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Technical Report

SEQUENCE GENERATORS AND FORMAL LANGUAGES

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Most of the results presented herein are drawn from our much longer paper which is to be published in the Proceedings of the Symposium on Recursive Function Theory held in New York City, April 6-7, 1961, and sponsored by the American Mathematical Society, the Association for Computing Machinery, and the Association for Symbolic Logic. The terminology and mode of development in the present paper differ considerably from that of the longer paper.

The fundamental concept is that of a sequence generator. An sequence generator is a finite, directed, labeled graph. Each node may or may not be labeled as a root R. Each node is labeled with a pair of truth values.

We are particularly interested in what we call the "behavior" of a sequence generator (see Fig. 2). The behavior of a sequence generator is a set of matrices of truth values. The pairs of truth values taken from the nodes along an infinite path of the graph constitute a two-by-omega matrix. Such a two-by-omega matrix belongs to the behavior of a sequence generator if and only if it is taken from a path of the graph which begins with a root. For example, the first element of Fig. 2 is taken from the path that starts with the root and always stays in the root, while the second element is taken from the path that starts with the root and oscillates back and forth between it and the other node.

This concept of behavior may be connected to the ordinary one by calling the upper truth value of a node an input and calling the lower truth value of a node an output. A sequence generator then determines a relation between infinite input sequences and infinite output sequences, which relation constitutes its behavior.

The inputs and outputs labeling nodes may be vectors of truth values rather than single truth values. Our results all apply to this more general case, but in the interest of simplicity we will present them in terms of the case where the input and output vectors are of length one.

Digital computers or finite automata are closely related to a special type of sequence generator called a deterministic sequence generator. Figure 3 is deterministic. A sequence generator is deterministic when it satisfies these two conditions:

(1) For each truth value, there is exactly one root having that as its input truth value.

(2) For each node and each truth value, there is exactly one node following the given node and having the given truth value as its input.

The deterministic sequence generator of Fig. 3 corresponds, in fact, to a binary counter.

Consider a graph derived from a digital computer or finite automaton in the following way. Each pair consisting of an input state and an internal state is a complete state. A node of the graph is provided for each complete state. The labels on a node give the input state and output state associated with the corresponding complete state of the automaton. Directed arrows of the graph indicate direct transitions between complete states. Any graph derived from a digital computer in this way is a deterministic sequence generator, and each deterministic sequence generator corresponds to a digital computer.

The rows of truth values in the infinite matrices introduced earlier define predicates on the natural numbers. This suggests the use of logical formulas to represent sequence generators. We do this in an interpreted system called the sequential calculus.

A formula of the sequential calculus is given in Fig. 4. The symbols of the sequential calculus are: individual variables, interpreted as ranging over the natural numbers; monadic predicate variables, interpreted as ranging over predicates of natural numbers; the individual constant zero; the successor function; truth-functional connectives; and quantifiers on both individual and predicate variables.

To each sequence generator there corresponds a formula of the sequential calculus, as illustrated in Fig. 4. The existential predicate quantifier "there is an s" asserts the existence of an infinite sequence of nodes satisfying four conditions: $s(o)$ says that this infinite sequence of nodes begins with the root R, $(\forall\tau)[s(\tau) \vee s(\tau')]$ that it is indeed a path of the graph; $i(\tau) \equiv s(\tau)$ and $o(\tau) \equiv \overline{s(\tau)}$ define the input and output predicates for the path. This correspondence between formulas and sequence generators is such that a pair of predicates of natural numbers $<i,o>$ satisfies the formula if and only if the corresponding two-by-omega matrix of truth values belongs to the behavior of the given sequence generator.

See next the normal form of Fig. 5. Every formula derived from a sequence generator can be put in the <u>normal form</u> $(\exists s)[R\&((\forall\tau)P)]$, where R and P are both truth functions, the root condition R being a truth function of $s(o)$, P being a truth function of $s(\tau)$, $s(\tau')$, $i(\tau)$, and $o(\tau)$. Conversely, one can derive from any normal form formula a corresponding sequence generator.

Thus formulas of the sequential calculus can be studied by means of sequence generators, to which combinatorial methods can be easily applied. We will next present a theorem which was arrived at by working with sequence generators but is more easily stated here as a theorem about formulas of the sequential calculus.

We introduce the following definition for a formula $\mathcal{C}(i,o)$ containing i and o as the only free variables. $\mathcal{C}(i,o)$ is <u>uniquely solvable</u> if and only if for each predicate i there is <u>exactly one</u> predicate o such that the pair

$\langle i,o \rangle$ satisfies $\mathcal{C}(i,o)$ in the intended interpretation. It is clear that when a formula $\mathcal{C}(i,o)$ is uniquely solvable, it *implicitly* defines a function $\mathcal{H}$ such that $o = \mathcal{H}(i)$ if and only if $\mathcal{C}(i,o)$. $\mathcal{H}$ is a function which maps the set of all predicates of natural numbers into the set of all predicates of natural numbers.

We are interested in a recursive definition of the type Church calls an unrestricted singulary recursion. As Fig. 6 shows, such a recursion gives:

$s(o)$ as a truth function of $i(o),\ldots, i(h)$

$s(\tau')$ as a truth function of $s(\tau)$; $i(\tau'),\ldots, i(\tau'+h)$

$o(\tau)$ as a truth function of $s(\tau)$,

where h is a constant.

Our *first theorem* is a definability theorem for uniquely solvable normal form formulas. The theorem is: for every uniquely solvable normal form formula $\mathcal{C}(i,o)$, there exists an equivalent formula $(\exists s)(\tau)\mathcal{U}(s,i,o,\tau)$, where $\mathcal{U}(s,i,o,\tau)$ is an unrestricted singulary recursion. Furthermore, the formula $(\exists s)(\tau)\mathcal{U}(s,i,o,\tau)$ can be constructed effectively from the given formula $\mathcal{C}(i,o)$.

The proof of the definability theorem has three steps. The first can be expressed if we think of the natural numbers as discrete times with τ being a time variable. Consider now a uniquely solvable normal form formula $\mathcal{C}(i,o)$ and the corresponding sequence generator with input i and output o. The output o at time τ may, contrary to physical reality, depend on input states which occur at a time later than τ. Let k be the number of nodes of the sequence generator. It turns out that the amount of anticipation is bounded by k^2, that is, the parameter h of the unrestricted singulary recursion is in fact k^2. Thus the first step of the proof of the definability theorem establishes the fact that for a uniquely solvable sequence generator with k nodes, the output o at time τ is independent of input states after $\tau+k^2$.

The second step of the proof of the definability theorem involves a function or operation on the class of sequence generators, called the subset sequence generator operation, and used by Myhill, Medvedev, Rabin, and Scott. Let Γ be a sequence generator and let Γ^* be *the subset sequence generator of* Γ. The nodes of Γ^* are sets of nodes of Γ. The arrows of Γ^* are placed according to the arrows of Γ in such a way as to take account of the fact that a binary relation induces on the subsets of its domain a function which is single-valued. There are two important facts about Γ^*. First, if Γ is uniquely solvable, then for each predicate i there is exactly one path of Γ^* giving rise to i. Second, Γ^* has the same behavior as Γ. Previous users of the subset sequence generator operation have shown this second fact to be so when behavior is based on finite sequences. This behavioral equivalence may be extended to our concept of behavior, which is based on infinite sequences, by means of König's infinity lemma.

As a consequence of steps one and two, our consideration of the normal form formula $\mathcal{C}(i,o)$ is reduced to a sequence generator Γ^* and a number k^2 having these two properties: first, for each predicate i there is a unique path in Γ^* giving rise to i; second, k^2 is a bound on the time dependence of the predicates s and o on the predicate i, since $s(\tau)$ and hence $o(\tau)$ are independent of the input states after $\tau+k^2$. To obtain the unrestricted singulary recursion for $\mathcal{C}(i,o)$ we must express $s(\tau')$ as a truth function of $s(\tau)$ and $i(\tau),\ldots,i(\tau'+h)$. For any given i we use the predicate s defined by the corresponding path through Γ^* as the s of the recursion, and we take h to be k^2. A simple argument based on the uniqueness of the path will show that $s(\tau')$ is a time-independent truth function of $s(\tau)$ and $i(\tau'),\ldots,i(\tau'+h)$. To write o as a restricted singulary recursion of i and s is now essentially a matter of an elaborate truth table.

Our definability result may be generalized to arbitrary formulas of the sequential calculus by means of some theorems about the sequential calculus established by J. R. Büchi. Büchi's main result is that there is a decision procedure for whether or not an arbitrary sentence of the sequential calculus is true in the intended interpretation.

An immediate corollary of this result is that there is a decision procedure for unique solvability, since an arbitrary formula $\mathcal{C}(i,o)$ is uniquely solvable if and only if the following formula of the sequential calculus is true:

$$(i)(\exists o)\left\{\mathcal{C}(i,o)\&(o_1)[\mathcal{C}(i,o)\supset(\tau)(o(\tau) \equiv o_1(\tau))]\right\}.$$

The generalization of our definability result involves the concept of a finitely anticipatory formula. A formula $\mathcal{C}(i,o)$ is <u>finitely</u> <u>anticipatory</u> if for each time τ there is a integer h such that the input states from time zero up to time $\tau+h$ determine the output state at time τ. It should be noted that in an unrestricted singulary recursion h is a constant, whereas in the definition of finitely anticipatory, h is a function of τ. Since $\leq$ can be defined in the sequential calculus, the statement that $\mathcal{C}(i.o)$ is finitely anticipatory can be expressed by a sentence of the sequential calculus. Therefore, the decidability of truth of sentences of the sequential calculus implies that the class of finitely anticipatory formulas is effectively decidable.

We can now state our general definability theorem. Let $\mathcal{C}(i,o)$ be any formula of the sequential calculus. The theorem states that the relationship between i and o can be expressed by an unrestricted singulary recursion if and only if $\mathcal{C}(i,o)$ is both uniquely solvable and finitely anticipatory. More precisely, let $\mathcal{C}(i,o)$ be any formula of the sequential calculus having i and o as the only free variables. There is an unrestricted singulary recursion $\mathcal{U}(s,i,o,\tau)$ such that $\mathcal{C}(i,o) \equiv (\exists s)(\tau)\mathcal{U}(s,i,o,\tau)$ if and only if $\mathcal{C}(i,o)$ is both uniquely solvable and finitely anticipatory. Moreover, there is an algorithm applicable to $\mathcal{C}(i,o)$ which yields an equivalent formula, $(\exists s)(\tau)\mathcal{U}(s,i,o,\tau)$, if one exists.*

The proof of the generalized definability theorem is too involved to be presented here, but we will make some remarks about it. The proof uses a normal form involving an infinite existential quantifier $(\exists^{\omega})$ (see Fig. 7). "$(\exists^{\omega})G$" means that there exist infinitely many natural numbers satisfying G. The infinite existential quantifier $(\exists^{\omega})$ is definable in the sequential calculus. Büchi has shown that, for any formula $\mathcal{C}(i,o)$ of the sequential calculus having the predicates i and o as the only free variables, there is an equivalent normal form formula. As shown in the top line of Fig. 7, this normal form is $(\exists s)[R\&(\exists^{\omega}\tau)G)\&((\forall\tau)P)]$, where R is a truth function of s(o), G is a truth function of $s(\tau)$, and P is a truth function of $s(\tau)$, $s(\tau')$, $i(\tau)$, and $o(\tau)$. This normal form is the same as the one we had before except for the addition of the conjunct $(\exists^{\omega}\tau)G$.

The generalized normal form of Fig. 7 corresponds to an extension of the idea of sequence generator in which each node may or may not be labeled as a goal G (see Fig. 8). The effect of the infinite quantifier $(\exists^{\omega})$ of the formula is obtained in a sequence generator with goals by considering only paths which pass through the goals infinitely many times. There is then a one-one correspondence between generalized normal form formulas and sequence generators with goals, which correspondence preserves behavior. Figure 8 gives an example of a sequence generator with goals and the equivalent generalized normal form formula.

It will be recalled that in the case of an ordinary uniquely solvable sequence generator with k nodes, the output o at time τ is determined by the input states from time zero up to time $\tau+k^2$. The same lemma holds for those sequence generators with goals which are both uniquely solvable and finitely anticipatory, so that, as before, the amount of anticipation is independent of time.

A complete presentation of the proof of the general definability theorem requires going inside Büchi's proof of decidability for the sequential calculus and making use of regular sets as defined by Kleene. We will not here present this proof but will conclude by summarizing the main result of this paper: there is an algorithm applicable to any formula of the sequential calculus with free input and output predicates. This algorithm produces an equivalent unrestricted singulary recursion, if one exists, and otherwise indicates that none exists.

*This theorem is not in the paper to be published in the Proceeding of the Symposium on Recursive Function Theory.

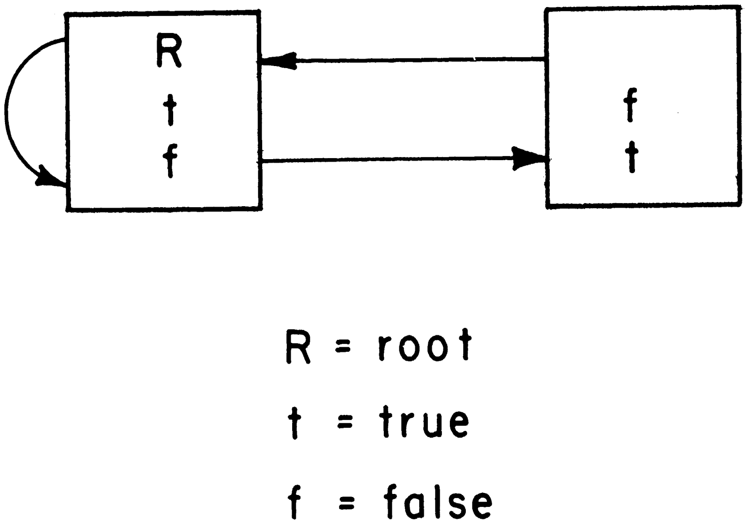


Fig. 1. Sequence Generator. Finite, directed, labeled graph.

```
R
| t t t t t t . . . |
| f f f f f f . . . |
R
| t f t f t f . . . |
| f t f t f t . . . |
```

Fig. 2. Two elements of the behavior of Fig. 1.

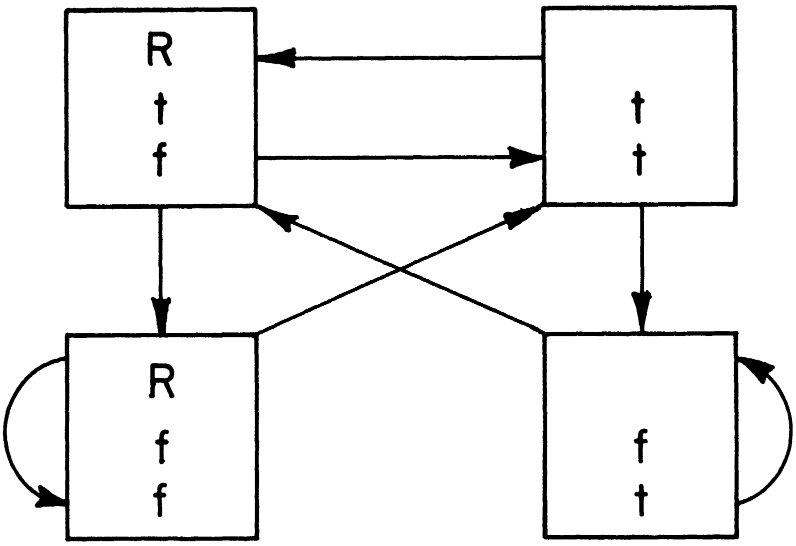
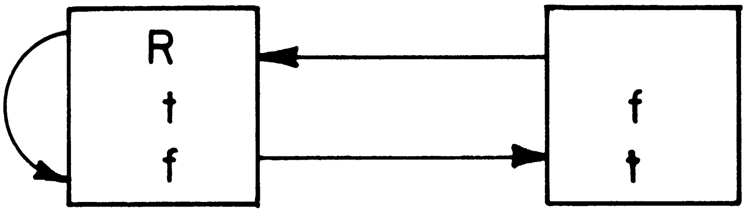


Fig. 3. Deterministic sequence generator for a binary counter.



$$
\begin{aligned}
&(\exists s)\ \{ s(o)\ \& \\
&\qquad (\forall \tau)\,[s(\tau) \vee s(\tau')]\ \& \\
&\qquad\qquad [i(\tau) \equiv s(\tau)]\ \& \\
&\qquad\qquad [o(\tau) \equiv \overline{s(\tau)}]\ \}
\end{aligned}
$$

Fig. 4. Sequence generator and formula of the sequential calculus.

$$
\begin{aligned}
&(\exists s)\ [R\ \&((\forall \tau)P)] \\
&R\ [s(o)] \\
&P\ [s(\tau),\ s(\tau'),\ i(\tau), o(\tau)]
\end{aligned}
$$

are truth functions

Fig. 5. Normal form.

$$
\begin{aligned}
&s(o) \equiv A\,[i(o), \ldots.\ i(h)]\ \& \\
&s(\tau') \equiv B\,[s(\tau);\ i(\tau'), \ldots.\ i(\tau' + h)]\ \& \\
&o(\tau) \equiv D\,[s(\tau)]
\end{aligned}
$$

A, B, D, are truth functions

Fig. 6. Unrestricted singulary recursion.

$$(\exists s)\left[R \,\&\, ((\exists^{\omega}\tau)\ G) \,\&\, ((\forall\tau)\ P)\right]$$

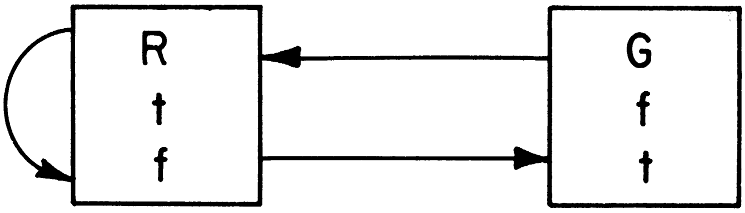
$$R\left[s(o)\right]$$

$$G\left[s(\tau)\right]$$

$$P\left[s(\tau),\ s(\tau'),\ i(\tau),\ o(\tau)\right]$$

are truth functions

Fig. 7. Generalized normal form.



$$(\exists s)\ \left\{s(o) \,\&\, (\exists^{\omega}\tau)\,\overline{s(\tau)} \,\&\, \right.$$
$$(\forall\tau)\left[s(\tau)\ \mathrm{v}\ s(\tau')\right] \,\&$$
$$\left.(i(\tau) \equiv s(\tau)) \,\&\, (o(\tau) \equiv \overline{s(\tau)})]\right\}$$

Fig. 8. Sequence generator with goals and equivalent generalized normal form formula.

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