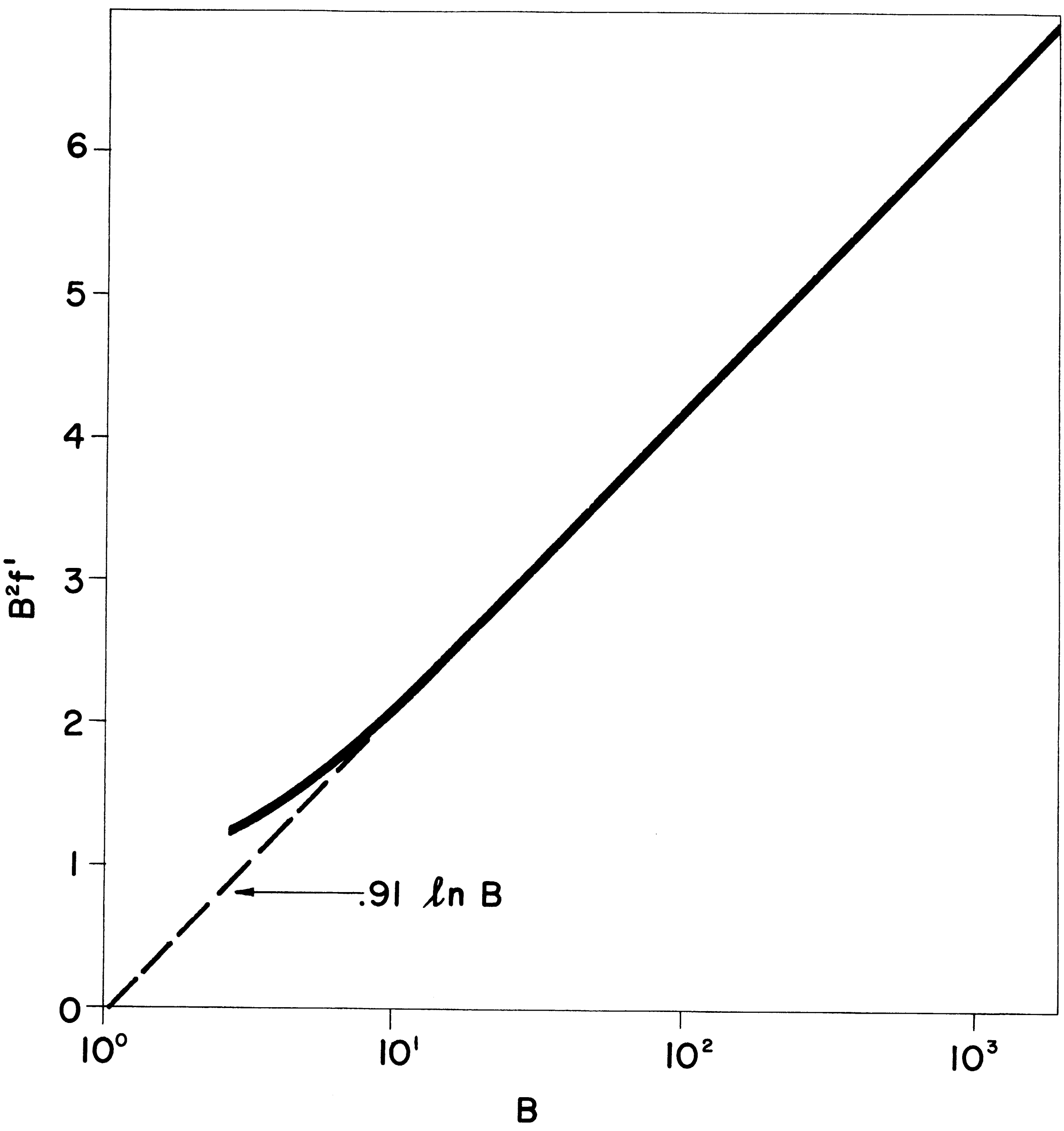

FIG 3: SCATTERING AMPLITUDE FOR VERY HIGH DENSITY

For a large electron density, the second term in equation (1) predominates, so that

$$k\bar{n} = (4\pi N r_o)^{1/2} \tag{40}$$

The electron line density, α, is

$$\alpha = \int_0^{2\pi} d\phi \int_0^{\bar{R}} r\, dr\, N = \pi \bar{R}^2 N \tag{41}$$

so that $k\,\bar{n}\,\bar{R}$ reduces to

$$k\,\bar{n}\,\bar{R} = 2(\alpha\, r_o)^{1/2} = B. \tag{42}$$

For large B, the attenuation factor is then simply

$$f' = B^{-1} \tag{43}$$

and the scattering amplitude

$$B^2 f' = B. \tag{44}$$

For two coaxial uniform cylinders, the procedure is analogous except that the r-integral is split into two portions (0 to $\bar{R}'$ and $\bar{R}'$ to $\bar{R}$) for each of which N and $\bar{n}$ are constants (denoted by subscripts 1 and 2 respectively). The denominator of equation (31) is always $(\alpha/2\pi)$, so

$$\alpha f'/2\pi = N_1 \int_0^{\bar{R}'} r dr \exp\left\{-2k\left[\bar{n}_1 \int_r^{\bar{R}'} dr' + \bar{n}_2 \int_{\bar{R}'}^{\bar{R}} dr'\right]\right\}$$

$$+ N_2 \int_{\bar{R}'}^{\bar{R}} r dr \exp\left[-2k\,\bar{n}_2 \int_r^{\bar{R}} dr'\right] \tag{45}$$

$$= (N_1\bar{R}'/2k\bar{n}_1)\left\{1-(2k\bar{n}_1\bar{R}')^{-1}\left[1-\exp(-2k\bar{n}_1\bar{R}')\right]\right\} \exp\left[-2k\bar{n}_2(\bar{R}-\bar{R}')\right]$$

$$+ (N_2\bar{R}/2k\bar{n}_2)\left[1-(2k\bar{n}_2\bar{R})^{-1}\right] - (N_2\bar{R}'/2k\bar{n}_2)\left[1-(2k\bar{n}_2\bar{R}')^{-1}\right]\exp\left[-2k\bar{n}_2(\bar{R}-\bar{R}')\right] .$$

In the limit of large density and extent for both regions (i.e. for equation (40) valid and the negative exponentials on the right-hand side of equation (45) all small), there results

$$B^2 f' = (4\pi r_o N_2)^{1/2} \bar{R} \tag{46}$$

Note that

$$\alpha = 2\pi \left[N_1 \int_0^{\bar{R}'} r dr + N_2 \int_{\bar{R}'}^{\bar{R}} r dr \right] = \pi \left[N_1 \bar{R}'^2 + N_2 (\bar{R}^2 - \bar{R}'^2) \right] = \pi N_2 \bar{R}^2 + \pi (N_1 - N_2) \bar{R}'^2 . \tag{47}$$

If the inner region has the higher electron density, there is thus a lower f' (for a given α) than in the uniform cylinder case.

For definiteness, the coaxial cylinder case will be specialized by making the proviso that each of the two regions contain the number of electrons that would be present in the corresponding part of the Gaussian. Then

$$\int_{\bar{R}'}^{\bar{R}} r dr \, N = N_2 (\bar{R}^2 - \bar{R}'^2)/2 = (\alpha / 2\pi) \left[\exp(-\bar{R}'^2/a^2) - \exp(-\bar{R}^2/a^2) \right] . \tag{48}$$

It was shown above that

$$\exp(-\bar{R}^2/a^2) = (K/B)^2 . \tag{49}$$

For convenience, write

$$\bar{R}'^2 = p\bar{R}^2 \tag{50}$$

where

$$0 < p < 1, \tag{50'}$$

so that

$$\exp(-\bar{R}'^2/a^2) = (K/B)^{2p} . \tag{51}$$

The value of N_2 can then be expressed simply by

$$(\pi/\alpha)N_2(1-p)\bar{R}^2 = (K/B)^{2p} - (K/B)^2 \tag{52}$$

so that equation (46) becomes

$$B^2 f' = B(1-p)^{-1/2} \left[(K/B)^{2p} - (K/B)^2\right]^{1/2} \quad . \tag{53}$$

As the discussion applies to high densities (B >> K), only the first term in the bracket need be retained since $p < 1$ (with some care that p not be too close to 1). Thus

$$B^2 f' = (1-p)^{-1/2} K^p B^{1-p} \tag{54}$$

Comparing now the high density limits of the three cases, we find that:

(1) for a given line density, the scattering amplitude is less for the two-region case (with inner region more dense) than for the uniform cylinder case, and still less for the Gaussian;

(2) the scattering amplitude for the two-region case is in fact, according to equation (46), just that which would occur if both regions had the density of the outer one;

(3) the scattering amplitude varies as the square root of the line density in the uniform region case, as a smaller positive power of the line density in the two-region case, and as its logarithm (still slower) in the Gaussian case.

The implication of these observations is that, for a region of radially decreasing high electron density, the scattering characteristics are predominantly determined by the outer portions of the super-critical density region (the core not being sufficiently penetrated by the radiation). The result from increasing the line density of electrons is primarily to push the effective scattering region outward, rather than to increase its density. This does result in an increase in the scattering because the surface area is increased (hence more electrons are accessible).

b. Critical Density Contour of Maximum Width

The electron distribution spreads out in time due to diffusion, and "a" increases. From equation (4) it can be deduced that the radius of the critical density region is given by

$$\bar{R}^2 = a^2 \ln(4\alpha r_o/k^2a^2) \tag{55}$$

The maximum value that it can attain (as a function of "a" - i.e., of time) is given by

$$\partial \bar{R}^2/\partial a^2 = \ln(4\alpha r_o/k^2a^2) - 1 = 0 \tag{56}$$

which means that

$$X = 1 \quad . \tag{57}$$

Heuristically, the widest critical density contour might be expected to yield the largest scattering return from the super-critical region, because it corresponds to the greatest number of electrons being exposed to an unattenuated incident field. This argument would be much weakened if it should turn out that f' (viewed as a function of X) has resonance-type oscillations.

Ideally, the maximum value of f' should be obtained by setting $\partial f'/\partial X = 0$, where f' is given by equation (35). Unfortunately, this is impractical because $\partial f'/\partial X$ includes a term in f' and also a term involving an integral like f' with an additional integral as a factor in the integrand, as well as a term independent of the f'-integral; the equation $\partial f'/\partial X = 0$ cannot be solved unless both f' and the somewhat more complicated companion integral are known as a function of X. Hence, there is no direct way of determining the maximum value of f' short of actually computing f' as a function of X. Since this must then be repeated for each B of interest, the computational effort required is quite large.

In what follows, it will be assumed that the maximum value of f' is indeed attained for X = 1. This is also the assumption in the metallic scatterer approximation, so there will be a direct comparison of results for the same configuration. The resultant special case of equation (35) is

$$f' = 2\left[1-e^{-1}\right]^{-1}\int_0^1 x\,dx\,\exp(-x^2)\exp\left\{-2B\int_0^1\left[\exp(-t^2)-e^{-1}\right]^{1/2}dt\right\} \quad (58)$$

This is plotted in Figure 4. The curve is well fitted by

$$f' \doteq 1.05\, B^{-.69} \quad . \quad (59)$$

So far, the "skin effect" attenuation and the phase relations around the critical density contour have been considered. There remains to examine the phase change along the trail axis due to the deviation of the index of refraction from unity. This consists of two contributions: The phase error in reaching the critical density contour (evaluated in Section 1), plus the error incurred in including the region inside the critical density region in the optical path

$$\phi' \doteq k\,\bar{R}\csc\theta \quad . \quad (60)$$

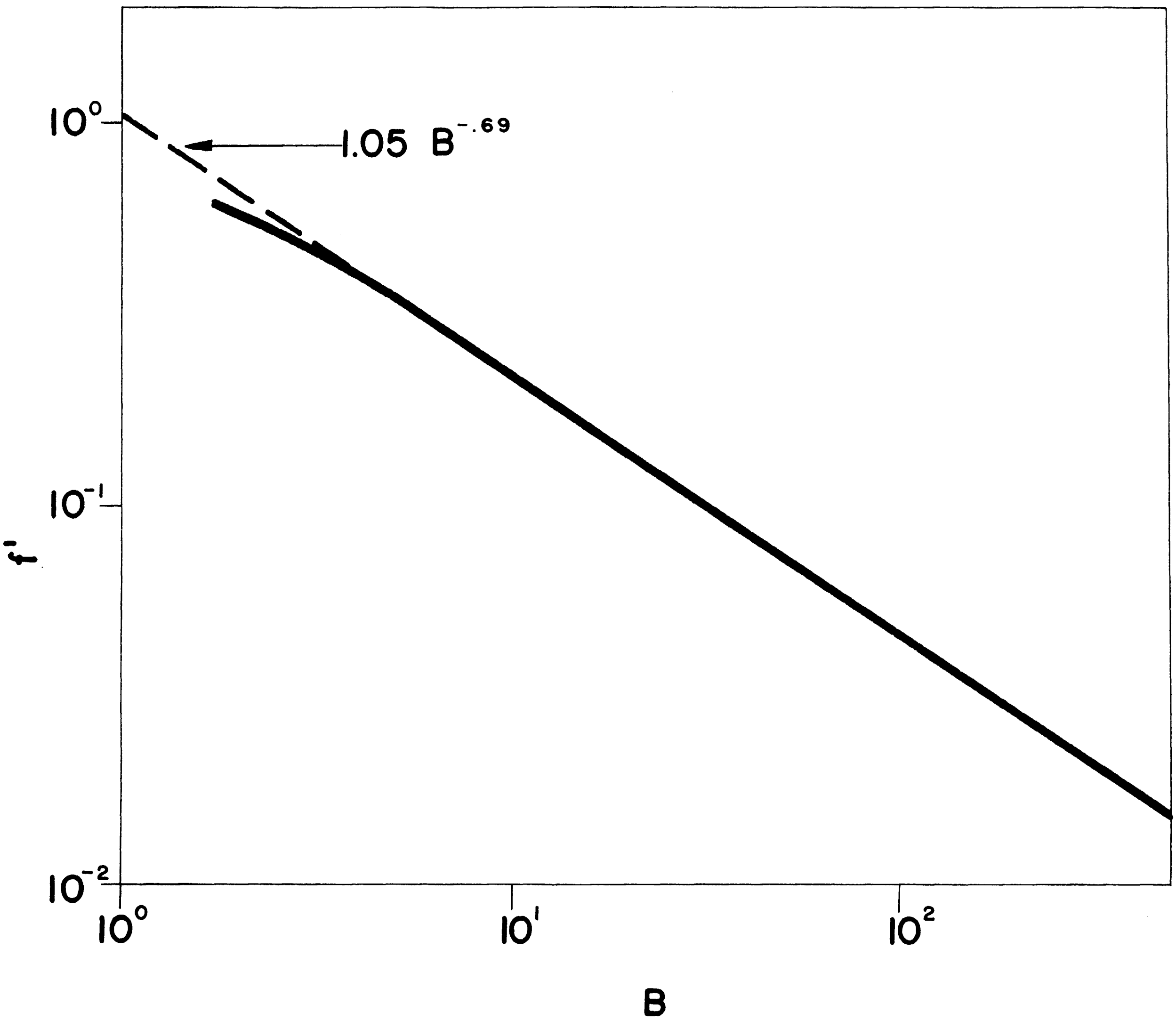

FIG 4: ATTENUATION FACTOR FOR CRITICAL DENSITY CONTOUR OF MAXIMUM EXTENT

During the growing phase (case a), the contribution of equation (14) dominates over that of equation (60) and leads to oscillations superposed on the scattering integrand. Near the maximum expansion of the critical density contour (case b), on the other hand, the two contributions are comparable and vary in opposite directions, so that the longitudinal phase variation is very slow.

Accordingly, for the maximum critical density contour the attenuation factor, instead of the exponential of Reference 1, is approximately the product of the phase reduction factor of equation (30) and the "skin effect" attenuation factor of (59). In (30), the relations for this maximum lead to

$$k\bar{R} \doteq ka \doteq K \doteq e^{-1/2} B \tag{61}$$

so that

$$f \doteq (2\pi e^{-1/2} B \csc \theta)^{-1/2} \, 1.05 \, B^{-.69} \doteq .52 \sin^{1/2} \theta \, B^{-1.19} \tag{62}$$

The maximum return is now given by

$$S \doteq .076 \csc \theta \, P \, GG' \lambda^3 (\alpha r_o)^{.81} (\hat{e} \cdot \hat{e}')^2 \Big/ 16 \pi^2 RR'(R+R') \tag{63}$$

or monostatically

$$S \doteq .038 \, P \, GG' \lambda^3 (\alpha r_o)^{.81} (\hat{e} \cdot \hat{e}')^2 \Big/ 16 \pi^2 R^3 . \tag{64}$$

This result is to be compared with that obtained by assuming the critical density region to scatter the electromagnetic radiation like a metallic cylinder, the return from the latter being computed by geometrical optics, i.e., in the limit $\bar{R} \gg \lambda$ [5]. The correct geometrical "cross section" for broadside backscattering from a metallic cylinder is

$$\sigma = \pi \bar{R} R \ . \tag{65}$$

This is a factor of two less than quoted by Greenhow [5], and it leads to

$$S = .048 \ P \ G \ G' \lambda^3 (\alpha r_o)^{\frac{1}{2}} (\hat{e} \cdot \hat{e}')^2 \Big/ 16 \ \pi^2 R^3 . \tag{66}$$

Figure 5 compares the maximum return from an overdense trail as obtained by the present "skin effect" model with that obtained by the "metallic cylinder" model. The Lovell-Clegg low-density result is also shown for orientation. For clarity of display exclusively, the overdense and underdense trail curves have been extended to meet; the temptation to bridge the transition by fairing in from one curve to the other should be resisted -- the intermediate region very probably does not behave that simply. The two overdense trail curves cross for relatively low α ($\sim 7 \times 10^{13}$ cm^{-1}). On the low-α side, they yield essentially undistinguishable predictions (especially taking into account the pile-up of theoretical errors in the transition region). On the high-α side, the "skin effect" model yields an increasingly larger result, as expected intuitively.

A feature deserving of comment is the specular nature of the return. In this respect, the present model leads to the same variation

5. J.S. Greenhow, "Characteristics of Radio Echoes from Meteor Trails: III The Behaviour of the Electron Trails After Formation", Proc. Phys. Soc., Vol. 65, Pt. B, 169-181 (1952).

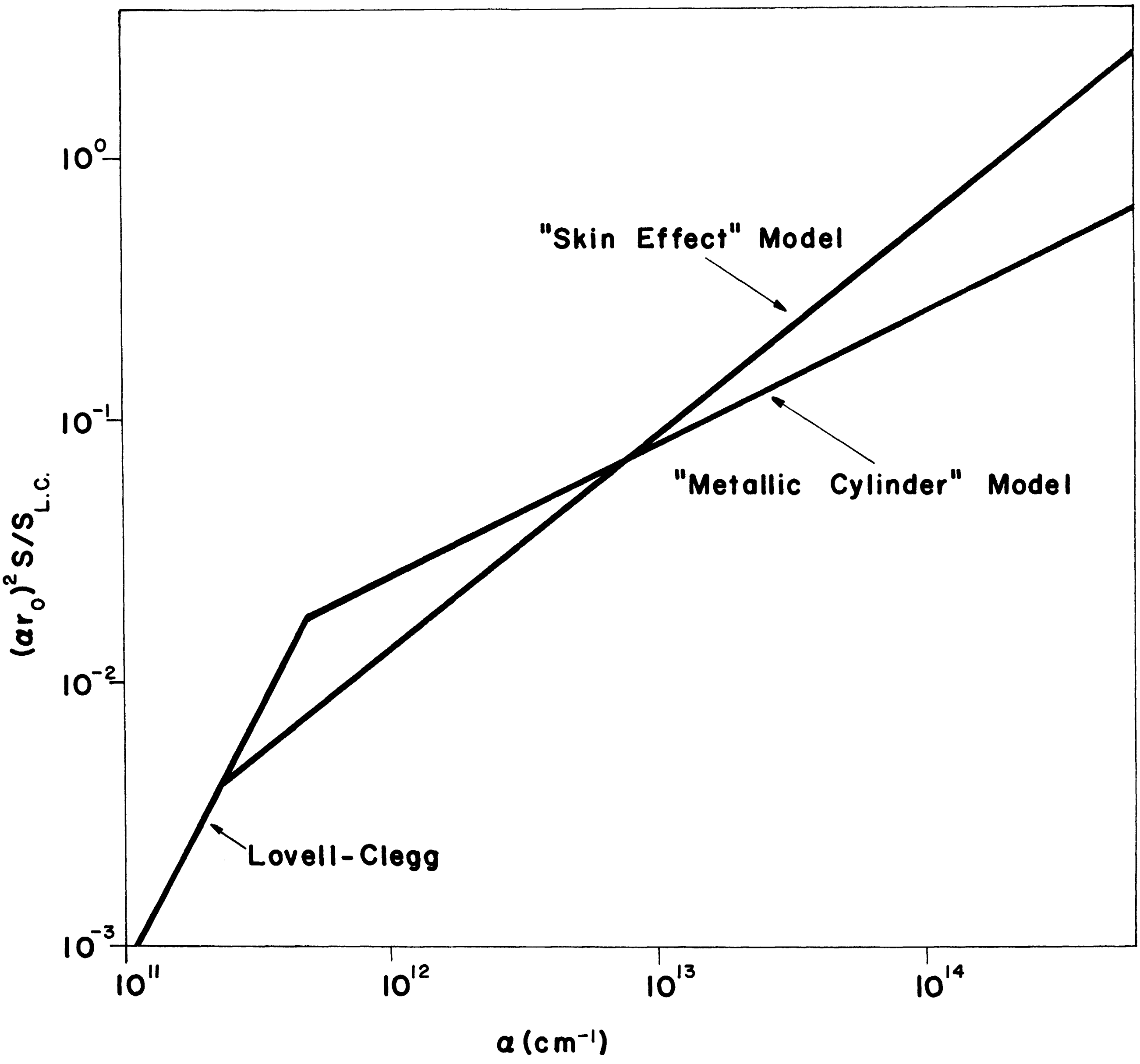

FIG. 5: COMPARISON OF "SKIN EFFECT" MODEL WITH "METALLIC CYLINDER" MODEL

as the underdense trail model. The metallic cylinder model of course shows a strong aspect dependence. On the other hand, a loss of specularity as the electron line density increases has been reported experimentally. Insofar as there is such a loss of specularity in the relatively early history of the trail (i.e. before enough time has elapsed for some form of turbulence to be invoked), it would appear that the basic assumption of a uniform line density of ionization underlying both models must be abandoned -- that, although a uniform line density can be satisfactorily assumed for underdense trails, it is an essential feature of scattering by overdense trails that the line density is markedly non-uniform.