Restructuring an Industry
During Transition
A Two-Period Model

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This paper develops a two-period model of enterprise shutdown during the post-Soviet transition, when enterprise performance is at best a highly noisy signal of its viability in the evolving market economy. The paper focuses at the industry level, taking a normative industrial policy perspective, and deriving an optimal shutdown-rule for choosing which firms should exit a shrinking industry when shutdown entails rapidly rising social and economic costs in its scale. The optimal policy is shown to be rather conservative, balancing at the margin the costs of shutdown against the expected costs of allowing a potentially value-destroying operation to continue. It is, however, also “front-loaded” in that it is optimal to be more aggressive in shutting down potentially bad firms in the first period, despite the fact that there is an ‘option value’ to waiting (learning from future performance). This is related, in part, to an ‘entry effect’ from the release of production factors and resources controlled by the closed enterprises. Results on the value of information and the timing of the shutdown decision in this model are also derived.
Restructuring an Industry during Transition: A Two-Period Model

Richard E. Ericson*

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1. Introduction.

This paper develops the two-period model of enterprise shutdown during transition that was introduced in Ericson (1994). Its purpose is to provide a formal basis for the study of what Kornai (1994) has called the "transformation of the real structure of the economy" during the transition process. It focuses at the industry level of restructuring, taking a normative, industrial policy, perspective.

Most of the discussion in the transition literature of the restructuring of industry deals with creating the institutional environment, and hence the appropriate incentives, for state firms to restructure themselves and for new firms to succeed and grow. Discussions of "liberalization," "stabilization," implementing "privatization," and fostering "competition," and the analysis of the consequences of such policies, have dominated the literature.¹ There is also a growing empirical literature on the key role played by the development of 'new firms,' frequently unrelated to any prior existing organizations in the economy.² Yet the focus in

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¹See, for example, the survey of economic transitions in Gelb et. al. (1996), McMillan (1995), and the Blanchard, Froot and Sachs (1994) collection of papers on the transition in Eastern Europe through 1993.

²As Peter Murrell (1996, p. 39) notes, "There is universal agreement that new private firms have been of great significance." For example, how and why they are important is explored
almost all of this literature is on the shifting of labor, on employment and unemployment and the barriers to smooth reallocation of labor between the state and private sectors. It is implicitly assumed that all other factors of production, all other aspects of industrial restructuring, will easily shift to accommodate an appropriate reallocation of labor and modernization of activity within the enterprise. A partial exception is the simple model of 'disorganization' in Lecture 1 of Blanchard (1995), where asymmetric information creates a coordination problem causing input disruptions and an excessive loss of output in the state sector.

Furthermore, the market institutions, in particular financial, that mobilize and reallocate real resources for the ongoing changes characteristic of a market economy are conspicuously underdeveloped and/or absent. Thus resources currently employed in existing institutions are largely immobile, tied up by those organizations having immediate physical control. This immobility of real resources is only aggravated by the weakness or absence of well defined and protected property rights that facilitate the reallocation/transfer of real assets. While appropriate reforms and the passage of sufficient time will ameliorate these problems as the transition proceeds, that transition's success depends in no small measure on the restructuring of production and other economic in some detail in Johnson and Loveman (1995) for the Polish case and in Richter and Schaffer (1996) and Earle and Rose (1996) for the Russian case. As I have argued elsewhere (Ericson, 1991), economic activity and organizations in the Soviet-type system were built, and functioned, without any consideration of real economic (opportunity) costs or benefits, other than those of the narrowly pursued priorities of the central authorities at the time of their construction. Thus there is no guarantee of any structure's viability, never mind profitability, in a market environment, and much of that structure is, indeed, net value destroying (see Ericson, 1996).

5 See Davis, Haltiwanger and Schuh (1996) for a seminal study of the dramatic, continual change in the U.S. economy.

6 Of course, alternative mechanisms have arisen in the private (shadow economy) sector, providing Gerschenkrohn substitutes for legal guarantees and enforcement of property and contractual rights. These substitutes are, of course, highly imperfect, carrying very high attendant 'transactions costs' as discussed in Leitzel (1995), Chapter 2.
activity, which in turn depends on the mobility of the underlying means of production. Indeed, calls for looser monetary policy to revive enterprise cash flow and thereby stimulate investment are, ultimately, addressing this need.\footnote{This can be seen in the policy prescriptions of Sergei Glaziev, the new Chief of the Security Council's Economic Security Administration interviewed in Trud, 26 September 1996, or the group of "Academicians" and their American advisors during the recent Russian presidential campaign. See, for example, the latter's manifesto "Novaia ekonomicheskaiia politika dlia Rossii," Nezavisimalna gazeta, 1 July 1996, pp. 1,4.}

One way to cut through this problem is to shut down net value destroying operations,\footnote{By "net value destroying" I mean that the market value of output produced is insufficient to cover the full costs of production, including the opportunity cost of the factors of land, labor, and capital.} forcing the release of those resources over which they have operational control (if not well defined ownership). That both limits the waste of social resources, and makes possible a greater inflow of new, more productive firms. In an increasingly marketized environment this requires allowing failing firms to shut down (bankruptcy), rather than supporting them with subsidies, credits, tax holidays, or tolerating growing payments, tax and/or wage arrears. Of course, there are substantial social costs to such closing of industrial enterprises, costs which are increasing in the scale of such activity. Were there full information on the viability of firms, there would be economically clear and easy, if politically very difficult, trade-offs to be made. However, in the unstable environment of the early to mid-transition, market performance may be only a very poor signal as to the potential viability of the firm. This uncertainty introduces an 'option value' to waiting and learning more about the firm's viability, as the decision to close the firm is not costlessly reversible.

As discussed in Ericson (1994), there are two types of mistakes possible: (i) shutting down a potentially viable firm based on weak early performance; and (ii) maintaining a net value-destroying firm. The optimal one-time (static) decision on maintaining money losing enterprises trades off the costs of these errors in choosing a rather conservative shutdown policy. Here we continue that investigation in a two-period extension of that model, developing the analysis of the factors that determine the optimal dynamic shutdown policy for apparently failing firms of a given industry. Again it is from the perspective of a weak industrial policy maker, who can withhold state support from firms in the industry, but cannot directly manage them or create new firms in the industry; new firms arise from entrepreneurial initiative responding to perceived opportunities.

The principle result of the model is a complete characterization of the opt-
mal shutdown policy under varying assumptions about information quality and decision horizon. The results support early decisive action in closing sufficiently “bad” firms, although the likelihood threshold (probability that a firm is viable) below which a firm is closed is still rather conservatively chosen. A longer horizon and better information about the ‘quality’ of the firm both raise the threshold for a firm to be allowed to survive. The most aggressive optimal policy occurs under full information with a long horizon. Earlier shutdown is also seen to dominate later, given that it should occur. However, sufficiently improved information in the future may lead to a more cautious initial policy, as that increases the value of waiting, although that effect is usually dominated by the increased marginal value of future shutdown possibilities.

In the next section we present the notation and structure of the model. In Section 3 we analyze the two-stage problem with discrete firms — the direct generalization of the prior model. Then in Section 4 we explore some policy relevant comparative statics. Finally we conclude with a discussion of some implications for industrial restructuring in the transition economies.


Consider an industry of \( N \) firms, indexed \( i = 1, \ldots, N \). The current net value produced by firm \( i \) is given by an i.i.d. random variable \( \eta_i \) with fixed finite support \( W = \{ w_1, \ldots, w_K \} \) and distribution \( P(\eta | \theta_i) \), where \( \theta_i \in \{ \theta, \bar{\theta} \} \) indicates whether the firm is ‘bad’ or ‘good’, i.e. able to survive in the coming market economy. Let

\[
\begin{align*}
p^+_{\eta} & \equiv P(\eta | \bar{\theta}); \quad p^-_{\eta} \equiv P(\eta | \theta); \\
y_i & = E(\eta | \bar{\theta}) > 0; \quad y_0 = E(\eta | \theta) < 0;
\end{align*}
\]  \hspace{1cm} (2.1)

and assume \( p^+ > p^- \) in the sense of first order stochastic dominance:

\[
\sum_{\eta \geq a} p^+_{\eta} \geq \sum_{\eta \geq a} p^-_{\eta}, \quad \forall a \in \mathbb{R}. \quad (2.2)
\]

As \( \theta_i \) is not observed, let \( \pi_i \in (0, 1) \) be the prior estimate that firm \( i \) is ‘good’ (of quality \( \bar{\theta} \)) and \( \bar{\pi}_i(\eta) \) be its Bayesian update on observing \( \eta \):

\[
\bar{\pi}(\eta) = \frac{\pi \cdot p^+_{\eta}}{\pi \cdot p^+_{\eta} + (1 - \pi)p^-_{\eta}}. \quad (2.3)
\]
Then the current expected net return to operating a firm with estimate of 'quality' \( \pi \) is:

\[
y_{\pi} = E\{\eta \mid \pi\} \equiv \pi \cdot (y_1 - y_0) + y_0.
\] (2.4)

If \( \pi < \bar{\pi} \equiv \frac{-y_0}{y_1 - y_0}, \) then \( y_{\pi} < 0 \) and the firm is expected net value destroying, i.e. in all likelihood that \( \theta = \bar{\theta}, \) while \( \pi \geq \bar{\pi} \) means that the firm is expected to contribute to social wealth. The expected net social returns to the functioning of all firms in the industry is then given by

\[
Y = \sum_{i=1}^{N} y_{\pi_i}.
\] (2.5)

This includes negative expected returns generated by those firms operating with \( \pi_i < \bar{\pi} \), and hence could be increased by shutting those firms down.

Closing firms has three impacts: (i) the loss of the (negative) net product of the closed firms (\( \sum y_k \)); (ii) the costs associated with closure, disruption of economic ties, and maintenance of the released factors of production are imposed; and (iii) the prospects for new firms entering the industry are enhanced, due to diminished competition and increased availability of released resources to support their operation. The second effect is modeled by a convex, increasing function, \( C(m) \), where \( m \) is the number (mass/volume) of firms shutdown in the period, and a function giving the continuing (social) cost of maintaining the unemployed resources of the industry, \( c(L) \), where \( L \) is the number (mass) of potential new firms that could be formed using the available resources. The final effect is captured in a probability measure over the entry of new firms, \( \nu^L \), which gives the probability that some number (mass), \( \ell \), of new entrants will operate in the period: \( \nu^L(\ell) \). In the simple discrete case it is given by:

\[
\forall L \geq 1, \quad \nu^L_\ell > 0; \\
(\ell - 1)\nu^L_{\ell - 1} \geq \ell \cdot \nu^L_\ell, \quad \ell \in \{2, \ldots, L\}; \\
\nu^L_0 = \left(1 - \sum_{\ell=1}^{L} \nu^L_\ell\right),
\] (2.6)

where \( \nu^L_\ell \equiv \nu^L(\ell) \).\(^9\) We assume that entry takes time, and hence entrants come out of preexisting resources; current shutdowns only add resources to the next

\(^9\)These conditions are satisfied by, among other distributions, a truncated Poisson distribution, \( p(x) = \frac{x}{\lambda^x} e^{-\lambda x} \), with \( \lambda < 2 \) and the mass of the tail placed on \( x = 0 \). Note that the tail conditions in the second line imply that the number of entrants is concave in \( L \) — the pressure for any bundle of resources to be reused is diminishing in the volume of unused resources.
period's entry pool. By the selection process that generates new entry, entrants must be expected to be 'good' [type $\bar{\theta}$], although they may actually turn out to be of type $\theta$. Hence we assume a prior quality estimate of $q > \bar{\pi}$ yielding expected firm payoff

$$y_q = q \cdot (y_1 - y_0) + y_0,$$

and expected net social payoff from new entrants,

$$Y_e = \sum_{t=1}^L v_t^L \cdot \ell \cdot y_q > 0$$

when the pool of available resources is $L$.

The objective of the industrial policy maker in this model is to maximize net social wealth over the course of the transition by a judicious choice of shutdowns—the order and timing of closure of (presumably) 'bad' firms. We assume that controlling new entry is beyond the capabilities of the policy maker, and hence arises exogenously from "the market." The policy maker can however 'fuel' entry by releasing resources to the market through shutdown (i.e., failure to sufficiently subsidize) existing operating enterprises. The general dynamic problem, given total social resources capable of supporting the operation of a set $N$ of firms, facing the policy maker is thus:

$$\max_{\{M_t\}_{t=0}} \left\{ \sum_{t=0}^T \beta^t E \left( \sum_{i \in (N \setminus \Lambda_t) \setminus M_t} \eta_i + \sum_{j \in \Lambda_t} \eta_j + |\Lambda_t| \cdot r - C(M_t) - c(L_t) \mid F_t \right) \right\},$$

where $M_t$ is the set of firms to be shut, $L_t$ — the set of potential ('latent') firms, and $\Lambda_t \subset L_t$ — the set of new entrants at time $t$, $r$ is the recovered cost of a latent firm that becomes active, $c(L) = c \cdot L$ is the continuing cost of maintaining the viability of the resource pool $L$, and $F_t$ is the information available at time $t$. The solution to this problem yields a value depending on the initial payoff-relevant parameters (state variables) of the problem: $V^T(\pi, N_0, L_0)$.\(^\text{10}\) This problem's solution also generates the optimal sequences $\{(M_t, \pi_t^*)_t, N_t, L_t\}_t$, where $\pi_t^*$ is the optimal cutoff expectation for shutting down firms (see below). Clearly $N_{t+1} = (N_t \setminus M_t) \cup \Lambda_t$, $L_{t+1} = (L_t \setminus \Lambda_t) \cup M_t$, and $N_t + L_t \equiv N$.\(^\text{11}\)

\(^{10}\) Note that $N_0$ is somewhat redundant as it is contained in the dimension of the vector of prior qualities, $\pi$. Indeed, in a full dynamic version of the model we set $\pi_i = 0$ for all $i \in L_t$, so that the vector $\pi \in [0, 1]^N$ contains all relevant information on the state of the industry.

\(^{11}\) For clarity we have assumed that the total resources available to the industry are fixed. It
3. Optimal Shutdown with Two Periods.

In the earlier paper (Ericson, 1994) we analyzed the discrete, single-period, case
\[ T = 0, \] with immediate new entry (using resources of the firms shutdown in the
same period) possible, and firms all of the same resource size (a 'standard firm'
assumption). The primary analytic result was a complete characterization of the
optimal shutdown policy, which we repeat here in the case where immediate reuse
of resources released by shutdown is not possible. The result is illustrated in
Figure 1.

**Proposition 3.1.** There exists a unique solution, \((m^*, \pi^*, \sigma^*)\), to the optimization problem (2.8) with \(T = 0\) such that

\[
\pi_{m^*} \leq \pi^* \leq \pi_{m^*+1},
\]

\[
-y_{r^*} = \Delta C(m^*),
\]

\[
\Delta W_{m^*} \geq 0 > \Delta W_{m^*+1},
\]

where

\[
W_m = E \left\{ \sum_{i=m+1}^{N_0} \eta_i + \sum_{j=1}^{L_0} \eta_j + \ell \cdot r - C(m) - c \cdot L \right\}
\]

\[
= \sum_{i=m+1}^{N_0} y_i + \sum_{j=1}^{L_0} j \cdot [y_0 + r] \cdot \nu_j - C(m) - c \cdot L
\]

\[
\Delta C(m) \equiv C(m) - C(m - 1) \text{ and } \Delta W_m \equiv W_m - W_{m-1}, \text{ implies } \sigma^* = 0.
\]

If

\[
\pi^* = \pi_{m^*+1} \text{ and } -y_{r^*} > \Delta C(m^*),
\]

then

\[
-y_{r^*} = (1 - \sigma^*) \cdot \Delta C(m^*) + \sigma^* \cdot \Delta C(m^* + 1),
\]

\[
(1 - \sigma^*) \cdot \Delta W_{m^*} + \sigma^* \cdot \Delta W_{m^*+1} = 0,
\]

and \(\sigma^* > 0\).

**Furthermore,** \(\pi^* < \bar{\pi}\) as \(\Delta C(\cdot) > 0\).

would change little to assume them to be growing or shrinking for exogeneous (to the policy of
the sector) reasons. Indeed, that will be the case for most industries in a transition economy
undergoing structural change.
Proof. See Propositions 1 and 2 in Ericson (1994).  

Remark 1. This is a simpler formulation than in Ericson (1994) as only resources lying fallow at the beginning can be drawn into new enterprises. Thus the impact of shutdown on new entry can only be felt in the future, reducing somewhat the incentive to close failing firms.

Remark 2. The first order conditions (3.1 and 3.2) follow immediately from placing the firms in increasing order of their likelihood of being good (π's); then the worst firms, those with the lowest indices, are the first candidates to be shut down. Thus \( W_m \) gives the expected net value of operation of the industry conditional on \( m \) firms being shut down.

Remark 3. The expected value of this problem is: \( V^0(\pi, N_0, L_0) = W_m \). Notice that only \( m^* \) firms are ever shut down, even when the conditions (3.2) are relevant; it is never desirable to even randomize over shutting down an additional firm, as that only adds to the expected cost. \( \sigma^* \) is just a measure of slack in the program due to its integer nature — the amount by which \( -y_{\pi^*} \) falls short of allowing another firm to be optimally shut. It is useful, as we shall see below, for characterizing the impact of changes in the state variables (\( \pi, N, L \)) on the optimal decision and hence on the value of the problem.

Remark 4. With linear shutdown costs (\( \Delta C(m) = C, \forall m \geq 1 \)) only the first order conditions (3.1) are relevant: \( -y_{\pi^*} = C \) and \( \pi^* = -\left( \frac{v_0 + C}{v_1 - v_0} \right) \).

Here we extend the analysis with 'standard firms' to two periods (\( T = 1 \)), with delayed entry and with an initial set of latent firms that might enter independently of the shutdown decision. That decision, however, affects entry in the next period. The results here provide a foundation for a general \( T \)-period analysis.

3.1. Terminal Period problem.

In the dynamic problem, the optimal initial decision of the policy maker must take into account what he will decide to do in the future in the situation created by that initial decision. Thus we must begin with an analysis of the static second period problem where there are \( N_1 = N \setminus M_0 \) operating firms with 'qualities' (likelihood of viability) \( \tilde{\pi} = (\tilde{\pi}_1, \ldots, \tilde{\pi}_{n_1}) \) arranged in increasing order, \( L_1 = L_0 \cup M_0 \) potential entrants each with expected quality \( q \), and \( c(L) = c \cdot L \).
Let us begin with the simplest case: no new entry is possible in the first period \((L_0 = \emptyset)\).\(^{12}\) \(N_1\) is then just the set of initial firms that were not shut down, i.e. such that \(\pi_i \geq \pi^*\) — the optimal first period cutoff expectation. The set of potential entrants, from which some random \(\ell\) standard firms will enter this (second) period, is built from the resources of the \(m^*\) firms optimally shut down in the first period. Thus the expected value of the problem for each possible realization of \(\hat{\pi}\) is given by

\[
V^0(\hat{\pi}, N-m^*, m^*) = \max_{0 \leq \mu \leq N-m^*} \left\{ \sum_{i=m^*+1}^{N} \hat{y}_i + m^* \sum_{j=1}^{m^*} j \cdot [y_q + r] \cdot \nu_j - C(m) - c \cdot m^* \right\}
\]

(3.3)

where \(\hat{\pi}\) contains updated likelihoods of being good for firms that are active, \(\hat{y}_i \equiv y_{\pi_i}\), \(N_1 = N - m^*\), and \(L_1 = m^*\).\(^{13}\) The solution to this problem, which we label \((\hat{m}, \hat{\pi}, \hat{\sigma})\), is given directly by Proposition 3.1:

\[
V^0(\hat{\pi}, N-m^*, m^*) = \sum_{i=m+1}^{N} \hat{y}_i + E\{\ell \mid m^*\} \cdot [y_q + r] - C(\bar{m}) - c \cdot m^*;
\]

(3.4)

where \(\Delta C(\bar{m} + 1) = -\hat{y}_{\bar{m}} \geq \Delta C(\bar{m})\) and \(\hat{\sigma} = 0, \pi_{\bar{m}} \leq \hat{\pi} \leq \pi_{\bar{m}+1}\) or \(\hat{\sigma} > 0, \hat{\pi} = \pi_{\bar{m}+1}\). Note that there only a finite number of such problems possible, one for each \((N-m^*)\)-tuple of \(\eta\)-realizations of net value. This characterization allows us to incorporate the discounted \(V^0(\hat{\pi}, N-m^*, m^*)\) in the first period problem, as it fully captures the impact of the first period shutdown decision on the industry’s future value.

The characterization becomes more complex when we begin with a non-trivial set of latent forms, \(L_0 \neq \emptyset\), from which new entrants, not subject to the first period’s shutdown threshold, might appear. There is then a similar, but different, problem for each realization of outcomes of surviving firms, for each possible number of new entrants, \(0 \leq \ell \leq L_0\), and for each realization of outcomes of the new firms. For any such problem, \(L_1 = L_0 + m^* - \ell\), \(N_1 = N_0 - m^* + \ell\), and \(\hat{\pi}\) contains an \((N_0 - m^*)\)-subvector of updated (from priors \(\pi_i\) — see equation (2.3)) expectations for the survivors and an \(\ell\)-subvector of updated (from prior \(q\) expected payoffs for the new entrants. Then the problem faced by the industry

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\(^{12}\)Thus reflects an assumption of initial socialist "overfull employment." There are initially no resources that are not fully committed to, and/or controlled by, existing enterprises.

\(^{13}\)With only slight abuse of notation, let \(N_i, L_i\) stand for the number of elements in those sets of firms, as well as the sets themselves.
planner is:

\[
\max_{m,n \geq 0} W(m, n; \bar{\pi}, N_1, L_1) = \sum_{i=m^*+m+1}^{N_0} \bar{y}_i + \sum_{j=n+1}^{\bar{\ell}} \bar{y}_j + [y_q + r] \cdot \sum_{h=1}^{L_0 + m^* - \bar{\ell}} h \cdot \nu_h^{L_1} - C(m + n) - c \cdot (L_0 + m^* - \bar{\ell}) \tag{3.5} \\
= \sum_{k=m+1}^{N_0 - m^* + \bar{\ell}} \bar{y}_k + [y_q + r] \cdot \sum_{h=1}^{L_0 + m^* - \bar{\ell}} h \cdot \nu_h^{L_1} - C(k) - cL_1.
\]

It is straightforward, if somewhat tedious, to derive the following

**Proposition 3.2.** Let \((\bar{\pi}, \bar{\ell})\) be the realization of expected productivities and new entrants at the beginning of the terminal period. Then there exists a unique solution, \((\bar{k}, \bar{\pi}, \bar{\sigma})\), \(\bar{k} = \bar{m} + \bar{n}\), to the problem (3.5) such that

\[
\begin{align*}
\bar{\pi}_k &= \max \{\pi_m, \pi_n\} \leq \bar{\pi} \leq \min \{\pi_{m+1}, \pi_{n+1}\}, \\
-y_{\pi} &= \Delta C(\bar{m} + \bar{n}), \\
\bar{\sigma} &= 0,
\end{align*}
\tag{3.6}
\]

or

\[
\begin{align*}
\bar{\pi} &= \min \{\pi_{m+1}, \pi_{n+1}\}, \\
-y_{\pi} &= (1 - \bar{\sigma})\Delta C(\bar{k}) + \bar{\sigma}\Delta C(\bar{k} + 1), \\
\bar{\sigma} &> 0.
\end{align*}
\tag{3.7}
\]

Furthermore, \(\bar{\pi} \geq \pi^*\) unless \(\bar{\ell} > m^*\) and \(\bar{\pi}_j < \bar{\pi}\) for at least \(m^* + 1\) of the \(\bar{\ell}\) new entrants, which will occur with probability no greater than

\[
\left( \sum_{k=m^*}^{L_1} \nu_k^{L_1} \right) \times \left[ \text{Prob} \left\{ \pi < \pi^* \mid \pi = q \right\} \right]^{m^* + 1}.
\tag{3.8}
\]

**Proof.** [Outline] The proof is virtually identical to that of Proposition 3.1 (see Ericson (1994) for more details), once both sets of firm expected values \((\bar{y}_i)'s\) are placed in increasing order. The marginal cutoff is clearly determined by \(\Delta C(k)\), so that firms are shut down from both sets starting with the lowest \(\bar{\pi}'s\), until the next shutdown (from either set) will cause the added cost to exceed the savings from that shutdown. Whether that occurs between firm expected values or at the
expected value of the next best firm determines the difference between (3.6) and (3.7).

Clearly, if fewer than \( m^* \) (necessarily new) firms have expected qualities below \( \pi^* \) then \( \Delta C \) for shutting down all those firms must be less than \( -y_{\pi^*} \), so new firms with higher \( \pi \)'s will be optimally shut down. Direct calculation then gives (3.8) as an upper bound on the probability that the quality cutoff will fall in the second period.

**Remark 5.** Note that the probability in (3.8) is extremely small; indeed it is often zero. It naturally depends on the structural probabilities in the problem and the relationship between \( q \) and \( \pi^* \). If the realizations of \( \eta \) are insufficient to drive the \( \tilde{\pi} \) derived from \( q \) to below \( \pi^* \) then that probability is zero. As that distance is quite large,

\[
q - \pi^* = \frac{y_q + \Delta C(m^*)}{y_1 - y_0},
\]

for \( \tilde{\pi} < \pi^* \) at least \( m^* + 1 \) firms must draw an \( \eta \) such that

\[
\left( \frac{p_{\eta}^q}{p_{\eta}^r} \right) > \frac{(1 - \pi^*) \cdot q}{\pi^* \cdot (1 - q)} = \frac{(y_1 + \Delta C(m^*)) (y_q - y_0)}{(y_1 - y_q) (-y_0 - \Delta C(m^*))}
\]

which can be, depending on the parameters of the problem, quite large. For example, if \( y_1 = -y_0 = 5 \), \( \Delta C(m^*) = 2 \) and \( y_q = 1 \), then \( \left( \frac{p_{\eta}^q}{p_{\eta}^r} \right) > 3.5 \) for the \( \eta \)'s drawn by \( m^* + 1 \) firms for the probability that \( \tilde{\pi} < \pi^* \) holds to be positive.

**Remark 6.** The notation has suppressed dependence of the state, \((\tilde{\pi}, N_1, L_1)\), on the prior shutdown decision, \( m^* \), although that dependence is obvious in the structure of (3.5). Thus it is clear that the current decision will depend non-trivially on \( m^* \): \( k(m^*), \tilde{\pi}(m^*), \tilde{\sigma}(m^*) \).

These results give, for each \((\tilde{\pi}, \hat{\ell})\) realization, a well defined value function for this terminal period problem from which we can derive the future impact of shutdown decisions in the preceding period:

\[
V^0(\tilde{\pi}, N_1, L_1) = \sum_{i=k+1}^{N_1} \tilde{y}_i + [y_q + r] \left( \sum_{j=1}^{L_1} j \cdot \nu_{j}^{L_1} \right) - C(\tilde{k}) - cL_1, \quad (3.9)
\]

where \( N_1 = N_0 - m^* + \hat{\ell}, L_1 = L_0 + m^* - \hat{\ell} \) and \((\tilde{k}, \tilde{\pi}, \tilde{\sigma})\) is the optimal decision in the terminal period. Notice that this reduces to (3.4) when \( L_0 = 0 \), implying that \( \hat{\ell} = 0 \).
3.2. Initial Period Problem.

We can now write the Bellman Equation for the first period problem \( (T = 1) \) when \( L_0 = 0 \),

\[
V^1(\pi, N, 0) = \max_{0 \leq m^* \leq N} \left\{ \sum_{i=m^*+1}^{N} y_i - C(m^*) + \beta E \left[ V^0(\tilde{\pi}, N - m^*, m^*) \mid \mathcal{F}_0 \right] \right\},
\]

and when \( L_0 > 0 \),

\[
V^1(\pi, N_0, L_0) = \max_{0 \leq m^* \leq N_0} \left\{ \sum_{i=m^*+1}^{N_0} y_i - C(m^*) - cL_0 + \sum_{j=1}^{L_0} \nu_j \left( j \cdot [y_0 + r] + \beta E \left[ V^0(\tilde{\pi}, N_0 - m^* + j, L_0 + m^* - j) \mid \mathcal{F}_0 \right] \right) \right\}
\]

Clearly the optimal policies for each of these problems are qualitatively similar, as (3.10) is just a special case of (3.11). This allows generalization to any finite horizon, as well as to the infinite horizon stationary problem.\(^{14}\)

The solution to this problem, an optimal first-period shutdown decision, must be based only on the information available in the first period. Thus iterated expectations must be calculated as \( V^0(\cdot) \) is a function of \( \mathcal{F}_1 \supseteq \mathcal{F}_0 \). Hence a useful property, a consequence of the Martingale property of Bayesian updating, is given by the following

**Lemma 3.3.** For \( \ell \) fixed, \( E \left[ V^0(\tilde{\pi}, N_0 - m^* + \ell, L_0 + m^* - \ell) \mid \mathcal{F}_0 \right] = V^0((\pi^0, q), N_0 - m^* + \ell, L_0 + m^* - \ell) \), where \( \pi^0 \) is the \((N_0 - m^*)\)-vector of initial firm likelihoods for surviving firms, and \( q \) is the \( \ell \)-vector of prior quality evaluations of new entrants.

**Proof.** For any firm \((\text{dropping the subscript } i)\) \( \tilde{\pi}(\eta) \) is given by equation (2.3). Let \( E_0 \) be the operator \( E \left[ \cdot \mid \mathcal{F}_0 \right] \) Then, if a firm is 'good' \( [\theta] \),

\[
E_0 \left( \tilde{\pi} \mid \theta \right) = \sum_{\eta} p^+_\eta \cdot \frac{\pi \cdot p^+_\eta}{(1 - \pi) \cdot p^-_\eta + \pi \cdot p^+_\eta} = \pi \sum_{\eta} \frac{(p^+_\eta)^2}{(1 - \pi) \cdot p^-_\eta + \pi \cdot p^+_\eta},
\]

\(^{14}\)The latter would only be interesting as an approximation if the decision and firm response periods were very short relative to the expected length of the "transition."
while if it is 'bad' \( \theta \) then

\[
E_0(\bar{\pi}|\theta) = \sum_\eta p_\eta^- \cdot \frac{\pi \cdot p_\eta^+}{(1-\pi) \cdot p_\eta^- + \pi \cdot p_\eta^+} = \pi \sum_\eta \frac{p_\eta^- \cdot p_\eta^+}{(1-\pi) \cdot p_\eta^- + \pi \cdot p_\eta^+}.
\]

Therefore,

\[
E(\bar{\pi}) = \pi \cdot E(\bar{\pi}|\theta) + (1-\pi) \cdot E(\bar{\pi}|\theta) = \pi \sum_\eta \frac{\pi (p_\eta^+)^2 + (1-\pi)p_\eta^+ p_\eta^-}{(1-\pi) \cdot p_\eta^- + \pi \cdot p_\eta^+} = \pi \sum_\eta p_\eta^+ = \pi.
\]

Thus \( E_0(\hat{y}_i) = y_i \) for any firm initially in operation, \( E_0(\hat{y}_i) = q \) for any entrant, and the expected future value, conditional on initial information and the number of new entrants, is unaffected by the output realizations at the end of the first period.

The solution to (3.10) and (3.11) clearly depends on how \( V^0(\cdot) \) varies in \( m \), the number of firms initially shut down, and in the latter case, on how that varies in the number of new entrants [See Remark 6]. Let \( V^0_\ell(m;\cdot) \) represent the optimized value in the terminal period [see (3.9)] as a function of the number of firms shut down in the prior period, \( m \), and the number of new firms, \( \ell \). That value is generated by the solution \( \bar{m}_\ell(m) \) to the terminal problem (3.5) given in Proposition 3.2.

**Lemma 3.4.** \( \bar{m}_\ell(m) + 1 \geq \bar{m}_\ell(m-1) \geq \bar{m}_\ell(m) \) for all feasible \((m,\ell)\), and \( \bar{m}_\ell(m) = \bar{m}_k(m) \) for all feasible \((m,\ell,k)\). Hence \( \bar{m}_\ell(m) \) is non-increasing in \( m \), and independent of \( \ell \).

**Proof.** The conditions determining the choice of \( \bar{m}_\ell(m) \) are in equations (3.6) and (3.7). As there is no dependence on the number of new firms operating in that period, the second assertion is obvious, and we can wlog drop \( \ell \) from our expressions here. Let \( \bar{m} = \bar{m}(m-1) \) be the number closed, and \( k(\bar{m}) \) be the index of that marginal firm. As the lowest expected value firms are shutdown first in any period, closing one more means that there is one less low \( \pi \), so that the same cutoff likelihood, \( \bar{\pi} \), can be reached with one fewer shutdown, i.e. at a saving of \( \Delta C(\bar{m}) \). Whether that is the optimal response to the extra initial shutdown depends on whether \( -y_{k(\bar{m})+1} \geq \Delta C(\bar{m}) \). If so, \( \bar{m}(m) = \bar{m} \); the same
number of firms are shut down, despite one more having been closed earlier. If not, then \( \tilde{m}(m) = \tilde{m} - 1 \), as it is too costly to close down another firm. Notice that a single extra prior shutdown will never lead to more than \( \tilde{m} \) closures in the second period, as optimization insures that \( \Delta C(\tilde{m} + 1) > -y_{k(\tilde{m})} \).

The different cases in this Lemma are illustrated in Figure 2. This allows us to readily characterize the impact of the prior choice of shutdowns on the subsequent value of the problem. Let \( \Delta V_I^0(m) = V_I^0(m; \cdot) - V_I^0(m - 1; \cdot) \) be the total impact of increasing prior shutdowns by one on the expected value in the terminal period, and let \( \Delta V_I^\pi(m) \) be the impact of increasing the number of latent firms by one through shutting down an m-th firm.

Lemma 3.5. The variation in the value function due to a unit increase in prior shutdown is given by:

1. \( \Delta V_I^\pi(m) = (L_0 - \ell + m) \nu^{L_1}_{(L_0+\ell-m)}[y_q + r] - c. \)

2. \( \Delta V_I^0(m) = -y_{\pi(m)} + \Delta V_I^\pi(m). \)

Proof. (1) follows from direct calculation:

\[
\Delta V_I^\pi(m) = [y_q + r] \sum_{j=1}^{L_0 - \ell + m} j \cdot \nu^{L_0-\ell+m}_{L_0-\ell+m} - (L_0 - \ell + m) c \\
- [y_q + r] \sum_{j=1}^{L_0 - \ell + m - 1} j \cdot \nu^{L_0-\ell+m-1}_{L_0-\ell+m-1} - (L_0 - \ell + m - 1) c \\
= (L_0 - \ell + m) \nu^{L_1}_{(L_0+\ell-m)}[y_q + r] - c.
\]

(2) is immediate from the first order conditions in Proposition 3.2, (3.6 or 3.7), where clearly \(-y_{\pi(m)} = \max \{-y_{\tilde{m}(m)+1}, \Delta C(\tilde{m}(m)) \}\).

Remark 7. While the above results are formulated for the problem (3.11), they all hold as well for (3.10) with \( L_1 = m \) (\( L_0 = \ell = 0 \)).

Remark 8. These are the key comparative statics results that we need. We could also readily calculate the impact of independent changes in \( \tilde{n} \) or \( N_1 \) or \( L_1 \) as done in Proposition 4 of Ericson (1994). The qualitative results would be the same.

Remark 9. Notice that \( \Delta V_I^0(m) \) is a decreasing function of \( m \), as both its components are such. This insures that second order conditions are automatically satisfied.
The main result of the two-period analysis is the derivation of an optimal policy that can be compared with other, perhaps institutionally constrained, approaches to restructuring of the industry. While by definition better than any other shutdown policy, it has a dynamic structure that is interesting to compare with that of other policies, for example, one in which all shutdown decisions must be made in the initial period (perhaps because politics will not allow a continuation). The key trade-offs revolve around three factors: 'bad' firms will continue destroying social wealth if not closed, new entry will not be forthcoming without early shutdowns, but the convexity of full shutdown costs implies that it is socially wasteful to shutdown too many firms too rapidly — those costs are minimized by spreading evenly.

To see this, define \( \Delta W_m \) to be the variation in \( W^1(\pi, N_0, L_0) \) as the \( m \)-th firm (starting from the lowest \( \pi \)) is shut down:

\[
\Delta W_m = W^1_m(\pi, N_0, L_0) - W^1_{m-1}(\pi, N_0, L_0)
\]

where

\[
W^1_m(\pi, N_0, L_0) = \sum_{i=m+1}^{N_0} y_i - C(m) - cL_0 + \sum_{j=1}^{L_0} \nu_j^L \left( j \cdot [y_0 + r] + \beta E \left[ V^0(\tilde{\pi}, N_0 - m + j, L_0 + m - j) \mid F_0 \right] \right). \tag{3.12}
\]

Pulling together and integrating the above results then gives us:

**Proposition 3.6.** The unique optimal policy for the two-period problem (3.11), \( \{(m^*, \pi^*, \sigma^*), (\tilde{k}(m^*), \tilde{\pi}(m^*), \tilde{\sigma}(m^*))\} \), gives total expected value

\[
V^1(\pi, N_0, L_0) = W^1_{m^*}(\pi, N_0, L_0)
\]

and satisfies the second period marginal conditions

\[
\pi_k^* \leq \bar{\pi} \leq \pi_{k+1}^*, \quad \bar{\sigma} = 0,
\]

\[
y_{\pi} = \Delta C(\tilde{k}),
\]

or

\[
\bar{\pi} = \pi_{k+1}^*, \quad \bar{\sigma} > 0,
\]

\[
y_{\pi} = (1 - \bar{\sigma})\Delta C(\tilde{k}) + \bar{\sigma}\Delta C(\tilde{k} + 1),
\]

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and the first period marginal conditions

\[ \pi_{m^*} \leq \pi^* \leq \pi_{m^*+1}, \quad \sigma^* = 0, \]

\[ -\left( y_{m^*} + \beta \left[ y_{m^*} - \xi_{m^*}(y_q + r) \right] \right) = \Delta C(m^*) + \beta c, \tag{3.13} \]
or

\[ \pi^* = \pi_{m^*+1}, \quad \sigma^* > 0, \]

\[ -\left( y_{m^*} + \beta \left[ y_{m^*} - \xi_{m^*}(y_q + r) \right] \right) = (1 - \sigma^*)\Delta C(m^*) + \sigma^*\Delta C(m^* + 1) + \beta c, \tag{3.14} \]

where \( \xi_m = \sum_{j=0}^{L_0} \nu_j \left( \nu_i \left( L_0 + m^* - j \right) \nu_{L_0 + m^* - j} \right)^{15} \), and \( \xi_{m^*}(y_q + r) \) is the prior expected value of the marginal entrant when \( m^* \) firms are shut. Furthermore, if there are any firms with \( \pi_i \in [\pi^*, \pi]\),

\[ \pi > E_0[\pi(m^*)] > \pi^*, \quad E_0[\pi(m^*)] > 0, \tag{3.15} \]

so that shutdown continues to increasingly high quality, albeit still expected value destroying, firms despite the prior elimination of the lowest quality (\( \pi_i \leq \pi^* \)).

**Proof.** The results follow from the arguments of Propositions 3.1 and 3.2 and the results of Lemmata 3.3, 3.4, and 3.5. The second period characterization is just that of Proposition 3.2. To derive the remaining results, notice that Lemma 3.3 gives

\[ W_{m}^{1}(\pi, N_0, L_0) = \sum_{i=m+1}^{N_0} y_i - C(m) - cL_0 + \]

\[ + \sum_{j=0}^{L_0} \nu^j \left( j \cdot [y_q + r] + \beta \cdot V^0((\pi, q), N_0 - m + j, L_0 + m - j) \right) \]

so that by Lemma 3.5 the first variation of the total payoff in the initial decision is:

\[ \Delta W_m = -y_m - \Delta C(m) + \beta \left( \sum_{j=0}^{L_0} \nu^j \left[ \Delta V^0_j(m) \right] \right) \]

\[ ^{15} \text{Clearly} \ \xi_m \text{is quite small as the probability of the coincidence of a number of unlikely (marginal) events [See equation (2.6)]. These events become oven less likely as } L_0 \text{ and/or} \]

\[ m \text{ increase. For example, if } \nu_i \text{ is truncated poisson, } L_0 = 3, \text{ and } m = 2, \text{ then } \xi_m \cong .004; \]

\[ L_0 = 0, \ m = 2 \Rightarrow \xi_m \cong .068; \ L_0 = 0, \ m = 4 \Rightarrow \xi_m \cong .0008. \]
\[
\begin{align*}
&= -y_m - \beta y_{\pi(m)} - (\Delta C(m) + \beta c) + \beta \left\{ \sum_{j=0}^{L_0} \nu_j^{L_0} \left[ \Delta V^e_j(m) \right] \right\} \\
&= -y_m - \beta y_{\pi(m)} - (\Delta C(m) + \beta c) + \\
&\quad + \beta [y_q + r] \left\{ \sum_{j=0}^{L_0} \nu_j^{L_0} (L_0 + m - j) \nu_{L_0+m-j}^{L_0+m-j} \right\} \\
&= -y_m - \beta y_{\pi(m)} - (\Delta C(m) + \beta c) + \beta [y_q + r] \xi_m.
\end{align*}
\]

As \( \Delta W_m \) is monotonically decreasing in \( m \), \( \Delta W_1 > 0 \), and \( \Delta W_m < 0 \) for any \( m \) such that \( \pi_m \geq \bar{\pi} \), the optimum occurs at \( m^* \) such that

\[
\Delta W_{m^*} \geq 0 > \Delta W_{m^{*}+1},
\]

that is,

\[
- \left( y_{m^*} + \beta \left[ y_{\pi(m^*)} - \xi_{m^*}(y_q + r) \right] \right) \geq \Delta C(m^*) + \beta c, \quad (3.18)
\]

\[
- \left( y_{m^{*}+1} + \beta \left[ y_{\pi(m^{*}+1)} - \xi_{m^{*}+1}(y_q + r) \right] \right) < \Delta C(m^* + 1) + \beta c.
\]

If there exists a \( \pi^* \in [\pi_{m^*}, \pi_{m^{*}+1}] \) such that

\[
- \left( y_{\pi^*} + \beta \left[ y_{\pi(m^*)} - \xi_{m^*}(y_q + r) \right] \right) = \Delta C(m^*) + \beta c,
\]

then we are in case (3.13). If not, then the relevant FOC's are given

by (3.14).

To see (3.15), first note that \( \Delta C(1) > 0 \) implies that, without
a future, no firm with positive expected payoff will ever be shut; indeed, no firm with \( -y_i < \Delta C(1) \) will be shut, unless there are sufficient future savings and expected value from
entry induced by that closing. That \( E_0[\pi(m^*)] \geq \pi^* \) follows immediately from Lemma 3.3 and the fact that the expected payoff to all new entrants is \( y_q > 0 \).

Thus the only firms that might be expected to be shut down are those with \( \pi_i \in [\pi^*, \bar{\pi}] \), so the expected cutoff must be in that region. Since \( \Delta C(m) \) is increasing in \( m \), and \( -y_{\pi^*} = \Delta C(m^*) \geq \Delta C(1) \), if there are any firms with \( \pi_i \in [\pi^*, \bar{\pi}] \) at least one of them must be optimally shut down. Hence \( \bar{\pi} > E_0[\pi(m^*)] > \pi^* \), \( E_0[\pi(m^*)] > 0 \).

The structure of the solution to this industrial policy problem is illustrated in

Figure 3, which shows how the second period solution builds in the first. This
proposition provides the basis for a number of comparative statics/dynamics
exercises, exploring the dynamic structure of an optimal industrial policy during
the transition. For example, it shows that the marginal shutdown likelihood, \( \pi^* \), is clearly greater when a future period is considered, than in its absence (compare
with Proposition 3.1 and Figure 1). We now turn to further exploration of such
comparisons in the next section.
4. Some Comparative Statics/Dynamics.

Here we explore the impact of varying information and the planning horizon on the nature of the optimal solution. The results are largely corollaries of the Propositions above, so we avoid full proofs.

4.1. The Value of Information.

How does better information affect the optimal shutdown policy and the 'option value' of delaying the shutdown decision? Intuitively, it is clear that the optimal policy must become more aggressive with better information, as mistakes are less likely to be made and there is less to learn from waiting. The extreme case of this situation is that of perfect information about the quality of all existing firms, although actual outcomes in any period remain uncertain and firms yet to enter are still of uncertain quality \( q > \bar{\pi} \). Then the initial \( N_0 \) firms can be divided into two sets: \( N_0^+ \) "good" firms with expected net return \( y_1 > 0 \), and \( N_0^- \) "bad" firms with expected net return \( y_0 < 0 \). In the terminal period there will be \( N_1 = N_0 - m^* + \hat{\ell} \) operating firms, where \( m^* \) is the number shutdown and \( \hat{\ell} \) is the number of new entrants from the initial resource pool \( L_0 \), and \( L_1 = L_0 + m^* - \hat{\ell} \) potential firms whose resources remain fallow. Clearly the problem faced by the planner is identical to that in equation (3.5) with \( y_i, i = 0, 1 \), in place of \( \hat{y}_k \), for 'bad' and 'good' firms respectively.

\[
\max_{k \geq 0} W(k; N_1, L_1, \hat{\ell}) = (N_0^+ + \hat{\ell}^+) y_1 + (N_0^- - m^* + \hat{\ell}^- - k)y_0 + \sum_{h=1}^{L_1} \nu_h \cdot h - C(k) - cL_1.
\]

The solution is a straightforward corollary of Proposition (3.2):

**Proposition 4.1.** Let \( \hat{\ell} = (\hat{\ell}^+, \hat{\ell}^-) \) be the realization of new entrants at the beginning of the terminal period. Then there exists a unique optimal solution, \( (k, \bar{\pi}, \bar{\sigma}) \), to the problem (4.1) such that

\[
\begin{align*}
\bar{\pi} & = 0 \\
-y_0 & = (1 - \bar{\sigma})\Delta C(k) + \bar{\sigma}\Delta C(k + 1) \\
\bar{\sigma} & \in [0, 1) \quad \text{if} \quad \tilde{k} < (N_0^- - m^* + \hat{\ell}^-).
\end{align*}
\]

If, however,

\[
y_0 \geq \Delta C(N_0^- - m^* + \hat{\ell}^-)
\]

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then

\[ \bar{\pi} = 0, \quad -y_0 \geq \Delta C(\bar{k}), \quad \bar{k} = (N^*_0 - m^* + \ell^-), \quad \text{and} \quad \bar{\sigma} = 0, \quad (4.3) \]

i.e. it is never optimal to close a “good” \([y_1]\) firm.

This solution is illustrated in Figure 4. It again gives a well-defined value function for the terminal period problem:

\[ V^0f(\bar{\pi}^f, N_1, L_1) = N_1^+ \cdot y_1 + (N_1^- - \bar{k})y_0 + [y_0 + r] \cdot \sum_{j=1}^{L_0} \nu_j^L \cdot j - C(\bar{k}) - cL_1 \quad (4.4) \]

where superscript \('f'\) indicates “full information” \([\bar{\pi}^f \in \{0, 1\}, \quad \forall i], \quad N_i^+ = N_0^+ + \ell^+, \quad N_i^- = N_0^- - m^* + \ell^-, \quad \text{and} \quad L_1 = L_0 + m^* - \ell^- - \ell^- \). This value, and its variation in \(m^*\), is a key input into solving the full two-period problem.

In the terminal period all remaining “bad” firms are closed unless the immediate incremental cost of doing so exceeds the maximum expected loss from the continued operation of such a firm. Thus only in the case of extraordinarily large incremental (social) costs of shutdown \([-y_0 < \Delta C(N_1^-)]\) will known “bad” firms be left in operation. It is also immediate that enterprise shutdowns will go deeper into the industry, closing some firms left in operation in the first period, unless there were none left and/or \(\ell^+ > m^*\). Thus the optimal policy with full knowledge of the quality of operating firms is substantially more aggressive than without such knowledge; there is no possibility of making a mistake. That is even clearer in the initial period, where any incentive to spread costs is dominated by the sure gains from eliminating value-destroying operations and the absence of any benefit to waiting.

As in the preceding section, the initial period’s Bellman Equation is

\[ V^1f(\pi^f, N_0, L_0) = \max_{0 \leq m^* \leq N_0^-} \left\{ N_0^+ y_1 + (N_0^- - m^*)y_0 - C(m^*) - cL_0 + \right. \]

\[ \left. + \sum_{j=1}^{L_0} \nu_j^L \cdot j : [y_0 + r] + \beta E \left[ V^0f(\bar{\pi}^f, N_0 - m^* + j, L_0 + m^* - j) \Big| \mathcal{F}_0 \right] \right\}, \quad (4.5) \]

since it never pays to shut down a “good” firm. As above, the solution can be easily characterized:

**Proposition 4.2.** Necessary and sufficient conditions for an optimal solution \(\{(m^f, \pi^f, \sigma^f), (\bar{k}^f(m^f), \bar{\sigma}^f(m^f))\}\) to the two-period full-information problem (4.5)
are given in Proposition 4.1 for the terminal period, and for the initial period by:

\[ \Delta C(m^I) + \beta c \leq -(y_0 + \beta [v_0 - \xi_{m^I}(y_q + r)]) < \Delta C(m^I + 1) + \beta c, \quad (4.6) \]

where \( \xi_m = \sum_{j=0}^{L_0} \{ \nu^{L_0}_{j} (L_0 + m - j) \nu^{L_0+m-j}_{L_0+m-j} \} \),

and \( v_0 = \max\{y_0, -\Delta C(\bar{k})\} \), \quad (4.7)

implying that

\[ \pi^I = 0, \]

\[ -(y_0 + \beta [v_0 - \xi_{m^I}(y_q + r)]) = (1 - \sigma^I) \Delta C(m^I) + \sigma^I \Delta C(m^I + 1) + \beta c, \]

and \( \sigma^I \in [0, 1), \quad \text{if } m^I < N_0^- \). \quad (4.8)

If, however,

\[ -(y_0 + \beta [v_0 - \xi_{N_0^-}(y_q + r)]) \geq \Delta C(N_0^-) + \beta c, \]

then

\[ m^I = N_0^- \quad \text{and} \quad \sigma^I = 0. \]

Again we can see that full information leads to optimal maximal shutdown of “bad” firms as illustrated (heuristically, ignoring integer constraints) in Figure 5. Indeed, it is clearly far more aggressive than in the single period case as \( -v_0 \gg c \), even without the possibility of new high quality entry whose marginal value is reflected in \( \xi(y_q + r) \).

These results give us a possible measure of the value of information to the industry policymaker. Clearly the fact that \( m^I > m^* \) and \( k^I \geq \bar{k} \) whenever there is the same number of “bad” firms left in the industry, shows that the information reduces the losses from the operation of “bad” firms and avoids the loss of net value from “good” firms, thereby raising the value of the industry to the planner. By virtue of optimization, the marginal costs of the additional shutdowns, \( \Delta C(\cdot) + \beta c \), are no greater than the marginal benefits, \(-y_0 - \beta v_0 + \xi(y_q + r)\), and this is reflected in the increase in the optimal value of the program. This is clear given full information about the quality of firms. From the perspective of a decision maker with only limited information, \( \pi \), about quality, however, that may not appear to be the case, as some “bad” firms may have high \( \pi \), thus decreasing \( V^1(\cdot) \) [the uninformed apparent value] when shut down, while some “good” firms may have low \( \pi \) and so reduce \( V^1(\cdot) \) when left in operation. Thus the difference in values, \( V^{1f} - V^1 \), cannot provide a reliable measure of the value of (in this case,
full) information to the industry planner. There is, however, a natural measure of
this value of information in this difference conditioned on the better information:¹⁶

\[ E \left\{ V^1(\pi', N_0, L_0) - V^1(\pi, N_0, L_0) \middle| \pi' \right\} = \]

\[ = (1 + \beta)(y_1 - y_0)m^{*+} + \sum_{j=m^{*+}+1}^{m'} (-y_0(1 + \beta) - \Delta C(j)) + \]

\[ + \beta \left\{ (y_1 - y_0)\tilde{k}^+ + \sum_{k=\tilde{k}+1}^{\tilde{k}'} (-y_0 - \Delta C(k)) \right\} \] (4.9)

> 0,

where superscript ‘+’ indicates the number of "good" firms shut under the uninformed policy, whenever \( N_0 \) and new entry are the same in both situations. Notice that the increase in costs due to extra shutdown is always dominated by the savings from eliminating a surely "bad" firm, while keeping a good firm is a solid gain from both the cost and benefit side.

We expect this increase value to hold for less extreme improvements in information; the better the information about the underlying quality of the firm, the greater the value of the problem to the planner. To pursue this, one needs a precise definition of "better" information. Here I propose, without exploring its full implications, one such definition.

**Definition 4.3.** For a given set of firms \( N = N^+ \cup N^- \), where \( N^+ \) contains firms with expected value \( y_1 \) and \( N^- \) contains those of expected value \( y_0 \), we say that a likelihood vector \( \pi' \) is more informative than \( \pi \) if the vector inequalities, \( \pi'^+ \geq \pi^+ \) and \( \pi'^- \leq \pi^- \), hold with at least one strict inequality, where the superscripts refer to the corresponding subsets of firms in \( N \).

The definition is illustrated in Figure 6. Note that this definition is consistent with an improvement in information under Bayesian updating of the likelihoods, \( \pi \); as the argument in Lemma 3.3 indicates, \( \bar{\pi} \) is increasing to 1 conditional on the firm being "good," while it is decreasing to 0 when the firm is "bad." It is

¹⁶Notice that, given identical initial conditions and realizations of \( \eta \), the carrying costs of latent firms and the impact of new entry is identical in both situations. Hence, conditioning on the true quality of operating firms, we need only consider the direct impact of differing shut down decisions.
immediately evident that there will be greater likelihood of shutdown of firms likely to be "bad" and a lesser likelihood of closing firms expected to be "good." This is reflected in an increase in the expected savings from the \( m^* \) firms to be shut down and an increase in the expected value created by those firms remaining in operation. It is always the case, conditional on the better information, that the increase in expected savings will compensate for the marginal cost of additional shutdown, thereby further increasing the value of the program \( V^1(\pi, N, L) \). This reflects the fact that the expected cost of each of the two types of errors balanced in the optimal policy is reduced by more information. It is just a matter of straightforward calculations, similar to those above, to show:

**Proposition 4.4.** For given sets, \( N, L \), of active and latent firms, if the likelihood distribution \( \pi' \) is more informative than \( \pi \), then \( V^1(\pi', N, L) > V^1(\pi, N, L) \), conditional on the information in \( \pi' \).

Thus we can clearly see the value of information in this model. Further, we see that improved information can make the optimal policy no less aggressive (in terms of likelihood cutoff for shutdown), so that the Bayesian learning that takes place with the passage of time will tend to stimulate further shutdowns, even if no further "bad" firms arise. Only the elimination of shutdown targets, i.e. those firms with \(-y_{\pi'} > C(1)\), will lead an optimal restructuring policy to avoid all further shutdowns.

### 4.2. Planning Horizon and Optimal Timing.

In Proposition 3.6 we saw that the optimal cut-off likelihood for shutting down weak firms was increasing in the length of the horizon \((T = 1 > 0)\). It is also the case that the marginal shutdown likelihood increases when there are two periods, but no shutdown is allowed in the second period. In that case, the marginal benefit to shutdown increases by a factor of \((1 + \beta)\) as expected savings from closing value-destroying firms now accrue over two periods, while the incremental carrying costs of unemployed resources are only \( \beta c < \beta C(1) < -\beta y_{\pi^*} \). Hence a greater marginal cost, \( \Delta C(\cdot) \), is justified, allowing more firms to be optimally shut down as can be seen by comparing Figure 1 and Figure 3.

But a more interesting comparison is, naturally, between the two-period situations with and without the possibility of second period shutdown. The latter might arise if, for example, a change in the political situation were to prevent any further shutdown in the second period. Both cases take into account the
fact that current shutdown of failing firms implies savings both now and in the future, but only the former case allows the optimal spreading of (convex) costs over both periods. Note that this implies that a single round of closures (if that were all that was politically, say, feasible) should never be put off until the later period; bearing the costs "up front" allows for two periods of savings, always exceeding the extra maintenance costs of unemployed resources $[C(1) \geq c]$, as well as for potential entry of new productive firms. Thus we might expect that the policy should be more conservative when there is an option for future shutdown. That this is indeed the case is easy to see, although optimal cumulative shutdown across two periods is naturally greater than the total that is optimal when only one round of shutdown of failing firms is possible.

If there is a future in which no further action can be taken, then the problem becomes

$$\max_{0 \leq m^* \leq N_0} \left\{ \sum_{i=m^*+1}^{N_0} y_i - C(m^*) - cL_0 + \sum_{j=0}^{L_0} i^0 \cdot j \cdot [y_q + r] + \right.$$

$$\left. + \beta \cdot \sum_{j=0}^{L_0} \nu_j^0 \cdot E \left[ \sum_{i=m^*+1}^{N_0} \tilde{y}_i + \sum_{k=1}^{i} \tilde{y}_{q_k} + [y_q + r] \left( \sum_{h=1}^{L_1} k \cdot \nu_i^1 \right) - L_1 \cdot c \left| F_0 \right. \right] \right\}$$

(4.10)

where $L_1 = L_0 + m^* - j$. Again let $\Delta W_m$ be the marginal gain from closing the $m$-th firm,

$$\Delta W_m = -(1 + \beta) y_m + \xi_m [y_q + r] - \Delta C(m) - \beta c,$$

where $\xi_m$ is defined in Proposition 3.6.\(^{17}\) Then, at the optimum, as in all the prior results (e.g. equation (3.18)),

$$-(1 + \beta) y_m \cdot \xi_m + \beta \xi_m \cdot [y_q + r] \geq \Delta C(m^*) + \beta c$$

$$-(1 + \beta) y_m \cdot \xi_m + \beta \xi_m \cdot [y_q + r] < \Delta C(m^* + 1) + \beta c$$

Comparing with equations (3.13) shows that the optimal first period cutoff, $\pi^*$, must be greater than $\pi^*$ when there is no second period opportunity to close firms: $-y_{\pi(m^*)} < -y_m \cdot \pi^* > \pi^* (3.15)$, so it is optimal be bear only a lesser cost of shutdown in the initial period. Further, $\pi^{**}$ can be no greater than $\pi^*(m^*)$ without violating the first order conditions. Thus, as again illustrated in Figure 3, the possibility of cost spreading gives

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\(^{17}\) $\xi_m$ is the same here, modulus the differing $m^*$, as it is a function only of the initial conditions and first period decision.
Proposition 4.5. The optimal first-period shutdown margin, $\pi^*$, is lower than $\pi^{**}$, and hence fewer firms are shutdown in the first period, when further shutdown is possible in the second period, than is the case when shutdown decisions can only be made in the first period.

Corollary 4.6. $m^* + \tilde{k}(m^*) > m^{**}$, i.e. more total shutdown optimally occurs when it can take place in both periods.

With two periods in which to make shutdown decisions the optimal policy is hence less aggressive in the first period, while eventually cutting deeper into the pool of firms likely to be “bad.” This result provides one sense in which “shock therapy” could be considered optimal when there is a narrow political ‘window of opportunity’ as has been argued in, among others, Aslund (1995).

Another comparison of interest deals with the optimal policy trajectory when there will be perfect information in the final, but significant uncertainty in the initial, decision period. The two-period problem is formally still given by (3.11), with $V^0$ replaced by $V^0f$ and $N^+_0$ and $N^-_1$ depend on the prior shutdown decision, $m^* = m^+ + m^-$, including mistaken shutdowns of ‘good’ firms: $m^+$.

The key to understanding how that dramatic improvement in information affects the first period optimal decision lies in the variational analysis, similar to that in Lemmata (3.4) and (3.5), of $V^0f(\cdot)$, as the initial period shutdown level, $m$, changes. It is straightforward to see:

Lemma 4.7. Let $\tilde{k}_{\ell^-}(m)$ be the optimal shutdown in the second (full information) period when $\ell^-$ “bad” firms enter, and $\Delta V^0f(\ell^-)(m)$ be the expected change in value for the entry of $\ell = \ell^+ + \ell^-$ new firms, when $m$ firms are shutdown in the first period. Then

1. $\tilde{k}_{\ell^-}(m) = k_{\ell^-}(m - 1)$ if $\Delta C(N_0^- + \ell^- - m + 1) > -y_0$;
   \[ \text{If} \Delta C(N_0^- + \ell^- - m + 1) \leq -y_0, \text{then} \tilde{k}_{\ell^-}(m) = k_{\ell^-}(m - 1) + 1; \]

2. $\Delta V^0f(\ell^-) = \Delta V^0(m)$ if $\Delta C(N_0^- + \ell^- - m + 1) > -y_0$;
   \[ \text{If} \Delta C(N_0^- + \ell^- - m + 1) \leq -y_0, \text{then} \]
   \[ \Delta V^0f(\ell^-)(m) = -\pi(m)y_1 + (1 - \pi(m)) \Delta C(\tilde{k}_{\ell^-}(m)) + \Delta V^0(m) \]
   \[ < -y_{\pi}(m) + \Delta V^0(\ell^-)(m) = \Delta V^0(m) \]

[see Lemma 3.5].

\footnote{This is a critical difference from the two-period full information case where there is never a mistake of shutting down “good” firms, so $m^* = m^-$ only.}
Proof. This follows almost immediately from Proposition 4.1, letting $\tilde{\pi}(m)$ be the cutoff likelihood when $m$ firms are closed in the first period. If the marginal cost of shutting all remaining bad firms is less than $-y_0$, it will be done. If an additional firm is then shut earlier, the cost of doing so later if it is “bad”, $\Delta C(\tilde{k}_t-(m-1))$, is saved, as $\tilde{k}_t-(m) = \tilde{k}_t-(m-1) - 1$. If, however, it was “good,” then there is an expected loss in value of output, $-y_1$, so the impact on program value (ex-ante) is determined by the prior probability that the firm is “good” (therefore not requiring shutdown). If, on the other hand, it is too costly to close all “bad” firms in the second period, then that will remain the case with the additional shutdown, $\tilde{k}_t-(m) = \tilde{k}_t-(m-1)$, and the impact on (ex-ante) expected value of the program is the same as with continuing imperfect information: with probability $\tilde{\pi}(m)$, $y_1$ is lost, and with probability $(1-\tilde{\pi}(m))$, $-y_0$ is gained. Finally, the impact on the expected value of new entry is unaffected by the improved information in the second period. \[\] This result allows us to give some conditions such that full information in the final period leads to greater caution earlier. They depend critically on how many “bad” firms there turn out to be, after shutdowns and new entry have taken place. Of course, in the first period the industry decision maker can never be sure how many “bad” firms there are, and so must probability-weight the two payoff relevant events analyzed in Lemma 4.7; his expectation of future value will depend not only on his beliefs about the quality of existing firms but on the distribution over the entry of ”bad” firms. To state our result, let $A(m)$ be the event that $N_t^-(m, \ell^-) > C^{-1}(-y_0)$, $N_t^-(m, \ell^-) = N_t^- + \ell^- - m + 1$, and $B(m)$ be its complement. The Bellman equation, as before, is

$$V^1(\pi, N_0, L_0) = \max_{0 \leq m \leq N_0} \left\{ \sum_{i=m+1}^{N_0} y_i - C(m) - cL_0 + \sum_{j=1}^{L_0} \nu_j^L \cdot j \cdot [y_q + r] + \beta \sum_{j=1}^{L_0} \nu_j^L \left( E \left[ V^0(\tilde{\pi}^f, N_0 - m + j, L_0 + m - j) \left| \mathcal{F}_0, j \right. \right] \right) \right\},$$

where terminal value is given by equation (4.4), and its expectation is taken with respect to the distribution of “good” $[N_t^+]$ and “bad” $[N_t^-]$ firms.

**Theorem 4.8.** An optimal solution $\{(m^0, x^0, \sigma^0), (\tilde{k}^0(m^0), \tilde{\sigma}^0(m^0))\}$ to the two period problem with second-period full information satisfies conditions (4.2) on event $A(m^0)$ and (4.3) on $B(m^0)$ for the terminal period. For the initial period,
the necessary and sufficient conditions are given by:

$$\pi_{m^0} \leq \pi^0 \leq \pi_{m^0+1}, \quad \sigma^0 = 0,$$

$$-(y_\pi^0 + \beta [y_{\pi^0} - \xi_{m^0}(y_q + r)]) = \Delta C(m^0) + \beta c,$$

or

$$\pi^0 = \pi_{m^0+1}, \quad \sigma^0 > 0,$$

$$-(y_\pi^0 + \beta [y_{\pi^0} - \xi_{m^0}(y_q + r)]) = (1 - \sigma^0)\Delta C(m^0) + \sigma^0\Delta C(m^0 + 1) + \beta c,$$

on event $A(m^0)$, i.e. when known “bad” firms are optimally left in operation in the second period. On event $B(m^0)$,

$$\pi_{m^0} \leq \pi^0 \leq \pi_{m^0+1}, \quad \sigma^0 = 0,$$

$$-(y_\pi^0 + \beta \left[\pi^0 y_1 - (1 - \pi^0)\Delta C(N_1^-) - \xi_{m^0}(y_q + r)\right]) = \Delta C(m^0) + \beta c,$$

or

$$\pi^0 = \pi_{m^0+1}, \quad \sigma^0 > 0,$$

$$-(y_\pi^0 + \beta \left[\pi^0 y_1 - (1 - \pi^0)\Delta C(N_1^-) - \xi_{m^0}(y_q + r)\right]) = (1 - \sigma^0)\Delta C(m^0) + \sigma^0\Delta C(m^0 + 1) + \beta c,$$

give the appropriate marginal conditions. Further, $\left(\pi y_1 - (1 - \pi)\Delta C(N_1^-)\right) > y_\pi^0 > v_0 \geq y_0$.

**Remark 10.** When some “bad” firms must optimally remain in operation in the second period [event $A(m^0)$], the marginal conditions are the same as those when the shutdown decision can only be made in the first period [see (4.11)]. However, if there is some slack [event $B(m^0)$], or even a positive probability of such slack, then the policy with full information in the future period becomes more conservative, $m^0 \leq m^\ast$ and $\pi^0 < \pi^\ast$.

Note that the uncertainty in the first period reduces the expected marginal return in the second period, despite full information at that time, because of the marginal likelihood, $\pi^0$, of shutting down a good firm by mistake. Thus this policy is more cautious than the full information first period optimal policy: $m^0 < m^f$. It will usually, however, not be more cautious in the first period than the optimal policy in the uninformed case (Theorem 3.6), which, as we have seen, is more
conservative than shutdown policy when there is not a second round. Whether it is so, depends on the relationship between \(y^*_m(m^*)\) and \((\pi^0 y_1 - (1 - \pi^0)\Delta C(N_1))\) and on the probability of \(B(m^0)\). If slack in the second period, implying low marginal costs of second period shutdown, is sufficiently likely, i.e. there are few “bad” firms and a low probability of entry of such, then it will be optimal to hold back in the first period, setting \(\pi^0 < \pi^*\), thereby spreading some of those costs across the two periods. In event \(A(m^0)\), however, \(m^0 = m^{**} > m^*\), so better information in the future encourages more aggressive initial restructuring.

5. Conclusions.

The analysis of this paper deals with the simplest formulation of a dynamic industry restructuring problem. It makes a number of quite strong assumptions in order to get an analytic handle on the complexities of enterprise shutdowns during the transition process. Among the assumptions that one would like to relax are those of the homogeneity of firms (the ‘standard firm’ assumption), the direct connection between the market generated signal of ‘viability’ and the social value of the firm, the lack of interdependence among firms, the static environment, and the inability of firms to alter their own viability through investment. These, as well as extending the analysis to a multiperiod frame work, provide directions for continuing research.

Despite these limitations, the model provides a basis for useful insights into some of the difficulties of the transition. It captures, in tractable ways, critical elements of the process — the randomness in current performance, the uncertainty about underlying viability, the learning from performance outcomes, the convex costs of closing industrial operations, the costs of keeping released resources usable in the future, and finally, the critical connection between shutdowns and new entry.

The results of the model highlight the necessary conservatism of the optimal policy: it balances the marginal costs of errors arising from the lack of information, leaving many firms that are likely to be net value-destroying in operation. This, in part, is a reflection of the option value of ‘waiting;’ the industry policy maker will know better in the future whether the firm is ‘bad’ or not. Despite this, however, the optimal policy is more aggressive in the early period, reflecting two factors. First, the expected savings to any closure in the first period accrue over the full horizon, and hence are worth almost twice savings in the second period. Second, shutdown in the first period allows for greater entry of valuable new firms in the second period. Both factors work to limit the desire of an optimizing industry
authority to spread the convex costs of necessary shutdowns over both periods, and indeed counteract, to some extent, the option value of waiting.

The second factor, highlighted in this model, has been, I believe, underappreciated in the transition literature. The release of resources from wasteful industrial operations acts as a countervailing option to that of waiting. By facilitating new start-ups with an expected viability much greater than that of the closed enterprises, the release of resources from shutdowns mitigates the hysteresis induced by the value of waiting. It must be emphasized that this is an effect that would be absent if the shutdown decision were being taken in a well functioning market economy where resources for new start-ups are available at their opportunity cost. In transition economies, and in particular the former Soviet Union, that is far from the case; resources, even if unemployed or seriously underemployed, are largely frozen in place by prior socialist overemployment policies, the lack of working factor markets (missing infrastructure and intermediaries), the lack of usable property rights, and transition policies that support existing industrial operations, strengthening their grip on existing resources and factors of production. One of the key messages of this paper is that shutting down existing industrial operations, by refusing to support them when the expectation that they are market-viable is sufficiently low, has an additional positive impact through new entry that must be considered in transition industrial policy.

Thus the optimal shutdown threshold, and number of optimally closed firms, is a complex function of many such factors that balances the countervailing forces introduced in the model. Overall, it supports something of a 'big bang' approach in the sense that earlier shutdown generally dominates, even when information is expected to improve dramatically in the future, and the number of firms optimally shut down declines rapidly over time. Convexity of costs and the possibility of mistakenly closing viable firms, however, limit the size of that initial 'bang,' as well as the number of future shutdowns, unless there is a dramatic improvement in information. Better information about the underlying quality of firms generally leads to a more aggressive first-period shutdown policy, as does a longer horizon, and full information maximizes the amount of shutdown that it is optimal to undertake. These results provide the first steps toward developing a general model of industrial transition, one which must include interdependencies among the firms and the ability of firms to make autonomous decisions about their own future, as well as the (non-)supporting role of central policy makers.

**********
References


Figure 1.
Case (a)
\[ \Delta C(\tilde{m}) > -y_{\tilde{m}(m)+1} \]
\[ \Rightarrow \tilde{m}(m-1) = \tilde{m}(m) \]

Case (b)
\[ \Delta C(\tilde{m}) \leq -y_{\tilde{m}(m)+1} \]
\[ \Rightarrow \tilde{m}(m-1) = \tilde{m}(m)+1 \]

Figure 2.
\( \Delta C_0 \) - marginal costs of shutdown from \( N_0 \) firms.
\( \Delta C_i \) - marginal costs of shutdown from \( N_i \) firms.

**Figure 3.**
Figure 4.
Figure 5.
--- perfect information about type.

--- $y_{\Pi_0}$ : information $\Pi_0$

+++ $y_{\Pi_1}$ : information $\Pi_1$

$\Pi_1$ "better" than $\Pi_0$

**Figure 6.**
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