Restructuring Investment in Transition: A Model of the Enterprise Decision

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Comments Welcome

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Abstract
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decision of the firm in the volatile environment of the post-Soviet
transition. Conditions favoring and limiting investment in fundamen-
tal restructuring are explored, where successful restructuring leads to
a substantial increase in expected profitability. In particular, the
impact of the availability of alternative uses for investment resources,
the degree of uncertainty and volatility in the economic environment,
and the cost of capital as affected by credit constraints and investment
subsidy policies is modeled and analyzed. It is shown that subsidizing
investment in the presence of significant outside opportunities for the
use of those funds, particularly when such opportunities are lost with
successful restructuring, can be counterproductive, delaying that re-
structuring. The paper also explores the interaction of restructuring
investment with investment in capacity, and shows that shrinking ca-
pacity can be an optimal alternative to restructuring investment in an
unfavorable environment, while successful restructuring is optimally
exploited by expanding capacity. Finally, there is an “unrestructured,”
low capacity trap, with investment effort largely directed toward out-
side activities. These results highlight some of the reasons for the
limited amount of restructuring investment in Russia and many of the
other former Soviet Republics.

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author’s responsibility. Comments, criticisms, suggestions and references are, of course,
extremely welcome.
Non-Technical Summary

This paper outlines a simple economic model of a firm's decision to invest in fundamental restructuring in the volatile environment of the post-Soviet transition. Conditions favoring and limiting investment in fundamental restructuring are explored, where successful restructuring leads to a substantial increase in expected profitability. In particular, the impact of the availability of alternative uses for investment resources, the degree of uncertainty and volatility in the economic environment, and the cost of capital as affected by credit constraints and investment subsidy policies is modeled and analyzed. It is shown that subsidizing investment in the presence of significant outside opportunities for the use of those funds, particularly when such opportunities are lost with successful restructuring, can be counterproductive, delaying that restructuring. The paper also explores the relationship between investment in restructuring and more traditional investment / disinvestment. It shows that disinvestment can be an optimal alternative to restructuring in an unfavorable environment, while successful restructuring is optimally exploited by expanding capacity. Finally, conditions are provided in which it is optimal to reduce capacity and invest largely in outside activities rather than in restructuring. These results highlight some of the reasons for the limited amount of restructuring investment in Russia and many of the other former Soviet Republics.
1 Introduction.

One of the puzzles of the transition process in Russia is the lack of substantial investment toward restructuring industry, despite apparent massive opportunities for improvement and the growing success of macroeconomic stabilization, price and activity liberalization, and privatization. There have been a number of explanations given for this phenomenon, ranging from willful theft and destruction of social value by rapacious "entrepreneurs," through lack of state support and counterproductive economic policies, to the lack of complementary supportive institutions such as functioning legal and banking systems. At the root of the explanations lie two competing hypotheses: (i) inability to restructure, despite knowing what needs to be done and wanting to do it, in the current economic situation; (ii) lack of desire to restructure, whether from perverse incentives or belief that it is wrong. The first is driven by a lack of access to needed resources at terms which would make restructuring profitable, raising equilibrium questions of the institutional constraints on access to resources, and on the determination of their terms of trade. Eschewing issues of beliefs about a "good" economic system, the second boils down to a question of the incentives faced by those able to make the restructuring decision. What are the relevant trade-offs and returns to different courses of action, including investment in the radical restructuring of the enterprise and its economic activity?

Of course, the answer to that question depends very much on who it is that is in the position to make that decision. Most of the literature assumes that it is the employees of the firm, who control (assumed positive) net revenues and face a risk of redundancy and loss of employment under restructuring, who must be convinced to restructure. Once 'insiders' have agreed to it, or an 'outsider' gets control, this literature assumes that restructuring is instantaneous and effortless.\(^1\) That seems to both substantially overstate the power of the workers collective in post-Soviet firms, where a small group of top managers have gained, through privatization, substantially full control in most cases (Blasi, et. al., 1997), and substantially understate the difficulties involved. While there seems a residual paternalism, it just adds directly to the costs of restructuring rather than creating a collective action problem. Hence the following analysis assumes that there is concentrated

\(^1\)A clear, concise analysis of such a situation, more applicable to east central Europe than to the former Soviet Union, is given in Blanchard (1997).
insider control that doesn’t take direct account of the risks and costs faced by the typical worker in enterprise restructuring.

This note begins a formal exploration of that investment decision by a rational enterprise in a simple model of a transition environment, characterized by high volatility and uncertainty. It takes a flow of funds perspective, assuming that any necessary resources are available at some cost, and investigates the trade-offs involved in using those resources. The critical alternative is investment in ‘restructuring’ rather than in some more immediately lucrative earning opportunity (e.g. the GKO market or what Ickes and Ryterman (1994) have called “short-run profit opportunities”), investigated under various assumptions about the cost of access to resources, technology of restructuring, volatility of the environment, and time preferences of the decision maker. The model also begins to explore the interaction between restructuring and altering the scale of operations (expansion vs. labor shedding [sometimes called ‘defensive restructuring;’ see Grosfeld and Roland (1994)]). In each case conditions are derived which might ‘rationalize’ an observed lack of restructuring, indicating where further reform would seem necessary for market-oriented agents to start investing in the true restructuring required for renewed growth of the Russian economy.

This paper thus takes a highly stylized look at the decision to implement physical change at the firm level subsequent to the liberalization, stabilization and privatization policies that launch a serious transition from a command to a market economic system. Such restructuring has been looked at in the literature largely through its reflection in labor flows\(^2\) or in the quality of ultimate performance (growth, productivity, labor shedding, etc.) as a function of observables such as ‘ownership,’ origin of the enterprise, hardness of financial constraints, and/or sector/branch (a measure of “opportunity”).\(^1\)

\(^2\)i.e. render consistent with existing incentives.

\(^3\)See for example Aghion and Blanchard (1994), Blanchard (1997), Commander and Coricelli (1995), Chada, Coricelli and Krajnyak (1993), and Layard and Richter (1995). Labor flows are sometimes treated as a proxy for changes in the whole optimal resource package, but there is an apparent assumption that appropriate restructuring of all complementary factors will be automatic as the labor force adjusts.

\(^4\)Among the papers relevant here are Djankov (1997), Konings (1997), Linz (1997a), Pohl, Djankov and Anderson (1996), and Pohl. Anderson, Claessens and Djankov (1997). Similar work on Russian firms can be found in Commander, Fan and Schaffer (1996). Some exceptions to this general approach, which however do not attempt to model the investment decision, are Ickes, Murrell and Ryterman (1997) focussing on barriers to investment. Blasi et al. (1996) focussing on the incidence and impact of variations in corporate governance.
These analyses appear fruitful for analyzing the process of restructuring that is occurring in East Central Europe where the development of market institutions has progressed much further than in the former Soviet Union. Yet they don't seem as relevant for understanding the lack of serious enterprise restructuring in Russia and the other CIS states.

Despite massive privatization, resulting in some 80% private ownership, there has been very little net investment or restructuring of capacities and interactions. Indeed, gross investment was down by over 7%, and contracted construction work by over 10%, in 1997, while industrial output grew by some 2% and National Income increased by about 1%.\(^5\) This has been occurring despite years of savings of over 20% of national income, claimed depreciation of 40-60% of capital value [Linz (1997b)], and apparently substantial (9-18%, varying by branch) "investment" use of revenues.\(^6\) Hence the question of this paper: Can this be explained as (individual enterprise) optimal economic behavior?

We pose the question by assuming present discounted value maximization and deriving optimal investment policies in a stylized transition environment. Then we investigate the impact of various environmental/policy factors on the optimal investment decisions of firms in order to identify factors inhibiting investment in transition and to begin the study of the dynamics of firm adjustment during transition. The basic structure of the model and some benchmark results are presented in Section 2, while Section 3 explores the kinds of assumptions that might lie behind the restructuring investment behavior observed in Russia. Section 4 concludes with some interpretation of the model's results and a discussion of potential extensions for future research.

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\(^6\) The figures can be found in the GosKomStat handbook Promyshlennost' Rossi 1996. Investment/sales ratios are calculated from the figures on pp. 6, 75, and 121-4. OECD (1997), p.245, shows savings (gross capital formation on the end-use side) to be over 21% of GDP since 1993.
2 The Model.

2.1 Basic Notation.

We begin with the simplest case where the firm cannot choose its scale of operation, $k$. The enterprise is either adjusted to the new environment ("good" = "restructured") or not. Restructuring is desirable as a restructured firm has substantially higher expected net revenues, $y$, although realized revenues in any period can vary widely. We assume that the firm knows its own state, although the government and outside financiers do not know it. Let the state ("type") of the firm be $\widetilde{\theta}$ if the firm is properly restructured, and $\vartheta$ if it is not. The firm can choose to invest an amount $x \geq 0$ in "restructuring"; if it is in state $\vartheta$ then the investment makes it possible (if not certain) to successfully restructure, i.e. move into state $\widetilde{\theta}$. If it is already 'restructured' ($\widetilde{\theta}$), then $x$ increases the probability that it will remain appropriately structured for the changing environment.

Under the assumption that firms know their own present state, this "restructuring technology" is given by the transition probability function $Q(\overline{x}, \underline{x})$, with potentially different investment in each state.

$$Q(\overline{x}, \underline{x}) = \begin{pmatrix} (1 - \delta e^{-a\overline{x}}) & \delta e^{-a\overline{x}} \\ (1 - e^{-a\underline{x}}) & e^{-a\underline{x}} \end{pmatrix}, \tag{1}$$

where $\delta$ represents the probability of "environmental drift" — the probability that the restructuring already achieved becomes inappropriate.\footnote{An alternate specification of the transition probabilities that is as tractable is:}

$$Q(\overline{x}, \underline{x}) = \begin{pmatrix} \frac{1 - \delta + a\overline{x}}{1 + a\overline{x}} & \frac{\delta}{1 + a\overline{x}} \\ \frac{a\overline{x}}{1 + a\overline{x}} & \frac{1}{1 + a\overline{x}} \end{pmatrix}.$$  \tag{2}

This allows somewhat greater response of transition probabilities to investment.

Remark 1 If $s = \begin{bmatrix} \pi, & (1 - \pi) \end{bmatrix}$ is the current distribution of types (say observed by a central authority), then

$$\mathbf{s} = sQ(\overline{x}, \underline{x})$$
\[
\begin{align*}
&= [1 - e^{-\alpha T}(1 - \pi(1 - \delta)) - e^{-\alpha(1-T)}(1 - \pi), \\
&\quad e^{-\alpha T}(1 - \pi(1 - \delta)) + e^{-\alpha(1-T)}(1 - \pi)]
\end{align*}
\]

is the future (next period) distribution. Similarly, if \( V = \begin{bmatrix} \bar{V} \\ \underline{V} \end{bmatrix} \) represents the current value of the two states, then

\[
\tilde{V} = Q(\bar{x}, \underline{x}) V = \begin{bmatrix} \bar{V} - \xi e^{-\alpha T}(\bar{V} - \underline{V}) \\ \underline{V} - e^{-\alpha(1-T)}(\bar{V} - \underline{V}) \end{bmatrix}
\]

is the expected value after transition under investment policies \((\bar{x}, \underline{x})\).\(^8\)

In this paper we want to explore the optimal decisions of firms in this environment under various "cost of capital" (for restructuring or other "investment") and "availability of other options" assumptions, including those derived from various policy considerations (subsidy arrangements). A critical assumption to the analysis is that firms face an "opportunity cost" to investing not just in terms of the need to raise capital but also in terms of foregone opportunities for the use (personal exploitation) of the funds. This captures outside or "rent-seeking" opportunities for the management of the firm that will be traded off against the reward/value to maximizing the PDV of expected net revenue flows.\(^9\) We will later also want to allow firms the opportunity to adjust the scale of their operations, to expand when they successfully restructure and shrink when they do not do so.

Formally, we have the following assumptions:

**Assumption 1:** The primary difference between the states of the firm is the distribution of net returns to operating in each state. Letting \( \Xi = \{ \eta \} \) be the set of possible net payoffs (cash flow) in a given period, expected

\(^8\)With only notational complication, the 'elasticity of restructuring with respect to investment' can be differentiated by state: \( \xi \) and \( \pi \) respectively.

\(^9\)One might also interpret \( R(z) \) as the return to exploiting "insider" network advantages to maintain viability in the face of the "transition shock." Such networks provide a survival mechanism, or safety net, for firms that choose not to, or cannot successfully, restructure (Ickes, Murrell, Ryterman (1997)). This return would then arise from maintaining old networks that must be broken up if true market-oriented restructuring is to take place. Under such interpretation, \( R(z) \) should be available only to an unrestructured firm that has never abandoned the network (successfully restructured). We pursue this interpretation in Section 3.3 below.
net returns to operating in that period are $\bar{y} = y(\bar{\theta}) = \mathbb{E} \{\eta \mid \theta = \bar{\theta}\}$ in state $\bar{\theta}$, and $\underline{y} = y(\underline{\theta}) = \mathbb{E} \{\eta \mid \theta = \underline{\theta}\}$ in state $\underline{\theta}$, where
\[
\bar{y} = \sum_{\eta \in \Xi} \eta \cdot p^+_{\eta} > 0, \\
\underline{y} = \sum_{\eta \in \Xi} \eta \cdot p^-_{\eta} < 0,
\]
and $p^+$ is the distribution of $\eta$ for ‘good’ ($\bar{\theta}$) firms and $p^-$ is the distribution for $\underline{\theta}$ firms.

**Assumption 2:** When available, the ‘outside’ use of resources, $z$, by the firm yields gross return $R(z)$, where $R(\cdot)$ is an increasing, bounded, and (eventually) concave function. This opportunity is available in any period with probability $\varepsilon$.

**Assumption 3:** The direct cost of capital used for either investment or ‘outside’ purposes is given by an increasing, convex function $c_\theta(x + z)$ which may depend on the “type” ($\theta$) of the firm.

These assumptions will be built on as we develop the full implications of this model for understanding the problems of investment in the Russian transition. Again, we do not yet formalize the investment required to alter the scale of operation of the firm.

### 2.2 The Linear Base Case.

In the simplest case, capital costs are linear and the same for both types (no credit constraints), the firm’s objective is to maximize the PDV of expected net revenues, whether from inside or outside sources, and firms of both types face the same outside opportunities, with $\varepsilon = 1$:

\[
V(\theta_0) = \max_{\{z_t, x_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ y(\theta_t) + R(z_t) - c(x_t + z_t) \right\} = \left\{ x_t \right\}_{t=0}^{\infty}.
\]

The firms face uncertainty with respect to the consequences of their decisions, both in terms of current payoffs and the future state of restructuring; $\theta_t$ is determined after (and by) $x_t$. This case provides an analytic benchmark.

The Bellman Equation for each of the two possible states is:

\[
\overline{V} \equiv V(\bar{\theta}) = \max_{z, x} \left\{ \bar{y} + R(z) - c \cdot (x + z) + \beta Q(x; \bar{\theta}) \bar{V} \right\}
\]

\[
\underline{V} \equiv V(\underline{\theta}) = \max_{z, x} \left\{ \underline{y} + R(z) - c \cdot (x + z) + \beta Q(x; \underline{\theta}) \underline{V} \right\}
\]
\[ V \equiv V(\theta) = \max_{\varepsilon, \bar{\varepsilon}} \left\{ y + R(z) - c \cdot (x + z) + \beta Q(x; \theta) V \right\} \]

where \( \bar{y} \) and \( y \) are the expected net returns to operating, with the given structure of production, in the current period, and \( Q(x; \theta) \) is the relevant row of the transition operator matrix (1). In this case a closed form solution to the firm’s decision problem can be obtained.

**Proposition 1** With constant marginal costs of capital the same for both types, and identical outside opportunities for the use of capital, firms of both types choose identical levels of ‘rent-seeking’ \( z^* \), and the ‘bad’-type firm optimally invests more than the ‘good.’ \( \bar{x} > \bar{\varepsilon} \).

\[
\begin{align*}
\bar{x} &= \max \left\{ \frac{1}{a} \ln \left( \frac{a \beta \delta \Delta V}{c} \right), 0 \right\}; \\
\bar{\varepsilon} &= \max \left\{ \frac{1}{a} \ln \left( \frac{a \beta \Delta V}{c} \right), 0 \right\}; \\
x - \bar{x} &= \frac{1}{a} \ln \frac{1}{\delta} \text{ if } \bar{x} > 0; \\
R'(z^*) &= c; \\
V &= \left( \begin{array}{c} \bar{y} + R(z^*) - c(x + z^*) + \beta(V - \frac{c}{\alpha \beta}) \\ \bar{y} + R(z^*) - c(x + z^*) + \beta(V - \frac{c}{\alpha \beta}) \end{array} \right) = \text{ (if } \bar{x} > 0) \\
\Delta V &= \bar{V} - V = \Delta y + \bar{x} - \bar{\varepsilon} = \Delta y + \frac{c}{a} \ln \frac{1}{\delta} \text{ if } \bar{x} > 0.
\end{align*}
\]

where \( \Delta y \equiv \bar{y} - y \).

**Proof.** Clearly the finite state, bounded discounted dynamic programming problem (3) has a solution that satisfies the Bellman equations (5,6). As the maximization problem that they pose is strictly concave, the first order conditions give the unique solution (7, 8):

\[-c + a \beta \delta \cdot \Delta V \cdot e^{-\alpha \bar{x}} \leq 0 \]
\[-c + a \beta \cdot \Delta V \cdot e^{-\alpha \bar{x}} \leq 0 \]
\[c + R'(z) \leq 0 \]

with appropriate complementary slackness. Substitution into equations (4) and (5) give the closed form for \( \Delta x, V, \) and \( \Delta V. \)
Remark 2 Note that the primary factor determining $\Delta V$, and hence when investment is positive, is $\Delta y$. As long as $\Delta y > \frac{c}{a} \left( \ln \delta + \frac{1}{\delta a} \right)$, $x > 0$.

Remark 3 This solution is a fixed point of (3) under the optimal investment policy. Notice that the exponential form of the investment impact means that the optimal policy equalizes across current states, the probability of being in the good state next period.

This allows complete comparative statics analysis, highlighting results to be expected in a well functioning economic environment. But first it is worth emphasizing: (i) Both types of firms engage in the same amount of optimal ‘rent-seeking’, $z^*$; (ii) The ‘bad’ (unstructured) type optimally invests more than the firm that has already restructured, narrowing the difference in the PDV of being in the two states; (iii) The optimal marginal value of both rent-seeking and investment is naturally $c$.

Corollary 2 The comparative statics are as follows:

- If investment is positive, $\Delta x$ is decreasing in $a, \delta$; $x$ is decreasing in $a, c$, and increasing in $\beta$; and $\bar{x} \gtrless x$ as $\delta \to 1$.
- If investment is positive, $\Delta V$ is decreasing in $a, \delta$ and increasing in $c$; $V$ is decreasing in $a, c, \delta$, and increasing in $\beta$; and $\Delta V \gtrless \Delta y$ as $\delta \to 1$.
- Outside investment $z$ is decreasing in the ‘cost of capital’, $c$.

Remark 4 Here we have assumed that the “elasticity of transition probability with respect to investment,” $a$, is the same in both states. It might be natural to suppose it greater in the state of “success,” $\bar{a} > a$; it is easier for a restructured firm to respond to changes in the environment. Such an assumption, however, still further reduces the incentive to invest in the restructured state, thus widening the discrepancy with what is observed in Russia.

A further consequence worth noting is the long run value to society of a firm pursuing the optimal policy derived above. This policy generates an ergodic Markov chain on the two states. While $V$ gives the value of the firm conditional on ‘type’, the long run value will be the average of those values under the stationary (limiting) distribution of the Markov chain, which here is fully characterized by the long run probability of being in the ‘good’
(restructured) state, $\bar{\pi}$. This can be calculated as a fixed point of (2), or as any row in the limit matrix $\lim_{n \to \infty} Q^n$, under the optimal investment policy. Clearly, $\bar{\pi}$ is increasing in investment, $x$, and in $\alpha$, and decreasing in $\delta$: the impact of other parameters depends on how they affect optimal $x$'s. Thus the long run value to society of a firm pursuing the optimal policy is

$$V^* = \bar{\pi} \bar{V} + (1 - \bar{\pi}) \bar{\nu}.$$  

These results derive from an assumption of full control over the operation and assets of the enterprise by value-maximizing decision makers, fully informed of the expected costs and benefits of their actions, and with access to perfect capital markets in which the marginal cost of capital is $c$. Here investment by a 'good' (restructured) firm is purely defensive to counter the possibility of a shift in the environment rendering its restructuring ineffectual. Thus the smaller $\delta$, the less the good firm will invest. The only other difference between the types of firm is in the current flow of net revenues that stands at the heart of the difference in long-run valuation; otherwise, all parameters have a natural and identical impact on both types of firms.

2.3 An Example.

Let the parameters in the model take the following values:

$$\begin{align*}
\bar{y} &= 1.0 \\
y &= -1.0 \\
\bar{R}(z) &= \ln(z + 1) \\
c &= 0.5 \\
a &= 1.0 \\
\beta &= 0.9 \\
\delta &= 0.8
\end{align*}$$

Then easy calculation gives $z^* = 1, \Delta V = 2.11157, \bar{x} \cong 1.3352193,$ and $\bar{x} \cong 1.1120757$. The marginal expected benefit from both investment and outside activities is 0.5, and the value vector is

$$V = \begin{bmatrix} \bar{V} \\ \bar{\nu} \end{bmatrix} \cong \begin{bmatrix} 1.3710933 \\ -0.740455 \end{bmatrix}.$$  

The optimal decision in both cases is illustrated in Figure 1. Notice that the "restructured" firm invests rather heavily as the environment is extremely unstable: $\delta = 0.8$.  

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Figure 1.
If $\delta = 0.3$, then $\Delta V = 2.60199$, $\underline{\kappa} \equiv 1.5440618$, $\bar{\kappa} \equiv 0.340089$, and

$$V = \begin{bmatrix} \bar{V} \\ \underline{V} \end{bmatrix} \approx \begin{bmatrix} 5.23103 \\ 2.62904 \end{bmatrix},$$

showing both the greater value of a stable environment to both types of firms and the shift in incentive to invest in restructuring from the already restructured to the 'bad' firm. This discrepancy is only aggravated if we let $\bar{a} = 1.5$.

In this example we can clearly see the impact of 'environmental volatility', $\delta$, on the value of the firm to society. When $\delta = .8$, $\bar{\kappa} = .73689952$, giving $V^* = .73689952 \times 1.3710933 + .26310048 \times (-.740455) = .815544$, while when $\delta = .3$, $\bar{\kappa} = .78649$, giving $V^* = .78649 \times 5.23103 + .21351 \times 2.62904 = 4.67548$. Thus lower volatility dramatically improves the quality of investment, despite its dramatic drop in the restructured state.\footnote{This is because the only productive use for investment in this model is to restructure, and in the more stable environment less investment is required to defend that achievement.}

### 2.4 More General Cases.

The first natural generalization is to consider a non-linear (convex) cost of capital function. As the acquisition of new resources for either investment or "outside enrichment" becomes more costly with the level of either activity, the trade-off between activities becomes sharper as do the differences between firms of different type. If both types of firm face the same convex cost of capital schedule, then the results are qualitatively the same as above, although a closed form solution cannot generally be calculated.

**Proposition 3** Let the cost of investible resources be given by the function $c(x + z)$, with $c(0) = 0$, and $c'(\cdot) > 0$, $c''(\cdot) > 0$ on $\mathbb{R}_+$. Then

$$\bar{\kappa} = \max \left\{ \frac{1}{a} \ln \left( \frac{a\beta \delta \Delta V}{c'(\bar{\kappa} + \bar{z})} \right), 0 \right\};$$

$$\underline{\kappa} = \max \left\{ \frac{1}{a} \ln \left( \frac{a\beta \Delta V}{c'(\underline{\kappa} + \underline{z})} \right), 0 \right\};$$

$$\kappa - \bar{\kappa} = \frac{1}{a} \ln \left( \frac{R'(\bar{z})}{\delta R'(\underline{z})} \right) \text{ if } \bar{\kappa} > 0;$$

$$R'(\bar{z}) = c'(\bar{\kappa} + \bar{z}); \quad R'(\underline{z}) = c'(\underline{\kappa} + \underline{z});$$
\[ V = \begin{cases} \bar{y} + R(\bar{z}) - c(\bar{x} + \bar{z}) + \beta(\bar{V} - \frac{c'(\bar{x} + \bar{z})}{a_i}) & \text{if } \bar{X} > 0; \\ y + R(z) - c(x + z) + \beta(\bar{V} - \frac{c'(x + z)}{a_i}) & \end{cases} \]

\[ \Delta V = \bar{V} - V = \Delta y + R(z) - R(\bar{z}) - [c(\bar{x} + \bar{z}) - c(x + z)] - a^{-1}[R'(\bar{z}) - R'(\bar{z})]. \]

**Proof.** Again the results follow immediately from the first order conditions.

This result is illustrated in **Figure 2.** Clearly the increasing opportunity cost of outside opportunities leads the 'bad' firm to cut back on rent-seeking, expanding the gap between its and the 'good' type's investment. The 'good' firm pursues a relatively more aggressive investment policy (of both types) as its marginal cost of capital is lower. Again, the good type only protects itself from a negative shift in the environment, and so engages substantially more in 'outside' profit opportunities. The other qualitative comparative statics remain as above (Corollary 2).

A further immediate generalization is to the case where \( \varepsilon < 1 \), i.e. there is some probability that outside opportunities will not persist into the future. This adds another, exogenously determined, dimension to the state space, so there are effectively four states indicating the "type" of the firm and whether the outside opportunities are available or not: \( \{\bar{V}_1, \bar{V}_0, \underline{V}_1, \underline{V}_0\} \), where the subscript '1' indicates the presence of the investment opportunity \( R(z) \). Since the availability of these outside opportunities is here assumed independent of the behavior of the firm, optimal behavior is easily characterized through the Bellman equations:

\[ \bar{V}_1 \equiv V_1(\bar{\theta}) = \max_{\bar{x}, \bar{z}} \left\{ \bar{y} + R(z) - c \cdot (x + z) + \beta Q(x; \bar{\theta}) [\varepsilon V_1 + (1 - \varepsilon)V_0] \right\} \]  

\[ \bar{V}_0 \equiv V_0(\bar{\theta}) = \max_x \left\{ \bar{y} - c \cdot x + \beta Q(x; \bar{\theta}) [\varepsilon V_1 + (1 - \varepsilon)V_0] \right\} \]  

\[ \underline{V}_1 \equiv V_1(\underline{\theta}) = \max_{\underline{x}, \underline{z}} \left\{ y + R(z) - c \cdot (x + z) + \beta Q(x; \underline{\theta}) [\varepsilon V_1 + (1 - \varepsilon)V_0] \right\} \]  

\[ \underline{V}_0 \equiv V_0(\underline{\theta}) = \max_x \left\{ y - c \cdot x + \beta Q(x; \underline{\theta}) [\varepsilon V_1 + (1 - \varepsilon)V_0] \right\} \]  

where \( V_r = \begin{bmatrix} \bar{V}_r \\ \underline{V}_r \end{bmatrix} \) for \( r \in \{0, 1\} \). Clearly \( V_0 \leq V_1 \), so in any state the potential gains to restructuring investment are less, while the returns to outside investment remain unchanged, when those opportunities are available. As the 'cost of capital' in these equations is assumed constant, this means that
Figure 2.
investment in restructuring in all states is depressed, while outside opportunities are fully exploited when available. That effect is clearly exaggerated when an increasing marginal cost of capital is considered: restructuring investment is further "crowded out" when rent-seeking opportunities are available, while their absence leads to an increase in restructuring investment.\textsuperscript{11} It also remains the case that firms that have yet to restructure still invest more in productive activity than those that have already made the transition.

It is important for all these results that both types of firms face the same credit conditions, and that they be motivated by maximizing PDV of the firm. Thus we see little of the perverse investment behavior so typical in the Russian transition. Such behavior becomes more evident in more substantial extensions of the model. In particular, we investigate the impact of differentiating the characteristics or opportunities by firm 'types', by allowing the government to differentiate between firms in providing access to investment capital, and by allowing firms to make decisions with respect to the scale of their operation, providing a possible alternative to fundamental restructuring in shrinking and awaiting government rescue.\textsuperscript{12}

3 Extensions of the Model.

There are three general extensions that we explore here. The most elaborate is to allow the firm to alter its scale of operation in response to economic difficulties as well as opportunities. This option is explored in a finite-state extension of the model, where the firm in any state of restructuring can choose (through 'extensive investment') its scale of operation in the succeeding period. As it is not possible to derive closed form solutions to the firm's optimal decisions and their impact on its fate, qualitative results are supplemented by using numerical simulations to analyze the comparative statics of optimal investment behavior. The second extension, undertaken with a fixed scale

\textsuperscript{11}This latter effect provides an incentive for the government to carry out reforms that reduce the opportunity for, and or return to, "outside investment opportunities." Measures lowering the yield on government financial paper (GKO's), reducing corruption and fighting organized crime, firming up property and contractual rights and protections, and generally restricting non-productive rent-seeking opportunities all would help.

\textsuperscript{12}The latter seems to have been the dominant behavior in much of Russian industry. Shrinking the work force and arguing for credit and investment funds to reverse the production decline has been a prominent part of "defensive restructuring" of those firms and industries that have been unable to cope with the new transition environment.
of operation for tractability, is to consider how differing cost functions affect the two ‘types’ of firms. I begin the analysis with this second extension, as its results follow directly from the prior analysis.

Then I turn to a third extension where “outside opportunities” are considered to be related to the inherited network of mutual support that firms have developed to cushion the shocks of marketization and avoid the pain of deep restructuring. As successful restructuring involves a substantial reorientation of firm activity, it breaks off those ties thus eliminating the opportunity to exploit them. Even if the restructuring later becomes inadequate, those ties remain broken, so an irreversibility is built into the restructuring process. This has a profound impact on the incentives to initiate restructuring that we begin to explore in this section.

3.1 Differing Capital Costs.

In a market environment we might expect that the “good” firm would have better access to bank credit and capital markets, and hence should face lower (average and marginal) costs of capital resources. It is easy to see that in such a case the qualitative nature of the above results does not change: the “bad” firm still invests more and does substantially less “rent-seeking” than the ‘good’ firm, as illustrated in Figure 3.a. As $c'_g(\cdot)$ lies everywhere above $c'_b(\cdot)$, and $R'(\bar{x})$ is the same for both types of firms. Optimal rent seeking, $\bar{x}$ is depressed even further relative to $\bar{z}$ while $\bar{x}$ may either grow or shrink relative to $\bar{z}$,

$$x - \bar{x} = \frac{1}{a} \ln \left( \frac{R'(\bar{z})}{\delta R'(\bar{z})} \right) \quad (12)$$

depending on how volatile the environment is (i.e. how close $\delta$ is to 1) as $R'(\bar{z}) > R'(\bar{z})$ [see Proposition 3]. However, relative to the case with identical costs, investment by the unrestructured firm, $\underline{x}$, will shrink. Thus higher marginal costs of capital for unrestructured firms have the salutary impact of restricting rent-seeking behavior. albeit also raising the marginal costs of investing in fundamental restructuring.

The situation is rather different, however, when the marginal cost of capital is lowered for the unrestructured firms. A policy often recommended as a way to encourage restructuring. Sufficiently lower, subsidized marginal costs for the unrestructured firm imply that

$$c'_b(\underline{x} + \bar{z}) = R'(\bar{z}) < R'(\bar{z}) = c'_b(\underline{x} + \bar{z})$$

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hence expanding both rent-seeking and productive investment, as can be seen in (12) and Figure 3.b. Again a closed form solution and analytic results do not seem derivable with general increasing cost functions. Assuming constant, if different, cost of capital to each type of firm allows a complete comparative statics analysis.

**Assumption 4:** Let \( \bar{c} = c \) be the constant marginal cost of capital to the restructured firm, while \( c = \alpha \cdot c, \alpha \in \mathbb{R}_{++} \) is the marginal cost of capital to the unstructured firm.

**Proposition 4** Under Assumptions 1, 2 and 4, the solution to each firm's optimization problem is characterized by:

\[
\bar{x} = \max \left\{ \frac{1}{a} \ln \left( \frac{a \delta \Delta V}{c} \right), 0 \right\};
\]

\[
\bar{c} = \max \left\{ \frac{1}{a} \ln \left( \frac{a \delta \Delta V}{\alpha c} \right), 0 \right\};
\]

\[
\bar{x} - \bar{c} = \frac{1}{a} \ln \left( \frac{1}{\alpha \delta} \right) \text{ if } \alpha \delta < 1 \text{ and } \bar{c} > 0;
\]

\[
\bar{c} - \bar{x} = \frac{1}{a} \ln \left( \alpha \delta \right) \text{ if } \alpha \delta > 1 \text{ and } \bar{c} > 0;
\]

\[
R'(\bar{z}) = c; \quad R'_{z}(\bar{z}) = \alpha c;
\]

\[
V = \left( \begin{array}{c}
\bar{y} + R(\bar{z}) - c(\bar{x} + \bar{z}) + \beta(V - \frac{\bar{c}}{\alpha \delta}) \\
y + R(\bar{z}) - \alpha c(\bar{x} + \bar{z}) + \beta(V - \frac{\bar{c}}{\alpha \delta})
\end{array} \right) \text{ if } \bar{x} \text{ and } \bar{c} > 0;
\]

\[
\Delta V = \bar{V} - V = \Delta y + R(\bar{z}) - R(\bar{z}) - [c(\bar{x} + \bar{z}) - \alpha c(\bar{x} + \bar{z})] - \frac{(1 - \alpha) c}{a}.
\]

The comparative statics in \( \alpha \) are as follows:

\[
\frac{\partial \bar{x}}{\partial \alpha} = 0; \quad \frac{\partial \bar{z}}{\partial \alpha} < 0; \quad \frac{\partial \bar{c}}{\partial \alpha} > 0; \quad \frac{\partial \bar{x}}{\partial \alpha} < 0;
\]

\[
\frac{\partial V}{\partial \alpha} < 0; \quad \frac{\partial V}{\partial \alpha} < 0; \quad \frac{\partial \Delta V}{\partial \alpha} > 0.
\]

Hence increasing the subsidization of capital (at the margin) increases both rent-seeking and investment by the unstructured firm, while lowering defensive investment by the restructured firm.

\[13\]If the marginal capital cost curves are sufficiently close it is possible that the optimal decision of the unstructured firm involves higher marginal costs, less rent-seeking, yet greater investment in restructuring than the optimal decision of the already restructured firm, reversing this inequality.
Proof. The impact on $z$’s is immediate, due to the fixed $R(z)$ function. The impact on $x$’s and $\Delta V$ must be calculated together due to their intimate connection, using the envelope theorem on the Bellman equation: the details are omitted as unenlightening. Using

$$
\bar{V} = (1 - \beta)^{-1} \left[ \bar{y} + R(z) - c(\bar{x} + \bar{z}) - \frac{c}{a} \right]
$$

gives $\bar{V}$ decreasing in $\bar{x}$, so $\Delta V$ increasing implies that $V$ must also be decreasing. ■

From this Proposition it is clear that a sufficient cost disadvantage can actually lead the unrestructured firm to invest less in restructuring than the firm that only needs to defend its position. Indeed, the worse the cost situation for the unrestructured firm, the more the ‘good’ firm will invest to prevent falling back into that undesirable state. On the other hand, the easier the access to capital by the unrestructured firm, the less the ‘good’ firm will try to defend its position and the proportionately more the unrestructured firm will spent on ‘outside’ opportunities. This is because the capital subsidy reduces the gain to restructuring, $\Delta V$, causing both firms to cut back on restructuring investment, while increasing the unrestructured firm’s incentive for all investment, in particular in outside opportunities that are unaffected by the fall in $\Delta V$.

In the base case example of Section 2.3, $\alpha = .6$ implies gives $\bar{z} = 1, \bar{x} \approx 2.333333333, \Delta V \approx 1.54892, \bar{\bar{x}} \approx 0.802204$, and $\bar{x} \approx 1.53617$. The marginal expected benefit from both investment and outside activities is 0.5 for the restructured firm and 0.3 for the other, and the value vector is

$$
V = \begin{bmatrix} \bar{V} \\ \bar{V} \end{bmatrix} \approx \begin{bmatrix} 2.92045 \\ 1.37153 \end{bmatrix}.
$$

Thus compared to the previous example we see $\Delta V$ decreasing from 2.11158 to 1.54892, $\bar{x}$ decreasing while $\bar{z}$ is increasing, and $\bar{V}$ and $\bar{V}$ both increasing as $\alpha < 1$. The decrease in productive investment is rather substantial in the restructured firm (28%), while the increase is limited for the unrestructured firm (15%) due to the drop in the expected gain to restructuring. Instead, the unrestructured firm vastly increases its investment in outside activities (233%). Hence a 40% unit subsidy on capital costs has only a limited impact on restructuring, being largely siphoned off into “outside” activities, while costing the subsidizer $c(1 - \alpha)[\bar{x} + \bar{z}] = 1.93475$. 

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This subsidy also has a negative impact on the long run distribution of states of a firm optimally taking advantage of it. Under the optimal policy, \( \bar{\pi} = .68633 \) (vs. .73690 in the same environment without subsidy; see Section 2.3 above). giving a private (post-subsidy) value \( V^* = .68633 \times 2.92045 + .31367 \times 1.37153 = 2.4346 \) and hence a net social value of \( .68633 \times 2.92045 + .31367 \times (1.37153 - 1.93475) = 1.82773 \) vs. .81554 in the case without investment subsidies. Thus, assuming that ‘outside’ investments generate a social value equivalent to their private value, there is a net social gain to the ‘insurance’ provided by subsidies in the ‘bad’ state.

### 3.2 Capital Constraints.

The preceding analysis has assumed that the only constraint on the use of capital for either restructuring or ‘outside’ investment was is cost; any desired amount is available at some cost. Introducing hard, binding capital constraints has the natural impact of increasing the marginal benefit relative to the marginal costs of investment at the binding constraint. That difference is reflected in the Lagrangian multipliers, \( \bar{\lambda} \) and \( \Lambda \), of the constraint in the first order conditions:

\[
\begin{align*}
-c(\bar{x} + \bar{z}) + a \beta \delta \cdot \Delta V \cdot e^{-a \bar{z}} - \bar{\lambda} & \leq 0, \\
-c(\bar{x} + \bar{z}) + R'(\bar{z}) - \bar{\lambda} & \leq 0, \\
-c(\bar{x} + \bar{z}) + a \beta \cdot \Delta V \cdot e^{-a \bar{z}} - \Lambda & \leq 0, \\
-c(\bar{x} + \bar{z}) + R'(\bar{z}) - \Lambda & \leq 0.
\end{align*}
\]

These are derived from solving the Bellman equations

\[
\begin{align}
\bar{V} & \equiv V(\bar{\theta}) = \max_{x, z, x + z \leq K} \left\{ \bar{y} + R(z) - c(x + z) + \beta Q(x; \bar{\theta})V \right\} 
\tag{15}
\end{align}
\]

\[
\begin{align}
V & \equiv V(\theta) = \max_{x, z, x + z \leq K} \left\{ y + R(z) - c(x + z) + \beta Q(x; \theta)V \right\} 
\tag{16}
\end{align}
\]

where \( K \) is the limit on available capital resources. Notice that \( \lambda(\theta) = \max \{0, R'(z(\theta)) - c'(K)\} \) so that \( R'(z(\theta)) = c'(K) + \lambda(\theta) \) whenever \( z(\theta) \) is optimally chosen. Hence, when the capital constraint is binding

\[
x(\theta) = \frac{1}{a} \ln \left( \frac{a \beta \delta \Delta V}{R'(z(\theta))} \right) \leq \frac{1}{a} \ln \left( \frac{a \beta \delta \Delta V}{c'(K)} \right).
\]

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This highlights the general result that the marginal cost of capital is not necessarily the cost of raising funds, but rather the "opportunity cost" of less "rent-seeking."

Capital constraints naturally hit the firm that wants to invest more, usually the unrestructured firm, the hardest, limiting both rent-seeking and restructuring investment. Indeed, political considerations aside, we might expect that the unrestructured firm should face tighter capital rationing, decreasing its probability of effectively restructuring, thereby increasing $\Delta V$ and the incentive to invest in restructuring, particularly for the less capital constrained firm. This situation is illustrated in Figure 4. It should be noted that the absence of capital constraints in the model above involves a certain level of subsidy for the unrestructured firm that is on average losing money each period $(y + R(z) - c(x + z) < 0)$ without having to finance or otherwise cover that cost.

Whether such constraints lead to a limitation of investment in restructuring clearly depends on how lucrative the 'outside opportunities' are at the margin. If $R'(z)$ is large relative to the marginal value of the first bit of restructuring investment, $a_\beta \theta \Delta V$, then the limited available capital will go largely to those 'outside activities,' and not toward restructuring. If, however, $R'(0) \leq a_\beta \theta \Delta V e^{-aK}$ as in Figure 4 then virtually all investment will be directed toward restructuring. Thus capital constraints, coupled with high marginal returns to "rent seeking," can generate the kind of low investment in fundamental restructuring that seems characteristic of unrestructured Russian enterprises. In addition, such a situation increases the relative investment by the already restructured firm which has an even greater incentive to avoid falling back into state $\theta$ where it will face capital constraints trapping it there.

3.3 Differential Outside Opportunities.

We have assumed so far that the outside investment opportunities of both types of firms are identical.\footnote{See footnote 9 above. We pursue its suggestion here.} While this may be true for some opportunities, e.g., playing the $GKO$ market, there are many others that are intrinsically tied to the inherited structures of the Soviet period as they depend on maintaining relationships, both political and economic, with partners of the old planned economy. These include privileged access to resources or products
Figure 4.
on non-market terms in return for favors, access to special licenses or distribution channels, or access to complex barter (nonmonetized exchange) networks where direct payment is neither required nor expected. Such earning opportunities are dramatically reduced, if they don’t entirely disappear, by successful restructuring which turns the firm’s efforts and activities away from those old interactions and toward new market-driven activities. And in many cases, that reorientation is irreversible; the old network must make do without the firm, and if successful in doing so will not generally accept the ‘traitor’ back should the restructuring prove unsuccessful [Ickes et al. (1997), p.8-9]. In addition, the withdrawal of an enterprise from the network may lead to the network’s collapse if that enterprise had played a crucial role in maintaining the network.

We model the situation where successful restructuring causes an irreversible change in the outside investment/earning opportunities by now interpreting \( R(z) \) as the differential earnings that can be generated only by an unrestructured firm exploiting its old networks.\(^{16}\) Once a firm has restructured, although that restructuring may fail, it can no longer return to its initial state (type); if restructuring proves inadequate, it enters a new ‘unrestructured’ state without the option of investing in network opportunities. \( R(z) \). This means that there are effectively three states (‘types’): effectively restructured – \( \overline{\theta} \); initially unrestructured – \( \theta_0 \); inappropriately restructured – \( \underline{\theta} \). As before, \( y(\overline{\theta}) = y \) and \( y(\underline{\theta}) = \overline{y} \), let \( y(\theta_0) = y \) also,\(^{17}\) and let \( R(z) \) be available only in state \( \theta_0 \). Assume further that any successful restructuring must involve breaking with old partners, and that even the attempt runs some risk of doing so, i.e. transitioning to the unconnected unrestructured state, \( \underline{\theta} \). Letting that probability be given by \( \nu \), the transition probability.

---

\(^{15}\)One of the key characteristics of the stabilization of production activity in 1997 was the dramatic growth of complex networks of mutual indebtedness and barter among Russian industrial and agricultural enterprises. These allowed stabilization of production, but not growth or investment, by generating sufficient commodity source flows without necessary or precise clearing of transactions, even in non-monetary terms. See IET (1997) for a detailed discussion.

\(^{16}\)As we have seen above that outside earning opportunities that are independent of the firm ‘type’ have no impact on behavior at the intensive margin, we drop their consideration to simplify the model.

\(^{17}\)This is to simplify the analysis, although it might be more reasonable to assume that \( y(\theta_0) < y(\overline{\theta}) \) as the intervening success in restructuring should have meant some progress.
function, $Q$, becomes

\[
Q(\bar{T}, x_0, \bar{z}) = \begin{bmatrix}
(1 - \delta e^{-ax}) & 0 & \delta e^{-a\bar{T}} \\
(1 - \nu) (1 - e^{-ax_0}) & e^{-ax_0} & \nu (1 - e^{-ax_0}) \\
(1 - e^{-a\bar{z}}) & 0 & e^{-a\bar{z}}
\end{bmatrix},
\]  

(17)

where subscript '0' indicates the initial, network connected, state. Then the expected future value of any state is given by

\[
QV = \begin{bmatrix}
\bar{V} - \delta e^{-a\bar{T}} \Delta V \\
\bar{V} - \nu (1 - e^{-ax_0}) \Delta V - e^{-ax_0} (\bar{V} - V_0) \\
\bar{V} - e^{-a\bar{z}} \Delta V
\end{bmatrix},
\]

and the Bellman equations become:

\[
\bar{V} \equiv V(\bar{\theta}) = \max_{x, z} \left\{ y - c(x) + \beta Q(x; \bar{\theta})V \right\},
\]

(18)

\[
V_0 \equiv V(\theta_0) = \max_{x, z} \left\{ y + R(z) - c(x + z) + \beta Q(x; \theta_0)V \right\},
\]

(19)

\[
\bar{V} \equiv V(\bar{\theta}) = \max_{x, z} \left\{ y - c(x) + \beta Q(x; \bar{\theta})V \right\},
\]

(20)

where the notation is as in Section 2.2 above.

The structure of $Q$ in (17) clearly implies that the initial state is transient unless there is optimally no investment in that state: $x_0 = 0$. Once, however, a transition has occurred to either other state, the structure of the decision problem is identical to that analyzed in Propositions 1 - 4 above. Hence the interesting issues raised by this formulation revolve around the decision to invest in the initial state and the expected time of exit from that state. In particular, we are interested in conditions (parameter values and configurations) such that $x_0 = 0$, so that the firm optimally chooses to eschew restructuring. These conditions can be found through analysis of the firm's first order conditions and the resulting (three point) value function in the Bellman equation above.

**Proposition 5** Assume that the outside opportunities indicated by $R(z)$ be available only in the initial state, $\theta_0$, and let the other assumptions of Proposition 1 hold. Then the optimal restructuring investment policies in the restructured and subsequently unrestructured states, $\bar{\theta}$ and $\theta$ respectively, remain as given in equations (7) and (8). In the initial unrestructured state, optimal
investment in $R(z)$ yields $P^* = R(z^*)$ where $z^*$ solves $R'(z^*) = c$. Optimal investment in the initial state, and the relevant value functions are given by:

$$x_0 = \max \left\{ \frac{1}{a} \ln \left( \frac{a \beta}{c} [\bar{V} - V_0 - \nu \Delta V] \right), 0 \right\}$$

$$\bar{V} = (1 - \beta)^{-1} [\bar{y} - c \bar{x} - c/a]$$

$$\bar{V} = y - c \bar{x} - c/a + \beta \bar{V}$$

$$\Delta V = \Delta y - c \cdot \Delta x$$

$$V_0 = y + P^* - cx_0 + \beta(\bar{V} - \nu \Delta V) - c/a \quad \text{if} \quad x_0 > 0$$

$$V_0 = (1 - \beta)^{-1}(y + P^*) \quad \text{if} \quad x_0 = 0$$

where $\Delta$ differences the variable in states $\bar{y}$ and $\bar{z}$ as above.

**Proof.** The result is immediate from the first order conditions of the finite state, bounded discounted dynamic programming problem.

**Corollary 6** Some properties of the optimal solution are as follows:

- Initial optimal restructuring investment, $x_0 = 0$ iff 

$$\frac{a \beta}{c} [\bar{V} - V_0 - \nu \Delta V] \leq 1.$$ 

This will be true for sufficiently small $a, \beta, \Delta y$ and/or for sufficiently large $c, \delta, \nu, R(\cdot)$.

- $x_0 < \bar{x}$ unless volatility is very low: $\delta + \nu \leq \frac{\bar{V} - V_0}{\Delta V} < 1$.

- If $x_0 = 0$, the enterprise will never restructure.

- If $x_0 > 0$, the enterprise will leave the initial state with probability one, thereby losing the $R(z)$ opportunities. The expected time of such transition is

$$\tau = \sum_{n=0}^{\infty} e^{-nx_0} > 1.$$ 

- If $\nu = 0$, $V_0 > \bar{V}$, so $x_0 < \bar{x}$; If $\nu = 1$, $x_0 > 0$ iff $\bar{V} - V_0 > c/a \beta$.

**Proof.** The results follow from analysis of the expression $\frac{a \beta}{c} [\bar{V} - V_0 - \nu \Delta V] = \frac{a \beta}{c} [1 - \nu(1 - \beta)] \Delta y - \frac{a \beta}{c} P^* - \beta [1 - \nu^2 (1 - \beta) \ln \delta] - \beta \ln [\Delta y - \frac{c}{a} \ln \delta]$.
units as the cost of capital, are given by a matrix $\kappa$, where $\kappa_{ij}$ is the net cost of shifting $k_i$ to $k_j$.\footnote{If $i = j$ then this is just the cost of maintaining capacity $k_i$. If $i > j$ it is the cost of maintaining and expanding the capacity to $k_j$, while $i < j$ is the net cost, including revenues from sale of assets, of reducing capacity to $k_j$. The latter will be negative when asset sales revenue exceeds the direct adjustment, including labor separation, costs.} $\kappa_{ij}$ is decreasing in $i$, increasing in $j$, convex in each argument, and independent of restructuring state $\theta$. The benefits (cash flow) from any capital stock, $k$, are $y(\theta) \cdot k$.

This means that maintaining capacity can be costly and is more so as capacity increases, increasing it is increasingly costly, and reducing it gives a one-time gain which diminishes at the margin if a larger drop in capacity is implemented. Under these assumptions, the optimization problem of the firm becomes:

$$V(\theta_{t_0}, k_{t_0}) = \max_{\{z_t, x_t, k_t\}} \mathbb{E}_{t_0} \sum_{t=0}^{\infty} \beta^t \left\{ y(\theta_t) \cdot k_t + R(z_t) - c(x_t + z_t) - \kappa_{x,x+1} \right\} \left\{ x_t \right\} \right.$$  \hspace{1cm} (21)

Again this is a standard finite-state stochastic discounted programming problem, the solution to which is given by the Bellman equation and the policy sequence generating it. That is, the optimal policy solves:

$$V(\theta, k) = \max_{z,x,k} \left\{ y(\theta)k + R(z) - c(x + z) - \kappa_{k,k} + \beta Q(x; \theta)k \right\}$$  \hspace{1cm} (22)

where $\hat{k}$ is the capacity optimally chosen for the next period, and $V(\cdot, \hat{k})$ is the 2-vector of optimal values in the different states of restructuring, given that capacity is chosen to be $\hat{k}$.

**Proposition 7** Under our assumptions, there exists a unique solution to the dynamic optimization problem (21) comprising the value function $V(\theta, k)$ and the optimal policy functions $\left\{ x(\theta, k), \hat{k}(\theta, k), z^* \right\}$.

**Proof.** The state space is finite, the action space can be easily compactified, the transition function is stochastically continuous, and the period payoff is bounded, hence standard theorems of discounted dynamic programming give existence. Uniqueness follows from the curvature properties of the $Q$ and $R$ functions and the discreteness of the state space, giving generic uniqueness of $\hat{k}$. $\blacksquare$
case $x_0 = 0$ and $V_0 = -1.96$. In such cases, the probability of restructuring remains zero forever: $\tau = \infty$. These then provide clear cases when a present-value maximizing firm would optimally choose not to restructure, or to delay restructuring, despite the increase in cash flow that successful restructuring promises.

### 3.4 The Option to Vary Capacity.

For a final extension of the model, let the firm have three possible uses for its resources: (i) the ‘outside’ opportunity returning $R(z)$ if $z$ invested; (ii) the ‘restructuring’ opportunity giving an increasing probability, $\pi(\theta, x)$, of successfully restructuring if $x$ is invested in state $\theta$; and (iii) a costly ‘capacity varying’ opportunity which alters the scale at which $\eta$ is earned in any period.

The new third opportunity gives the firm an alternative to restructuring in that it might shrink the scale of its operation, selling off assets and reducing proportionately its losses while unrestructured, as well as its gains from restructuring. It thereby reduces the incentive for restructuring, while freeing resources for the more profitable outside activities. It also allows the firm to take greater advantage of restructuring by expanding operations, thereby enhancing both the returns to successful restructuring and the incentive to make that effort. Thus there is a new, highly complex, interaction between the restructuring investment decision, the ‘outside’ use of funds, and the expansion or contraction of the capacity of the firm. Here I report on the exploration of that interaction through numerical simulations under a number of simplifying assumptions used to make the analysis tractable.

In any period, the state (type) of the firm is a pair $(\theta, k)$ where, as above, $\theta \in \{\bar{\theta}, \underline{\theta}\}$, and $k \in \{k_0, k_1, k_2\}$, $k_0 < k_1 < k_2$. The low capital state, $k_0$, represents a substantial shrinking of capacity (labor shedding and capital asset disposal), and the high capital state reflects substantial capital accumulation and new some hiring. The middle state represents the inherited capacity, perhaps after shedding seriously redundant labor. I retain the assumptions on current returns (expected operating cash flow, $y(\theta)$), the return to outside investment $R(z)$, and the impact of restructuring investment on its state, $Q(\bar{x}, z)$. To simplify analysis, I begin with the assumption that all firms face similar costs of capital and face the same costs/benefits of altering and maintaining capacity, and that expected returns are linear in capacity.

**Assumption 5:** The full costs of altering capacity, measured in the same
these 'solutions' are fed back into the Bellman equation giving a new value function, \(\hat{V}(\cdot)\), which is then used to recalculate the optimal decisions. This iterative procedure converges quite rapidly, yielding:

- a value function, \(V(\theta, k)\);
- an optimal restructuring investment function, \(x(\theta, k)\);
- an optimal 'outside investment' decision, \(z^*\);
- an optimal capacity decision, \(\hat{k}(\theta, k)\);
- the long-run (steady-state) probability of being appropriately restructured, \(\pi^*\), and its associated long-run capacity, \(k^*(\theta)\).

By varying the underlying parameters of the model, \(a, \beta, c, \delta, y(\theta), \{k_i\}, \kappa\),\(^{20}\) one can see their impact on the optimal decisions and value of the transition firm in this model. Some preliminary qualitative comparative statics given by this exercise are presented in Table 1 below. A number of analytic results are also available, despite our inability to solve for \(V(\theta, k)\) explicitly.

**Proposition 8** Under the assumptions of Proposition 1 and Assumption 4, the solution to (22) has the following properties:

- \(\pi(\hat{k}) - \pi(\tilde{k}) < 0\) for all \(\delta < 1\), decreasing in \(\delta\) and increasing in \(\Delta y\) and \(\Delta V\);
- \(\pi(\tilde{k})\) is increasing in \(\beta, \delta,\) and \(\bar{y}\), and decreasing in \(c, y,\) and \(a\) when \(a\bar{\pi} > 1\);
- \(\pi(\tilde{k})\) is increasing in \(\beta\) and \(\bar{y}\), and decreasing in \(c, \delta, \bar{y},\) and \(a\) when \(a\bar{\pi} > 1\);
- \(z^*\) is decreasing in \(c\), and independent of all other parameters;
- \(\Delta V(k)\) is increasing in \(k\), hence \(x(\theta)\) is also increasing in \(k\);
- The long-run stationary probability, \(\pi^*(\bar{\theta})\), is increasing in \(k\).

\(^{20}\)Or, indeed, by substituting various functional forms for \(c_\theta(\cdot), Q(\cdot), R(\cdot)\), etc., appropriately parametrized.
To get a feel for the optimal policies, I use the assumptions of Proposition 1, giving the following first order consequences for the investment variables:\(^{19}\)

\[
\bar{x}(\hat{k}) = \frac{1}{a} \ln \left( \frac{a \beta \Delta V(\hat{k})}{c} \right) \quad \text{or} \quad \bar{x}(\hat{k}) = 1 \left( \sqrt{\frac{a \beta \Delta V(\hat{k})}{c}} - 1 \right);
\]

\[
\bar{x}(\hat{k}) = \frac{1}{a} \ln \left( \frac{a \beta \Delta V(\hat{k})}{c} \right) \quad \text{or} \quad \bar{x}(\hat{k}) = \frac{1}{a} \left( \sqrt{\frac{a \beta \Delta V(\hat{k})}{c}} - 1 \right);
\]

\[
\bar{x}(\hat{k}) - \bar{x}(\hat{k}) = \frac{1}{a} \ln \frac{1}{\delta} \quad \text{or} \quad \bar{x}(\hat{k}) - \bar{x}(\hat{k}) = \frac{1 - \sqrt{\delta}}{a} \sqrt{\frac{a \beta \Delta V(\hat{k})}{c}};
\]

\[
\bar{x} = \bar{x} = R^{-1}(c).
\]

Note that even in this highly simplified case there is a complex feedback between the choice of next period's capacity and current optimal investment, \(x\), as \(\Delta V\) depends on the capacity to be in use, while the choice of \(\hat{k}\) depends on the value it produces in each restructuring state and the probabilities of those states, which in turn are determined by \(x\). While general arguments show that a solution \((z^*, x^*, k^*)\) must exist, characterizing it analytically seems quite difficult, except for \(z^*\) which, under the uniform environment assumption of this case, is independent of the other optimal decisions.

One approach to exploring the optimal capacity decision, as well as the comparative statics of the solution to this investment problem, is through numerical simulation. The problem is solved by using the contraction property of the Bellman operator. First the explicit solution for optimal investments (23) for any given \(V(\cdot)\), is used to search over the finite state space for the best (highest \(V(\hat{k})\)) capacity decision and its associated investments. Then

\(^{19}\)Simulations were run for both functional forms. The only substantial difference in results was that the logarithmic form makes \(Q(\bar{x}, \bar{\theta}) = Q(\hat{x}, \bar{\theta})\), while the other leaves the optimal success probability from the high state larger.
The impact of $\beta$ seems to depend critically on the level of volatility in the environment. When $\delta$ is high, increasing $\beta$ (lowering the discount rate) unambiguously increases both restructuring investment and the value of the firm in both states. It also encourages, ceterus paribus, increasing capacity in order to take advantage of the greater future value of being restructured. When volatility of the environment is lower, however, increasing $\beta$ seems to increase value in both states when capacity is large, to decrease value when capacity is small, and when $\beta$ is sufficiently high, to increase only the value of the restructured firm when capacity is at its initial level. In all cases, however, increasing $\beta$ raises investment in restructuring.

The value of the unrestructured firm $[V]$ sometimes responds non-monotonically to capacity, while the value of the restructured firm increases in capacity. That nonmonotonicity is tied to (codetermined with) the optimal choice of capacity and restructuring investment by the unrestructured firm. One case is when circumstances lead the firm to optimally choose a high steady state capacity because of the high likelihood of successfully restructuring. Then the value function may reverse the (monotonic) ordering of current returns in capacity in the unrestructured state. Nonmonotonicity typically occurs when there is a bifurcation of the optimal steady state capacity choices as a function of initial conditions. This may generate a "low-capacity trap" when a high-capacity optimum is possible, or in some circumstances a situation where the firm never chooses to change its capacity, restructured or not, due to the costs of adjustment. Some of the kinds of capacity-investment configurations, and the circumstances which generate them, are discussed below.

Of particular interest are the circumstances when firms do not engage in restructuring investment, as this reflects a typical situation in Russian industry. Simulations of the model have isolated a number of circumstances when this seems to systematically occur as a consequence of PDV maximization by firms. One such circumstance is when the outside opportunity is sufficient valuable [say, $2R(z)$], the cost of capital is relatively high $[c = 1]$, and environmental volatility sufficiently low $[\delta = .3]$ so that it doesn’t pay for the already restructured firm to defend its position with further investment; the unrestructured firm still invests and the restructured firm will begin investing at higher volatility. If there is a choice of capacity, however, even the unre-

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22It is typically decreasing in capacity due to the negative current return, $y$, per unit capacity.
Proof. These follow, with some calculation, from the general properties of
the solution and the functions defining optimal investment behavior. ■

Further properties of the value function, the optimal capacity choice and
the long run stationary distribution could not be derived, but were explored
through numerical simulation. The results are presented in Table 1 and
discussed below it. The table shows the impact of the row variable on the
column variable. \( \partial (SS - k) \) refers to the states that can be supported as
optimal choices as the firm responds to the true value function, while \( \partial V(k) \)
refers to the value response at any \( k \). Similarly, the \( \partial y(\theta) \) row reflects the
increment of the expected cash flow in either state \( \theta \).

<table>
<thead>
<tr>
<th>( \frac{\partial}{\partial x} )</th>
<th>( V(k) )</th>
<th>( V(k) )</th>
<th>( SS - k )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>( \beta )</td>
<td>?</td>
<td>?</td>
<td>-</td>
</tr>
<tr>
<td>( c )</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( \delta )</td>
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<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( k )</td>
<td>+</td>
<td>?</td>
<td>na</td>
</tr>
<tr>
<td>( y(\theta) )</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>( \Delta y )</td>
<td>+</td>
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</tbody>
</table>

Qualitative Comparative States

The definite signs show results that recurred without deviation in all
simulation runs.\(^\text{21}\) An increase in the responsiveness of restructuring to in-
vestment brings uniform improvement; even the optimal capacity (weakly,
given discreteness) increases. And both a greater cost of capital and greater
environmental volatility decrease both firm valuations and the desired level
of capacity. Similarly, increased current cash flow in any state raises valua-
tions and the desired level of capacity, and a greater difference in cash flow
between states raises the value of being in the 'restructured' state.

Where the response isn't uniform, it seems to have a natural explanation
in the interaction of two or more parameters. The effect of increasing the
difference in current payoffs, \( \Delta y \), depends on how that increase is achieved.
If it comes from increasing \( y \) then it is positive on both \( V \) and "\( SS - k \)",
while if it comes from decreasing \( y \) then the opposite impact occurs.

\(^{21}\)Simulation base parameter values are \( a = 1, \beta = .9, c = .5, \delta = .8 \) or \( .3, k = \{1.4, 1.4\}, y = \{1, -1\}, \) and \( \kappa = \begin{bmatrix} .12 & 0 & -2 \\ .4 & .1 & -15 \\ 1.75 & 1.2 & .04 \end{bmatrix} \).
time at $\bar{S}$ in Figure 5a. In ‘unfavorable’ conditions the opposite outcome obtains: the firm shrinks capacity to $k_0$ and invests largely in the ‘outside’ opportunity, spending most of its time as $\bar{S}$ in Figure 5a.

There are, however, a number of interesting cases in which the initial state determines the optimal long-run capacity: high and low capacities reproduce themselves as optimal, while the intermediate level shifts to either one or the other. The only situation in which the middle capacity became an attractor was with low volatility and exceptionally high returns to outside investment [$\geq 2R(z)$]; greater capacity requires higher optimal investment to maintain the ‘good’ state which produces less value than maintaining smaller capacity and taking large rents on the side.\(^\text{23}\)

In a moderately unstable [$\delta \leq .7$], unfavorable payoff [$\bar{y} + y < 0$] environment with high responsiveness of the transition probability to investment [$a > 1.2$], $k_1, k_2 \rightarrow k_2$, while $k_0 \rightarrow k_0$. When volatility becomes sufficiently high, so it is harder to maintain restructuring, then $k_1 \rightarrow k_0$, although a high capacity firm will maintain that capacity. With a balanced current payoff environment, the same effect is achieved when $a$ is sufficiently small. Indeed, there are cases when $k_1$ can go either way: $(\bar{\theta}, k_1) \rightarrow k_2$ and $(\bar{\theta}, k_1) \rightarrow k_0$. If $a$ is large enough, the firm chooses to expand capacity to $k_2$ in a moderate or low volatility environment, and shrinks to $k_0$ in the face of high volatility. The same variations in choice of optimal capacity can be induced by altering the discount factor, $\beta$, and the cost of capital, $c$: as $\beta$ falls or $c$ rises the “basin of attraction” of $k_2$ shrinks while that of $k_0$ increases. Various “basins of attraction” are illustrated in Figure 5b.

This richer framework, although it ignores the possibility of irreversible loss of initial network opportunities, allows a potentially useful categorization of firms through their optimal choice of capacity. Because greater capacity exaggerates the returns to each ‘type’, firms choosing to expand capacity will invest more in restructuring, regardless of ‘type,’ than those who find it optimal to shrink or just maintain initial $k$. However, the underlying structure of opportunity in this model means that $\bar{\theta}$-firms still invest more than $\theta$-firms with the same capacity. Indeed, most imperfections modeled here tend to cut off defensive investment by the restructured firm, while unrestructured firms still strive to change their ‘type.’ In the 2-type model, only severe capital constraints coupled with significant (high marginal return) outside

\(^{23}\)Of course, every capacity can be made an attractor by imposing sufficiently high costs of changing capacity, $\kappa$. 

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structured firm may prefer shrinking and exploiting its outside opportunities to investing in attempting to restructure, even at relatively high volatility \([\delta = .8]\) of the environment.

Subsidizing the capital costs of the unrestructured firm gives it an incentive to invest in both restructuring and the outside opportunity, while undercutting the incentive of the restructured firm to maintain that state. This reduces, rather rapidly, \(\Delta V\), thus undercutting the incentives of both types of firms to invest in restructuring, and hence making the outside opportunity relatively more valuable to both. Such subsidies also seem to stimulate a reduction in capacity to minimize current losses in the unrestructured state, further reducing incentives to put resources into restructuring. Thus the policy of subsidizing investment, at least in the face of substantial ‘rent-seeking’ opportunities appears counterproductive.

A high discount rate \([\beta \leq .5]\) tends to discourage both investment to maintain restructuring and the maintenance of capacity. As the future is relatively unimportant, the firm optimally shrinks in the unrestructured state, even if it would choose to maintain capacity if it were restructured. Furthermore, low investment in the restructured state insures that the firm spends most periods unrestructured and hence with low capacity, an absorbing state for the optimal policy. This is true whether the volatility of the environment is high or low, although high volatility stimulates positive investment in the restructured state, when it would be zero with low volatility.

A final point of interest from these simulations relates to the dynamics of firm states, \((\theta, k)\), under the optimal investment policies. As long as investment is positive there will be positive probability of the firm being in both \(\theta\)-states infinitely often, with the limiting probabilities calculated from the optimal investment as \(\lim_{n \to \infty} Q^n(\bar{\gamma}, \bar{x})\). Only if \(\bar{\tau} = \bar{z} = 0\) will the process converge, and then only to \(\bar{\theta}\).

The level of optimal investment depends, however, on the simultaneous choice of optimal capacity. Thus the long-run probability of a firm’s type depends on the sustainable capacity for any type. In all simulations, the optimal choice of capacity converged rapidly to some fixed level, sometimes depending on the initial state. Most patterns of such convergence could be generated by appropriate choice of parameter values. When conditions are ‘favorable’ \([\text{high } a, \beta, \bar{\gamma}, \Delta y; \text{ low } c, \delta]\) the firm optimally chooses a high capacity, \(k_2\), and invests strongly in restructuring to avoid the losses that would occur in the unrestructured state. Thus the firm spends most of the
opportunities for its investible resources leads the unrestructured firm to invest less in restructuring than its less constrained restructured counterpart.

4 Conclusion.

These results should be understood as a benchmark for approaching an understanding of the investment behavior of a firm in a post-communist transition. They explore the impact of standard economic factors on a firm pursuing the standard objective of maximizing its economic value. They thus give an indication of how much of investment behavior in transition economics can be explained in standard neo-classical terms. Where they fail to replicate observed patterns of behavior, they help clarify where different assumptions about behavior of, and constraints faced by, post-communist firms need to be made.

The basic model confirms our intuition about value-maximizing behavior in a well-functioning market environment: despite the availability of outside opportunities (rent-seeking) unrestructured firms optimally invest substantially in achieving the more profitable state. Restructured firms work harder to preserve that state as environmental volatility increases, and all firms invest more as the future becomes more important and certain ($\beta$ increases). This virtuous behavior is reinforced when restructured firms face easier credit conditions ($\alpha > 1$). None of this, however, helps explain the lack of investment in restructuring in most transition economies of the former Soviet Union.

Some insights in that lack of restructuring do appear when we incorporate some stylized peculiarities of the post-Soviet economic environment. Easy credit/subsidies for unrestructured firms can reduce, and even eliminate, investment in defending restructuring as it reduces the difference in value of the types. It also thereby makes investment in outside opportunities relatively more valuable, cutting into all restructuring investment, although the net effect of marginal investment subsidies is still to increase investment in restructuring by the subsidized firm. Another situation in which the unrestructured firm holds back on restructuring investment is when the marginal returns to outside use of investment resources is high, while those resources are limited. Then capital constraints create a "crowding out" of restructuring investment by more lucrative forms of rent seeking. These results are somewhat enriched by allowing the firm to affect the variance in its net
Steady States and Values

patterns of attraction

Figure 5.
an additional incentive for generating outside income (If this is actually done for restructuring purposes?) to cover investments when capital costs are increasing or access to capital is constrained. This might provide still another reason for investment in restructuring to be limited in favor of rent-seeking in unrestructured firms.

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References


income by choice of capacity. This shows that shrinking capacity can be an optimal alternative to restructuring investment in an “unfavorable” environment, while successful restructuring is optimally exploited by expanding capacity in a “favorable” environment. Furthermore, greater capacity justifies greater investment in restructuring as it increases the cash flow difference between types. Finally, there is an “unstructured,” low capacity trap, with investment effort largely directed toward outside activities.

But perhaps the most plausible of the economic environments restraining firms from investing in restructuring is that investigated in Section 3.3. There the outside opportunities are tied to inherited network relations and so are substantially reduced (eliminated) once the firm moves to seriously restructure. Once restructuring has occurred, a shift in the economic environment can render it useless, indicating a need for further restructuring, but cannot ressurrect the lost network opportunities. This irreversibility dramatically reduces the incentive to invest in restructuring and, if network opportunities are sufficiently lucrative, may eliminate it completely despite a dramatic difference in profitability between restructured and unrestructured firms. We have not yet analyzed how this situation is affected by the introduction of an option to alter capacity, but believe that doing so would add to the model’s ability to explain post-Soviet restructuring investment behavior.

There are a number of other extensions of the basic model that recommend themselves. One involves letting the discount factor, $\beta$, depend on type. This would capture differing decision horizons, reflecting for example the greater uncertainty of survival for the unrestructured firm. If $\beta < \bar{\beta}$ then equations (7) clearly show that the unrestructured firm will invest less in achieving restructuring than the restructured firm in defending it. It also would be of interest to explore the impact of general differences in $R(z)$ between types, and in $c_\phi(x, z)$ between the uses of capital — there may be systematic differences in the cost of resources for restructuring vs. “exploiting network opportunities.”\footnote{Indeed, barter and the possibility of using non-monetary resources (and tax avoidance) can make the opportunity cost of investing in the network or other outside opportunities much lower than that of attempting to restructure. See the discussion in IET (1997) and Ickes et. al. (1997).}

Finally, a variation that is worth investigating involves a capital constraint that depends on the market success, $\eta$, or its average, $\bar{y}(\theta)$, in that both investments and net (of outside investment income) losses must be paid for at the cost of capital. This would seem to require no change in analysis if the marginal cost of capital is constant, but to provide


