THE WILLIAM DAVIDSON INSTITUTE
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# *Agency in Project Screening and Termination Decisions:*

## *Why Is Good Money Thrown After Bad?*

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We construct an agency model in which the planner (agent) makes project starting and termination decisions on behalf of the state (principal) to reflect the practice of socialist economies. The model shows that asymmetric information between the state and the planner regarding the quality of projects started leads to the persistence of unprofitable projects. Since in the model it is assumed that the state's objective is to maximize economic profit and the state has full power to dictate and enforce the optimal contract, the finding of the model has the implication that hardening budget constraints in socialist economies is difficult even under an "ideal" setting when these economies are free of social considerations and political frictions.

JEL classification: P51, D82
Key Words: Ex Post Inefficiency, Agency, Project Screening [and Termination], Information.

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Abstract*

We construct an agency model in which the planner (agent) makes project starting and termination decisions on behalf of the state (principal) to reflect the practice of socialist economies. The model shows that asymmetric information between the state and the planner regarding the quality of projects started leads to the persistence of unprofitable projects. Since in the model it is assumed that the state's objective is to maximize economic profit and the state has full power to dictate and enforce the optimal contract, the finding of the model has the implication that hardening budget constraints in socialist economies is difficult even under an "ideal" setting when these economies are free of social considerations and political frictions.

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## 1. Introduction

Since Kornai's (1980) seminal work, it has been widely recognized that socialist economies are flawed in allocating investment funds and suffer from the "soft-budget" problem.[1] Particularly puzzling is the question of why money-losing projects are not terminated.[2] The purpose of this paper is to argue that the phenomenon of not terminating unprofitable projects can be explained by agency problems in the sequential decision process of project screening and termination. In this introduction, we first briefly describe the main idea of our argument and then compare it with explanations offered by others.

We model an economy in which there are many projects, each requiring an initial and a subsequent investment before the return can be realized. The projects have continuously distributed returns, some of them *ex ante* (before the initial investment) profitable, others *ex ante* unprofitable but *ex post* (after the initial investment) profitable, and still others unprofitable both *ex ante* and *ex post*.[3] The principal in the model is the state, and the agent is the planner (the bureaucrat). The agent first examines the profitability of the projects, the only action that requires an effort, and then decides which projects to undertake. The number of unprofitable projects successfully identified increases with the agent's screening effort. Once the projects to

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[1]The problem caused enormous inefficiency in former socialist economies and contributed greatly to the eventual political collapse of many of them. As a significant economic phenomenon, the problem has not disappeared with the reforms undertaken in socialist economies. In China, for example, after 17 years of market-oriented reforms, poor financial performance continues to plague the state sector. In recent years, fully one-third of state-owned enterprises persistently lost money. Another one-third are believed to have made money solely due to their receipt of various forms of state subsidies.

[2]Kornai (1980, p.197) described the problem in the following words: "From the claimant's point of view investment is a long campaign with many battles. But the whole campaign has only one life-and-death battle and that is at the beginning, since approval must be obtained for starting the investment. Once started, it will end in some way and at some time. That is exactly why it is possible to underestimate, without much hesitation, expected costs, and to forget about complementary investments. If costs are higher, or if investments above the plan are necessary, money will surely be raised in one way or another. Perhaps the claimant will be blamed for erroneous calculations, perhaps work will slow down for a while to wait for financial cover, but an investment project that has been started will not be stopped for good."

[3]The terms "*ex ante*" and "*ex post*" here have the same meaning as in Dewatripont and Maskin (1995). A project is *ex ante* profitable if its return is greater than the total of the initial and the second investment. It is *ex post* profitable if its return is greater than the second investment.

be undertaken have been chosen, and the initial investments made, the agent obtains perfect information about the quality of all started projects; the principal does not. The agent then decides which of these projects to terminate. Projects that are not terminated will receive the second investment and, after that, realize returns. The main question of interest is: What should the state's contract with the planner say about project termination decisions?

If the principal does not impose restrictions on the agent's project-termination decisions, and ties the agent's income only to aggregate net output, then the agent will terminate all *ex post* unprofitable projects (projects that have a return smaller than the second investment) to maximize net output. However, as we will show, by requiring the agent to continue some *ex post* unprofitable projects in the *ex ante* incentive contract, the principal can expect a higher profit. The reason for this is that, if an *ex post* unprofitable project is not screened out, it costs the agent more if it can not later be terminated than if it can. Therefore, requiring the agent to continue some *ex post* inefficient projects has the effect of increasing the marginal benefit of effort to the agent.[4] This effect relaxes the incentive constraint for the agent's effort and, given the sharing rule, induces a higher effort. The higher effort leads to the benefit of more unprofitable projects being identified in the initial screening so that investments are not wasted on them. Restricting the agent's flexibility in project termination is desirable as long as the cost of continued financing of *ex post* unprofitable projects is smaller than the benefit of the resulting higher screening effort. This is usually true in the case of infinite projects with continuously distributed returns because of the envelope theorem.

Another prediction of the model is that, if the expected return of an unexamined project is greater than the total investment costs, then the principal should also impose restrictions on project starting so that the agent has to forego some projects that he would otherwise start. The intuition for this result is similar to that for restricting project termination: As effort decreases,

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[4]Another way of understanding the result is as follows. When the agent's effort is higher, more bad projects are found by the agent before the initial investment and quality of projects that are started is higher. As a result, the cost of continuing an *ex post* unprofitable projects is lower. Therefore, imposing restrictions on project termination costs the agent less when effort is higher.

the agent identifies fewer bad projects so that, if he is restricted from starting projects, he has to forego more profitable projects and is therefore hurt more. The model also shows that the principal should mandate the division of investment funds into that for starting projects and that for supporting ongoing projects.

At the first glance, our explanation for the persistence of unprofitable projects seems rather counterintuitive; what we often observe in a hierarchically organized economy is that agents keep requesting for ever more funds and mount (political and/or other) pressures on their principals (superiors) to meet their requests. The principals, on the other hand, often seem reluctant to provide all funds requested by agents. This leaves the impression that soft budget constraint results from a principal's giving in under pressure to an agent's request for more funds. However, once we look closer to see how the principal and the agent disagree on whether investment funds should be used to start new or support old projects beyond the fact on the surface that the agent is always requesting for more funds, the empirical relevance of our model becomes more apparent. Evidence from China's reform experience shows that, in many cases, agents prefer to start more new projects, while the principal takes the opposite position and introduce measures to force agents to start fewer new projects and better fund existing projects, just as our model predicts.

Previous explanations of the soft budget problem have focused largely on social and political grounds. Kornai (1980) attributes it to the "paternalistic" role of the socialist state. Bardhan (1993) attributes it to the lower tolerance of socialist than capitalist societies for mobility and unemployment. Li (1996) suggests that employment stability has a social value taken into account by socialist but not by private capitalist employers. Also, the reports that workers of a shipyard in an East European country struck to force the government to back down from a decision to reduce subsidies suggest that soft budget constraints can be a result of political bargaining. The problem exists because socialist governments are responsible for overseeing project performance, but politically too weak to commit to terminating money-losing projects.[5]

Dewatripont and Maskin (1995) provide an economic rationale for the soft budget constraint by focusing on an informational feature of a centralized system. In their model, managers have the *ex ante* information of the profitability of projects and propose projects to be undertaken. Projects approved by the creditor, which is a government agency in socialist economies, are undertaken. Faced with limited opportunities, managers have an incentive to propose projects that are *ex ante* unprofitable, but *ex post* profitable. The creditor can try to discourage managers from proposing *ex ante* unprofitable projects by threatening not to refinance them. The threat, however, is not credible because, when the creditor learns about the quality of the project, the initial investment in the project is already sunk. It is then in the creditor's self interest to finish projects that are *ex post* profitable. Schaffer (1989) and Segal (1993) also offer explanations based on time-inconsistency of the central planner.

We see these different explanations as complements rather than substitutes to each other, each of them offering some insight, but, by itself, explaining only part of the complex phenomenon of the soft budget constraint. For example, while the "preference" of the socialist state (paternalism or employment preference) can sometimes lead the state to "bailout" money-losing projects, one could also argue that improved project performance under hard budget constraints would only enhance the state's ability to play its paternalistic role. The degree to which the government or people in a socialist country might want to trade the greater long-term instability and higher unemployment associated with a less efficient economy for increased short-term stability and employment is also not clear.[6] Similarly, while the bargaining story is plausible in some cases, it does not seem to explain why soft budget constraints also prevailed in

[5]Note that the commitment problem here is due to the government's lack of political power, which is different from that in Dewatripont and Maskin (1994), Schaffer (1989), and Segal (1993) where the failure is due to time inconsistency in the government's own behavior.

[6]Indeed, since communist governments do not face elections every few years, one could quite reasonably argue that they are better able to take measures that reflect the long-term interests of the people even though these measures may cause short-term political damage due to a higher level of short-term unemployment.

socialist economies during the times of Stalin, Mao or Deng when the KGB or troops would be sent to crush any resistance to the leader's will. Dewatripont and Maskin's work and those of Schaffer (1989) and Segal (1993) address the important question of why in socialist economies *ex ante* inefficient projects are started and refinanced. However, since these explanations are based on the idea that the government can not commit to not pursuing *ex post* efficiency, they do not address the question as why *ex post* inefficient projects are not terminated.[7]

Our approach of abstracting from political considerations enables us to focus on a most fundamental feature of socialism: socialist economies are hierarchically managed with the means of production publicly owned (practically state owned).[8] This ownership arrangement precludes the trading of productive equities and, thereby, eliminates the price mechanism for information about project quality. With an alternative mechanism to secure information yet to be found, asymmetric information exists between the state and the planner regarding the quality of ongoing projects.[9] A valuable insight of our model is that this asymmetric information problem in the hierarchically managed socialist system is sufficient for the soft budget problem to arise. By assuming that the state's objective is to maximize profit and it can dictate optimal contracts, our model studies the soft budget problem in a socialist economy that is "perfect" except for asymmetric information between the state and the planner. The result obtained under these assumptions shows that the soft budget constraint is unavoidable in socialism even under an ideal setting when it is free of social considerations and political frictions. It is very suggestive of the limited extent to which the socialist system can improve the efficiency of investment fund

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[7]In these models, the government can pursue *ex post* efficiency because *ex post* there is no asymmetric information between the government and managers. In contrast, in our explanation, the state's behavior is time consistent. Asymmetric information regarding the profitability of projects persists after the initial investment. As we will discuss in detail later, because of this problem of *ex post* asymmetric information, renegotiation does not qualitatively alter the result of our model.

[8]Public ownership of the means of production is one of the most fundamental teachings of Marxism.

[9]Price signals from equity markets may not provide perfect information on project quality. Also, it is not clear if alternative institutions exist that can provide equally good or better information of ongoing projects. Practically, however, it seems that a more effective institution than the equity market is yet to be found.

allocation without doing away with some of its most fundamental features.

Our focus on the informational problem in socialist economies and the result derived from the model echo a view emphasized by Hayek (1945), who argued that a main problem of "designing an efficient economic system" is how to utilize "knowledge not given to anyone in its totality." He then suggested that it is in utilizing information "initially dispersed among all the people" through a price system that a decentralized economy enjoys a merit not shared by centralized planning. Following Hayek, Stiglitz (1994) also sees informational problems in socialism as "perhaps the most important reason" for failure (p.198).[10] How to better utilize information in an economy is also a focal point in the discussion of market socialism, both historically and most recently.[11]

The plan for the remainder of the paper is as follows. The model is introduced in Section 2. Section 3 discusses how the planner's effort and the restrictions on the planner's project starting- and termination-decision affect the expected gross value-added of the projects. The main result of the model is derived in Section 4. Section 5 presents evidence supporting the empirical relevance of our model. Section 6 discusses some of the assumptions made in the model. Section 7 concludes the paper with some additional remarks.

## 2. The Model

The principal in the model is the socialist state and the agent the planner (bureaucracy). We assume that there are a continuum of projects.[12] Without loss of generality, we assume that the projects are uniformly distributed on the unit square $[0,1]\times[0,1]$, represented by area OCKH in Figure 1. The expected revenue (*type*) of a project, denoted by $\alpha$, is given by the horizontal

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[10]On the role of price in designing an efficient economic system, Stiglitz (1994, p.202) points out that "...while the price system may be imperfect, it performs a number of vital roles." One of such roles is that "...prices...provide the basis of an incentive structure and a selection mechanism" in an economy.

[11]See Bardhan and Roemer (1993) for a review of the history and a collection of recent works on market socialism.

[12]We will discuss the case in which there are finite projects in section 4.

coordinate of the point representing it. Therefore, $\alpha$ is uniformly distribution on [0,1] and there are infinitely many projects of each type.

The relationship lasts two periods, given by time points t = 0, 1, 2. The projects have the technological feature that, to realize the return of a project, two investment are needed: investment of amount $\$ c$ in the first period, and $\$ i$ in the second. For simplicity, we assume that the interest rate is zero. We call the difference between the revenue and the investment costs the value-added of a project. If the project is completed, its expected value-added is $\alpha - c - i$. If it is terminated after the first investment, then its value-added is $-c$. A project is *ex ante* (at time 0) profitable if its expected value-added is non-negative, that is, $\alpha \geq c + i$. In Figure 1, area BCKJ is the set of *ex ante* profitable projects. Projects with $\alpha \in [i, c+i)$ are *ex ante* unprofitable because $\alpha < c + i$, but they are *ex post* (at time 1) profitable because $\alpha \geq i$ --- by then the first investment $c is sunk. In Figure 1, area ABJI is the set of *ex ante* unprofitable but *ex post* profitable projects. Finally, projects with $\alpha \in [0, i)$ (in area OAIH in Figure 1) are unprofitable both *ex ante* and *ex post*.

The timing of events is given in Figure 2.

| | |
|---|---|
| before time 0 and | The principal receives project proposals, hires the agent, signs an incentive contract with him. |
| time 0 | The agent examines the projects and decides which projects to fund. |
| first stage | The first-stage investment $c is made. |
| time 1 | The profitability of funded projects is revealed to, and the termination decision is made by, the agent. |
| second stage | The second-stage investment $i is made. |
| time 2 | Returns to the projects are realized and the agent is rewarded. |

Figure 2

Before time 0, the principal hires an agent to screen the projects and make investment decisions on the principal's behalf. The principal signs an incentive contract with the agent to maximize net revenue, which is the gross revenue net of investment costs and payment to the agent.

At time 0, the planner examines the project proposals.[13] We assume that each proposal is reviewed with probability e, which is normalized to be the planner's effort level. The range of e is of course [0,1]. When a project is reviewed, its true profitability is identified. The set of such projects is area OCDG in Figure 1. No new information is acquired about projects that are not reviewed (in area GDKH in Figure 1).[14]

After the initial screening, the planner decides which projects to start, and invests $\$ c$ in each of them. Let $s$ be the number of projects started.[15]

At time 1, the profitability of all the projects that are funded in the first stage is revealed to the planner, but not to the principal. Some of the projects are *ex post* unprofitable, as expected. The planner then chooses to terminate some of them. Let $t$ be the number of projects terminated. s and $t$ are public information.[16]

In the second stage, an additional investment of $i is required for each project that is not terminated at time 1 in order to realize its return.

At time 2, the returns of the retained projects are realized and the planner is rewarded

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[13]In this paper, we assume that the managers under the administration of the planner are each endowed with a project of given profitability. As long as a project is started, the manager will receive a positive *private* benefit that cannot be taken away. We also assume that the manager has no personal wealth and thus cannot be penalized for proposing a poor project. Under these assumptions, the manager always proposes the project to the planner, regardless of its quality.

These assumptions about the managers are made so that we can focus on the planner's agency problem. In a companion paper, we add to our consideration the manager's agency problem in project searching effort and the possibility of penalizing the manager.

[14]Such a screening technology is very similar to that used by many to study the monitoring problem in hierarchies, e.g., Calvo and Wellizs (1978) and Qian (1994), called "imperfect supervision and monitoring but perfect observation".

[15]Since the set of projects is not countable, the term "number" is abused. The more proper terms are "the probability measure" or "the proportion". However, we ask the reader to tolerate this abuse of the term because "the probability measure" is a mouthful expression and “the proportion” can be confusing due to changes in the reference population over time.

[16]Starting or terminating a project is usually a high profile public event.

according to the incentive contract signed with the principal before time 0.

The expected gross value-added thus depends on screening effort e, projects started, s, and projects terminated, t. We denote it by $y(e,s,t)$. To compute $y(e,s,t)$, let us look at Figure 3. In the figure, NS is the set of projects that are not started, T is the set of projects that are terminated at time 1, and C is the set of projects that are completed. NS, T, and C are mutually exclusive and collectively exhaustive. Let $m$ denote the probability measure in the space of projects (square OCKH in Figure 1). The measure (area) of T is $m(T) = t$ and the measure (area) of C is $m(C) = s - t$. Then,

$$y(e,s,t) = \int_{(\alpha,\beta)\in C} \alpha d\alpha d\beta - (c + i)m(C) - cm(T).$$

Here, the expression of $y(e,s,t)$ is provided to help conceptual understanding. Detailed algebraic definition will be provided in Section 3, where we will see that the shapes of NS and T depend on s and t, and can be different from those depicted in Figure 3.

We assume that the gross value-added is given by

$$x = y(e,s,t) + \theta,$$

where $\theta$ is a random variable with mean 0 and probability density function $g(\theta)$. The value of $\theta$ is realized at time 2. The principal can observe the realization of $x$, but not that of $\theta$ or $y(e,s,t)$.[17]

Besides $x$, $s$, and $t$, the principal observes the total investment budget, denoted by $b$. However, since $b$ is uniquely determined by $s$ and $t$ --- $b = sc + (s-t)i$, the principal only needs to include $x$, $s$, and $t$ in the incentive contract; that is, the payment to the planner is $w(x,s,t)$.[18]

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[17] $\theta$ is a common shock to all projects under the planner's jurisdiction. Without this common shock, the first-best effort of the planner can be implemented by "selling the economy to the planner", i.e. demanding a certain payment from the planner and giving him all the residual value-added. When there is no uncertainty, this arrangement is efficient even when the planner is risk averse or/and has limited liability. Idiosyncratic shocks are not sufficient, because the number of projects is large and, by the law of large numbers, idiosyncratic shocks will be averaged out.

[18] When there is more than one planner, the state can use relative performance evaluation, as discussed by Lazear and Rosen (1981) and Holmstrom (1982), among others. Our results are robust to this possibility. According to Holmstrom (1982), the optimal incentive scheme depends on a planner's performance alone if the $\theta$'s are

Assume that the planner's utility function is

$$v(w) - d(e),$$

where $w$ is the income of the planner and $d(e)$ the disutility of effort $e$. Assume that

$$v' > 0,\ v'' < 0,\ d' > 0,\ d'' > 0,\ d'(0) = 0, \text{ and } d'(1) = \infty.$$

Let $w(x,s,t)$ be the principal's wage offer to the agent. Then the expected utility of the agent and the expected net revenue of the principal are, respectively,

$$u(e,s,t) = E_\theta\{v[w(x(e,s,t,\theta),s,t)] - d(e)\}$$

and

$$\pi = E_\theta\{x(e,s,t,\theta) - w(x(e,s,t,\theta),s,t)\}.$$

Our assumption that the profitability of the projects is not revealed to the principal after the initial investment is critical. Otherwise, the principal would be able to deduce the effort of the agent from the observed profitability and base a contract on the effort level. Then, by mandating the first-best screening effort and paying a fixed wage in the incentive contract, the principal can achieve first-best efficiency. The contract leads to efficient project-starting and -termination decisions. In summary, we have,

<u>Proposition 1</u>: *If the principal knows the profitability of the projects at time 1, then she will know the agent's effort in screening the projects and can implement first-best screening effort and efficient project-starting and -termination decisions.*

We now proceed to study and characterize the optimal contract when there is asymmetric information about the profitability of the projects at time 1. We first show that the optimal contract can take a special form.

The general form of the contract is for the principal to offer a wage function $w(x,s,t)$. Given this wage function, the agent chooses his effort, e, the number of projects to start at time 0,

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independent across planners. Relative performance evaluation will not change the qualitative features of our conclusion as long as $\theta$ has a component that is idiosyncratic to individual planners. Note that the idiosyncrasy here is one reason why the state cannot observe $\theta$, and is different from that in the last footnote, where it is with respect to individual projects, as opposed to individual planners.

s, and the number of projects to terminate at time 1, t, to maximize his expected utility. Since s and t are verifiable, the principal can mandate $s=\tilde{s}$ and $t=\tilde{t}$, for some $\tilde{s}$ and $\tilde{t}$. Together with a wage function $w(x)$, they constitute a wage contract of the following special form:

$$w(x,s,t)=\begin{cases} w(x) & \text{if } s=\tilde{s} \text{ and } t=\tilde{t}; \\ -\infty & \text{otherwise.} \end{cases} \tag{SF}$$

Given such a contract, the agent chooses the effort level e to maximize his utility. We have the following standard result:

_Proposition 2_: *Restricting the wage contracts to the special form given in (SF) does not reduce the principal's optimal payoff; that is, the optimal contract can be chosen to be of the special form.*[19]

Proof: Suppose the optimal wage contract is $w^*(x,s,t)$, and under this contract, $e^*$, $s^*$, and $t^*$ are induced. If the principal mandates $s^*$ and $t^*$ and offers a wage contract

$$\hat{w}(x)=w^*(x,s^*,t^*),$$

then the agent chooses e to maximize

$$\max_e u(e,s^*,t^*)=\max_e E_\theta\{v[w^*(x(e,s^*,t^*,\theta),s^*,t^*)]-d(e)\}.$$

By the definition of $e^*$, $s^*$, and $t^*$, the agent's optimal effort is $e^*$. That is, mandating $s^*$ and $t^*$ and offering $\hat{w}(x)$ induces the same effort as offering $w^*(x,s,t)$. The principal gets

$$\pi=E_\theta\{x(e^*,s^*,t^*,\theta)-w(x(e^*,s^*,t^*,\theta),s^*,t^*)\},$$

which is the net revenue that the principal gets under the optimal contract $w^*(x,s,t)$. That is, the maximum net revenue that the principal can attain by choosing the optimal contract from all possible contracts is attained by choosing one,

$$w(x,s,t)=\begin{cases} \hat{w}(x) & \text{if } s=s^* \text{ and } t=t^*; \\ -\infty & \text{otherwise,} \end{cases}$$

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[19] $w$ will depend on $t$ smoothly if the state only observes an imperfect signal of $t$, and/or if the value of $\theta$ is at least partially realized before the project-termination decision is made. A similar remark applies to $s$.

from the special class of contracts. Q.E.D.

Corollary 1: The amount of funds to be used to start projects, $sc$, and that to be used to continue projects, $(s-t)i$, are fixed in the *ex ante* optimal incentive contract.

The result of Proposition 2 makes it much easier to set up the principal's problem of choosing the optimal incentive contract. Since $x = y(e,s,t) + \theta$ and the probability density function of the random variable $\theta$ is $g(\theta)$, the probability density function of $x$ is

$$f(x;e,s,t) \equiv g(x - y(e,s,t)).$$

Given $s$, $t$, and $w(x)$, the agent's utility is

$$u(e,s,t) = \int v(w(x)) f(x;e,s,t)dx - d(e).$$

Then, the principal's problem can be reformulated as

$$\max_{s,t,e} \Pi(s,t,e) \tag{1}$$

where,

$$\begin{aligned} \Pi(s,t,e) = \max_{w(x)} & \int [x - w(x)] f(x;e,s,t)dx \\ \text{s.t.} \quad & \int v(w(x)) f(x;e,s,t)dx - d(e) \geq \\ & \int v(w(x)) f(x;e',s,t)dx - d(e') \quad \text{for all } e', \quad \text{(IC)} \\ \text{and} \quad & \int v(w(x)) f(x;e,s,t)dx - d(e) \geq 0 \qquad \text{(IR)} \end{aligned} \tag{2}$$

$\Pi(s,t,e)$ is the maximum expected net revenue when the principal chooses to induce the effort level $e$ and mandates s and t. (IC) is the incentive compatibility constraint. It requires that the wage contract chosen by the principal must make it optimal for the agent to choose the intended $e$. (IR) is the agent's individual rationality constraint; i.e., the wage contract must be acceptable to the agent.

## 3. Effort and Project Starting and Termination Decisions.

In this section, we analyze the optimal project-starting decision given the screening effort e, the number of projects to start at time 0, s, and the number of projects to terminate at time 1, t.

When t=s, all projects started at time 0 are to be terminated at time 1 and therefore the firm does not get any return from its investment. This is obviously not the optimal choice of s and t and thus we require t<s. We also assume that e>0 and use y(e, s, t) to denote the maximum expected value-added.

Given $e$, the type of some projects is identified. Let $\hat{s}$ be the number of projects that are identified to be profitable; $\hat{s} = e(1-c-i)$. The set of these projects is area $\hat{s}$ in Figure 1. The area above the horizontal line $e$ represents the sets of projects with unidentified profitability.

In Figures 5 and 6, the shaded area represents the set of projects started by the agent. Let s be the size of this area. Suppose that, of the screened projects, the agent chose to start those with profitability higher than $c+i-w$. Since $c+i-w$ can be from 0 to 1, $c+i-1 \le w \le c+i$. Note that $w$ could be negative. The agent may also choose to start some projects that have not been screened and, therefore, have unidentified profitability. Let $h$ be the number of such projects, $0 \le h \le 1-e$. Define $\Delta s \equiv s - \hat{s}$. Then $we + h = \Delta s$. Given $e$, $s$, and $t$, the agent will choose $h$ and $w$ to maximize the expected value-added, subject to the constraints:

$$we + h = \Delta s \tag{c1}$$

$$0 \le h \le 1-e \tag{c2}$$

$$c+i-1 \le w \le c+i \tag{c3}$$

Note that w could be negative. Rearranging constraints (c1)-(c3), we have,

$$(c1) \Leftrightarrow h = \Delta s - ew$$

$$(c2) \Leftrightarrow 0 \le \Delta s - ew \le 1-e \Leftrightarrow \frac{\Delta s + e - 1}{e} \le w \le \frac{\Delta s}{e}$$

$$(c3) c+i-1 \le w \le c+i$$

Then $\underline{w} = \max\left\{c+i-1, \frac{\Delta s + e - 1}{e}\right\}$ is the lower bound, and $\overline{w} = \min\left\{\frac{\Delta s}{e}, c+i\right\}$ is the upper bound, of w; (c2) and (c3) are equivalent to $\underline{w} \le w \le \overline{w}$.

The expected value-added, y, is determined by e, s, t, h, and w, and has two possible forms depending on the number of projects that need to be terminated, t, relative to the number of projects started without known profitability, h.

Case I: $t \le (c+i-w)h$.

In this case, t is sufficiently small so that, at time 1, only the least profitable projects in h need to be terminated; no projects with their profitability identified before they were started at time 0 are terminated. Note that in this case, all terminated projects have profitability less than $c+i-w$. Figure 5 illustrates this case. The expected gross value-added is

$$\begin{aligned} y_1 &= \int_{\frac{t}{h}}^{1}\int_{e}^{e+h}\alpha d\beta d\alpha + \int_{c+i-w}^{1}\int_{0}^{e}\alpha d\beta d\alpha - s(c+i) + ti \\ &= \int_{\frac{t}{h}}^{1} h\alpha d\alpha + \int_{c+i-w}^{1} e\alpha d\alpha - s(c+i) + ti \end{aligned}$$

$y_1$ is defined for $h > 0$ or $w < \frac{\Delta s}{e}$.

<u>Lemma 1</u>: *$y_1$ is concave with a unique maximum point, $w_2$, which is the solution to*

$$t = \sqrt{(2c+2i-1-2w)(\Delta s - ew)}$$

*in the range* $w \in \left(-\infty, \min\left\{\frac{\Delta s}{e}, c+i-\frac{1}{2}\right\}\right)$.

The proofs of all lemmas are provided in the appendix.

Case II: $t > (c+i-w)h$.

In this case, t is large so that some projects with profitability greater than $c+i-w$ also need to be terminated. This means that some projects with identified profitability at time 0 also need to be terminated at time 1. Figure 6 illustrates this case. In the figure, $k$ is defined by $kh + e(k-c-i+w) = t$. Therefore,

$$k = \frac{t + e(c+i-w)}{h+e}.$$

The expected gross valued-added is

$$\begin{aligned} y_2 &= \int_{k}^{1}\int_{0}^{e+h}\alpha d\beta d\alpha - s(c+i) + it \\ &= \int_{k}^{1}(e+h)\alpha d\alpha - s(c+i) + it \end{aligned}$$

Given e, s, t, h, and w, the expected value added is then

$$y = \begin{cases} y_1 & \text{as } t \le (c+i-w)h \\ y_2 & \text{as } t > (c+i-w)h. \end{cases}$$

The agent's optimal choices of h and w, $h^*$ and $w^*$ respectively, are characterized by Lemma 1.

<u>Lemma 2</u>: *Suppose 0<t<s. Let y(e, s, t) be the maximum expected value-added. There exist* $e^*$ *and* $e^{**}$ *with* $1-s<e^*<e^{**}$ *such that,*

*(i) as* $0<e<e^*$, $w^*=w_2$, $h^*=\Delta s-ew_2 \in (0,1-e)$, *and* $y=y_1$;

*(ii) as* $e^*<e<e^{**}$, $w^*=\underline{w}=c+i-\frac{1-s}{e}$, $h^*=1-e$, *and* $y=y_1$; *and*

*(iii) as* $e>e^{**}$, $w^*=\underline{w}=c+i-\frac{1-s}{e}$, $h^*=1-e$, *and* $y=y_2$.

Lemma 2 makes clear that the agent will always start some projects with unidentified profitability at time 0, i.e., $h^*>0$. This is somewhat surprising. The intuition for it is that, when the agent is to terminate some projects (t>0), if the agent starts a project with unidentified profitability, he has the option to terminate the project as the profitability turns out to be low, or to retain the project and terminate some other project as the profitability turns out to be high. This option has a positive value if so few projects with unidentified profitability are started that some projects started with certain profitability will have to be terminated at time 1. For example, suppose that the agent is to terminate one project at time 1 and that at the margin, the agent has the choice of starting a project with identified profitability 0.9 or starting a project whose profitability is uniformly distributed between 0 and 1. Suppose the agent also starts another project with certain profitability 0.91. If the agent chooses the second strategy, then the agent has the option to terminate project 0.91 when the profitability of the unidentified project turns out to be higher than 0.91 or to terminate the initially unidentified project when the realization is lower than 0.91. If the agent chooses the first strategy, however, the agent will not have the option. In this example the option value is positive with probability 0.09.

Lemma 2(i) says that when effort is very low (as $e<e^*$), only part of the unscreened projects are started at t=0 ($h^*<1-e$) and no screened projects need to be terminated at t=1. This belongs to Case I above in which $y=y_1$. The reason is that, in this case, there are too many unscreened projects for the agent to start all of them. If a screened project was started at t=0 and needed to be terminated at t=1, then it would be better to start an unscreened project instead because the latter has an option value.[20] As e increases a little (as $e^* < e < e^{**}$), the lower bound of w (and the upper bound of h) becomes binding ( $w^* = \underline{w}$) and all unscreened projects will be started at time 0 ( $h^* = 1 - e$), but there is still no need to terminate screened projects. Therefore, we are still in Case I in which $y = y_1$. This gives rise to Lemma 2(ii). As e becomes very large (as $e > e^{**}$), the lower bound of w (and the upper bound of h) becomes such a severe constraint that even screened projects need to be terminated at time 1. This gives us Case II above in which $y = y_2$ and Lemma 2(iii).[21]

Now that we have analyzed the optimal choice of projects to start and thus found the value of $y$ for different cases, we can start to discuss the properties of $y$. Regarding the relationships between y, e, and t, we have,

*<u>Lemma 3</u>: (i)* $\frac{\partial y}{\partial t}$ *is continuous in e,*

*(ii)* a. $\frac{\partial y}{\partial e} > 0$ *and* $\frac{\partial^2 y}{\partial e \partial t} < 0$ *as* $0 < e < e^*$;

b. $\frac{\partial y}{\partial e} > 0$ *and* $\frac{\partial^2 y}{\partial e \partial t} < 0$ *as* $e^* < e < e^{**}$;

c. $\frac{\partial y}{\partial e} = 0$ *and* $\frac{\partial^2 y}{\partial e \partial t} = 0$ *as* $e > e^{**}$.

*(iii) Let* $\hat{t}$ *be the number of ex post unprofitable projects at time 1. Then* $\left.\frac{\partial y}{\partial t}\right|_{t=\hat{t}} = 0$.

*(iv)* $\frac{\partial y}{\partial s}$ *is continuous in e and*

$$\frac{\partial y}{\partial s} = \begin{cases} \frac{1}{2} + \frac{t^2}{2h^2} - (c+i) & \text{as } e < e^*; \\ \frac{1-s}{e} - (c+i) & \text{as } e^* < e < e^{**}; \\ t + 1 - s - (c+i) & \text{as } e > e^{**}. \end{cases}$$

[20] Consider the limiting case of e approaching 0.
[21] Consider the extreme case of e approaching 1.

*Furthermore,* $\frac{\partial^2 y}{\partial s^2} < 0$.

*(v)* $$\frac{\partial^2 y}{\partial s \partial e} = \begin{cases} > 0 & \text{as } e < e^*; \\ < 0 & \text{as } e^* < e < e^{**}; \\ = 0 & \text{as } e > e^{**}. \end{cases}$$

*(vi) Let* $\tilde{s}$ *be the number of projects to start that is optimal after project examination. Then* $\left.\frac{\partial y}{\partial s}\right|_{s=\tilde{s}} = 0$.

Part (ii) of the lemma tells us how the expected gross value-added, $y$, changes with the screening effort. When effort is high, the agent has better information about the profitability of the projects. Given s and t, better information leads to at least equal, and possibly higher, expected gross value-added. Therefore, $\frac{\partial y}{\partial e} \geq 0$. The inequality is not strict only when the agent's screening effort is already so high relative to s and t that any improvement in the information about the projects is no longer valuable.

Part (ii) of the lemma also says that the marginal cost of retaining (or the marginal benefit of terminating) *ex post* unprofitable projects, $\frac{\partial y}{\partial t}$, decreases with the effort level. When the agent's effort is higher, more unprofitable projects are discovered by the agent before the first investment is made and the quality of projects started is higher. As a result, the cost of retaining an *ex post* unprofitable project is lower. An alternative interpretation of the result is that restricting project termination raises the marginal benefit of effort, $\frac{\partial y}{\partial e}$. The reason is that an *ex post* unprofitable project that the agent fails to screen out costs more if he cannot later be terminated than if he can. Therefore, requiring the agent to retain some *ex post* inefficient projects has the effect of increasing the marginal benefit of effort.

Part (iii) of the lemma is not as trivial as it seems; t affects the project-starting decision and therefore may have an indirect effect on the maximum expected gross value-added, y(e, s, t). However, since the project-starting decision is made optimally, part (iii) of the lemma can be derived by using the envelope theorem.

4. Persistence of Ex Post Inefficient Projects

In this section, we answer the question when it is optimal for the principal to impose restrictions on project termination. As a result, some *ex post* inefficient projects persist. To highlight the idea, we consider the case where there are two effort levels, $e_h$ and $e_l$.[22] We show that when it is more efficient for the principal to induce the lower effort, $e_l$, it is also optimal for the principal to set s and t so that project-starting and -termination decisions are all *ex post* efficient.[23] However, when it is optimal for the principal to induce the higher effort, $e_h$, the principal should not only choose a different wage schedule, but also mandate $t^* < \hat{t}(e_h, s^*)$ so that the agent must continue some of the *ex post* unprofitable projects in the second stage.[24]

Let us first consider the case when the low effort is optimal. Since the principal is risk neutral and the agent is risk averse, the least costly way for the principal to induce the low effort is to offer the agent a fixed wage subject to the (IR) constraint.[25] Note that the choice of s and t does not affect the (IC) constraint in (2) when the wage is fixed. Therefore, in this case, *ex ante* efficient s and t coincide with *ex post* efficient of s and t. Consequently, all and only *ex post* unprofitable projects should be terminated.

Now consider the principal's optimal incentive contract for inducing the high effort, $e_h$. The principal's problem is

$$\max_{s,t} \Pi(s,t) \qquad (3)$$

where,

[22]We discuss the continuous-effort case in Section 5.

[23]In the continuous-effort case, it is never optimal to choose an effort that does not need incentives to induce.

[24]See the discussion after the Corollary at the end of this section.

[25]We assume that, when indifferent among some actions himself, the agent will choose the action that is best for the principal.

$$
\begin{aligned}
\Pi(s,t) = \max_{w(x)} & \int [x - w(x)] f(x;e_h,s,t)dx \\
\text{s.t.} & \int v(w(x)) f(x;e_h,s,t)dx - d(e_h) \geq \\
& \int v(w(x)) f(x;e_l,s,t)dx - d(e_l) \qquad \text{(IC)}\ (\mu) \\
\text{and} & \int v(w(x)) f(x;e_h,s,t)dx - d(e_h) \geq 0 \qquad \text{(IR)}\ (\lambda)
\end{aligned}
\tag{4}
$$

We denote the solution to (3) as $(s^*,t^*)$.

The main result of the paper is that the *ex ante* optimal number of terminations, $t^*$, is less than the *ex post* optimal number of terminations, $\hat{t}$; that is, to maximize expected net revenue, the principal should require the agent to retain some *ex post* unprofitable projects. The intuition of the result is as follows. In order to improve *ex ante* efficiency, the principal wants to make it easier to induce the higher effort by relaxing the incentive compatibility constraint, (IC). This can be achieved by restricting project termination by the agent at time 1. Since the agent's income increases with the gross value-added, as we will show later in a standard exercise, he is hurt by not being able to terminate all *ex post* unprofitable projects, regardless of the effort level. However, when the agent's effort is lower, the quality of started projects is poorer and the cost of retaining *ex post* unprofitable projects is higher, as is shown and discussed in Lemma 3(ii). Therefore, restricting project termination hurts the agent more if he exerts the lower effort, making it easier to induce the higher effort by relaxing the incentive compatibility constraint, (IC). Of course, retaining *ex post* inefficient projects has its costs. However, the envelope theorem implies that the costs of a marginal deviation from the *ex post* optimum are negligible when compared to the benefit from relaxing constraint (IC). Consequently, it is optimal for the principal to restrict the agent's freedom to terminate *ex post* unprofitable projects. In the rest of this section, we formalize the above argument.[26]

To characterize the solution to the problem, we make the assumption that the density function of $\theta$, $g(x)$, satisfies the monotone likelihood ratio condition:

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[26] We should point out that this is not a sufficient statistic argument as in Holmstrom (1979), because here $t$ is a choice variable, which is different from the additional signal in his model.

$$\frac{g(x+z)}{g(x)} \text{ is decreasing in } x \text{ for all positive } z. \qquad \text{(MLRC)}$$

(MLRC) is a common assumption in the principal-agent literature and is satisfied by normal distributions and t-distributions, among others. Lemma 4 states some properties of optimization problem (4).

Lemma 4: *(i) The feasible set of the principal's optimization program (4) is non-empty if and only if* $y(e_l,s,t) < y(e_h,s,t)$. *If the feasible set of program (4) is non-empty, then: (ii) the constraint (IC) has a positive Lagrange multiplier,* $\mu$, *and thus is binding; (iii) the optimal wage function,* $w(x)$, *is strictly increasing.*

The proof of the lemma is given in the appendix. Intuitively, if $y(e_l,s,t) = y(e_h,s,t)$, then the higher effort cannot be induced; constraint (IC) cannot be satisfied. If $y(e_l,s,t) < y(e_h,s,t)$, however, (IC) can be satisfied by choosing $w(x)$ so that its slope is large enough. $y(e_l,s,t) < y(e_h,s,t)$, together with the monotone likelihood ratio condition, implies that

$$\frac{f(x;e_l,s,t)}{f(x;e_h,s,t)} = \frac{g(x-y(e_l,s,t))}{g(x-y(e_h,s,t))}$$

is decreasing in $x$. That is, the likelihood of $e_h$ with respect to that of $e_l$ increases with $x$. It is then natural that under these conditions the agent should be paid more when the value of $x$ is higher.

Now, we are ready to present the main result of the paper.

Proposition 3: *Suppose it is optimal for the principal to induce the high effort,* $e_h$. *Let* $(s^*,t^*)$ *be the optimal combination of* $(s,t)$ *and* $\hat{t} = \hat{t}(s^*,e_h)$ *be the number of ex post inefficient projects at time 1 given* $s = s^*$ *and* $e = e_h$. *Then* $t^* < \hat{t}$ *or* $\frac{\partial \Pi}{\partial t}(s^*,\hat{t}) < 0$.

Proof: By Lemma 3(ii), $\frac{\partial y}{\partial e} \geq 0$. Therefore,

$$y(e_l,s^*,\hat{t}) \leq y(e_h,s^*,\hat{t}). \qquad (5)$$

Case I: If inequality (5) is not strict, i.e., if $y(e_l,s^*,\hat{t}) = y(e_h,s^*,\hat{t})$, then $\frac{\partial^2 y}{\partial e \partial t} \le 0$ (shown in Lemma 3(ii)) implies that $y(e_l,s^*,t) \ge y(e_h,s^*,t)$ for all $t > \hat{t}$, which in turn, by Lemma 4(i), implies that program (4) is not feasible for any $t \ge \hat{t}$. Therefore, for program (4) to be feasible (i.e., for it to be possible to induce $e_h$), the optimal $t^*$ must be chosen to be less that $\hat{t} = \hat{t}(s^*,e_h)$.

Case 2: If inequality (5) is strict, i.e., $y(e_l,s^*,\hat{t}) < y(e_h,s^*,\hat{t})$, then by Lemma 4(i), program (4) is feasible. By the envelope theorem,

$$\frac{\partial \Pi}{\partial t} = \frac{\partial L}{\partial t}(s,t;w^*_{s,t}(x)),$$

where

$$L = \int [x - w(x)] f(x;e_h,s,t)dx + \lambda \left\{ \int v(w(x)) f(x;e_h,s,t)dx - d(e_h) \right\}$$
$$+ \mu \left\{ \int v(w(x)) f(x;e_h,s,t)dx - d(e_h) - \int v(w(x)) f(x;e_l,s,t)dx + d(e_l) \right\}$$

is the Lagrangian of program (4),

$$\frac{\partial L}{\partial t} = \int [x - w_{s,t}(x)] f_t(x;e_h,s,t)dx + \lambda \int v(w_{s,t}(x)) f_t(x;e_h,s,t)dx$$
$$+ \mu \left\{ \int v(w_{s,t}(x)) f_t(x;e_h,s,t)dx - \int v(w_{s,t}(x)) f_t(x;e_l,s,t)dx \right\},$$

and $w_{s,t}(x)$ is the optimal wage function given $s$ and $t$. By the definition of $f(x;e,s,t)$,

$$f_t(x;e,s,t) = -g'(x - y(e,s,t)) \frac{\partial y}{\partial t}(e,s,t). \tag{6}$$

By Lemma 3(iii),

$$\frac{\partial y}{\partial t}(e_h,s^*,\hat{t}) = 0. \tag{7}$$

Substituting equations (6) and (7) into $\frac{\partial L}{\partial t}$ yields

$$\left.\frac{\partial L}{\partial t}\right|_{s=s^*,t=\hat{t}} = \mu \frac{\partial y}{\partial t}(e_l,s^*,\hat{t}) \int v(w_{s^*,\hat{t}}(x)) g'(x - y(e_l,s^*,\hat{t}))dx.$$

Integration by parts yields

$$\left.\frac{\partial L}{\partial t}\right|_{s=s^*,t=\hat{t}} = -\mu \frac{\partial y}{\partial t}(e_l,s^*,\hat{t}) \int v'(w_{s^*,\hat{t}}(x)) w'_{s^*,\hat{t}}(x) g(x - y(e_l,s^*,\hat{t}))dx,$$

because $g$ is a probability density function. Recall that $\hat{t} = \hat{t}(s^*, e_h)$. Lemma 4 implies $\mu > 0$, $w'(x) > 0$. Therefore, $\left.\frac{\partial L}{\partial t}\right|_{s=s^*, t=\hat{t}}$ has the opposite sign as $\frac{\partial y}{\partial t}(e_l, s^*, \hat{t})$. Lemma 3(ii) implies the diagrams in Figure 15. By Figure 15a, $y(e_l, s^*, \hat{t}) < y(e_h, s^*, \hat{t})$ implies that $e_l < e^{**}(s^*, \hat{t})$, which in turn, together with Figure 15b and (7), implies that $\frac{\partial y}{\partial t}(e_l, s^*, \hat{t}) > \frac{\partial y}{\partial t}(e_h, s^*, \hat{t}) = 0$. Consequently, $\frac{\partial \Pi}{\partial t}(s^*, \hat{t}) < 0$.

Q.E.D

Since we don't know the global properties of function $\Pi(s,t)$, Propositions 3 does not directly mean that $t^* < \hat{t}(s^*, e_h)$. However, If $\Pi(s,t)$ is concave with respect to $t$, Proposition 3 does imply that $t^* < \hat{t}(s^*, e_h)$. Therefore, we have,

<u>Corollary 2</u>: *If $\Pi(s,t)$ is concave with respect to $t$, then under the ex ante optimal contract, the number of projects terminated in the second stage is less than the number of ex post unprofitable projects existing at time 1.*

In our verbal discussion on the desirability for the principal to restrict project termination and its intuition, we implicitly assumed the concavity of $\Pi(s,t)$ with respect to $t$. The assumption simplified the discussion by a great deal.[27]

<u>Proposition 4</u>: *Suppose it is optimal for the principal to induce the high effort, $e_h$. Let $(s^*, t^*)$ be the optimal combination of (s,t) and $\tilde{s} = \tilde{s}(t^*, e_h)$ be the number of projects to start that is optimal after project examination, given $t = t^*$ and $e = e_h$. Then $s^* < \tilde{s}$ or $\frac{\partial \Pi}{\partial s}(\tilde{s}, t^*) < 0$ if $c + i < \frac{1}{2}$.*[28]

The proof of Proposition 4 uses Lemma 3(iv), (v), and (vi), and is very similar to that of Proposition 3. The difference is that $\frac{\partial^2 y}{\partial s \partial e}$ is not always negative as $e < e^{**}$. However, if

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[27]We realize that it is desirable to express the condition about $\Pi(s,t)$ in the Corollary in terms of primitive parameters. Unfortunately, the complexity of the problem makes it intractable to do so.
[28]Not providing enough funds for all projects is known as "rationing" and has been studied by Stiglitz and Weiss (1981), among others, in the context of creditor-borrower transactions in a capital market. What we study is the intra-organizational resource allocation problem when the market is suppressed.

$c+i<\frac{1}{2}$, $\frac{\partial y}{\partial s}=\frac{1}{2}+\frac{t^2}{2h^2}-(c+i)$, for $e<e^*$, is always positive, which is enough for the result. Figure 16 illustrates the situation.

We wish to point out that the condition for the result can be relaxed. We can show that, in many cases, there exists some $\alpha^*>\frac{1}{2}$, such that the result still holds when $c+i<\alpha^*$.

<u>Corollary 3</u>: *If $\Pi(s,t)$ is concave with respect to $s$, then under the ex ante optimal contract, the number of projects started in the first stage is less than that which is optimal after project termination.*

Corollary 3 says that the principal should also impose restrictions on project starting so that the agent has to forego some projects that he would otherwise start. The intuition for the result is similar to that for restricting project termination: As effort decreases, the agent identifies fewer bad projects so that, if he is restricted from starting projects, he has to forego more profitable projects and is therefore hurt more.

5. The Empirical Relevance

At the first glance, our explanation for the persistence of unprofitable projects seems rather counterintuitive; what we often observe in a hierarchically organized economy is that agents keep requesting for ever more funds and mount (political and/or other) pressures on their principals (superiors) to meet their requests. The principals, on the other hand, often seem reluctant to provide all funds requested by agents. This leaves the impression that the soft budget constraint problem results from a principal's giving in under pressure to an agent's request for more funds.

We acknowledged in the Introduction that there are certainly cases in which the superiors would like to terminate a project but failed to do so because the project is protected by interest groups.[29] In the Introduction, we also commented that political or other pressure does not quite

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[29]Note that $t^*>0$ is perfectly consistent with our model.

explain why the soft budget constraint was also common in times of ruthless strong rulers like Stalin and Mao. In fact, once we look closer to see how the principal and the agent disagree on whether investment funds should be used to start new or support old projects beyond the fact on the surface that the agent is always requesting for more funds, the empirical relevance of our model becomes more apparent.

Evidence from China's reform experience shows that, in many cases, agents prefer to start more new projects, while the principal takes the opposite position and introduce measures to force agents to start fewer new projects and better fund existing projects.

In the 1980s and the first half of the 1990s, economic reform in China's state sector concentrated on implementing two major policies. One is to increase "management autonomy" to allow state-owned enterprise (SOE) managers and their local supervisors (e.g., local industrial bureaus) greater decision-making power. The other is to introduce a "contract responsibility" system to provide explicit monetary incentives for managers to improve the financial performance of the SOEs, i.e., to increase the profit.

As management autonomy increased, however, it became increasingly clear that local agents (local government bureaucrats and SOE managers) would start more projects than the principal (national policy makers) was willing to fund.[30] Throughout this period, the policy makers had to battle with local agents their tendency to start too many project. Year after year, the government documents criticized local agents for starting too many projects and urge them to reduce the number of new project starts.[31]

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30Wang (1991) shows that competition among local agents for resources controlled by the central government leads to over-investment in fixed capital. But this does not imply that investment will be heavily in new projects. Another explanation could be that local agents over invest for "empire building". Again, the explanation cannot count for investments being mostly in new projects.

31For example, in his 1995 economic report to the People's Congress, the legislative body of China, Cheng Jinghua, the director of the State Planning Committee, which is the highest government agency in charge of economic affairs in China, said that: The rapidly increasing fixed capital investment must be strictly controlled, and "[t]the emphasis is to continue to strictly control new projects starts. No large or median-sized new projects should be started in any region by any government institutions [in that year]. Small project starts should also be strictly controlled. Investment funds must be strictly managed and supervised so that the problem can be attacked from the source. (Almanac of China's Economy, 1995, p.14-22. )

While they were very enthusiastic with starting new projects, local agents at the same time moved funds away from existing SOEs, leaving many existing projects not finished, SOEs' technology not updated, and SOEs generally severely short of liquidity (known as "circulating capital" in Chinese accounting system). To battle this tendency, the government strictly prohibits using banks' circulating capital loans to start new projects.[32] In more recently years, the state banks also increased the proportion of loans designated to be invested in existing SOEs, known as "technological innovation and renovation" funds.[33] Between 1990 and 1993, the proportion of such designated funds ranged from 25.5 percent to 28.7 percent. Local agents can be severely punished if these funds are not used for the designated purposes.[34]

Our model is consistent with the above evidence. We show that the principal makes the agent to start fewer, and terminate fewer, projects than the agent would like to. Therefore, the agent has incentives to move fund away from supporting existing projects to start new projects. To counter this tendency, the principal mandates the uses of funds in starting new projects versus supporting existing ones. These are just what we observe in China's economic reform.

## 6. Discussion of the Model

### A. The Number of Projects

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[32]Although we used the term "loans" to talk about allocating financial resource in China's state sector, we are not talking about capital market transactions. In the 1980s, the state banks took over some of the financial ministry's role in financial resource allocation. The long-term goal is to make these state banks behave more like commercial banks. So far, however, like the financial ministry, the state banks have largely followed government directions in allocating funds to various agents. Until very recently, almost all banks in China were state owned. About 60 to 80 percent of their loans have been circulating capital loans for existing SOEs'.

[33]In September, 1991, the Central Committee of the Chinese Communist Party held a working meeting on improving the performance of the large and median sized state-owned enterprises (SOEs). Among the measures adopted are: Increase the funding for technological updating and renovation of, continue to supplement the circulation capital for, and further reduce interest and tax rates for, the existing SOEs. (Almanac of China's Economy, 1992, p.730-31. )

[34]A conventional wisdom to explain government's efforts to guarantee funds for existing SOEs is its ideological bias. The Chinese government insists that China must remain a socialist country and that a strong state sector is essential for a socialist market economy. However, without the agency problem studied in this paper, it seems that the objective of maintaining a large state sector can be implemented by requiring most new projects to be state-owned without restricting local agents' freedom to choose new or old projects.

In our model, we assume that there are infinite many projects. If there are only a finite number of projects under the administration of the planner, the law of large numbers no longer applies so that $\hat{s}$ and $\hat{t}$ become stochastic. Because of this, the *ex ante* optimal $t^*$ may not be always less than $\hat{t}$, but $t^*$ is still less than the expected value of $\hat{t}$. Another change is that the cost of retaining *ex post* unprofitable projects becomes discontinuous so that we cannot use the envelope theorem argument. Consequently, this cost may not be dominated by the benefit of retaining *ex post* unprofitable projects that it makes it easier to induce effort. Therefore, we can only conclude that $t^* \leq \hat{t}$. Whether or not the inequality is strict depends on the parameters of costs and revenue.

B. Independent Monitor

We have assumed there is only one agent, the planner. One might wonder what happens if there are two or more agents? There are several possibilities here.

First, if the two agents have exactly the same contracts and work on the same block of projects, then the problem is also exactly the same as the one we have just studied. The same result would, of course, be obtained.

The principal could also divide the project proposals into many small portions and hire many agents to each work on one portion. If a portion is large and contains many projects, then the same problem that an agent would not spend enough effort to inspect all of them would arise. If the portions are each very small, then there would be a large number of agents, raising the issue of the principal's span of control. If for any reason (not modeled in this paper) the principal can not directly monitor so many agents so that another tier of agents need to be hired, then the problem becomes one of optimal hierarchy. The problem is beyond the scope of this paper, but it suffices to point out that a supervisor's problem with her subordinates in an incentive hierarchy is essentially the same as the planner's problem with project proposals at time 0 in this paper. In these hierarchies, it is in general optimal to have imperfect monitoring and declining efforts down the hierarchies.[35]

Still another possibility is that the principal could hire a second agent to monitor the planner. One way for the second agent to monitor the planner's effort is to inspect project quality at time 1. As Proposition 1 shows, one can perfectly infer the planner's effort level from project quality at time 1. If the second agent can get perfect information about project quality at time 1 at a very low cost and provide the information to the principal, i.e., he has access to information about project quality , does not have an agency problem himself so that he will not collude with the first agent or cheat in any other way, then no ex post unprofitable projects would be refinanced because the first best effort, project starting and termination decisions can all be achieved. However, we can probably more reasonably expect the second agent to have an incentive problem of his own so that the principal has to sign a separate incentive contract with him. We have discussed the case in which the second agent has the same incentive contract with the first one. If the second agent has a different incentive contract, he is unlikely to get the same information about project quality as the planner. This prevents the principal from obtaining perfect information about project quality at time 1. If this is the case, the asymmetric information problem between the principal and the planner will continue to exist and, thereby, the results of our model continue to hold.

C. Continuous Effort

In Section 4, we assumed that there were two effort levels. We have also proved results similar to Propositions 3 and 4 for the continuous-effort case using the first-order condition approach.[36] If we assume that the marginal disutility of effort is zero at $e = 0$, then it is optimal to induce a positive effort level (e>0) and thus also optimal to set $t^* < \hat{t}$ rather than $t^* = \hat{t}$. In contrast, in the two-effort case we considered, it may be optimal to select the lower effort, in

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[35]See, among others, Calvo and Wellizs (1978) and Qian (1994) for studies of incentive hierarchies with endogenously determined degree of supervision and monitoring.

[36]It is difficult to show the validity of the first-order condition approach for this model.

which case there is no benefit in setting $t^* < \hat{t}$ because the lower effort requires no incentive to implement.

D. Renegotiation

One might think that, after the planner has made the effort, a Pareto efficient new contract can be signed through renegotiation, as the original contract leads to *ex post* inefficient project-starting and -termination decisions and does not provide full insurance to the planner. Presumably, a new contract that specifies a fixed wage for the agent would induce *ex post* efficient project-starting and -termination decisions and also fully insure the planner against any risk associated with output fluctuations.[37]

Several considerations suggest that our results are robust with respect to renegotiation. First, if the game is repeated, renegotiation in earlier stages damages the principal's reputation to maintain incentives for screening effort in later stages. Such reputation concerns restrain the principal from renegotiating the wage contract with the agent. The idea can be formally modeled by considering an infinitely repeated game between the same principal and generations of different agents. When the principal's discount factor is sufficiently large, she will not renegotiate with any agent in equilibrium.[38] Such formalization is a standard exercise and thus is not elaborated in this paper.

Second, even for the one-shot game we model, adding a renegotiation stage does not change the equilibrium allocation if the renegotiation rule requires that the agent propose the new contract, as specified by Ma (1994). The key to this is that the principal has less information about project profitability and thus about the agent's effort than the agent has. The agent's proposal of removing restrictions on project-starting and -termination gives the principal reason

[37]We assume that when the planner is indifferent between different actions, he takes the action that is best for the principal. Alternatively, the payoff to the planner should increase slightly with the total surplus.

[38]In Dewatripont and Maskin (1995), renegotiation is not prevented even in a repeated game. The reason is that, in our model, the number of closed projects is public information, but in their model, the profitability of a project is not public information.

to believe that the agent has chosen a lower effort initially. Given such a belief, the principal will adopt a different sharing rule; i.e., one that maximizes profit, given the lower effort level. Having made the higher effort, the agent is worse off and therefore should not propose to renegotiate the original contract.[39]

Critical for this argument is the assumption that only the agent can propose to renegotiate the contract. The assumption, however, is not unreasonable in this context. Fudenberg and Tirole (1990) consider the alternative renegotiation rule that the principal proposes a new contract after the agent has taken the action. They find that the set of implementable actions is much smaller and as a result, the outcome of the game is less efficient than in the case where only the agent proposes to renegotiate. Thus, without modeling, we can think of a larger game played between the state and the planner to choose the renegotiation rule before they start the contracting and the renegotiation game. If the game leads to the result that the more efficient rule is chosen, the result should be that only the planner can propose to renegotiate the contract. An institution may also be established to safeguard the rule.[40]

This paper offers an explanation of the soft budget constraint by focusing on the moral hazard problem in the agent's project screening effort. We conjecture that the agent's concern about the reputation regarding his ability also leads to continuations of *ex post* inefficient projects, as terminating projects sends an unfavorable signal about his ability. In the latter context, there is no ground for renegotiation.[41]

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[39]We have formally shown that Ma's results can be adapted to this model. The proof is not included in the paper but is available upon request.

[40]One might further ask if the rule itself is subject to renegotiation and the institution to change. The question is out of the scope of this paper. Suffice to point out that, first, changing the rule and institution may be more difficult than changing a contract itself, and, second, our understanding of institutional changes is generally more limited than contract renegotiation.

[41]The learning problem here is similar to that of Holmstrom and Ricart i Costa (1986). The problem of optimal incentive when moral hazard and adverse selection problems both exist is studied by McAfee and McMillan (1991) and Picard and Rey (1990). It is also a basic assumption in Laffont and Tirole's (1993) study of the government procurement problem.

## 7. Summary and Concluding Remarks

We have shown that, when there is agency in project screening and termination decisions, it is in general optimal for the principal in the *ex ante* incentive contract to set restrictions upon the agent's freedom to terminate *ex post* unprofitable projects. The result suggests that the widespread soft budget problem in socialist economies has a very profound informational reason. It is thus likely to persist even when a socialist state strives to maximize economic profit. The result of the model is driven by the idea that, by reducing an agent's flexibility in getting away from a problem, the agent will be induced to make a greater effort to avoid the problem. It is thus quite robust to alternative technical assumptions.

Some people may be concerned about whether the kind of complex contract between the state and the planner we studied can be found in real world socialist economies. One possible response to this concern is that a model should be judged in "as if" terms, i.e., by its predictions. More importantly, as already mentioned in the Introduction, the strategy of this paper is to study the cause of the soft budget problem by constructing an "ideal" socialist economy in which the state is only concerned with efficiency in terms of economic profit, while asymmetric information about project quality exists between the state and its agent. In doing so, we have necessarily abstracted from many observable real world institutions, e.g., the state may have other concerns than economic profit, or it may not have the full bargaining power to dictate the optimal contract. The most important insight of our model is that the soft budget constraint in socialist economies has a very profound informational reason. It is hard to believe that adding specific institutions of socialist economies like those mentioned above to the "ideal" socialism modeled in this paper would alter the result of our model.

It is worthwhile to point out that the agency problem in projects requiring sequential decisions is also common in capitalist market economies. For example, in the problem of employment decisions. an academic department of a university, a plant of a business company, or an office of a government may, on behalf of the university, the company, or the government, respectively, review job applicants, make hiring decisions, train new employees, and then make retention and separation decisions about them. Since central planning is also a feature of internal governance in many organizations in market economies, our model suggests that restrictive termination rules should also be expected in these organizations. Consequently, subunits are forced to retain some of the unproductive projects (employees) and continue to finance them. This provides an explanation of tenure in universities, the "no-layoff" rule in large Japanese and also some American companies, and other restrictive layoff rules:[42] they have the benefit of inducing higher effort in initial screening and thereby reducing costly initial investment in bad projects (unproductive employees).[43]

Given agency in sequential decision problems and persistence of *ex post* unprofitable projects in both capitalist and socialist economies, an interesting and important question is: What differences exist between the two settings?

One most important and obvious difference is the lack or underdevelopment of equity markets in socialist economies. In capitalist economies, the market for equity shares of a publicly owned firm generates information about its performance. In the extreme case of an equity market generating perfect information regarding project quality, the first best allocation can be achieved, as shown in Proposition 1. In the less extreme case in which the principal gets limited signals about the agent's effort, in addition to final output and the number of closed projects, the soft budget constraint problem continues to exist, but is less severe.[44] It is true that in capitalist economies many firms are privately owned and their shares not publicly traded.

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[42]Carmichael (1988) offers an explanation of the tenure rule in universities. He points out that, in the academic job market, universities have to rely upon incumbent professors for information about job candidates' quality. Tenure is needed to protect the job security of incumbent professors so that they would truthfully report the quality of job candidates to the university, enabling it to hire the best candidates. This explanation for tenure deals with the issue of incumbent professors' willingness to report true information, but not with the issue of optimizing effort needed to obtain that information.

[43]Although comparative static results of our model are difficult to derive, it seems reasonable to predict more restrictive layoff rules and greater screening efforts in recruiting in companies that invest more in employee training.

[44]Holmstrom (1979) shows that an informative signal about effort can always improve the efficiency of the optimal principal-agent contract. Furthermore, if we use second-order stochastic dominance to rank the accuracy of signals, then the result about sufficient statistics in Holmstrom (1982) implies that a more accurate signal leads to a more efficient outcome.

However, in capitalist economies where equity markets play a critical role in resource allocation, a firm's ownership structure is a matter of choice rather than one of imperative as in socialist economies; While investors in capitalist economies have the freedom to choose either public or private ownership, in socialist economies, state ownership is decreed. The critical importance of this difference becomes rather apparent when we think of likely different types of information that need different types of institutions to utilize. Titman and Subrahmanyam (1996), for example, make the observation that some information is serendipitous in nature, i.e., it is obtained costlessly and purely by chance. They show that, when information is readily available through deliberate effort, e.g., research and auditing, there is an advantage associated with limiting the number of active investors. Concentrated ownership would presumably do well in this case. However, when the influence of serendipitous information on the firm's value is strong, information regarding project values can be best obtained when stocks are traded on a market with the largest possible number of active investors. Presumably, when investors have the freedom to choose the ownership structure, firms can be sorted into public or private ownership depending on which one can better generate information regarding its performance to help alleviate agency problems. In contrast, when a certain type of ownership, e.g., state ownership, is decreed, it is unlikely to be optimal for all firms.

Recent discussion of market socialism has revealed an increased recognition of the importance of the equity market with some fairly detailed proposals as to how such a market might be organized. (See Bardhan and Roemer, 1993.) Hardening the budget constraint will depend upon the degree to which the capitalist system's equity market can be mimicked; the extent to which this can be done under market socialism, however, remains an open question. This paper does not directly address this question and interested readers are referred to Bardhan and Roemer (1992, 1993) and Shleifer and Vishny (1994) for opposing views. However, the result of this paper does suggest that, in order to understand the efficiency of market socialism relative to a capitalist economy, it is important to understand how equity markets under the two systems generate information on managers' efforts as well as on the performance of individual firms (projects). In socialist market economies, the state is the dominant shareholder, while in capitalist market economies, ownership tends to be more diffused. On the one hand, diffused ownership creates the problem of free-riding and discourages small shareholders from monitoring management, as argued by Shleifer and Vishny (1986). On the other hand, the existence of liquidity traders among firm owners gives speculators incentives to collect information, as pointed out by Holmstrom and Tirole (1993).

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Appendix:[1]

<u>Lemma 1</u>: *$y_1$ is concave with a unique maximum point, $w_2$, which is the solution to*

$$t=\sqrt{(2c+2i-1-2w)}(\Delta s-ew)$$

*in the range* $w\in\left(-\infty,\min\left\{\frac{\Delta s}{e},c+i-\frac{1}{2}\right\}\right)$.

The proofs of Lemma 1 and 2 are rather messy and tedious and the reader might want to skip this part in the first reading.

Proof: Let $k_1=\frac{t}{h}=\frac{t}{\Delta s-ew}$, Then

$$y_1=\int_{k_1}^{1}(\Delta s-ew)\alpha d\alpha+\int_{c+i-w}^{1}e\alpha d\alpha-s(c+i)+ti.$$

Therefore,

$$\begin{aligned}\frac{dy_1}{dw}&=\int_{k_1}^{1}-e\alpha d\alpha-\frac{dk_1}{dw}(\Delta s-ew)k_1+e(c+i-w)\\&=-e\frac{1}{2}\alpha^2\Big|_{k_1}^{1}-\frac{d}{dw}\left(\frac{t}{\Delta s-ew}\right)t+e(c+i-w)\\&=\frac{1}{2}e\alpha^2\Big|_{1}^{k_1}-t^2\frac{e}{(\Delta s-ew)^2}+e(c+i-w)\\&=\frac{1}{2}e\frac{t^2}{(\Delta s-ew)^2}-\frac{1}{2}e-\frac{et^2}{(\Delta s-ew)^2}+e(c+i-w)\\&=-\frac{1}{2}e-\frac{1}{2}\frac{et^2}{(\Delta s-ew)^2}+e(c+i-w)\\&=e(c+i-\frac{1}{2}-w)-\frac{1}{2}\frac{et^2}{(\Delta s-ew)^2}\end{aligned}$$

Take another derivative, we have,

$$\frac{d^2y_1}{dw^2}=-e-\frac{et^2}{2}\frac{2e}{(\Delta s-ew)^3}=-e-\frac{e^2t^2}{(\Delta s-ew)^3}=-e-\frac{e^2t^2}{h^2}<0,$$

that is, $y_1$ is a concave function of $w$. Therefore, $y_1$ has a unique maximum which is determined by $\frac{dy_1}{dw}=0$ if $\frac{dy_1}{dw}=0$ has a solution. $\frac{dy_1}{dw}=0$ if and only if

[1] An alternative proof with more graphs and less algebra is available upon request.

$$t^2 = (2c+2i-1-2w)(\Delta s - ew)^2.$$

At the solution to the equation, $w < c+i-\frac{1}{2}$. Furthermore, for $y_1$ to be defined, $w < \frac{\Delta s}{e}$.

Therefore, the solution is in the range $\left(-\infty, \min\left\{\frac{\Delta s}{e}, c+i-\frac{1}{2}\right\}\right)$. In this range,

$\sqrt{(2c+2i-1-2w)}(\Delta s - ew)$ is a strictly decreasing function of w. Therefore the solution is uniquely defined, denoted by $w_2$, as illustrated by Figure 7. Note that we do not require $w_2 \geq \underline{w}$ at this moment.

Lemma 2: *Suppose 0<t<s. Let y(e, s, t) be the maximum expected value-added. There exist $e^*$ and $e^{**}$ with $1-s<e^*<e^{**}$ such that,*

*(i) as* $0<e<e^*$, $w^* = w_2$, $h^* = \Delta s - ew_2 \in (0, 1-e)$, *and* $y = y_1$;

*(ii) as* $e^* < e < e^{**}$, $w^* = \underline{w} = c+i-\frac{1-s}{e}$, $h^* = 1-e$, *and* $y = y_1$; *and*

*(iii) as* $e > e^{**}$, $w^* = \underline{w} = c+i-\frac{1-s}{e}$, $h^* = 1-e$, *and* $y = y_2$.

Proof: The boundary of case 1 and case 2 is defined by $t = (c+i-w)(\Delta s - ew)$. The right hand side, $(c+i-w)(\Delta s - ew)$, is a quadratic function of w, the two roots of which are $w_1' = c+i$, $w_2' = \frac{\Delta s}{e}$. Therefore, in the range $w \leq \min\{w_1', w_2'\} = \overline{w}$, $(c+i-w)(\Delta s - ew)$ decreases with $w$. Let $w_1$ be defined by $t = (c+i-w)(\Delta s - ew)$ in the range $w \leq \min\{w_1', w_2'\} = \overline{w}$. $w_1$ is uniquely defined, as illustrated by Figure 8. Note that we do not require $w_1 \geq \underline{w}$ at the moment and that $w_1$ could be negative if t is sufficiently large or $\Delta s \leq 0$. It is easy to see that $w_1(t)$ decreases with $t$. From Figure 8, $t \leq (c+i-w)h \Leftrightarrow w \leq w_1(t)$. Then the two cases we discussed above become

| | | |
|---|---|---|
| Case I: | $w \leq w_1(t)$ | $(\Leftrightarrow t \leq (c+i-w)h)$ |
| Case II: | $w > w_1(t)$ | $(\Leftrightarrow t > (c+i-w)h)$ |

Figure 9 illustrates the situation.

The expected gross value-added, $y$, is equal to $y_1$ when $w \leq w_1(t)$ and is equal to $y_2$ when $w > w_1(t)$, i.e.,

$$y = \begin{cases} y_1 & \textit{for} \quad w \leq w_1(t) \\ y_2 & \textit{for} \quad w > w_1(t) \end{cases}.$$

At $w = w_1(t)$, or $t = (c+i-w)h$, $y_1 = y_2$. Therefore $y$ is continuous in $(e,s,t)$.

We have already considered Case I in Lemma 1 and found that $y_1$ is concave with a unique maximum point, $w_2$. Now, we analyze the realtionship between $w_1$ and $w_2$. $w_1 \le \min\left\{\frac{\Delta s}{e}, c+i\right\}$ implies $c+i-w_1 \ge 0$ and $w_2 \le \min\left\{\frac{\Delta s}{e}, c+i-\frac{1}{2}\right\}$ implies $c+i-w_2 \ge 0$. When $(c+i-w) \ge 0$, $(c+i-w) \ge \sqrt{2(c+i-w)-1}$. Therefore

$$w_2 \le w_1.$$

This is illustrated by the two diagrams in Figure 10. The first disgram is for the case $\frac{\Delta s}{e} \le c+i-\frac{1}{2}$, and the second diagram is for the case $\frac{\Delta s}{e} > c+i-\frac{1}{2}$.

Next, we consider Case II.

$$\begin{aligned}
\frac{dy_2}{dw} &= \int_1^k -e\alpha d\alpha - \frac{dk}{dw}(e+h)k \\
&= e\frac{1}{2}\alpha^2\Big|_1^k - \frac{d}{dw}\left[\frac{t+e(c+i-w)}{e+\Delta s-ew}\right][t+e(c+i-w)] \\
&= \frac{1}{2}ek^2 - \frac{1}{2}e - [t+e(c+i-w)]\frac{-e(e+\Delta s-ew)+[t+e(c+i-w)]e}{(e+\Delta s-ew)^2} \\
&= \frac{1}{2}ek^2 - \frac{1}{2}e - ek^2 + ek \\
&= -\frac{1}{2}e - \frac{1}{2}ek^2 + ek = -\frac{1}{2}e(1+k^2-2k) \\
&= -\frac{1}{2}e(k-1)^2 \le 0
\end{aligned}$$

The Inequality is strict unless

$$k=1 \Leftrightarrow t+e(c+i-w) = e+\Delta s - ew \Leftrightarrow t = e(1-c-i)+\Delta s = \hat{s}+\Delta s = s,$$

i.e. all started projects are terminated at time 1, which we have assumed away. Therefore, $y$ is a decreasing function in the range $w > w_1(t)$.

Combining the two cases, we find that the graph of $y$ is as illustrated in Figure 11. Recall that

$$\overline{w} = \min\left\{\frac{\Delta s}{e}, c+i\right\}$$

is the upper bound of $w$, and

$$\underline{w} = \max\left\{c+i-1, \frac{\Delta s + e - 1}{e}\right\} = \max\left\{c+i-1, c+i-\frac{1-s}{e}\right\}$$

is the lower bound of $w$.[2] Therefore, the optimal choice of $w$ is,

$$w^* = \begin{cases} \underline{w} & \text{if} \quad \underline{w} > w_2 \\ w_2 & \text{if} \quad \underline{w} \le w_2 \end{cases} \qquad (1)$$

We discuss two separate cases:

Case 1: $1-s \ge e$. Then $\underline{w} = c+i-1$. Since $\frac{\Delta s}{e} = \frac{s}{e} - 1 + c + i > c+i-1$,

$$\underline{w} = c+i-1 < \min\left\{c+i-\frac{1}{2}, \frac{\Delta s}{e}\right\}$$

and,

$$\begin{aligned} &\sqrt{2c+2i-1-2\underline{w}}(\Delta s - e\underline{w}) \\ &= \sqrt{2c+2i-1-2c-2i+2}(s-e+ce+ie-ce-ie+e) = s > t \end{aligned}$$

Thus, by Figure 7, $\underline{w} < w_2$. Then , $y = y_1$, and $h^* = \Delta s - ew_2 < \Delta s - e\underline{w} = s \le 1-e$. Meanwhile, by the definition of $w_2$, $h^* = \frac{t}{\sqrt{2c+2i-1-2w_2}} > 0$.

Case 2: $1-s < e$. Then $\underline{w} = c+i-\frac{1-s}{e} = \frac{\Delta s}{e} + 1 - \frac{1}{e} < \frac{\Delta s}{e}$. Therefore, $y_1$ is defined at $w = \underline{w}$.

Evaluate the derivative of $y_1$ at $w = \underline{w}$. Then,

$$\begin{aligned} \left.\frac{dy_1}{dw}\right|_{w=\underline{w}} &= \frac{1}{2}e\left\{[2c+2i-1-2c-2i+\frac{2(1-s)}{e}] - \frac{t^2}{(s-e(1-c-i)-(c+i)e+(1-s))^2}\right\} \\ &= \frac{1}{2}e\left\{\frac{2(1-s)}{e} - 1 - \frac{t^2}{(1-e)^2}\right\} = \frac{e}{2(1-e)^2}\left\{(1-e)^2[\frac{2(1-s)}{e} - 1] - t^2\right\} \end{aligned}$$

Since $y_1$ is concave in $w$ and $\left.\frac{dy_1}{dw}\right|_{w=w_2} = 0$ (Figure 11), we have,

$$\underline{w} < w_2 \qquad \text{when} \quad \left.\frac{dy_1}{dw}\right|_{w=\underline{w}} = \frac{e}{2(1-e)}\left\{(1-e)^2[\frac{2(1-s)}{e} - 1] - t^2\right\} < 0$$

$$\underline{w} > w_2 \qquad \text{when} \quad \left.\frac{dy_1}{dw}\right|_{w=\underline{w}} > 0$$

[2]It has been checked that $\underline{w} < \overline{w}$ so that the feasible set is not empty.

Let $\eta(e,s)=(1-e)^2[\frac{2(1-s)}{e}-1]$, for $e\geq 1-s$. As $e=1-s$, $\eta(e,s)=s^2\geq t^2$. As $e=1$, $\eta(e,s)=0$. $\eta=2(1-s)(e-2+\frac{1}{e})-(1-e)^2$. Therefore, $\eta_e=2(1-s)(1-\frac{1}{e^2})-2(e-1)$. $\eta_e=0$ has three solutions:

$$e_1=1,\ \ e_2=\frac{1-s-\sqrt{(1-s)^2+4(1-s)}}{2}<0 \ \text{ or } \ e_3=\frac{1-s+\sqrt{(1-s)^2+4(1-s)}}{2}.$$

Then, the graph of $\eta(e)$ is as that given in Figure 12. In both cases in Figure 12, there exists a unique $e^*$, s.t. $t^2=\eta(e,s)$. Furthermore,

$$\underline{w}>w_2 \ \text{ as } \ e>e^* \ \text{ and } \ \underline{w}<w_2 \ \text{ as } \ e\leq e^*.$$

Therefore, (1) becomes

$$w^*=\begin{cases}\underline{w} \text{ if } e>e^*\\ w_2 \text{ if } e\leq e^*\end{cases} \qquad (2)$$

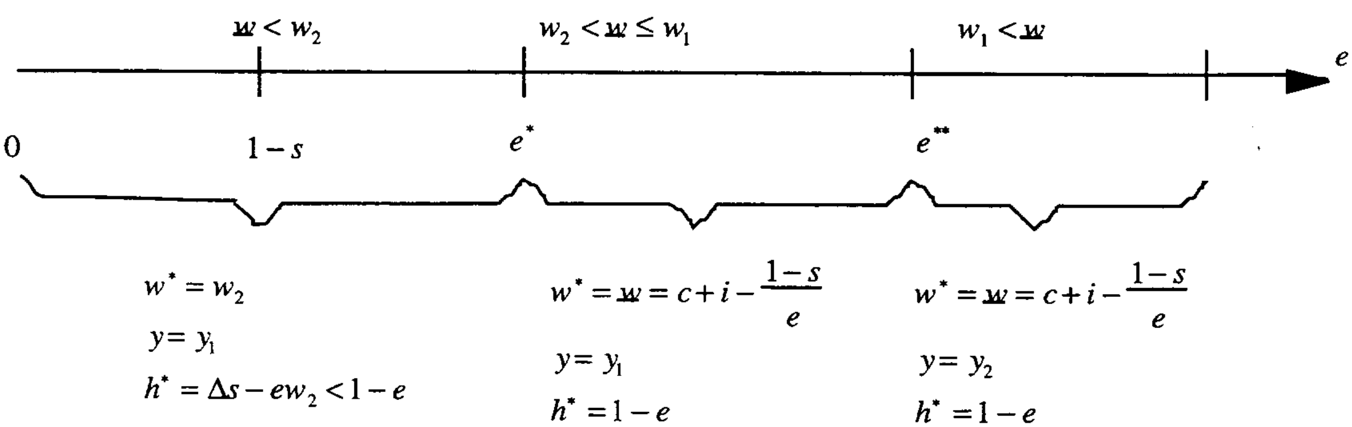
When $e\leq e^*$, $w^*=w_2$ and $y=y_1$. When $e>e^*$, $w^*=\underline{w}$, and $y=y_1$ or $y_2$ depending on $\underline{w}<w_1$ or $\underline{w}\geq w_1$. Using Figure 8, we now compare $\underline{w}$ with $w_1$ for the case $e>e^*>1-s$. In this case,

$$\underline{w}=c+i-\frac{1-s}{e}=\frac{\Delta s}{e}+1-\frac{1}{e}<\min\left\{c+i,\frac{\Delta s}{e}\right\}.$$

Therefore, by Figure 8, $\underline{w}<w_1$ if and only if $(c+i-\underline{w})(\Delta s-e\underline{w})>t$.

$$(c+i-\underline{w})(\Delta s-e\underline{w})=\frac{1-s}{e}(s+ce+ie-e-ce-ie+1-s)=\frac{(1-s)(1-e)}{e},$$

which is illustrated in Figure 13. Therefore, there exists $e^{**}$, such that, $\underline{w}<w_1$ if and only if $e<e^{**}$. Since at $e=e^*$, $\underline{w}=w_2\leq w_1$, $e^*\leq e^{**}$. In summary, we have




Note that $h^*>0$, which means that the manager should always start some projects with unidentified profitability. The reason for this result is that, when the manager is to terminate some projects $(t>0)$, if the manager starts a project with unidentified profitability, he has the option to terminate the project as the profitability turns out to be low, or to retain the project and terminate some other project as the profitability turns out to be high. This option value is positive if $h$ is so small that some projects started with identified profitability will have to be terminated at time 1. For example, suppose that the manager is to terminate one project at time 1 and that at the margin, the manager has the choice of starting a project with identified profitability 0.9 or starting a project whose profitability is uniformly distributed between 0 and 1. Suppose the manager also starts another project with certain profitability 0.91. If the manager chooses the second strategy, then the manager has the option to terminate project 0.91 when the profitability of the unidentified project turns out to be higher than 0.91 or to terminate the initially unidentified project when the realization is lower than 0.91. If the manager chooses the first strategy, however, the manager will not have the option. In this example the option value if positive with probability 0.09.

*Lemma 3: (i)* $\frac{\partial y}{\partial t}$ *is continuous in e, (ii)*

a. $\frac{\partial y}{\partial e}>0$ *and* $\frac{\partial^2 y}{\partial e \partial t}<0$ *as* $0<e<e^*$;

b. $\frac{\partial y}{\partial e}>0$ *and* $\frac{\partial^2 y}{\partial e \partial t}<0$ *as* $e^*<e<e^{**}$;

c. $\frac{\partial y}{\partial e}=0$ *and* $\frac{\partial^2 y}{\partial e \partial t}=0$ *as* $e>e^{**}$.

*(iii) Let* $\hat{t}$ *be the number of ex post unprofitable projects at time 1. Then* $\left.\frac{\partial y}{\partial t}\right|_{t=\hat{t}}=0$.

*(iv)* $\frac{\partial y}{\partial s}$ *is continuous in e and*

$$\frac{\partial y}{\partial s}=\begin{cases}\frac{1}{2}+\frac{t^2}{2h^2}-(c+i) & \text{as } e<e^*;\\ \frac{1-s}{e}-(c+i) & \text{as } e^*<e<e^{**};\\ t+1-s-(c+i) & \text{as } e>e^{**}.\end{cases}$$

*Furthermore,* $\frac{\partial^2 y}{\partial s^2}<0$.

(v) $$\frac{\partial^2 y}{\partial s \partial e} = \begin{cases} >0 & \text{as } e<e^*; \\ <0 & \text{as } e^*<e<e^{**}; \\ =0 & \text{as } e>e^{**}. \end{cases}$$

*(vi) Let $\tilde{s}$ be the number of projects to start that is optimal after project examination. Then* $\left.\frac{\partial y}{\partial s}\right|_{s=\tilde{s}} = 0.$

Proof: We first compute $\frac{\partial y}{\partial e}$.

Case a: $e<e^*$. In this case $w^* = w_2$, at which $\frac{dy}{dw} = \frac{dy_1}{dw} = 0$.

$$y(e,s,t) = \int_{\frac{t}{h^*}}^{1} h^* \alpha d\alpha + \int_{c+i-w^*}^{1} e\alpha d\alpha - s(c+i) + ti$$

$$= \int_{\frac{t}{\Delta s - ew^*}}^{1} (\Delta s - ew^*)\alpha d\alpha + \int_{c+i-w^*}^{1} e\alpha d\alpha - s(c+i) + ti$$

Using the envelope theorem,

$$\frac{\partial y}{\partial e} = \int_{\frac{t}{\Delta s - ew^*}}^{1} [-(1-c-i)-w^*]\alpha d\alpha - t^2 \frac{(1-c-i)+w^*}{(\Delta s - ew^*)^2} + \int_{c+i-w^*}^{1} \alpha d\alpha$$

$$= \int_{\frac{t}{h^*}}^{1} [c+i-w^*-1]\alpha d\alpha - t^2 \frac{1-(c+i-w^*)}{h^{*2}} + \int_{c+i-w^*}^{1} \alpha d\alpha$$

$$= [c+i-w^*-1](\frac{1}{2} - \frac{1}{2}\frac{t^2}{h^{*2}}) + t^2 \frac{(c+i-w^*)-1}{h^{*2}} + \frac{1}{2} - \frac{1}{2}(c+i-w^*)^2$$

$$= \frac{1}{2}[c+i-w^*-1] + \frac{1}{2}\frac{t^2}{h^{*2}}(c+i-w^*-1) + \frac{1}{2} - \frac{1}{2}(c+i-w^*)^2$$

$$= \frac{1}{2}[c+i-w^*] - \frac{1}{2}(c+i-w^*)^2 + \frac{1}{2}\frac{t^2}{h^{*2}}(c+i-w^*-1)$$

$$= \frac{1}{2}(c+i-w^*)(1-(c+i-w^*)) + \frac{1}{2}\frac{t^2}{h^{*2}}(c+i-w^*-1)$$

$$= \frac{1}{2}(c+i-w^*-1)[\frac{t^2}{h^{*2}} - (c+i-w^*)] = \frac{1}{2}(c+i-w^*-1)^2 > 0$$

Because by the definition of $w^* = w_2$,

$$t = \sqrt{2c+2i-1-2w_2}(\Delta s - ew_2) = \sqrt{2c+2i-1-w_2}h^*$$

Case b: $e^*<e<e^{**}$. In this case $w^* = \underline{w} = c+i-\frac{1-s}{e} < w_1$, $h^* = 1-e$

$$y(e,s,t) = \int_{\frac{t}{1-e}}^{1} (1-e)\alpha d\alpha + \int_{\frac{1-s}{e}}^{1} e\alpha d\alpha - s(c+i) + ti$$

$$= (1-e)[\frac{1}{2} - \frac{1}{2}\frac{t^2}{(1-e)^2}] + e[\frac{1}{2} - \frac{1}{2}\frac{(1-s)^2}{e^2}] - s(c+i) + ti$$

$$= \frac{1}{2} - \frac{1}{2}\frac{t^2}{(1-e)} - \frac{1}{2}\frac{(1-s)^2}{e} - s(c+i) + ti$$

Differentiate this with respect to $e$, we have,

$$\frac{\partial y}{\partial e} = -\frac{t^2}{2}\frac{1}{(1-e)^2} + \frac{1}{2}\frac{(1-s)^2}{e^2} = \frac{1}{2}[\frac{(1-s)^2}{e^2} - \frac{t^2}{(1-e)^2}]$$

By Figure 13, $e < e^{**} \Rightarrow t < \frac{(1-s)(1-e)}{e} \Rightarrow \frac{t}{1-e} < \frac{1-s}{e}$. Therefore $\frac{\partial y}{\partial e} > 0$.

Case c: $e > e^{**}$. In this case $w^* = \underline{w} = c + i - \frac{1-s}{e} > w_1$, $h^* = 1 - e$.

$$y(e,s,t) = y_2(e,s,t) = \int_k^1 (e + h^*)\alpha d\alpha - s(c+i) + ti$$

where $k = \frac{t + e(c+i-w)}{h+e} = t + e\frac{1-s}{e} = t + (1-s)$. Therefore, $y(e,s,t) = \int_{t+(1-s)}^1 \alpha d\alpha - s(c+i) + ti$, which is independent of $e$. Thus $\frac{\partial y}{\partial e} = 0$.

In summary,

$$\frac{\partial y}{\partial e} > 0 \text{ for } e < e^{**} \text{ and } \frac{\partial y}{\partial e} = 0 \text{ for } e > e^{**}.$$

Now we consider $\frac{\partial^2 y}{\partial e \partial t}$. We will show that

$$\frac{\partial^2 y}{\partial e \partial t} < 0 \text{ as } e < e^{**}; \ \frac{\partial^2 y}{\partial e \partial t} = 0 \text{ as } e > e^{**}.$$

We again consider three separate cases.

Case a: $e < e^*$

$$\frac{\partial y}{\partial e} = \frac{1}{2}(c + i - w^* - 1)^2$$

Where $w^* = w_2$ is defined by (see Figure 7)

$$\sqrt{2c + 2i - 1 - 2w_2}(\Delta s - ew_2) = t$$

and $h^* = \Delta s - ew_2$. Substitution and rearrangement yield

$$\Delta s - ew_2 = s - \hat{s} - ew_2 = s - (1-c-i)e - ew_2 = s - e + (c + i - w_2)e.$$

Let $x = c + i - w_2 - 1$. Then $h^* = s + xe$, $t = \sqrt{2x+1}(s + xe)$, and $\frac{\partial y}{\partial e} = \frac{1}{2}x^2$. Differentiate $t = \sqrt{2x+1}(s + xe)$ with respect to $t$. Then,

$$1=[\frac{1}{2}\frac{2}{\sqrt{2x+1}}(s+xe)+\sqrt{2x+1}e]\frac{dx}{dt}$$
$$=[\frac{h^*}{\sqrt{2x+1}}+\sqrt{2x+1}e]\frac{dx}{dt}.$$

Therefore $\frac{dx}{dt}>0$. Then $\frac{\partial^2 y}{\partial e\partial t}=x\frac{dx}{dt}<0$ because $x=c+i-w_2-1<0$.

Case b: $e^*<e<e^{**}$.

$$\frac{\partial y}{\partial e}=\frac{1}{2}[\frac{(1-s)^2}{e^2}-\frac{t^2}{(1-e)^2}].$$

Differentiate it with respect to $t$. Then,

$$\frac{\partial^2 y}{\partial e\partial t}=-\frac{t}{(1-e)^2}<0.$$

Case c: $e>e^{**}$

$$\frac{\partial y}{\partial e}=0\Rightarrow\frac{\partial^2 y}{\partial e\partial t}=0.$$

Now, consider $\frac{\partial^2 y}{\partial e\partial s}$. We will show that

$\frac{\partial^2 y}{\partial s\partial e}>0$ as $e<e^*$, $\frac{\partial^2 y}{\partial s\partial e}<0$ as $e^*<e<e^{**}$, and $\frac{\partial^2 y}{\partial s\partial e}=0$ as $e>e^{**}$.

Case a: $e<e^*$

$$\frac{\partial y}{\partial e}=\frac{1}{2}x^2,$$

where $x$ is defined by $t=\sqrt{2x+1}(s+xe)$. Differentiate the above equation with respect to $s$.

Then,

$$0=\left\{\frac{1}{2}\frac{2}{\sqrt{2x+1}}(s+xe)+\sqrt{2x+1}e\right\}\frac{dx}{ds}+\sqrt{2x+1}.$$

Therefore, $\frac{dx}{ds}<0$. Then $\frac{\partial^2 y}{\partial e\partial s}=x\frac{dx}{ds}>0$.

Case b: $e^*<e<e^{**}$.

$$\frac{\partial y}{\partial e}=\frac{1}{2}[\frac{(1-s)^2}{e^2}-\frac{t^2}{(1-e)^2}].\text{ Then }\frac{\partial^2 y}{\partial e\partial s}=-\frac{1-s}{e^2}<0.$$

Case c: $e>e^{**}$.

$$\frac{\partial y}{\partial e}=0 \qquad \Rightarrow \frac{\partial^2 y}{\partial e \partial s}=0$$

In summary, the parameters of the problem are divided into three regions:

$$\text{region } I=\left\{(s,t): s<1-e \quad or \quad t^2<(1-e)^2[\frac{2(1-s)}{e}-1]\right\},$$

$$\text{region } II=\left\{(s,t): t^2>(1-e)^2[\frac{2(1-s)}{e}-1] \quad and \quad t<\frac{(1-s)(1-e)}{e}\right\}, \text{ and}$$

$$\text{region } III=\left\{(s,t): t>\frac{(1-s)(1-e)}{e} \quad and \quad s>1-e\right\},$$

In region I, $0<e<e^*$,

$$w^*=w_2>\underline{w},\ y=y_1,$$

$$h^*=\Delta s-ew_2=\frac{t}{\sqrt{2c+2i-1-2w_2}}>0,$$

$$\frac{\partial h^*}{\partial e}<0,\ \frac{\partial y}{\partial e}>0,\ \frac{\partial^2 y}{\partial e \partial t}<0,\ \frac{\partial^2 y}{\partial s \partial e}>0.$$

In region II, $e^*<e<e^{**}$,

$$w^*=\underline{w}=c+i-\frac{(1-s)}{e}\in(w_2,w_1),$$

$$y=y_1,\quad h^*=1-e,$$

$$\frac{\partial y}{\partial e}>0,\ \frac{\partial^2 y}{\partial e \partial t}<0,\ \frac{\partial^2 y}{\partial s \partial e}<0.$$

In region III, $e>e^{**}$,

$$w^*=\underline{w}=c+i-\frac{(1-s)}{e}>w_1,$$

$$y=y_2,\quad h^*=1-e,$$

$$\frac{\partial y}{\partial e}=0,\ \frac{\partial^2 y}{\partial e \partial t}=0,\ \frac{\partial^2 y}{\partial s \partial e}=0.$$

Now, we wish to show that $\frac{\partial y}{\partial t}$ is continuous in $e$.

Case a, $e<e^*$. By the envelope theorem,

$$\frac{\partial y}{\partial t}=-\frac{t}{\Delta s-ew^*}+i=i-\frac{t}{h^*}=i-\sqrt{2c+2i-1-2w_2}.$$

Case b. $e^*<e<e^{**}$.

$$\frac{\partial y}{\partial t}=-\frac{t}{1-e}+i=-\frac{t}{h^*}+i$$

At $e = e^*$, $\sqrt{2c+2i-1-2w_2} = \sqrt{2c+2i-1-2\underline{w}} = \sqrt{\frac{2(1-s)}{e} - 1} = \frac{t}{1-e}$. Therefore, $\frac{\partial y}{\partial t}$ is continuous at $e^*$.

Case c. $e > e^{**}$.

$$\frac{\partial y}{\partial t} = i - [t + (1-s)].$$

At $e = e^{**}$, $t = \frac{(1-s)(1-e)}{e}$ $\Leftrightarrow$ $\frac{t}{1-e} = t + (1-s)$. Therefore, $\frac{\partial y}{\partial t}$ is continuous at $e^{**}$.

The graph of $\frac{\partial y}{\partial t}$ is shown in Figure (15b). y has a similar graph, given by Figure (15a).

At last, we show that $\left.\frac{\partial y}{\partial t}\right|_{t=\hat{t}} = 0$, where $\hat{t}$ is the number of ex post inefficient projects at time 1. Note that $y(e,s,t) = \tilde{y}(e,s,t,w^*(e,s,t))$, where $\tilde{y}(e,s,t,w)$ is the expected gross value-added given $e,s,t$, and possibly sub-optimal $w$. Therefore,

$$\left.\frac{\partial y}{\partial t}\right|_{t=\hat{t}} = \left.\frac{\partial \tilde{y}}{\partial t}\right|_{t=\hat{t}} + \left.\frac{\partial \tilde{y}}{\partial w}\right|_{t=\hat{t}} \left.\frac{\partial w^*}{\partial t}\right|_{t=\hat{t}}$$

By the definition of $\hat{t}$, the first term is zero. In region I where $0 < e < e^*$, the optimal $w$ is an interior solution and thus $\frac{\partial \tilde{y}}{\partial w} = 0$. In regions II and III, the optimal w does not depend on t and thus $\frac{\partial w^*}{\partial t} = 0$. Therefore $\left.\frac{\partial y}{\partial t}\right|_{t=\hat{t}} = 0$ for all possible cases.

The proof of (iv), (v), and (vi) is very similar to that of (i), (ii), and (iii).

<u>*Lemma 4*</u>*: (i) The feasible set of the principal's optimization program (4) is non-empty if and only if $y(e_l,s,t) < y(e_h,s,t)$. If the feasible set of program (4) is non-empty, then: (ii) the constraint (IC) has a positive Lagrange multiplier, $\mu$, and thus is binding; (iii) the optimal wage function, $w(x)$, is strictly increasing.*

Proof: (i) By Lemma 3(ii), $y(e_l,s,t) \le y(e_h,s,t)$. If $y(e_l,s,t) = y(e_h,s,t)$, then the definition of $f(x;e,s,t)$ implies that $f(x;e_l,s,t) = f(x;e_h,s,t)$. With this equality, constraint (IC) becomes $-d(e_h) \ge -d(e_l)$, which contradicts $d(e_h) > d(e_l)$. Therefore, the feasible set of program (4) is empty. If $y(e_l,s,t) < y(e_h,s,t)$, however, (IC) can be satisfied by choosing $w(x)$ so that its slope is large enough.

(ii) The Lagrangian of program (4) is

$$L = \int [x - w(x)] f(x; e_h, s, t) dx + \lambda \left\{ \int v(w(x)) f(x; e_h, s, t) dx - d(e_h) \right\}$$
$$+ \mu \left\{ \int v(w(x)) f(x; e_h, s, t) dx - d(e_h) - \int v(w(x)) f(x; e_l, s, t) dx + d(e_l) \right\}.$$

Pointwise optimization of the Lagrangian with respect to the sharing rule, $w(x)$, and rearrangement yield,

$$\frac{1}{v'(w(x))} = \lambda + \mu \left[ 1 - \frac{f(x; e_l, s, t)}{f(x; e_h, s, t)} \right]. \qquad \text{(FOC-w)}$$

Both $\lambda$ and $\mu$ are non-negative.

If $\mu = 0$, then $w(x)$ is constant. Let $w(x) = w_0$. Then constraint (IC) becomes

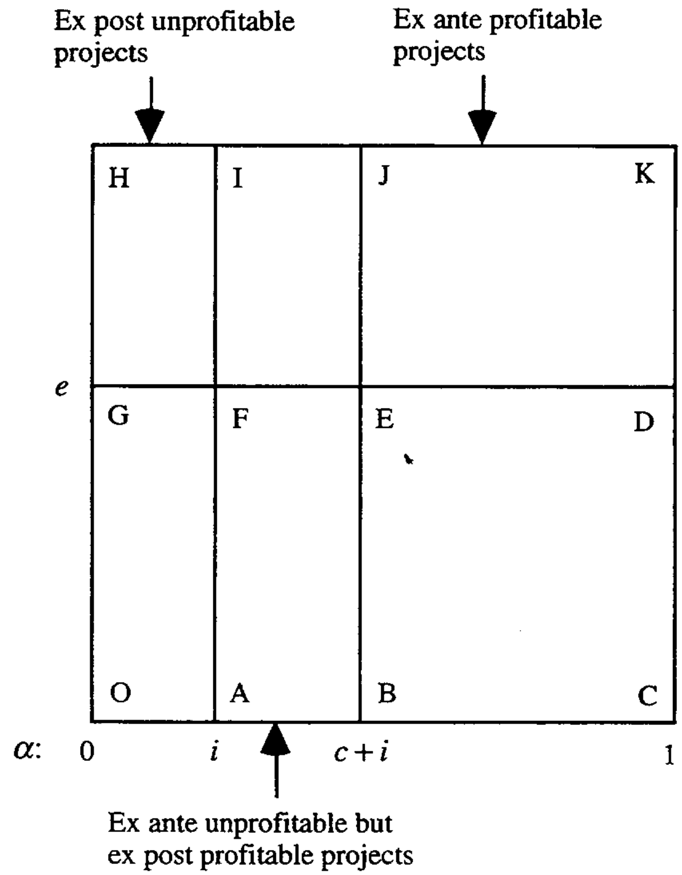
$$v(w_0) - d(e_h) \geq v(w_0) - d(e_l),$$

which contradicts $d(e_h) > d(e_l)$. Therefore, $\mu$ must be positive.

(iii) By the definition of $f(x; e, s, t)$,

$$\frac{f(x; e_l, s, t)}{f(x; e_h, s, t)} = \frac{g(x - y(e_l, s, t))}{g(x - y(e_h, s, t))}.$$

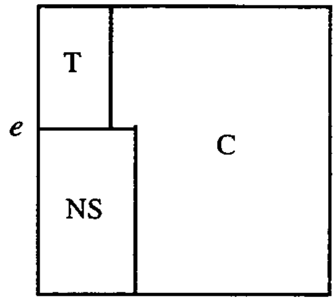
Since $y(e_l, s, t) < y(e_h, s, t)$, the monotone likelihood ratio condition implies that the right hand side of the above equation is a decreasing function of $x$. Since $\mu > 0$, $v'' < 0$, and by (FOC-w), the optimal wage function, $w(x)$, increases with $x$. Q.E.D.




*The horizontal axis, $\alpha$, indicates the profitability of a project. The vertical axis, $\beta$, is a dummy variable. Projects with different $\beta$ but the same $\alpha$ are identical to each other.*

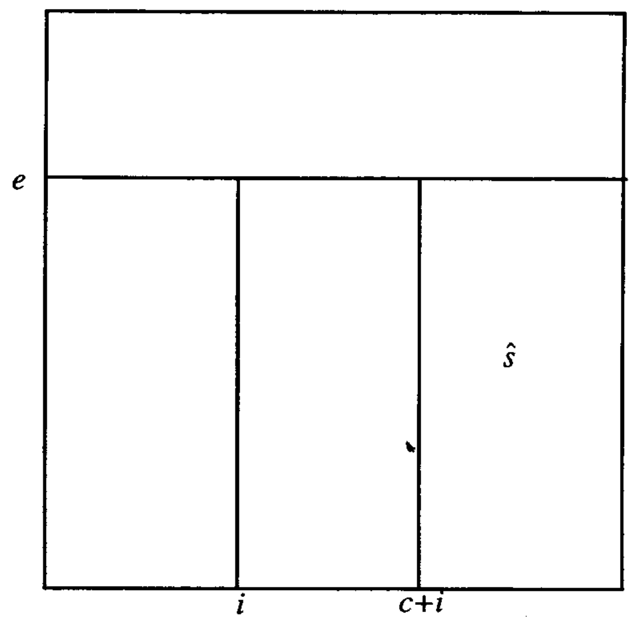
*Area OAIH is the set of ex post unprofitable projects*
*Area ABJI is the set of ex ante unprofitable but ex post profitable projects*
*Area BCKJ is the set of ex ante profitable projects*
*Area OCDG is the set of projects that are reviewed at time 0*
*Area OBEG is the set of ex ante unprofitable projects that are identified at time 0*

Figure 1




*NS is the set of projects that are not started, T is the set of projects that are terminated at time 1, and C is the set of projects that are completed.*

Figure 3




*The horizontal axis,* $\alpha$*, indicates the profitability of a project. The vertical axis,* $\beta$*, is a dummy variable. Projects with different* $\beta$ *but the same* $\alpha$ *are identical to each other.*

Figure 4

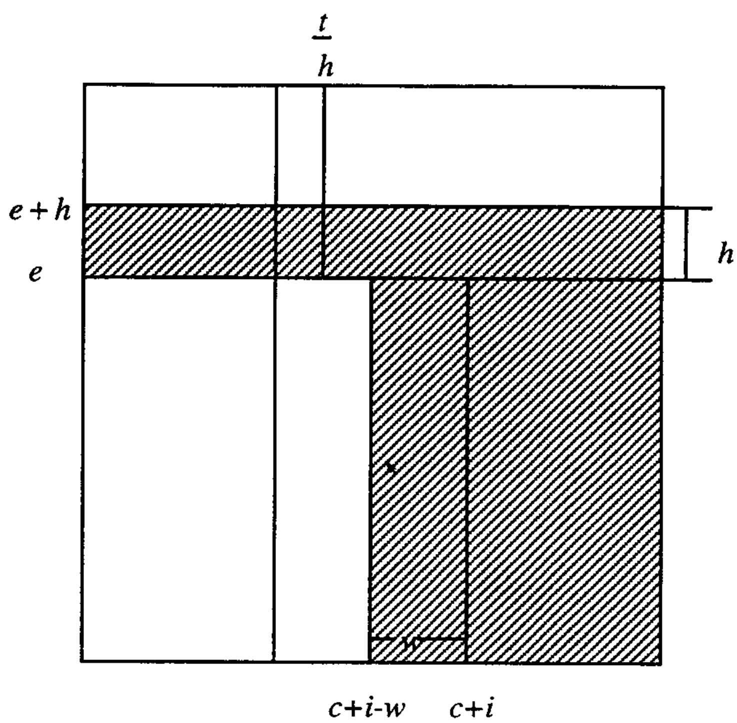



Figure 5

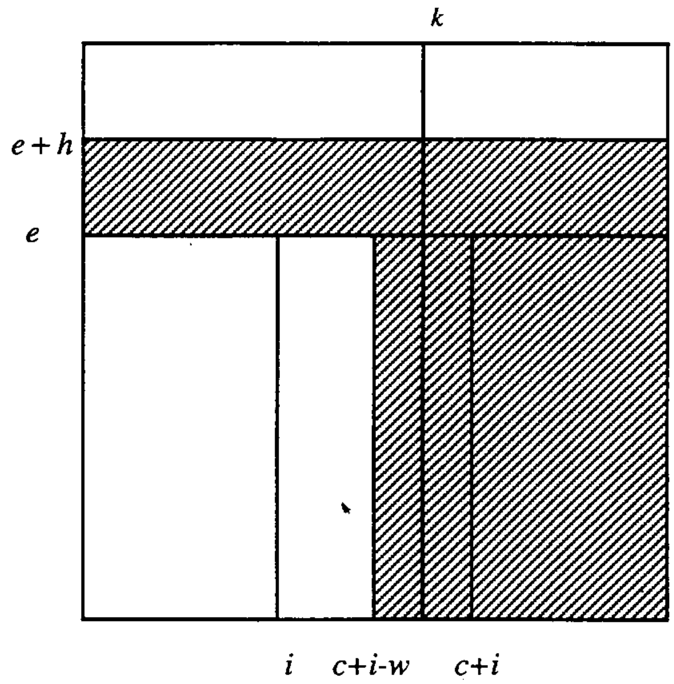



Figure 6

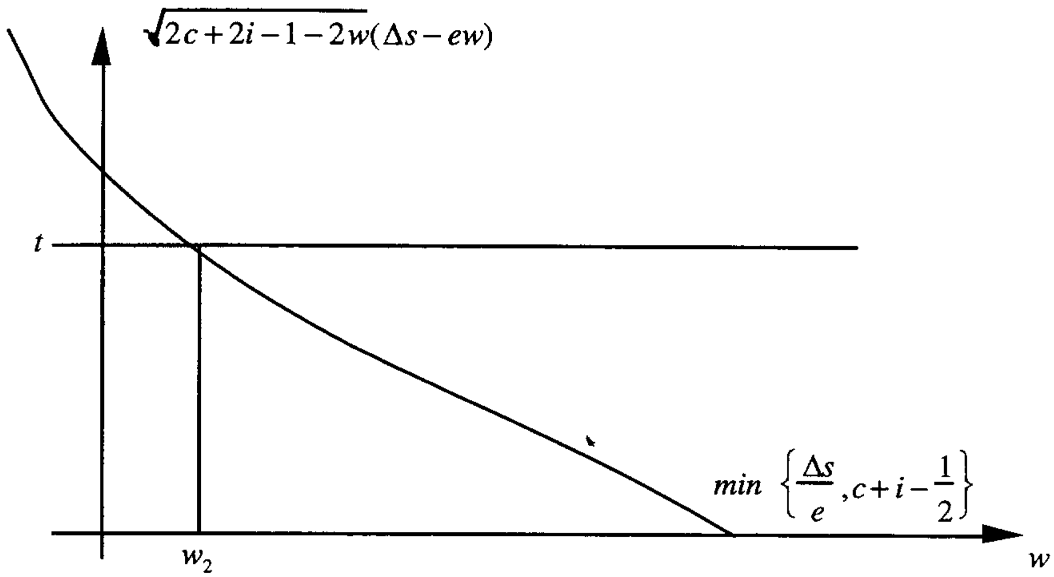

$\sqrt{2c+2i-1-2w}(\Delta s - ew)$
$t$
$min\ \left\{\frac{\Delta s}{e}, c+i-\frac{1}{2}\right\}$
$w_2$
$w$

Figure 7

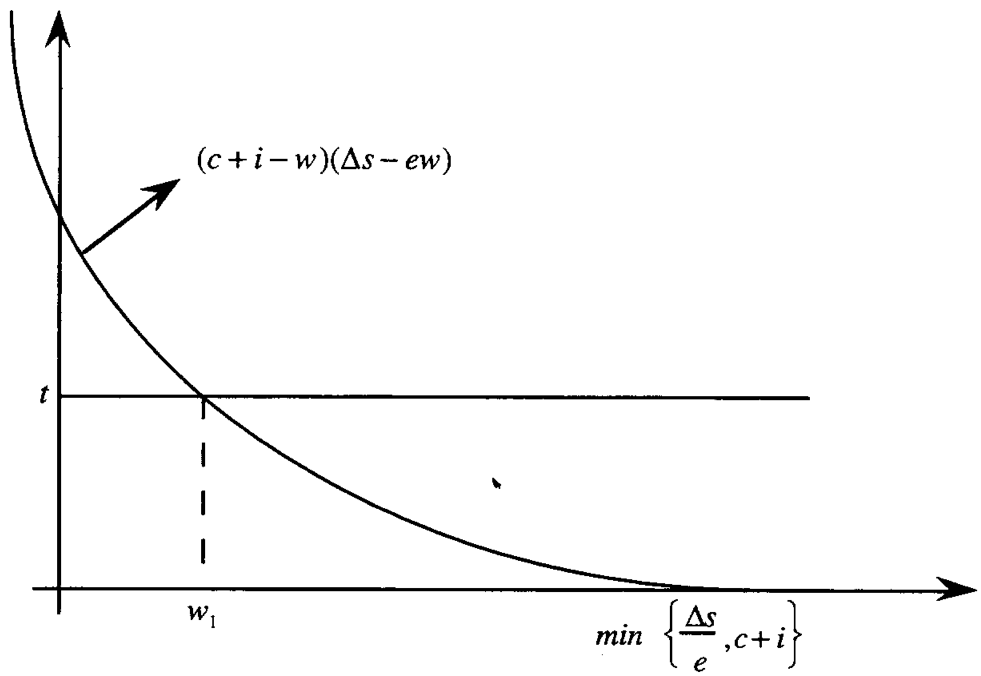



Figure 8


Figure 9

Case 1: $\frac{\Delta s}{e} \leq c + i - \frac{1}{2}$

Case 2: $\frac{\Delta s}{e} > c + i - \frac{1}{2}$

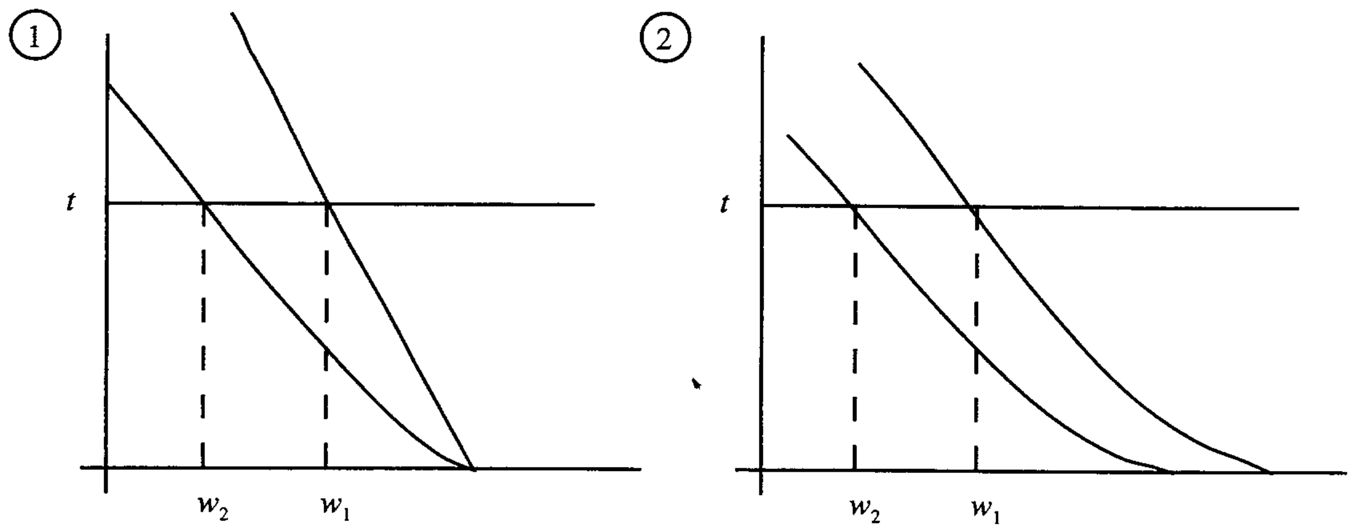



Figure 10

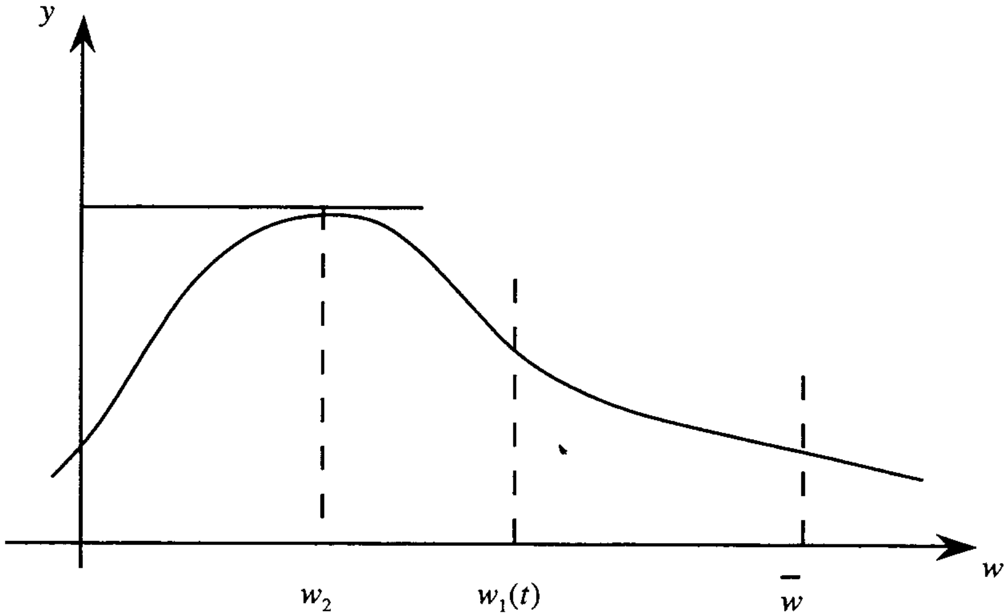



Figure 11

as $1-s<\frac{1}{2}$ or as $1-s>\frac{1}{2}$

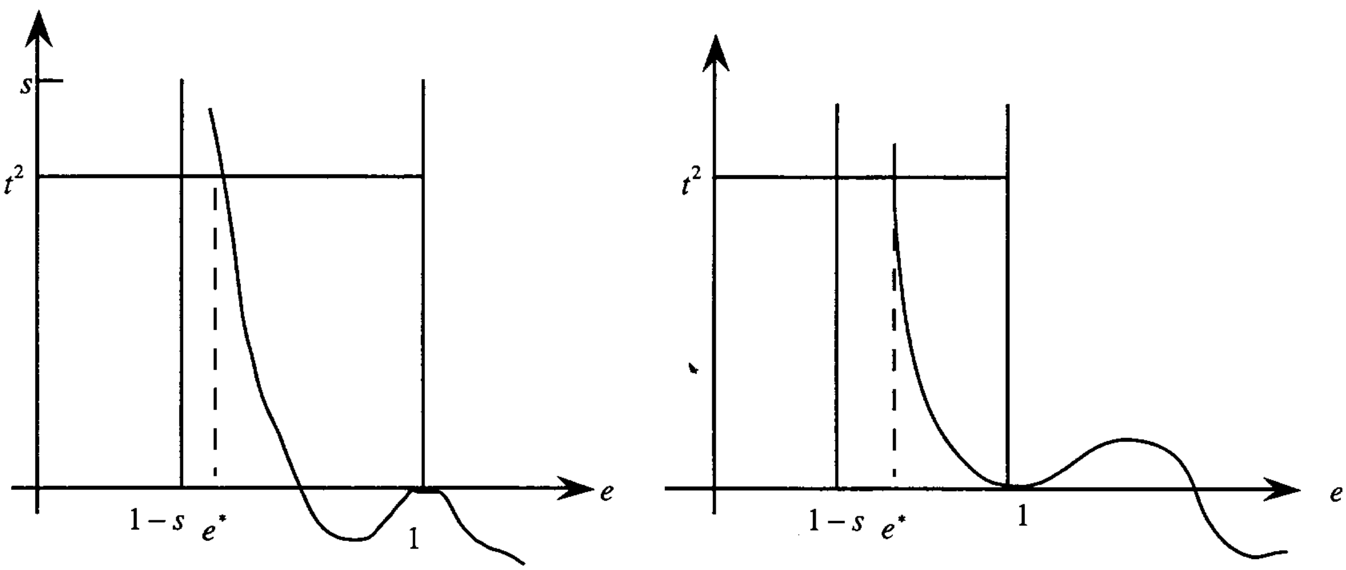



Figure 12

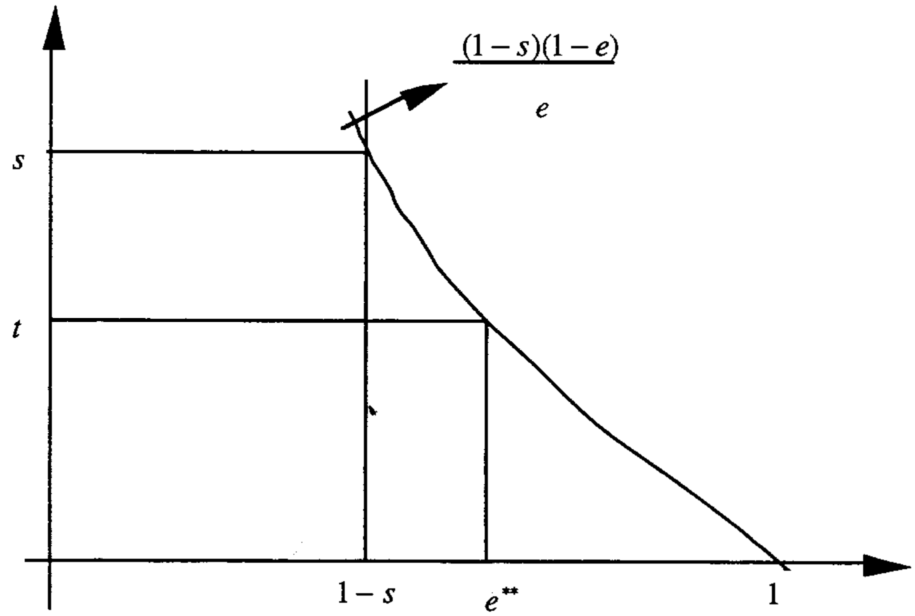



Figure 13

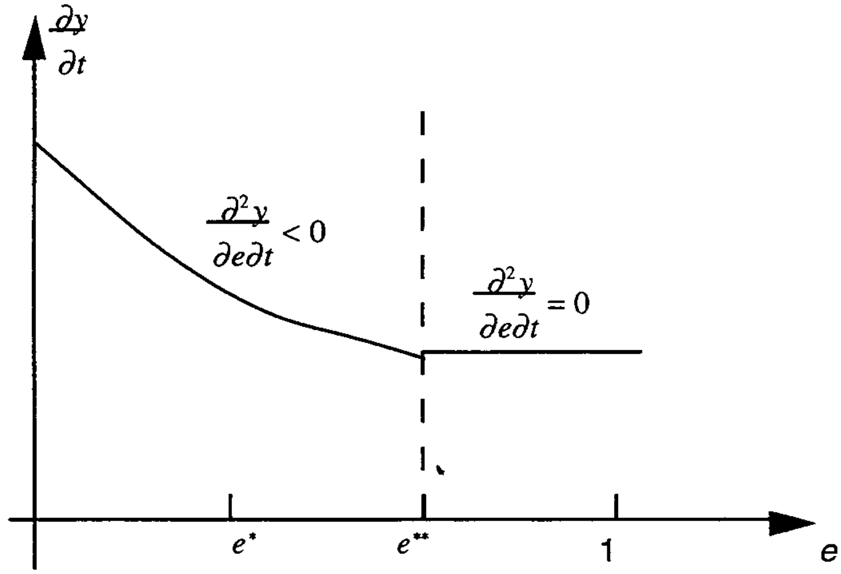

$\frac{\partial y}{\partial t}$
$\frac{\partial^2 y}{\partial e \partial t} < 0$
$\frac{\partial^2 y}{\partial e \partial t} = 0$
$e^*$
$e^{**}$
1
$e$

Figure 14

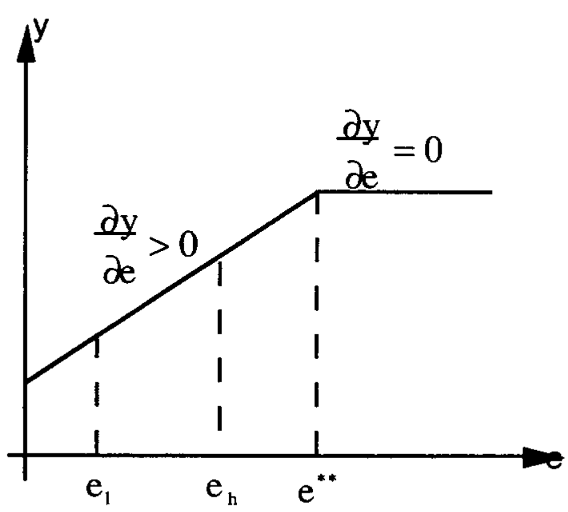



Figure 15a

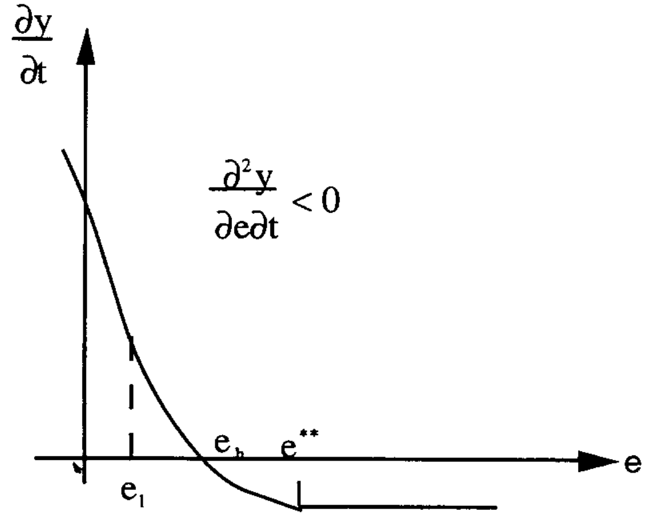



Figure 15b

Figure 15

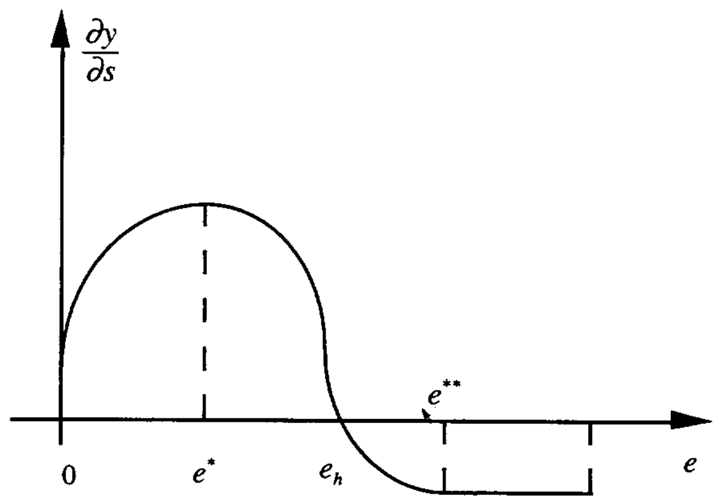



Figure 16