Output and Unemployment Dynamics in Transition

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OUTPUT AND UNEMPLOYMENT DYNAMICS IN TRANSITION

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ABSTRACT

In this paper, we present a simple but rigorous model of the dynamics of output and unemployment in transition. We consider a worker-entrepreneur, who is "locked in" to her current production technique, with a choice of continuing to work with it or search for a better technique. If she succeeds, she jumps to the "cutting edge" frontier; if not, she becomes unemployed and searches in the next period. We model a movement to a market economy as a discontinuous jump in the technological frontier and analyze the transitional dynamics. We are able to contrast the difference between a fully anticipated versus an unanticipated policy shock. After an unanticipated shock, output falls immediately and unemployment spikes, as agents search for better techniques. Output is higher in the new steady state, but is approached by dampening oscillations. The results become more interesting when the reform is fully anticipated. Unemployment falls and output stagnates in anticipation of the reform. Surprisingly, output exhibits cycles before and after the reform. Therefore, announcing a reform in advance may have unintended negative consequences.
1. Introduction

Analyzing the dynamics of industry restructuring is one of the key questions in the field of transition economics, and it has spawned a large body of theoretical literature. One of the stylized facts that these models attempt to capture is the large and long-lasting drop in output, and corresponding increase in unemployment, that followed the initial policy reform. Our goal in this paper is to present a rigorous model of the transition, in which the dynamics of output and unemployment are endogenously determined. This phenomenon is of more than purely theoretical interest. The short-term economic hardships associated with a major reform may lead to a policy reversal before it has had a chance to be effective. Further, expectations of short-term hardship may prevent the reform from being introduced in the first place, particularly if reforms develop a reputation for being reversed. Evidently, understanding the dynamic forces at work during a transition is a key ingredient in developing sound policy advice on how to implement a reform and ensure its success.

Transition dynamics are difficult to analyze. Models with intertemporal optimization in an equilibrium context quickly become intractable analytically. Therefore, transition dynamics are usually modeled using numerical techniques (cf. King and Rebelo, 1993; Boucekkine, Licandro, and Paul, 1997; and Cooley, Greenwood and Yorukoglu, 1997). Such techniques have the advantage that they allow the economist to analyze more realistic models than can be handled analytically. They sometimes have the disadvantage, however, of obscuring the intuition behind the results and the causal mechanisms driving the model.

In this paper, rather than develop a "realistic" model, we analyze a barebones model -- almost an extended example -- of transition. What we will lose in realism, we hope to gain in intuition: by utilizing a mix of analytical and numerical techniques, we believe we can persuade the reader that our results are of some interest, and show exactly

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where they are coming from. Further, we are able to contrast the difference in transition dynamics between two extreme versions of expectations: a completely unanticipated versus a fully anticipated reform. Most models of transition dynamics are only able to handle the former. We can handle both and show that there are important differences in their implications.

Of the literature that precedes us, the paper which is closest in spirit to ours is Atkeson and Kehoe (1997). They, too, build a search-theoretic model in which transition dynamics are endogenously derived, and focus on the effects of a productivity-enhancing policy change. There are important differences, however. While they abstract from unemployment and focus on the dynamics of output, and are accordingly able to deploy a more complex model, we wish to consider both output and unemployment dynamics in the context of our simpler model. In addition, Atkeson and Kehoe build a model with closed economy market equilibrium (relative prices and the interest rate are endogenous), whereas we model a small economy (relative prices and the interest rate are pinned down by world market conditions). While Atkeson and Kehoe consider only the case of a fully anticipated reform, we contrast the differences in the implications of fully anticipated versus fully unanticipated reforms. Like them, we also abstract from political economy considerations, which are extensively dealt with elsewhere in the literature.²

We analyze a worker-entrepreneur searching for a production technique. The worker-entrepreneur has a probability of finding a “cutting edge” technique, but only if she is searching full time, i.e., is unemployed. The quality of the cutting edge technique improves at a constant and exogenous rate. Once the entrepreneur finds a technique, she can continue to use it, but its productivity never improves (and falls over time relative to the frontier technique). Therefore, the search cost acts like a fixed cost in an [S,s] model. In our model, there is an optimal tenure rule, which can be viewed analogously to the range of inaction.³ We solve for a unique steady state, where the number of people

³ This search model differs from the traditional Lucas-Prescott search model (cf. Stokey and Lucas, 1989) in that we solve for an optimal tenure rather than an optimal reservation wage. Gomes, Greenwood and Rebelo (1997) and Violante (1997) solve for both the optimal
quitting their jobs equals the number of people finding jobs. In steady state there will be a natural rate of unemployment and a self-replicating, uniform distribution of tenure.

We model a reform as a one-time discontinuous outward shift in the frontier. This could be viewed as a “freeing up” of the economy which increases the possible menus of production technologies available. Nevertheless, state-owned enterprises are locked into a particular way of doing things and cannot implement the new technologies. For example, they might inherit a stock of fixed capital from the old planning regime that is incompatible with modern production techniques. Therefore, the only way the new technologies can come into being is that an entrepreneur quits her job with the state-owned enterprise and searches for one of the new techniques on her own.

We find that, when the reform is unanticipated, output falls immediately following the reform, because many people decide to search for the frontier technique. Over time, however, as people find techniques/jobs, output increases due to a level effect. The reform actually leads to dampening oscillations, and, therefore, business cycles. Many people get jobs shortly after the reform, which creates a “spike” in the tenure distribution. Therefore, many people will exhaust their optimal tenure at the same time at some point in the future. Consequently unemployment will go up and output will go down as these people quit their jobs. The length of the optimal tenure determines the wavelength of the business cycle. As these people get jobs, however, the economy will recover. It will continue to cycle until it reaches a steady state. The cycles dampen because the “spike” in the tenure distribution gets “spread out” every time these agents become unemployed, by the variance in the duration of unemployment.

When the reform is fully anticipated, the results become more interesting. If people know that the reform is coming shortly, they wait for the reform before quitting their jobs to search for a better technique: why quit your job today to search for a technique that you know will shortly be made obsolete? Because no one is quitting her job, the pool of unemployed people quickly dries up. Consequently, no one is finding quitting rule and an optimal reservation wage.

4 This might be analogous to “X efficiency” gains in the context of economic development. 5 These dampening oscillations are reminiscent of so-called “echoes” in the vintage capital
new production techniques and output growth stagnates. After the reform occurs, the volatility in output is larger than in the anticipated case, because everyone who has been waiting to quit her job does so. These results suggest that announcing a reform ahead of time may have unintended consequences.

We were surprised to find that an anticipated reform leads to cycles before the reform. What we believe is happening is that people seek to time their searches so that they are searching just as the reform hits — when the opportunity cost of searching is the least. Consequently, people “bunch up” in terms of when they quit, which leads to cycles. We believe these results are novel to the transition economics literature.

The rest of the paper is organized as follows. Section 2 sets up the model. Section 3 considers the analytics of the unanticipated case, and section 4 contains the corresponding numerics. Section 5 sets up the analytics of the anticipated case, and section 6 contains the corresponding numerics. Section 7 concludes.

2. Setting up the Model

Consider a small, open economy, in which each individual can borrow and lend freely at the world interest rate, assumed constant. Suppose that time is discrete, and indexed by \( t = 0, 1, \ldots, \infty \).

Suppose that all output is manufactured by worker-entrepreneurs, each of whom has access to a Ricardian-type labor-only technology.\(^4\) The technological frontier is shifting out exogenously, but a worker is “locked in” to the technology that is at the cutting edge when she begins producing.\(^7\) Thus, an individual produces a flow of output (or, equivalently, earns a wage) that remains constant in real terms for the duration of her use of that production technique (or, equivalently, of her employment by the owner of

\( \text{literature (cf. Boucekkine, Germain, and Licandro, 1997).} \)

\(^4\) The Ricardian assumption ensures that output equals the competitive wage, so that it is equivalent to assume that the agent receives a wage from the owner of the production technique or that she owns it herself and sells the output.

\(^7\) This specification is similar in spirit to Violante (1997) and Dwyer (1998).
that production technique).

Because an individual's technological level becomes obsolete over time, she has an incentive to abandon the technique (or, equivalently, to quit) and search for a better production technique (or, equivalently, search for a better job). However, there is a cost to doing so. If someone abandons her production technique (quits her job) in period \( \tau \), she becomes unemployed, and searches for a new technique (job). She has a probability \( p \) of finding a new technique (job), which has a productivity (pays a wage) of \( \gamma' \) (define \( w' = w' \gamma' \), with \( \gamma > 1 \)) and a probability \( (1-p) \) of failing to find a (technique) job, in which case she remains unemployed for that period, and continues her search next period. When she does begin producing again, she is assumed to jump to the technological frontier prevailing at the date she begins producing with the new technique.

This is a rather routine dynamic programming problem. Clearly the agent will not want to keep the same job forever, but also will not want to quit her job every period. Therefore, the optimal decision rule should take the form of picking optimal tenure. Given the log-linearity of the problem, it is also likely that the optimal tenure is time-invariant. In the text we will develop a conjecture that this is indeed the case, and prove it by construction. More formally, we verify that the optimal decision rule is a time-invariant tenure rule in the appendix, using Theorem 9.2 of Stokey and Lucas (1989) (appendix will be included in our next draft).

Let \( V_E(w_{\tau}, w_{\tau}) \) denote the value of being employed with a wage of \( w_n \) when the cutting edge wage is \( w_{\tau} \). Note that \( \tau \) indexes when the individual was initially employed (i.e., after her last spell of unemployment). Let \( V_U(w_{\tau}) \) be the value of being unemployed when the cutting edge wage is \( w_{\tau} \). Obviously,

\[
V_U(w_{\tau+1}) = V_U(\gamma w_{\tau}) = \gamma V_U(w_{\tau})
\]  

(1)

and

\[
V_E(w_{\tau+1}, w_{\tau+1}) = V_E(\gamma w_{\tau}, \gamma w_{\tau}) = \gamma V_E(w_{\tau}, w_{\tau})
\]

(2)

(because \( V_U \) and \( V_E \) are linear operators in \( w_{\tau} \) and \( (w_{\tau}, w_{\tau}) \), respectively). Given the way that the problem has been set up, the individual's only decision is to determine whether
she should quit, and, if so, when. The individual's optimal policy is to select a duration of employment, after which she will always quit and search for a new technique. Furthermore, it turns out that this optimal policy will be time-invariant (as noted above).

We will initially characterize the value functions pertaining to the individual for an arbitrary duration of employment, and then use that information to determine the optimal duration of employment. Thus, suppose that an individual waits \( x \) periods to quit. Then,

\[
E(w_t, w_t) = \gamma^t \left( 1 + \beta + \ldots + \beta^{x-1} \right) + \beta^x V_U(w_{t+x})
\]  

(3)

or

\[
V_E(w_t, w_t) = \gamma^t \frac{1 - \beta^x}{1 - \beta} + \gamma^x \beta^x V_U(w_t),
\]  

(4)

where \( \gamma \) is the trend growth rate of the technological frontier and \( \beta \) is the individual's discount factor. Of course, in order for the value function to be finite we will require that \( \gamma < \beta \). Note that we have repeatedly invoked (1). The value of being unemployed in period \( t \) is given by:

\[
U(w_t) = pV_E(w_t, w_t) + p(1 - p)\beta \gamma V_E(w_t, w_t) + p(1 - p)^2(\beta \gamma)^2 V_E(w_t, w_t) + \ldots
\]  

(5)

where \( p(1 - p)^{r-1} \) is the probability of finding a new technique (getting a new job) in period \( r \), and \( (\beta \gamma)^{r-t} V_E(w_t, w_t) \) is the present discounted value of a new technique (job) in period \( r \). Note that we have repeatedly invoked (2).

Therefore,

\[
V_U(w_t) = \frac{p}{1 - \beta \gamma(1 - p)} V_E(w_t, w_t).
\]  

(6)

Substituting (6) into (4):

\[
V_E(w_t, w_t) = \gamma^t \frac{1 - \beta^x}{1 - \beta} + \frac{p \gamma^x \beta^x}{1 - \beta \gamma(1 - p)} V_E(w_t, w_t)
\]  

(7)

or
\[ V_E(w_t, w_r) = \gamma^t \left( \frac{1 - \beta^x}{1 - \beta} \right) \left( \frac{1}{1 - \frac{\beta^x \gamma^x p}{(1 - \beta \gamma (1 - p))}} \right), \]

which yields the value of being employed in period \( t \) with the "cutting edge" wage as a function of \( x \). The rational agent chooses \( x \) at date \( t \) to maximize \( V_E(w_t, w_r) \), which is equivalent in this model to maximizing lifetime utility, given the assumptions of a constant world interest rate and the fact that the individual is employed in period \( t \) (which we assume). Let \( x^* \) denote optimal tenure. It is evident that maximizing the value expression above will yield a time-invariant optimal solution, thus confirming our conjecture by construction. Then, \( \gamma^{-x^*} \) is the proportion of the frontier wage at which someone will abandon her technique (quit her job).

At any given point in time, therefore, the state space, of dimension \( x^* + 2 \), can be expressed as \( S = \{0,1,2,\ldots,x^*,x^* + 1\} \). An individual in time period \( t \) who was employed in time period \( t - \tau \) is in element \( t - \tau \) of \( S \), or, if unemployed, in element \( x^* + 1 \) of \( S \).

We assume that there is a continuum of individuals, of measure unity. Accordingly, we define \( \delta_t \), a \( 1 \times (x^* + 2) \) row vector, in which a given element represents the proportion of individuals at the corresponding element in the state space, at time period \( t \). The elements of this vector sum to unity, and it bears the interpretation of the distribution of individuals across types at period \( t \). For instance, the final element of \( \delta_t \) represents the proportion of individuals who are unemployed at period \( t \).

By construction, the stochastic process governing the evolution of the state space is first-order Markov. The matrix of transition probabilities is defined as:
\[ \Lambda = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & 0 & 1 & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 1 & 0 \\ p & 0 & \cdots & 0 & 1 - p \\ p & 0 & \cdots & 0 & 1 - p \end{bmatrix} \] (9)

where \( \Lambda \) is an \( x^* + 2 \times x^* + 2 \) matrix.

Evidently, the transition equation is given by:
\[ \delta_{t+1} = \delta_t \Lambda \] (10)

One can verify that \( \sigma = \{\alpha, \alpha, \ldots, \alpha, \theta\} \) is a steady state if
\[ \alpha = \frac{1}{\frac{1 - p}{p} + (x^* + 1)} \] (11)

and
\[ \theta = \frac{(1 - p)}{p} \alpha, \] (12)

because, by construction,
\[ \sigma = \alpha \Lambda. \] (13)

Further, this steady state is unique, because it is the solution to two linear equations in two unknowns.

The output of the economy at time \( t \), \( y_t \), is given by:
\[ y_t = \gamma' \begin{bmatrix} 1 & \gamma^{-1} & \gamma^{-2} & \cdots & 0 \end{bmatrix} \delta_t. \] (14)

This just says that total output is the weighted sum of all existing techniques, with the weight on a given technique being given by the proportion of individuals using it.

Unemployment at time \( t \) is given by:
\[ u_t = \{\delta_t\}_{x^* + 2}, \] (15)

recalling that the final element of the vector \( \delta_t \) represents the unemployment rate.

The model has two key parameters, \( \gamma \) and \( p \). The parameter \( p \) can be interpreted as a measure of the cost of upgrading. A smaller \( p \) will result in a larger range of inaction and less frequent upgrading. However, a smaller \( p \) will also increase the duration of
unemployment. Therefore, it is not clear \textit{a priori} whether or not a smaller $p$ will result in more unemployment.
3. Simulating the Optimal Tenure Decision Rule and Steady State

It is computationally cumbersome to obtain an analytical solution for $x^*$, the optimal tenure decision rule. Instead, we solve for it numerically. We let $\beta = 0.975$, so that the interpretation of a time period is a quarter. Then, we obtain the following results:

Table 1: Optimal tenure, $x^*$, as a function of $\gamma$ and $p$

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>1.001</th>
<th>1.005</th>
<th>1.009</th>
<th>1.013</th>
<th>1.017</th>
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</table>

Table 2: The range of inaction ($y^*$) as a function of $\gamma$ and $p$

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<th>$\gamma$</th>
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</table>

Table 3: Steady state unemployment as a function of $\gamma$ and $p$

Note that a larger $\gamma$ will imply a larger range of inaction but more frequent quitting/upgrading. Therefore, a larger $\gamma$ will imply more unemployment. A smaller $p$ acts like a larger fixed cost: the range of inaction gets bigger, and the optimal tenure becomes longer.

Note that $p = 0.3$ and $\gamma = 1.005$ imply a steady state rate of unemployment of 6.4 percent, which is not inconsistent with historical US experience. The value for $\gamma$ of 1.005 implies that the rate of productivity growth and output growth is 2 percent per annum (recall that the interpretation of a time period is a quarter). We will use these parameter values to simulate the reform.
4. Simulating the Transition: An Unanticipated Reform

We now wish to simulate the process of transition. We model the transition as a discontinuous, level jump in the frontier technology available to the economy, not as a change in its long run growth rate. This is similar to the specification adopted by Atkeson and Kehoe (1997), with one important difference. While they assume that the transition economy’s technological frontier smoothly catches up with the outside world, according to an exponential relationship, we assume that the jump is discontinuous. We would suggest that our specification better matches the intuition of a sudden “freeing up” of production techniques, as developed in our introduction, and thus better fits our story. In our simulations, we start the economy in a steady state corresponding to its lower, pre-transition technological frontier. Then, in time period $t=0$, we suppose that the transition is ushered in, and there is a discontinuous jump in the technological frontier. We suppose for the moment that this change is unforeseen, to keep the analysis simple. Later we consider the possibility that the technological jump is foreseen.

Suppose then, that the frontier shifts out unexpectedly due to a reform at $t=0$:

$$w^*_t = \begin{cases} y^* & \text{if } \tau \geq 0 \\ y^* - \alpha \tau & \text{if } \tau < 0 \end{cases}$$

(16)

where $\alpha$ denotes the magnitude of the reform. We would expect that a large number of individuals will search for a new job, of which some fraction will get a new job, the others not. In the short run, output can go down because many people are unemployed. In the long run, however, there must be a level effect on output due to the technological improvement. Therefore, after a period of stagnation and perhaps decline, the economy’s growth will accelerate as more and more individuals get the new and better jobs at the cutting edge productivity level. Furthermore, there will be cyclical dynamics in the initial stages of the transition — there will be many individuals getting the cutting edge jobs shortly following reform; therefore, $x^*$ periods after the reform many individuals will become unemployed, and so on.
Once the optimal tenure decision, \( x^* \), is found then the model's dynamics are completely determined, and can be analytically represented. We select \( \gamma = 1.005 \) and \( p = 0.3 \) for the aforementioned reasons. We select \( ref = 15 \), which corresponds to a 7.7 percent increase in the cutting edge technology.

Figures 1 and 2 present a long and short time series of output before and after the reform. Figure 3 presents a time series of unemployment before and after the reform. (These figures are in 11098#1.xls.) Output falls immediately following the reform as expected. Further, the economy exhibits dampening oscillations. The oscillations have a wavelength of 33 quarters, which corresponds to the optimal tenure. The oscillations are long-lasting: they are discernible 50 years (200 quarters) after the reform. While output falls immediately following the reform, it does not fall for very long: within three quarters it has reached its pre-reform level. As expected unemployment increases dramatically following the reform.

5. Modeling An Anticipated Reform

We now consider the case of an anticipated reform (we will continue with \( w_\pi \) as defined in (16)). This is a more difficult problem, technically, than the unanticipated case we have already considered, and as before we rely on a mix of analytical and numerical approaches. We present analytical results in this section, and simulations in the next.

It is convenient to define a new variable, \( t \), which denotes the number of "notches" that a given person is away from the frontier wage in a given time period. Therefore,

\[
    i_\pi = \begin{cases} 
        t - \tau & \text{if } t \geq 0 \text{ and } \tau \geq 0 \text{ or } t < 0 \text{ and } \tau < 0 \\
        t - \tau - ref & \text{if } t \geq 0 \text{ and } \tau < 0 
    \end{cases}
\]

Further,

\[
    i_{\pi+1} = \begin{cases} 
        i_\pi + 1 & \text{if } t \neq 0 \\
        i_\pi + 1 + ref & \text{if } t = 0 
    \end{cases}
\]

To keep the notation sparse, define

\[
    \bar{V}_E(i, t) = V_E(w_{t-i}, w_t).
\]
At period $t \geq 0$, the decision problem is identical to that in the unanticipated case, and thus the optimal decision rule is as before: the agent will quit if $i > x^*$. At period $t = -1$, however, the optimal decision rule changes. The agent will take into account that getting a job is not as valuable in a relative sense, because the frontier wage jumps in the next period. The optimal decision rule can be solved for by backward recursion.

First, we must solve for the value of a job in period 0, as well as the value of being unemployed.

$$V_E(i,0) = \gamma^{-i} \left( \frac{1 - \beta^x}{1 - \beta} \right) + (\beta \gamma)^{-i} V_U(w_0)$$  \hspace{1cm} (20)

and

$$V_U(w_0) = \left( \frac{p}{1 - \beta \gamma (1 - p)} \right) \left( \frac{1 - \beta^x}{1 - \beta} \right) \left( \frac{1}{1 - \beta \gamma (1 - p)} \right)$$  \hspace{1cm} (21)

In period $t = -1$, the agent will quit her job, only if the value of quitting exceeds the value of keeping it.

$$\psi_E^{\text{noquit}} (i,-1) = \begin{cases} \gamma^{-i-1-\text{ref}} + \beta V_U(w_0) & \text{if } i + 1 + \text{ref} \geq x^* \\ \gamma^{-i-1-\text{ref}} + \beta V_E(i + 1 + \text{ref},0) & \text{otherwise} \end{cases}$$  \hspace{1cm} (22)

$$\psi_E^{\text{quit}} (i,-1) = \begin{cases} p \gamma^{-1-\text{ref}} + \beta V_U(w_0) & \text{if } 1 + \text{ref} \geq x^* \\ p \left( \gamma^{-1-\text{ref}} + \beta V_E(1 + \text{ref},0) \right) + (1 - p) \beta V_U(w_0) & \text{otherwise} \end{cases}$$  \hspace{1cm} (23)

Of course,

$$\psi(i,-1) = \text{Max} (\psi_E^{\text{quit}} (i,-1), \psi_E^{\text{noquit}} (i,-1)).$$  \hspace{1cm} (24)

Further,

$$V_U(w_{-1}) = p \left( \gamma^{-1-\text{ref}} + \beta \psi(\text{ref} + 1,0) \right) + (1 - p) \beta V_U(w_0).$$  \hspace{1cm} (25)
We now state a number of results that allow us to characterize the anticipated case.

**Result 1:**

There exists an \( x^* \) such that

\[
\begin{align*}
\bar{\nu}_E^{\text{quit}} (i,-1) &\geq \bar{\nu}_E^{\text{noquit}} (i,-1) \quad \text{if } i \geq x^* \\
\bar{\nu}_E^{\text{quit}} (i,-1) &< \bar{\nu}_E^{\text{noquit}} (i,-1) \quad \text{if } i < x^*
\end{align*}
\]  

(26)

**Proof:**

The value of quitting is independent of \( i \). The value of not quitting is a strictly decreasing function of \( i \). Further, as \( i \) approaches infinity, the current wage approaches zero.

Quitting results in a finite probability of earning a finite wage in the current period.

Therefore, for a sufficiently large \( i \), quitting will be preferable to not quitting and a finite \( x^* \) will exist. Q.E.D.

We now wish to characterize the optimal decision rule, which we do through the next result.

**Result 2:**

Provided \( r_{ef} \geq x^*-1 \), \( x^*_{-1} \geq x^* \), i.e., the range of inaction gets bigger in anticipation of the reform.

**Proof:**

It is sufficient to show that

\[
\bar{\nu}_E^{\text{noquit}} (x^*-1,-1) - \bar{\nu}_E^{\text{quit}} (x^*-1,-1) > 0.
\]  

(27)

The reasoning is as follows. If (27) holds, then \( x^*_{-1} > x^* - 1 \), by Result 1. Therefore (27) is sufficient to show Result 2, because if (27) holds then \( x^*_{-1} \geq x^* \), because \( x^*_{-1} \) must be an integer.

We know that

\[
\bar{\nu}_E^{\text{noquit}} (x^*-1,0) - \bar{\nu}_E^{\text{quit}} (x^*-1,0) > 0,
\]  

(28)
and we know that this agent will definitely quit in the next period. This implies that:

$$\gamma^{-x^*+1} - p - p\betay(\bar{V}_E(1,0),-V_U(w_0)) > 0.$$  \hspace{1cm} (29)

Note that $\gamma^{-x^*+1} - p$ is the difference between the current wage and the expected current period earnings if the agent quits. And, $p\beta y(\bar{V}_E(1,0),-V_U(w_0))$ is the expected gain in the next period associated with the possibility of getting a job in the current period if the agent quits in the current period.

We also know that:

$$\bar{V}_E^{\text{noquit}}(x^* - 1, -1) - \bar{V}_E^{\text{quit}}(x^* - 1, -1)$$

$$= \gamma^{-x^* - \text{ref}} - p\gamma^{-1 - \text{ref}} - p\beta(\bar{V}_E(\text{ref} + 1,0) - V_U(w_0))$$  \hspace{1cm} (30)

Hence, it is sufficient to show that

$$\gamma^{-x^* - \text{ref}} - p\gamma^{-1 - \text{ref}} - p\beta(\bar{V}_E(\text{ref} + 1,0) - V_U(w_0)) > 0,$$  \hspace{1cm} (31)

or

$$\gamma^{-x^* + 1} - p - p\gamma^{+1} p\beta(\bar{V}_E(\text{ref} + 1,0) - V_U(w_0)) > 0.$$  \hspace{1cm} (32)

By (29), it is also sufficient to show that

$$\gamma^{-x^* + 1} - p - p\gamma^{+1} p\beta(\bar{V}_E(\text{ref} + 1,0) - V_U(w_0)) >$$

$$\gamma^{-x^* + 1} - p - p\beta y(\bar{V}_E(1,0),-V_U(w_0))$$

or

$$\gamma^{+1} (\bar{V}_E(\text{ref} + 1,0) - V_U(w_0)) < (\bar{V}_E(1,0) - V_U(w_0)).$$  \hspace{1cm} (33)

This condition must hold because $\bar{V}_E(\text{ref} + 1,0) = V_U(w_0)$, recall that we assumed that ref + 1 $\geq x^*$. Q.E.D.

The value function in period $t = -2$ can be defined recursively:

$$\bar{V}_E^{\text{noquit}}(i,-2) = \begin{cases} 
\gamma^{-i-2 - \text{ref}} + \beta V_U(w_{i+1}) & \text{if } i \geq x^*_{i+1} - 1 \\
\gamma^{-i-2 - \text{ref}} + \beta \bar{V}_E(i + 1, -1) & \text{otherwise}
\end{cases}$$  \hspace{1cm} (34)
\[ \mathcal{P}^{\text{qull}}_{E}(i,-2) = p(\gamma^{-2}\text{-ref} + \beta \mathcal{V}_{E}(1,-1)) + (1-p)\beta \mathcal{V}_{U}(w_{-1}) \]  

(35)

Of course, \( \bar{V}_{E}(i,-2) = \max\{\bar{V}_{E}^{\text{qull}}(i,-2),\bar{V}_{E}^{\text{qull}}(i,-2)\} \).

Conjecture 3:

There exists an \( x_{-2}^{*} \) such that

\[ \mathcal{P}^{\text{qull}}_{E}(i,-2) \geq \mathcal{P}^{\text{noqull}}_{E}(i,-2) \quad \text{if } j \geq x_{-2}^{*} \]  

\[ \mathcal{P}^{\text{qull}}_{E}(i,-2) < \mathcal{P}^{\text{noqull}}_{E}(i,-2) \quad \text{if } i < x_{-2}^{*} \]  

(36)

Conjecture 4:

Further, \( x_{-2}^{*} < x_{-1}^{*} \), i.e., the range of inaction gets smaller the further away you get from the reform

We can use backward induction to define \( \bar{V}_{E}^{\text{qull}}(i,t), \bar{V}_{E}^{\text{qull}}(i,t), \bar{V}_{E}(i,t) \) for \( t < -1 \):

\[ \mathcal{P}^{\text{noqull}}_{E}(i,t|t < -1) = \begin{cases} 
\gamma^{-i-t}\text{-ref} + \beta \mathcal{V}_{U}(w_{t+1}) & \text{if } i \geq x_{t+1}^{*} - 1; \\
\gamma^{-i-t}\text{-ref} + \beta \mathcal{V}_{E}(i+1,t+1) & \text{otherwise} 
\end{cases} \]  

(37)

\[ \mathcal{P}^{\text{qull}}_{E}(i,t|t < -1) = p\left(\gamma^{-i} + \beta \mathcal{V}_{E}(1,t+1)\right) + (1-p)\beta \mathcal{V}_{U}(w_{t+1}); \]  

(38)

\[ \bar{V}_{E}(i,t) = \max\{\bar{V}_{E}^{\text{qull}}(i,t),\bar{V}_{E}^{\text{noqull}}(i,t)\}. \]  

(39)

Conjecture 5:

There exists an \( x_{t}^{*} \) such that

\[ \mathcal{P}^{\text{qull}}_{E}(i,t) \geq \mathcal{P}^{\text{noqull}}_{E}(i,t) \quad \text{if } i \geq x_{t}^{*} \]  

\[ \mathcal{P}^{\text{qull}}_{E}(i,t) < \mathcal{P}^{\text{noqull}}_{E}(i,t) \quad \text{if } i < x_{t}^{*} \]  

(40)

Further, \( \lim_{t \to \infty} x_{t}^{*} = x^{*} \), i.e., as you get sufficiently far from the reform, the reform does not impact the optimal quitting rule.

(Proofs of Conjectures 3, 4, and 5 will be included in the next draft. They are not
necessary for the simulations in section 6 below.)

6. Simulating an Anticipated Reform

We now turn to a simulation of the anticipated case. We assume that the reform occurs in period 0 and that it impacts the optimal tenure rule for \( t \in \{-200,-199,\ldots,-1\} \). We assume that the economy starts in a steady state in \( t = -210 \). The evolution of the economy can be described as in Section 2, except the transition matrix is now a function of time when \( t \in \{-200,-199,\ldots,-1\} \) (i.e., \( \Lambda_t \) is an \( x_t^* + 2 \times x_t^* + 2 \) matrix).

Figures 3 and 4 depict the optimal decision rules in the unanticipated case, for the same sets of parameter values as before. The analytical results are confirmed — the optimal quitting interval is highest just before the reform occurs, and declines as we move backward in time. Interestingly, it displays non-monotonic behavior when the reform is in the relatively distant future. As expected, it appears to approach the decision rule in the unanticipated case as the reform recedes into the ever more distant future (as in Conjecture 5). Figure 5 depicts the wage relative to the frontier wage that will induce a quit — its behavior reflects the change in the optimal quitting rule.

Figure 6 depicts the dynamics of output and unemployment, detrended by the long-run rate of output growth, and Figure 7 depicts the dynamics of output per capita. (These figures are in 11098#2.xls.) Evidently, the anticipation of reform induces anticipatory cycles and lead to economic stagnation before the reform. The model thus endogenously generates business cycles in anticipation of economic reform. Figure 8 contrasts the path of output when the reform is anticipated versus unanticipated. The recession following the reform is more severe and longer lasting when the reform is anticipated. One could take this as one argument against delaying reforms into the future.

Our intuition for the non-monotonicity in the optimal tenure rule and the endogenous cycles in anticipation of the reform is as follows. First, consider an agent with a tenure of 31 (at \( t = -18 \)). If the agent does not anticipate the reform she will wait two periods to quit. If the agent anticipates the reform, then she finds it optimal to quit. Suppose this agent were to deviate from the optimal strategy in period -18 by not
quitting, but then follow it in the following periods. She would not quit until the actual reform. (These facts follow from the table corresponding to Figure 4.) Therefore, the option value of waiting to quit is smaller (at $t = -18$) when the reform is anticipated, because the option will be exercised in the more distant future. Consequently, the range of inaction is smaller (at $t = -18$) when the reform is anticipated. The non-monotonicity in the range of inaction causes agents to be "in sync" in anticipation of the reform: When the range of inaction is small many agents quit their jobs and consequently unemployment is high. This leads to cycles in anticipation of the reform.

7. Conclusion

This paper presents a dynamic model of a worker-entrepreneur who searches for a production technique. A reform is modeled as an abrupt increase in the quality of production techniques available. Following the reform a large number of people quit their jobs in order to search for the new high productivity techniques. Initially unemployment goes up, because not all entrepreneurs find new techniques right away. As the entrepreneurs start to find the new techniques, however, growth increases rapidly. The reform causes a larger number of entrepreneurs to be "in sync", in the sense that they find their technique at the same time. Consequently, their techniques will become obsolete at the same time and they will all quit at the same time. The model yields dampening cycles following the reform.

If the reform is anticipated, people wait to quit until the reform. With a sufficiently large anticipated reform, because no one quits before the reform, there is no productivity growth and the economy stagnates in anticipation of the reform. Further, agents seek to time their quitting so that they quit when the opportunity cost of quitting is smallest, which is when the reform occurs. Therefore, people tend to start quitting "in sync" long before the reform. Consequently, the model yields endogenous cycles in anticipation of the reform. This can be taken as an argument against announcing a delayed reform: the losses in output implied by the endogenous cycles would be attenuated if the reform were unanticipated, i.e., announced at a given date and
implemented with immediate effect.

This is a very simple model with no “equilibrium features”. Following the reform there is a large drop in output, but prices do not go up because the small country is selling its goods on international markets. Further, many people are unemployed and therefore borrowing goes up. Nevertheless, the interest rate does not respond (once again because of the small country assumption).

Of course these are simplifying assumptions of the model. We believe that adding equilibrium features to the model will spread out the impact of the reform. For example, suppose the economy faces a downward-sloping demand curve for its product. If everyone becomes unemployed all at once, the price of the country’s product will rise rapidly. Consequently, everyone will not become unemployed at once. Those with the least to gain by quitting will wait until the price goes back down. We expect that this will tend to reduce the increase in unemployment following the reform but tend to increase the duration of the high unemployment rate (it would reduce the amplitude and increase the wavelength of the cycle). Adding such elements to the model would make it more realistic and more interesting and is the subject for future research.
REFERENCES


Unemployment. Mimeo, University of Rochester.


Figure 1: Output before and after the unanticipated reform
Figure 2: Output before and after the unanticipated reform
Figure 3: Unemployment before and after the unanticipated reform
Figure 4: Optimal tenure for the anticipated vs. unanticipated reform
Figure 5: Range of inaction when the reform is anticipated vs. unanticipated
Figure 6: Normalized output and unemployment before and after the reform.
Figure 7: Stagnation before the reform
Figure 8: Output when the reform is anticipated vs. unanticipated