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*Modelling Stock Returns in the G-7 and in Selected CEE Economies:  
A Non-linear GARCH Approach*

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# Modelling stock returns in the G-7 and in selected CEE economies:

## A non-linear GARCH approach

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### ABSTRACT

This paper investigates conditional variance patterns in daily return series of stock market indices in the G-7 and 6 selected economies of Central and Eastern Europe. For this purpose, various linear and asymmetric GARCH models are employed. The analysis is conducted for Canada, France, Germany, Italy, Japan, the UK and the US for which the TSX, CAC-40, DAX-100, BCI, Nikkei-225, FTSE-100 and DJ-30 indices are respectively considered over the period 1987 to 2002. Furthermore, the official indices of Czech, Hungarian, Polish, Russian, Slovak and Slovene stock markets are also studied, i.e. the PX-50, BUX, WIGI, RFS, SAX-16 and SBI, respectively, over 1991/1995 to 2002. The estimation results reveal that the selected stock returns for the G-7 can be reasonably well modelled using linear specifications whereas the overwhelming majority of the stock indices from Central and Eastern Europe can be much better characterised using asymmetric models. In other words, stock markets of the transition economies exhibit much more asymmetry because negative shocks hit much harder these markets than positive news. It also turns out that these changes do not occur in a smooth manner but happen pretty brusquely. This corroborates the usual observation that emerging stock markets may collapse much more suddenly and recover more slowly than G-7 stock markets.

**JEL:** C52, G10, P52

**Keywords:** volatility modelling, conditional variance, non-linearity, asymmetric GARCH, G-7, transition economies

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# I. Introduction

The most prominent features of financial time series such as volatility clustering, excess kurtosis and fat-tailedness have been long attracting considerable interest of both market professionals and academic researchers working in the field of finance. The seminal ARCH process proposed by Engle (1982) to model this phenomenon has given a huge impetus to both econometric model building and applied research.

Recently, the traditional linear ARCH model has been found inappropriate to describe financial time series mainly because of the presence of non-linearity in the series. For instance, Franses et al. (1998) show in general that non-linear GARCH models characterise volatility of the AEX, DAX, DJI, FTSE and the NIKKEI stock returns far better than traditional GARCH model. Also, Koutmos (1998) present results according to which asymmetric models perform better for stock market indices in industrialised countries such as Australia, Belgium, Canada, France, Germany, Italy, Japan, UK and the US. Fornari and Mele (1997) employ, for instance, the asymmetric GARCH model proposed by Glosten, Jagannathan and Runkle (1993) (GJR) and volatility switching GARCH (VS-GARCH) for selected American French, Japanese and Italian stock market returns. Using daily series, the Volatility Switching GARCH process is found to capture asymmetries better than the GJR model. Omran and Avram (2000) also consider these two models and argue that the GJR model outperforms VS-GARCH for all stock returns but the S&P 500.

Not only returns observed in financial markets of highly industrialised countries appear to exhibit volatility clustering but asset returns in emerging and transition countries turn out to be described correctly by conditional volatility. As regards transition economies of Central and Eastern Europe, Poshakwale and Murinde (2001) find empirical evidence for the presence of conditional heteroscedasticity in Hungary and Poland. Using daily data from 1994 to 1998, they show that the returns of the official indices of the Budapest and the Warsaw stock exchanges, i.e. BUX and WIG-20, can be modelled using a GARCH model. However, the baseline GARCH model fails to account for the entirety of heteroscedastic conditional volatility in the return series. Kasch-Haroutounian and Price (2001) argue that this is due to the presence of asymmetry and non-linearity in the series. And this is evidenced for the Czech Republic, Hungary, Poland and Slovakia over the period 1992/1994 to 1998 employing a variety of asymmetric models to the data.

The ambition of this paper is to contribute in three aspects to this debate. First, we propose to study and compare daily stock returns of the G-7 and 6 selected Central and Eastern European economies with functioning stock markets, namely the Czech Republic, Hungary, Poland, Russia, Slovakia and Slovenia. Second, we investigate longer time periods than done other studies in the literature, and this especially for the CEE economies, i.e. from 1991 to 2002. Finally, we compare results obtained using linear and non-linear GARCH models.

The roadmap of the paper is the following: Section II provides a general picture of recent developments regarding asymmetric GARCH models. Section III deals with data issues. Section IV focuses on the testing procedure and presents the estimation results for the G-7 and the 6 selected CEE economies. Finally, Section V gives some concluding remarks.

## II. A quick overview of the theoretical literature

Autoregressive conditional heteroscedastic (ARCH) models, introduced by Engle (1982), have proved to be very popular and, more importantly, very useful in modelling financial time series. In such models, the mean equation is given, in the baseline scenario, by an AR(p) process:  $x_t$ , the stock returns series, is regressed on its past values. Then, the conditional variance is regressed on a constant and lagged values of the squared error term obtained from the mean equation. This baseline model was extended by Bollerslev (1986) leading to the class of generalised ARCH models (GARCH), in which the conditional variance depends not only on the squared residuals of the mean equation, but also on its own past values. For simplicity, only the GARCH(1,1) model is shown here.

$$(1) \quad x_t = \rho \cdot x_{t-1} + \varepsilon_t, \quad \varepsilon_t \text{ being such that } E(\varepsilon_t / \varepsilon_{t-1}) = 0 \text{ and } V(\varepsilon_t / \varepsilon_{t-1}) = \sigma_t^2 \text{ where } \varepsilon_t = \varepsilon_{t-1}, \varepsilon_{t-2}, \dots$$

$$(2) \quad \sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \text{ and } V(\varepsilon_t / \varepsilon_{t-1}) = \sigma_t^2 \text{ where } \varepsilon_t / \varepsilon_{t-1}$$

Parameters  $\omega$  and  $\alpha$  should take values higher than 0 and  $\beta$  is to be positive so as to ensure that the conditional variance  $\sigma_t^2$  be nonnegative. In addition, it is necessary that  $\alpha + \beta < 1$ . This condition secures covariance stationarity of the conditional variance. A straightforward interpretation of the estimated coefficients in (2) is that the constant term  $\omega$  is the long-term average volatility, i.e. conditional variance, whereas  $\alpha$  and  $\beta$  represent how volatility is affected by current and past information, respectively.

In accordance with the extensive body of empirical literature aimed at investigating returns of financial assets such as stocks, GARCH models proved successful in taking account of prominent features of return series, namely volatility clustering, i.e. heteroscedasticity in the mean equation's residuals and the leptokurtosis in the empirical distribution. In contrast, these models fail to account for asymmetry and non-linearity in the conditional variance. This problem, also referred to as the leverage effect, has given rise to an array of asymmetric models. The simplest asymmetric GARCH model is that proposed by Glosten, Jagannathan and Runkle (1993)(GJR henceforth). In this model, not only the size but also the sign of the residual obtained from the mean equation, determine the conditional variance, which is tantamount to capture asymmetry as in (3):

$$(3) \quad \sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \lambda \varepsilon_{t-1}^2 S_{t-1} + \beta \sigma_{t-1}^2$$

where  $S_{t-1}$  takes the value of 1 if  $\varepsilon_{t-1} < 0$  and 0 if  $\varepsilon_{t-1} \geq 0$ . Put differently, the impact of negative shocks/news on the conditional variance ( $\alpha + \lambda$ ) is higher than that of positive shocks/news ( $\alpha$ ) provided  $\lambda$  is significantly different from 0. Note that for the conditional variance to be positive, the coefficients have to be non-negative, i.e.  $\omega > 0; \frac{\alpha + \lambda}{2} \geq 0; \beta > 0$ . Furthermore, covariance stationarity is secured by  $\frac{\alpha + \lambda}{2} + \beta < 1$ . In a very similar setup, Zakoian (1994) uses the conditional standard deviation instead of the conditional variance as shown in (4) below. This model is called the Threshold GARCH (TGARCH) process.

$$(4) \quad \sigma_t = \omega + \alpha |\varepsilon_{t-1}| + \lambda \varepsilon_{t-1} S_{t-1} + \beta \sigma_{t-1}$$

Similarly to the GJR model, the parameters should be equal to or higher than zero whilst the sum of them excluding the intercept should be lower than 1. The Exponential GARCH (EGARCH), developed by Nelson (1991), is based on the log-transformed conditional variance. Hence, the asymmetric effect is exponential instead of being quadratic as in the GJR model:

$$(5) \quad \log(\sigma_t^2) = \omega + \alpha \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \lambda \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} - \sqrt{\frac{2}{\pi}} \right| + \beta \log(\sigma_{t-1}^2)$$

Whereas stationarity is ensured by  $\beta < 1$ , the positivity constraint of the parameters is lifted in this model. The Quadratic GARCH (QGARCH), pioneered by Sentana (1995) can be viewed as the approximation of a second order Taylor expansion series of an unknown conditional variance:

$$(6) \quad \sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \lambda \varepsilon_{t-1} + \beta \sigma_{t-1}^2$$

with non-negativity being respected by  $\omega \geq 0; \alpha \geq 0; \text{and } \beta \geq 0$  and  $\lambda < 4\alpha\omega$ . The volatility switching GARCH (VS-GARCH) introduced by Fornari and Mele (1996) is able to detect the mean reversion of the conditional variance as shown in (7):

$$(7) \quad \sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \lambda S_{t-1} \nu_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

such that  $S_t=1$  if  $\varepsilon_t > 0$ ,  $S_t=0$  if  $\varepsilon_t=0$  and  $S_t < 1$  when  $\varepsilon_t < 0$ . Note that the term  $\nu_{t-1} = \varepsilon_{t-1}^2 - E(\sigma_{t-1}^2)$  and thus it measures the degree of persistence and mean reversion in the conditional variance. The Logistic Smooth Transition GARCH (LST-GARCH) developed by Hagerud (1996) and Gonzales-Rivera (1998) considers two regimes in which the conditional variance can be described by a different GARCH(1,1) process:

$$(8) \quad \sigma_t^2 = \alpha_0 + (\alpha_1 + \alpha_1 F(\varepsilon_{t-1})) \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

where  $F(\cdot)$  is the transition function that describes the path from one regime to another. Note that  $0 \leq F \leq 1$  and  $F(\varepsilon_{t-1}) = \frac{1}{1 + \exp(-\theta \varepsilon_{t-1})}$  with  $\theta > 0$ . Asymmetry is controlled for by  $\theta$ . The two regimes can be described by introducing  $c$ , the threshold value for  $\varepsilon_{t-1}$ :

$$\text{If } \varepsilon_{t-1} \rightarrow -\infty \text{ then } F(\varepsilon_{t-1}) \rightarrow 0$$

$$\text{If } \varepsilon_{t-1} \rightarrow c \text{ then } F(\varepsilon_{t-1}) \rightarrow \frac{1}{2}$$

$$\text{If } \varepsilon_{t-1} \rightarrow \infty \text{ then } F(\varepsilon_{t-1}) \rightarrow 1$$

The positivity condition is respected if  $\alpha_0 > 0; \alpha_1 \geq 0; \beta_1 \geq 0; \text{and } \alpha_1 \geq \frac{1}{2} |\alpha_2|$ . For covariance stationarity

to hold, the following condition is to be fulfilled:  $(\alpha_1 - \frac{1}{2} |\alpha_2| + \max(\alpha_2, 0)) + \beta_1 < 1$ .

### III. Data Issues

In this paper, we consider major stock market indices of the G-7 countries, that is for Canada, France, Germany, Italy, Japan, the UK and the US, on the one hand, and stock market indices of 5 selected Central and Eastern European (CEE) transition economies, i.e. the Czech Republic, Hungary, Poland, Slovakia and Slovenia, on the other hand. Because of its importance, Russia is also

taken under into consideration. The daily return series ( $S_t$ ) used for the estimations are of daily frequency and are constructed from the price series,  $P_t$ , as follows:  $S_t = \ln(P_t) - \ln(P_{t-1})$ .

The series for the G-7 countries start shortly after the 1987 crash, i.e. in late-1987 and end in June 2002. The stock indices considered are the TSX (Toronto), CAC-40 (Paris), DAX-100 (Frankfurt), BCI (Milan), Nikkei-225 (Japan), FTSE-100 (UK) and the Dow Jones (USA). As regards countries from Central and Eastern Europe, the time span studied is largely determined by the date of re-opening of the stock exchanges and the introduction of an official stock market index during the early 1990s. So, the period under study runs from 1991 to 2002 for Hungary and Poland, from 1993 to 2002 for Slovakia, from 1994 to 2002 in Slovenia and in the Czech Republic and finally from 1995 to 2002 in Russia. The indices we consider in this paper are these: PX-50 (Prague), BUX (Budapest), WIG-20 (Warsaw), SAX-16 (Bratislava), SBI (Ljubljana) and RFS (Moscow). All data series are drawn from Datastream.

## IV. Testing procedure

Evaluating the adequacy of and estimating the symmetric and asymmetric GARCH models presented earlier involves a score of interrelated steps. A general overview is given below and we shall develop them in more detail in what follows.

- 1.) Descriptive statistics and unit root tests as a check for stationarity in the return series
- 2.) Estimation of the mean equation and the use of preliminary tests
  - a.) Testing for asymmetry in the residuals: Sign and size bias tests
  - b.) Testing for linear and non-linear ARCH effects in the residuals
- 3.) Estimation of the volatility models
- 4.) Specification tests on the standardised residuals issued from the volatility models
  - a.) Testing for remaining asymmetry in the residuals: Sign and size bias tests
  - b.) Model misspecification: remaining ARCH, higher order ARCH/GARCH etc.
  - c.) Parameter stability
  - d.) Skewness, kurtosis, normality test

## IV.1. Descriptive statistics and unit root tests

Table 1 hereafter provides a general overview of the data used and gives preliminary descriptive statistics. The first striking feature is that the mean of daily returns is significantly higher in Hungary, Poland and Russia when compared to the G-7 countries, and higher returns go hand in hand with higher standard deviation. This fits into the picture on emerging markets, i.e. higher returns come at cost of higher risk. But this is not the case for the Czech Republic, Slovakia and Slovenia where the mean return is in line to that in the G-7. By contrast, the standard deviation is relatively high in these countries. In addition to this, maximum and minimum returns are much higher in all CEE countries relative to those of the G-7. All this is not surprising in Hungary, Poland and Russia. In the Czech Republic, Slovakia and Slovenia, the fact that the standard deviation and minimum and maximum values are higher than what could be explained by returns might be due to the low liquidity of the stock markets, entailing higher structural volatility. All series are, without exception, highly leptokurtic and exhibit strong skewness, mostly to the left. This suggests the presence of asymmetry towards negative values. As a result, the Jarque and Bera test rejects the null of the normality, as shown in Table 1.

**Table 1.** Data overview and descriptive statistics for the return series

	N. of OBS	Period	Mean	SD	Max	Min	Skew	Kurt	Jarque-Bera
<b>G-7</b>									<i>(p-value)</i>
DJ	3745	14:12:1987- 19:04:2002	0.00044	0.0097	0.0486	-0.0745	-0.591	9.008	5849.9 (0.00)
CAC-40	3748	09:12:1987- 19:04:2002	0.00047	0.0122	0.0680	-0.0767	-0.207	5.554	1044.9 (0.00)
DAX-100	3393	25:04:1989- 25:04:2002	0.0002	0.0051	0.0288	-0.0610	-0.947	14.859	19387.6 (0.00)
BCI	3393	25:04:1989- 25:04:2002	0.0002	0.0127	0.0630	-0.0847	-0.447	6.472	1815.4 (0.00)
T'SX	3764	01:12:1987- 03:05:2002	0.00025	0.0084	0.0460	-0.0846	-0.820	11.996	13110.0 (0.00)
Nikkei-225	3754	25:04:1989 – 25:04:2002	-0.0003	0.0146	0.1241	-0.0720	0.268	6.984	2281.0 (0.00)
FTSE-100	3753	01:12:1987- 18:04:2002	0.00031	0.0095	0.0544	-0.0588	-0.119	5.154	734.8 (0.00)
<b>CEEs</b>									
BUX	3000	02:01:1991- 03:07:2002	0.00067	0.0169	0.1361	-0.1803	-0.867	18.311	29681.7 (0.00)
SAX16	2297	14:09:1993- 03:07:2002	0.00003	0.0176	0.2755	-0.1245	2.378	42.276	149417.5 (0.00)
SBI	2217	03:01:1994- 03:07:2002	0.00045	0.0131	0.1091	-0.1132	-0.434	14.827	12931.7 (0.00)
PX-50	1783	06:04:1994- 03:07:2002	-0.00042	0.0122	0.0582	-0.0707	-0.180	5.617	625.3 (0.00)
RFS	1783	01:09:1995- 03:07:2002	0.00232	0.0347	0.2154	-0.7200	-4.358	97.495	786845.2 (0.00)
WIGI	2926	16:04:1991- 03:07:2002	0.0009	0.0223	0.1478	-0.1134	0.006	8.560	3769.6 (0.00)

*Note:* p-values in parentheses for the Jarque-Bera test statistics.



As a second step, unit root tests are applied to the stock index series in log levels (not return series) in accordance with the Dickey and Pantula (1987): the conventional ADF (Augmented Dickey-Fuller) and PP (Phillips-Perron) tests are applied to series in second differences, then to first differences and finally to the series in level. The results reported in Table 2 suggest that, except for the Czech Republic, all the series contain a unit root in levels but are stationary in first and second differences: in other words, the price series are i.e. integrated of order 1.<sup>1</sup> So, from this point of view, stock markets appear weakly efficient with the exception of the Prague Stock Exchange for which weak efficiency cannot be established.<sup>2</sup>

**Table 2.** ADF and PP unit root tests

	Second differences		First differences		Levels	
	ADF	PP	ADF	PP	ADF	PP
<b>G-7</b>						
DJ	-47.052**	-191.630**	-28.488**	-60.544**	-0.889	-0.947
CAC-40	-47.280**	-186.353**	-27.782**	-58.634**	-1.385	-1.507
DAX-100	-45.356**	-193.235**	-26.796**	-60.702**	-1.388	-1.386
BCI	-43.627**	-163.454**	-25.026**	-52.115**	-0.604	-0.543
TSX	-46.742**	-168.099**	-27.268**	-54.387**	-0.933	-0.915
Nikkei-225	-44.821**	-191.878**	-26.849**	-59.546**	-1.615	-1.569
FTSE-100	-46.659**	-181.238**	-28.355**	-57.973**	-1.624	-1.665
<b>CEEs</b>						
BUX	-40.741**	-164.022**	-23.886**	-49.670**	-0.761	-0.752
SAX16	-36.515**	-146.725**	-15.561**	-45.138**	-1.494	-1.411
SBI	-36.273**	-102.645**	-20.152**	-36.001**	-1.159	-1.164
PX-50	-32.744**	-123.371**	-18.947**	-40.701**	-3.799**	-3.786**
RFS	-31.622**	-114.502**	-20.948**	-38.117**	-0.909	-0.878
WIGI	-38.836**	-151.861**	-22.174**	-46.675**	-1.906	-1.918

*Note:* \* and \*\* denote the acceptance of the alternative of no unit root in the series at the 5% and 1% levels, respectively. The lag length of 4 and 7/8 is used for the ADF and PP tests, respectively.

<sup>1</sup> Only the model including a drift, and no trend is tested since there is no theoretical reason to think that stock prices contain a deterministic trend. The results reported in Table 1 are robust against different lag lengths.

<sup>2</sup> This result conflicts with Kasch-Haroutounian and Price (2001) who find weak efficiency for the Czech Republic, Hungary, Poland and Slovenia at the 1% level.

## IV. 2. Preliminary tests

### The mean equation

Before jumping into the preliminary tests, some developments should be done on the mean equation. Throughout this paper, we assume that the return series can be modelled as an autoregressive process. For each country, an AR(p) process is specified for which the lag length is obtained using the Akaike information criterion (AIC) and that eliminates serial correlation from residuals of the mean equation. The same AR(p) is then used for a given country when estimating different GARCH models for the conditional variance in the remainder of the paper (See Table 3 below). For the FTSE-100, Nikkei-225 and the Polish WIGI, the lag length is zero, which lends support in favour of the market efficiency hypothesis. For the other countries, the lag length amounting up to 5 shows some serial correlation in the return series for up to one week.

**Table 3.** Order of AR(p) used in the paper

<b>DJ</b>	<b>CAC-40</b>	<b>DAX-100</b>	<b>BCI</b>	<b>TSX</b>	<b>FTSE-100</b>	<b>Nikkei-225</b>
AR(3)	AR(1)	AR(5)	AR(3)	AR(2)	AR(0)	AR(0)
<b>BUX</b>	<b>SAX-16</b>	<b>SBI</b>	<b>PX-50</b>	<b>RFS</b>		<b>WIGI</b>
AR(1)	AR(5)	AR(4)	AR(4)	AR(5)		AR(0)

### Sign and size bias tests

Several diagnostic tests are employed in order to have a general idea on whether a linear GARCH model should be appropriate or rather a non-linear model should be used instead for the stock returns under study.

For this purpose, a battery of diagnostic tests are employed, namely sign and size bias tests and tests for linear and non-linear ARCH/GARCH effects in the residuals. The *sign bias test*, put forward by Engle and Ng(1993), consists in testing for asymmetry by regressing the squared residuals on the dummy variable  $S_{t-1}^-$ :

$$(9) \quad \varepsilon_t^2 = \phi_0 + \phi_1 S_{t-1}^- + \xi_t \text{ where } \xi_t \text{ is a white noise}$$

The dummy  $S_{t-1}^-$  takes the value of 1 if  $\varepsilon_{t-1} < 0$  and is zero otherwise. The sign bias test aims to analyse whether the squared residual and consequently the conditional variance depend on the sign of the lagged residual. If the coefficient  $\phi_1$  is found statistically significant, the sign of the lagged residual does matter for the conditional variance. A modified version of equation (9) leads us to the *negative size bias* tests that can be written as follows:

$$(10) \quad \varepsilon_t^2 = \phi_0 + \phi_1 S_{t-1}^- \varepsilon_{t-1} + \xi_t$$

Based on the term, the *negative bias test* examines whether not only the sign but the size of the negative shock significantly impacts on the squared residual, and thus the conditional variance. Analogously, the *positive size bias test* is based on Eq. (10) but regresses the squared residual on  $S_{t-1}^+ \varepsilon_{t-1}$  instead of  $S_{t-1}^- \varepsilon_{t-1}$  where  $S_{t-1}^+ = 1 - S_{t-1}^-$ :

$$(11) \quad \varepsilon_t^2 = \phi_0 + \phi_1 S_{t-1}^+ \varepsilon_{t-1} + \xi_t$$

**Table 4.** Asymmetry tests (p\_values)

	SB test	NSB test	PSB test
<b>G-7</b>			
DJ	<b>0.004</b>	<b>0.001</b>	<b>0.013</b>
CAC-40	<b>0.002</b>	<b>0.003</b>	0.090
DAX-100	0.630	0.058	0.500
BCI	0.420	0.550	0.550
TSX	0.520	0.350	0.300
Nikkei-225	0.900	0.270	0.950
FTSE-100	0.613	0.270	0.720
<b>CEE countries</b>			
BUX	0.104	0.550	0.080
SAX16	0.790	0.160	0.270
SBI	0.930	<b>0.000</b>	<b>0.000</b>
PX-50	0.910	<b>0.000</b>	<b>0.000</b>
RFS	0.090	0.170	0.056
WIGI	0.360	0.680	0.920

*Note:* SB stands for sign bias, NSB and PSB represent negative and positive size bias, respectively. The tests are conducted on the residuals obtained from an AR(p). The lag length is determined using the Akaike information criterion. The LM statistics follows an asymptotic  $\chi^2(3)$ . Asymmetry is accepted at the 5% level if the p-value is lower than 0.05.

The sign and size bias tests are applied to the G-7 and the 6 economies from Central and Eastern Europe. The sign bias tests, reported in Table 5 indicate the presence of asymmetry for only part of the stock returns considered in this paper. The null of no asymmetry is rejected only for the Dow Jones and the CAC-40 at the 5% level. The result of the positive bias test is very similar. In addition to the DJ and CAC-40, the negative size bias test is able to reject the null of symmetry and thus to accept negative asymmetry at the 10% level for the DAX-100.

For the CEE stock returns, the negative and the positive bias tests accept the presence of asymmetry at the 5% level for the Ljubljana and the Prague stock exchanges, whereas asymmetry appears significant only at the 10% at the Budapest and Moscow stock exchanges. In these countries, the size effect appears more important than the sign effect.

## Linear and non-linear ARCH effects

A second type of test is the Lagrange Multiplier (LM) test of Engle (1982) that investigates the presence of ARCH effects. Using the generalised form of an ARCH(q) as in Eq (12) below, the rejection of the null hypothesis of  $\alpha_1 = \alpha_2 = \dots = \alpha_q = 0$  suggests the presence of heteroscedasticity in the return series, and this ARCH effect could be described by a linear ARCH model. It should be noted that in accordance with Lee (1991), the LM test with an alternative hypothesis of a GARCH(p,q) is tantamount to an LM test including an alternative of an ARCH(q).

$$(12) \quad \sigma_t^2 = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2$$

In order to introduce the possibilities of nonlinear ARCH effects, an extension of the linear ARCH test given by the LM test of Sentana (1995) that considers the null of homoscedasticity against the alternative of a QARCH(q) as shown hereafter:

$$(13) \quad \sigma_t^2 = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^q \lambda_i \varepsilon_{t-i}$$

The rejection of the null hypothesis  $H_0 : \alpha_0 = \dots = \alpha_q = \lambda_1 = \dots = \lambda_q = 0$  provides evidence in favour of the presence of QARCH effects in the residuals. In addition to this, Hagerud (1997) proposes a Smooth Transition ARCH (ST-ARCH) process as described in (14) so as to formulate two test statistics: the null of homoscedasticity is tested for against an alternative of an ST-ARCH process:

$$(14) \quad \sigma_t^2 = \omega + \sum_{i=1}^q \left( \alpha_i \varepsilon_{t-i}^2 (1 - F(\varepsilon_{t-i})) + \lambda_i \varepsilon_{t-i}^2 F(\varepsilon_{t-i}) \right)$$

The use of a logistic function  $F(\cdot)$  corresponds to an LST-ARCH model whereas employing an exponential function leads to an EST-ARCH specification. The test statistics follow  $\chi^2$  with  $2q$  degrees of freedom.

According to the results reported in Table 5., ARCH, QGARCH, LSTGARCH and ESTGARCH models are accepted against the null of homoscedasticity for the cases of the Dow Jones and the CAC-40 at the 1% level, irrespective to the choice of the lag length. The same result holds true for the TSX and the DAX-100, but to a lesser extent since the null cannot be rejected for small lags. By contrast, the null cannot be rejected for the cases of the Nikkei-225 and the FTSE-100. The Milan stock exchange constitutes an intermediate case since the alternative hypothesis is accepted only at lower significance levels and when using higher lag length. What we obtain for stock exchange returns in Central and Eastern Europe is more clear: for all series but one, namely the WIGI, the three asymmetric GARCH models are to be preferred to the homoscedasticity assumption. This result holds at the 1% level and is independent of the lag chosen.

To summarise the results of the preliminary tests, substantial nonlinear and asymmetric ARCH effects appear in the residuals obtained from the mean equation for only two of the G-7 stock returns whereas they are found very strong in all returns of the transition economies with the exception of Poland. In these cases, asymmetric models described in Section II may be preferred to the linear ones.

**Table 5.** Testing for linear and non-linear ARCH

Q	LM(A)				LM(Q)				LM(L)				LM(E)			
	1	2	5	10	1	2	5	10	1	2	5	10	1	2	5	10
<b>G-7</b>	<i>p-values</i>				<i>p-values</i>				<i>p-values</i>				<i>p-values</i>			
DJ	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>
CAC-40	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>
DAX-100	0.807	0.947	<b>0.002</b>	<b>0.035</b>	0.953	0.854	<b>0.000</b>	<b>0.001</b>	0.958	0.998	<b>0.000</b>	<b>0.000</b>	0.961	0.998	<b>0.000</b>	<b>0.000</b>
BCI	0.667	0.840	<b>0.024</b>	<i>0.080</i>	0.902	0.980	0.183	<b>0.001</b>	0.902	0.981	0.214	<b>0.023</b>	0.903	0.982	0.218	<i>0.063</i>
TSX	0.642	<b>0.000</b>	<b>0.001</b>	<b>0.009</b>	0.856	<b>0.000</b>	<b>0.000</b>	<b>0.004</b>	0.874	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	0.879	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>
Nikkei-225	0.781	0.925	0.995	1.000	0.952	0.994	1.000	1.000	0.951	0.995	1.000	1.000	0.953	0.996	1.000	1.000
FTSE-100	0.840	0.960	0.999	1.000	0.960	0.997	1.000	1.000	0.960	0.974	1.000	1.000	0.998	0.999	1.000	1.000
<b>CEECs</b>	<i>p-values</i>				<i>p-values</i>				<i>p-values</i>				<i>p-values</i>			
BUX	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>
SAX16	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>
SBI	<b>0.037</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<i>0.054</i>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.010</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.008</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>
PX-50	<b>0.025</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.027</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.002</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.002</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>
RFS	<b>0.020</b>	<i>0.051</i>	0.136	0.210	<b>0.012</b>	<b>0.038</b>	<b>0.002</b>	<b>0.007</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.002</b>
WIGI	0.969	0.998	1.000	1.000	0.979	0.998	1.000	1.000	0.999	1.000	1.000	1.000	0.999	1.000	1.000	1.000

Note: LM(A): standard ARCH LM test, LM(Q): LM test with the alternative of a QGARCH, LM(L): LM test with the alternative of an LSTGARCH, LM(E): LM test with the alternative of a ESTGARCH

## IV. 3. Estimating and diagnosing the volatility models

### The benchmark GARCH(1,1) model

Let us now turn to the estimation of the linear and non-linear GARCH models. First, we estimate a GARCH(1,1), which will serve as benchmark model in what follows. By comparing estimations of nonlinear models to this benchmark, we will see to what extent non-linear models perform better in terms of absorbing skewness and kurtosis. A first analysis of the results presented in Tables 6a and 6b reveals the following features: (a) First, as already mentioned, different mean equation specifications, i.e. AR models of different order appear, necessary for different countries. (b) Second, while parameters  $\omega$ ,  $\alpha$  and  $\beta$  turn out to be significant for the majority of stock indices, this finding does not hold for Tokyo, London and Warsaw. (c) Finally, the coefficients  $\alpha$  and  $\beta$  are correctly signed without exception. As expected, the sum of  $\alpha_1$  and  $\beta_1$  is smaller than unity in all cases except for Russia and Slovenia. Nonetheless, the size of the coefficients differ substantially among countries. The sum is low in Germany and in Italy. By contrast, it is particularly high in the Czech Republic, Hungary and Slovakia. This implies that in these countries, shocks to the conditional variance are highly persistent and especially in CEE countries.

**Table 6a. AR(p)-GARCH(1,1), G-7 countries**

	DJ	CAC-40	DAX-100	BCI	TSX	Nikkei-225	FTSE-100
	AR(3)	AR(1)	AR(5)	AR(2)	AR(2)	AR(0)	AR(0)
$p_0$	0.002 (3.860)	0.0002 (2.63)	-0.035 (-0.97)	-0.050 (-1.03)	-0.06 (-1.35)	-0.0654 (-1.15)	-0.065 (-1.453)
$p_1$	0.026 (1.44)	0.050 (2.83)	-0.004 (-0.005)	-0.008 (-0.007)	-0.006 (-0.02)		
$p_2$	-0.006 (-0.36)		-0.008 (-0.06)	0.040 (1.21)	0.035 (1.68)		
$p_3$	-0.03 (-1.973)		0.060 (2.31)				
$p_4$			-0.004 (-0.006)				
$p_5$			0.057 (2.63)				
$\omega$	0.000 (5.82)	0.000 (8.03)	0.085 (3.13)	0.064 (9.32)	0.024 (3.21)	0.000 (0.002)	0.002 (0.000)
$\alpha$	0.059 (10.20)	0.0725 (7.72)	0.004 (2.43)	0.008 (5.72)	0.000 (3.21)	0.000 (0.000)	0.000 (0.000)
$\beta$	0.923 (140.40)	0.896 (67.5)	0.680 (6.71)	0.811 (40.26)	0.938 (49.12)	0.911 (0.024)	0.912 (0.024)

*Note:*  $p_0 \dots p_p$  are the autoregressive coefficients in the mean equation, i.e. in the AR(p). t-statistics are reported in parentheses,

**Table 6b. AR(p)-GARCH(1,1), CEE economies**

	<b>BUX</b>	<b>SAX-16</b>	<b>SBI</b>	<b>PX-50</b>	<b>RFS</b>	<b>WIGI</b>
	AR(4)	AR(5)	AR(5)	AR(5)	AR(5)	AR(0)
$p_0$	0.0003 (1.779)	-0.006 (-3.089)	2203.9 (13.210)	1782.86 (14.34)	-0.002 (3.875)	-0.006 (-0.232)
$p_1$	0.183 (8.642)	-0.008 (-0.499)	-0.092 (-0.875)	-0.102 (-0.977)	-0.166 (5.875)	
$p_2$		-0.022 (-1.217)	0.395 (28.620)	0.358 (26.985)	0.069 (2.626)	
$p_3$		0.044 (2.212)	-0.090 (-0.883)	-0.101 (0.975)	-0.002 (-0.127)	
$p_4$		0.067 (3.608)	-0.238 (17.45)	0.259 (20.69)	0.112 (4.830)	
$p_5$		0.054 (3.693)			-0.024 (-0.950)	
$\omega$	0.000 (19.39)	0.000 (14.08)	8895.21 (16.14)	6763.53 (13.56)	0.000 (12.17)	0.006 (0.55)
$\alpha$	0.211 (34.87)	0.099 (24.41)	0.0579 (23.24)	0.069 (20.16)	0.309 (24.61)	0.000 (0.59)
$\beta$	0.757 (116.80)	0.878 (205.54)	0.929 (461.70)	0.914 (252.68)	0.735 (89.51)	0.889 (4.44)

*Note:* see Table 6a.

## Specification tests

It is indispensable to check whether the GARCH(1,1) specification performs reasonably well for the countries under study. Therefore, as announced at the beginning of the section, tests for remaining ARCH and for higher order ARCH and GARCH effects in the residuals of the model are carried out in accordance with Lundbergh and Teräsvirta (1998) and Bollerslev (1986). Rejecting the null hypothesis implies that the residuals are still heteroscedastic and that a higher order ARCH and GARCH model would be more appropriate.

As evidenced in Tables 7a and 7b, remaining ARCH is found among the G-7 countries only for Canada, and for the Czech Republic, Hungary, Slovakia and Slovenia. With regard to the higher order ARCH and GARCH tests, it appears that beside the Canadian TSX, a higher order ARCH/GARCH would be more appropriate for the DAX. This finding confirms the preliminary tests conducted on the residuals issued from the mean equation, because the tests reject the null only for higher lag length. When analysing the case of the CEECs, the results for the Czech, Hungarian, Russian, Slovak and Slovene stock indexes evidence problems both in term of remaining ARCH and higher order ARCH/GARCH effects.



**Table 7a.** Diagnostic tests for higher order ARCH and GARCH-G7 countries

Lag	DJ	CAC-40	DAX-100	BCI	TSX	Nikkei-225	FTSE-100
<b>Test for remaining ARCH (<i>p-values</i>)</b>							
1	0.680	0.530	0.540	0.580	0.320	0.990	0.780
2	0.860	0.800	0.190	0.590	<b>0.001</b>	0.990	0.900
3	0.840	0.620	0.260	0.480	<b>0.004</b>	1.000	0.940
4	0.930	0.780	0.200	0.630	<b>0.009</b>	1.000	0.950
5	0.960	0.840	0.053	0.710	<b>0.018</b>	1.000	0.950
<b>Test for higher order ARCH (<i>p-values</i>)</b>							
1	0.780	0.240	0.490	0.550	0.320	0.990	0.780
2	0.890	0.370	0.094	0.520	<b>0.002</b>	0.990	0.900
3	0.330	0.290	0.180	0.450	<b>0.008</b>	1.000	0.940
4	0.480	0.440	<b>0.026</b>	0.580	<b>0.016</b>	1.000	0.950
5	0.460	0.560	<b>0.010</b>	0.640	<b>0.031</b>	1.000	0.960
<b>Test for higher order GARCH (<i>p-values</i>)</b>							
1	0.230	0.240	0.490	0.550	0.320	0.620	0.970
2	0.350	0.370	0.090	0.520	<b>0.002</b>	0.870	0.990
3	0.290	0.290	0.180	0.450	<b>0.008</b>	0.960	0.990
4	0.420	0.440	<b>0.026</b>	0.580	<b>0.010</b>	0.990	0.990
5	0.550	0.560	<b>0.017</b>	0.640	<b>0.031</b>	0.990	1.000

Note: lag in the first column refers to the order of the ARCH model and stands for p and q in the case of the GARCH model

**Table 7b.** Diagnostic tests for higher order ARCH and GARCH- CEE economies

Lag	BUX	SAX-16	SBI	PX-50	RFS	WIGI
<b>Test for remaining ARCH (<i>p-values</i>)</b>						
1	<b>0.006</b>	<b>0.036</b>	<b>0.001</b>	<b>0.000</b>	0.790	0.950
2	<b>0.010</b>	0.072	<b>0.000</b>	<b>0.000</b>	0.880	0.990
3	<b>0.040</b>	0.100	<b>0.000</b>	<b>0.000</b>	0.950	0.990
4	0.050	0.150	<b>0.000</b>	<b>0.000</b>	0.960	1.000
5	<b>0.040</b>	0.240	<b>0.000</b>	<b>0.000</b>	0.140	1.000
<b>Test for higher order ARCH (<i>p-values</i>)</b>						
1	<b>0.000</b>	<b>0.018</b>	<b>0.000</b>	<b>0.000</b>	0.060	0.990
2	<b>0.000</b>	0.057	<b>0.000</b>	<b>0.000</b>	0.180	0.990
3	<b>0.000</b>	0.053	<b>0.000</b>	<b>0.000</b>	0.220	0.990
4	<b>0.000</b>	0.100	<b>0.000</b>	<b>0.000</b>	0.340	1.000
5	<b>0.000</b>	0.170	<b>0.000</b>	<b>0.000</b>	0.290	1.000
<b>Test for higher order GARCH (<i>p-values</i>)</b>						
1	<b>0.000</b>	<b>0.019</b>	<b>0.000</b>	<b>0.000</b>	<b>0.060</b>	0.950
2	<b>0.000</b>	<b>0.057</b>	<b>0.000</b>	<b>0.000</b>	0.180	0.990
3	<b>0.000</b>	<b>0.053</b>	<b>0.000</b>	<b>0.000</b>	0.220	0.990
4	<b>0.000</b>	0.100	<b>0.000</b>	<b>0.000</b>	0.340	1.000
5	<b>0.001</b>	0.172	<b>0.000</b>	<b>0.000</b>	0.290	1.000

Note: see Table 7a.

When it comes to analysing the battery of other specification tests, the misspecification of a GARCH(1,1) against an alternative of a QGARCH(1,1) and LSTGARCH(1,1) indicated that a QGARCH is preferred to a linear GARCH specification for the Dow Jones, the CAC-40 among the G-7, and for the Czech (PX-50) and the Slovene (SBI) stock markets. Furthermore, in the case of the CAC-40, an LSTGARCH seems superior to a GARCH. In addition to this, the sign and size bias tests also clearly indicate that the GARCH model is not able to take into account the asymmetries for these four indexes. This seems also be the case for the Hungarian BUX and ,to a lesser, extent for the Russian RFS. ( tables 8a and 8b).

**Table 8a.** Sign and size bias and parameter stability tests, G-7 countries

	DJ	CAC-40	DAX-100	BCI	TSX	Nikkei-225	FTSE-100
p-values							
QGARCH	<b>0.000</b>	<b>0.000</b>	0.339	0.744	0.890	0.393	0.893
LSTGARCH	0.336	<b>0.002</b>	0.213	0.690	0.520	0.478	0.986
SB	<b>0.004</b>	<b>0.002</b>	0.630	0.425	0.552	0.887	0.660
NSB	<b>0.013</b>	<i>0.084</i>	0.510	0.550	0.305	0.954	0.729
PSB	<b>0.001</b>	<b>0.004</b>	<i>0.058</i>	0.558	0.351	0.242	0.332
General	<b>0.007</b>	<b>0.014</b>	0.268	0.661	0.202	0.584	0.704

**Table 8b.** Sign and size bias and parameter stability tests, CEE 6 economics

	BUX	SAX-16	SBI	PX-50	RFS	WIGI
p-values						
QGARCH	0.266	0.186	<b>0.001</b>	<b>0.038</b>	0.352	0.597
LSTGARCH	0.217	0.135	0.845	0.257	0.840	0.953
SB	0.135	0.796	0.424	0.532	<i>0.091</i>	0.458
NSB	<b>0.018</b>	0.273	<b>0.000</b>	<b>0.000</b>	<i>0.059</i>	0.947
PSB	0.556	0.162	<b>0.000</b>	<b>0.000</b>	0.178	0.869
General	<i>0.092</i>	0.182	<b>0.000</b>	<b>0.000</b>	0.205	0.903

## Testing Parameter Stability

Finally, the parameter constancy test is meant to check whether or not the estimated parameters of the model vary over time. Lin and Teräsvirta (1994) and Lundbergh and Teräsvirta (1998) put forward that parameters may have regime changing dynamics with an exponential or logistic transition function where the transition parameter is the time  $t$ . Franses and van Dijk (2002) show

parameter stability tests where a constant parameter GARCH(1,1) is tested against the following alternative:

$$(15) \quad \sigma_t^2 = \alpha_1 + \alpha_2 \cdot \varepsilon_{t-1}^2 + \beta_1 \cdot \sigma_{t-1}^2 + (\alpha_3 + \alpha_4 \cdot \varepsilon_{t-1}^2 + \beta_2 \cdot \sigma_{t-1}^2)F(t)$$

Based on this, the stability of the intercept can be tested as follows  $H_0 : \alpha_1 = \alpha_3$  (Test1). Alternatively, one may want to test for the stability of the ARCH parameters,  $H_0 : \alpha_2 = \alpha_4$  and  $\beta_1 = \beta_2$  (Test2). Finally, the stability of the intercept and the other parameters can be also checked:  $H_0 : \alpha_1 = \alpha_3$  and  $\alpha_2 = \alpha_4$  and  $\beta_1 = \beta_2$ . (Test3). The test statistics follows a  $\chi^2$  distribution with  $(p+1)$  degree of freedom.

These three tests are applied to the estimated coefficients of the 13 stock returns. Table 9a reveals that major stability problem is found only for the CAC-40. The additional normality tests show that the GARCH model could not eliminate the problem of non-normality. Only the test statistic for the DAX-100 decreased considerably indicating that the GARCH model was able to capture some skewness and kurtosis in the series.

With regard to the transition economies of Central and Eastern Europe (table 9b), the parameter constancy tests provide evidence in favour of time varying coefficients for the BUX and the SBI at the 5% significance level and for the PX-50 and the RFS at the 10% level. Similarly to the findings for the G-7, the absence of normality remains a problem as documented in Table 9b. However, when skewness, kurtosis and the Jarque-Bera test statistics in Table 9a are compared to those in Table 1, it appears that the GARCH model reduces a large chunk of non-normality.

All in all, these findings strongly corroborate what is found using preliminary diagnostic checks applied to the residuals of the mean equation. The GARCH (1,1) model turns out to be inadequate to describe conditional variance for the Dow Jones, the CAC-40, BUX, PX-50, SBI and possibly the RFS.

**Table 9a.** Parameter stability and normality tests, G-7 countries

	DJ	CAC-40	DAX-100	BCI	TSX	Nikkei-225	FTSE-100
Parameter stability				<i>p-value</i>			
Test1	0.85	<b>0.027</b>	0.407	0.364	0.220	0.0846	0.605
Test2	0.263	<b>0.051</b>	0.572	0.262	0.188	0.626	0.274
Test3	0.164	<b>0.082</b>	0.634	0.421	0.066	0.209	0.545
				<i>test statistics</i>			
Skewness	-0.811	-0.450	-14.790	-11.45	-11.06	-14.65	-11.34
Kurtosis	8.78	5.75	236	137.8	130	228.6	137.8
Jarque-Bera normality	5065	1308	7763	2632	2604	8092	2919

**Table 9b.** Parameter stability and normality tests, CEE 6 countries

	BUX	SAX-16	SBI	PX-50	RFS	WIGI
Parameter stability				<i>p-value</i>		
Test1	<b>0.000</b>	0.648	<b>0.004</b>	0.058	0.912	0.211
Test2	<b>0.019</b>	0.917	<b>0.096</b>	0.251	0.053	0.965
Test3	<b>0.009</b>	0.751	<b>0.011</b>	0.164	0.052	0.446
				<i>test statistics</i>		
Skewness	-0.45	-0.24	3.018	2.652	-1.76	-26.73
Kurtosis	11.09	12.54	20.03	16.60	27.90	783.60
Jarque-Bera normality	8277	14830	5300	3339	5293	6.34*10 <sup>7</sup>

## Non-linear GARCH models

Bearing in mind the estimation results presented in the previous section, non-linear GARCH models have to be estimated for some of the stock market returns. However, for comparison purposes and in order to check the robustness of the results, the non-linear models are estimated for all stock returns under investigation. First, the GJR model and subsequently the QGARCH model are assessed. With regard to the G-7, results for the GJR model are reported in Table 10a: this model can be verified only for the Dow Jones and the CAC-40. For these two series, the constraints to respect positivity and covariance stationarity are fulfilled. As far as the remaining indexes go, the model appear to perform rather poorly. Not only that the  $\frac{\alpha + \lambda}{2} + \beta < 1$  constraint is not respected, but the estimates of parameter  $\lambda$  are not statistically significant at the standard 5% level.

Let us now turn to the estimation results of the QGARCH model (table 11a). The coefficients appear statistically significant and respect the positivity and stationarity constraints in all cases. However, the  $\lambda$  coefficient for the German, Italian and Canadian series is found to be not significant.

**Table 10a. AR(p)-GJR GARCH(1,1), G-7 countries**

	DJ	CAC-40	DAX-100	BCI	TSX	FTSE-100	Nikkei-225
	AR(3)	AR(1)	AR(5)	AR(3)	AR(2)	AR(0)	AR(0)
p <sub>0</sub>	0.00017 (2.520)	0.0012 (1.534)	-0.034 (-1.460)	-0.050 (-1.396)	-0.059 (-1.430)	-0.065 (-1.653)	-0.039 (-2.051)
p <sub>1</sub>	0.0326 (1.748)	0.054 (3.026)	-0.0047 (-0.009)	-0.007 (-0.0023)	-0.009 (-0.350)		
p <sub>2</sub>	-0.0017 (-0.098)		-0.0087 (-0.084)	-0.008 (0.290)	0.043 (3.095)		
p <sub>3</sub>	-0.0289 (-1.165)		0.061 (2.862)	0.044 (2.977)			
p <sub>4</sub>			-0.0048 (-0.009)				
p <sub>5</sub>			0.056 (2.290)				
ω	0.000 (7.38)	0.000 (8.352)	0.032 (6.491)	0.039 (5.128)	0.0435 (3.310)	0.048 (4.535)	0.029 (2.278)
λ	0.0236 (4.911)	0.0183 (2.408)	1.133 (1.547)	3.076 (1.420)	3.362 (1.472)	2.247 (1.223)	9.580 (1.814)
A	0.099 (14.105)	0.0972 (12.656)	0.004 (6.707)	0.0076 (11.760)	0.008 (13.645)	0.007 (5.66)	0.004 (2.278)
B	0.916 (134.18)	0.909 (104.15)	0.858 (41.457)	0.852 (81.540)	0.852 (83.500)	0.870 (60.970)	0.083 (30.543)

**Table 11a. AR(p)-QGARCH(1,1), G-7 countries**

	DJ	CAC-40	DAX-100	BCI	TSX	FTSE-100	Nikkei-225
	AR(3)	AR(1)	AR(5)	AR(3)	AR(2)	AR(0)	AR(0)
$p_0$	0.000 (1.54)	0.001 (1.559)	-0.035 (1.033)	-0.05 (-0.997)	-0.059 (-1.284)	-0.037 (-2.048)	-0.018 (-1.649)
$p_1$	0.040 (2.231)	0.054 (3.051)	-0.004 (-0.004)	-0.007 (-0.010)	-0.009 (-0.025)		
$p_2$	0.006 (0.360)		-0.008 (-0.036)	-0.008 (-0.018)	0.043 (1.066)		
$p_3$	-0.020 (-1.192)		0.060 (1.359)	0.045 (1.041)			
$p_4$			-0.004 (-0.020)				
$p_5$			0.058 (1.498)				
$\omega$	0.000 (5.620)	0.000 (8.652)	0.039 (4.83)	0.012 (4.74)	0.07 (6.931)	0.129 (28.67)	0.0385 (32.35)
$\lambda$	-0.0004 (-15.13)	-0.000 (-7.998)	0.023 (0.381)	0.045 (1.082)	0.054 (1.791)	0.130 (51.79)	0.078 (58.62)
$\alpha$	0.049 (13.74)	0.055 (10.443)	0.011 (2.361)	0.012 (4.740)	0.010 (5.423)	0.009 (32.51)	0.005 (35.62)
$\beta$	0.936 (0.021)	0.91 (115.37)	0.084 (22.69)	0.847 (46.33)	0.808 (27.97)	0.701 (67.20)	0.852 (252.36)

Let us now examine the results for the transition economies ( table 10b et 11b). The picture that emerges from Tables 10b and 11b largely correspond to the preliminary and diagnostic tests conducted for the linear GARCH model. That is to say, the Polish WIGI stock returns is the only series for which none of the non-linear models, i.e. GJR and QGARCH are found to be at work. The coefficients are systematically insignificant and some of the pre-imposed constraints turn out to be violated. In contrast with this finding, both the GJR and the QGARCH models seem to correctly describe the BUX and the SAX-16 returns. Moreover, the Czech, Slovene and Russian returns can be characterised by means of the QGARCH model. This provides strong empirical evidence in favour of the fact that stock returns in transition economies, with the exception of Poland, exhibit strong non-linear and asymmetric behaviour, which occur in an abrupt ways rather than in a smooth manner.

**Table 10b. AR(p)-GJR GARCH(1,1), CEE countries**

	<b>BUX</b>	<b>SAX-16</b>	<b>SBI</b>	<b>PX-50</b>	<b>RFS</b>	<b>WIGI</b>
	AR(1)	AR(5)	AR(4)	AR(4)	AR(4)	AR(1)
P <sub>0</sub>	0.0002 (0.908)	-0.0003 (-1.261)	220.10 (10.955)	1800.07 (10.60)	0.0014 (2.468)	-0.0063 (-0.479)
P <sub>1</sub>	0.187 (8.792)	-0.002 (0.097)	-0.097 (-0.907)	-0.106 (-0.994)	0.164 (5.760)	0.0009 (0.010)
P <sub>2</sub>		-0.0183 (-0.832)	0.408 (25.780)	0.3637 (24.620)	0.087 (3.467)	
P <sub>3</sub>		0.0345 (1.536)	-0.092 (0.902)	-0.101 (-0.971)	-0.0009 (-0.041)	
P <sub>4</sub>		0.069 (3.962)	0.250 (18.330)	0.266 (19.720)	0.1145 (5.024)	
P <sub>5</sub>		0.0544 (3.156)				
Ω	0.00001 (18.841)	0.00001 (12.994)	10062.05 (20.551)	72842.6 (17.84)	0.00004 (13.12)	0.0069 (2.434)
Λ	0.1758 (15.966)	0.1257 (19.347)	0.0747 (18.676)	0.0797 (18.33)	0.2295 (10.296)	5.766 (1.940)
A	0.252 (29.32)	0.379 (26.38)	0.000 (0.000)	0.0008 (0.867)	0.429 (19.126)	0.0009 (0.450)
B	0.751 (102.42)	0.874 (181.99)	0.930 (489.9)	0.921 (292.034)	0.701 (74.176)	0.850 (17.002)

**Table 11b. AR(p)-QGARCH(1,1), CEE countries**

	<b>BUX</b>	<b>SAX-16</b>	<b>SBI</b>	<b>PX-50</b>	<b>RFS</b>	<b>WIGI</b>
	AR(1)	AR(5)	AR(4)	AR(4)	AR(5)	AR(0)
$p_0$	0.00302 (1.32)	-0.0009 (-3.41)	224.46 (10.22)	1795.22 (10.63)	-0.0007 (0.056)	-0.006 (-0.133)
$p_1$	0.185 (8.70)	-0.067 (2.797)	-0.101 (-1.125)	-0.108 (-1.177)	0.062 (0.284)	
$p_2$		-0.023 (-0.925)	0.427 (28.48)	0.36 (24.47)	0.017 (0.074)	
$p_3$		-0.024 (-11.554)	-0.086 (-1.193)	-0.097 (-1.146)	-0.001 (-0.024)	
$p_4$		0.53 (24.09)	-0.26 (18.53)	0.278 (11.54)	-0.006 (-0.024)	
$p_5$		-0.37 (-17.35)			-0.021 (-0.84)	
$\omega$	0.000 (5.87)	0.00 (0.00)	16647.5 (9.911)	8294.85 (11.47)	0.016 (0.488)	0.035 (0.294)
$\lambda$	-0.0005 (-2.115)	0.003 (11.38)	659.65 (10.87)	301.56 (9.66)	-0.007 (-0.331)	0.021 (0.393)
$\alpha$	0.210 (10.135)	0.379 (26.38)	0.036 (16.205)	0.039 (15.242)	0.005 (0.054)	0.001 (0.788)
$\beta$	0.757 (31.9)	0.78 (169.24)	0.93 (524.15)	0.93 (305.22)	0.099 (0.054)	0.393 (0.190)

## V. Concluding Remarks

The aim of this paper was to analyse features of conditional variance in daily return series of stock market indices in the G-7 and 6 selected economies of Central and Eastern Europe, namely the Czech Republic, Hungary, Poland, Russia, Slovakia and Slovenia. For this purpose, various linear and asymmetric GARCH models have been applied to the TSX, CAC-40, DAX-100, BCI, Nikkei-225, FTSE-100 and DJ-30 returns in the G-7 over the period 1987 to 2002 and the PX-50, BUX, WIGI, RFS, SAX-16 and SBI returns in the CEECs from 1991/1995 to 2002. The estimation results reveal



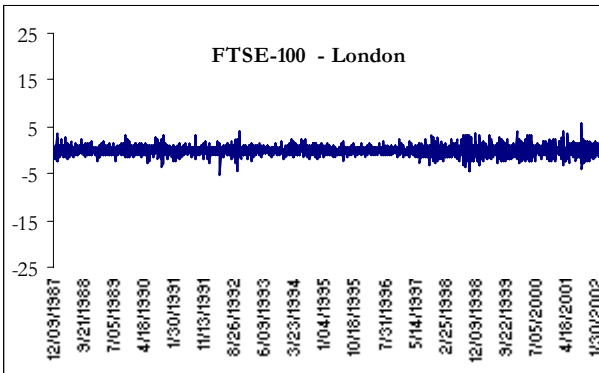
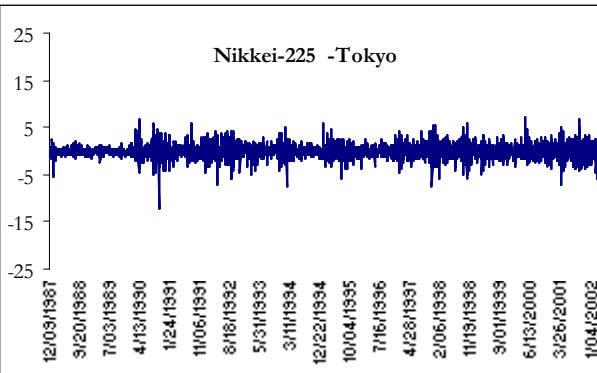
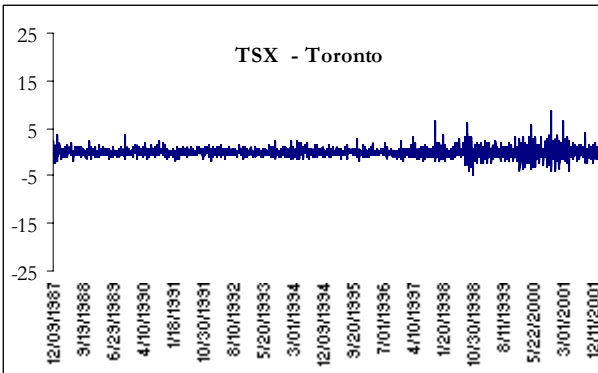
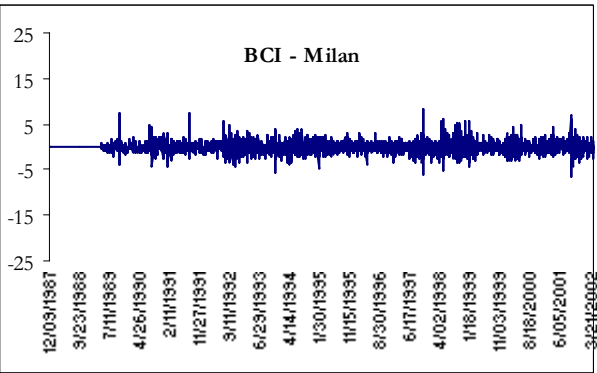
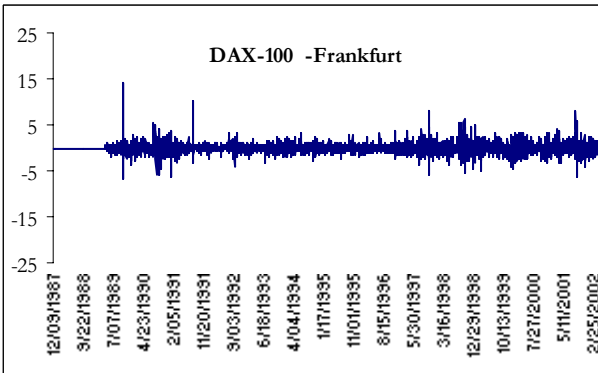
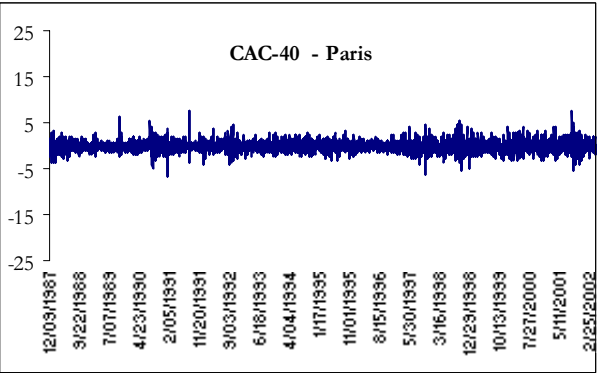
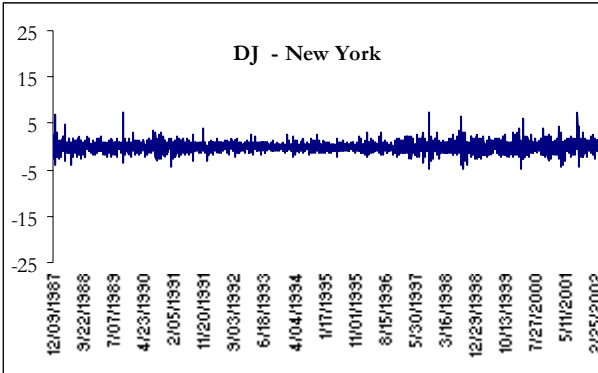
that the selected stock returns for the G-7 can be reasonably well modelled using linear specifications except the Dow Jones and the CAC-40 whereas the stock indices from Central and Eastern Europe can be much better characterised using asymmetric models. An exception is the Polish series WIGI. In other words, stock markets of the transition economies exhibit much more asymmetry because negative shocks hit much harder these markets than positive news. It also turns out that these changes do not occur in a smooth manner but happen pretty brusquely. This corroborates the usual observation that emerging stock markets may collapse much more suddenly and recover more slowly than G-7 stock markets.

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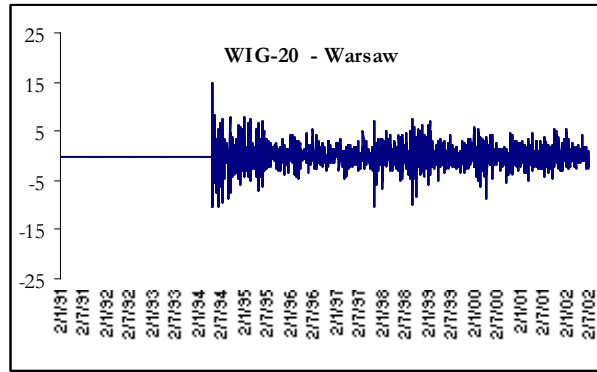
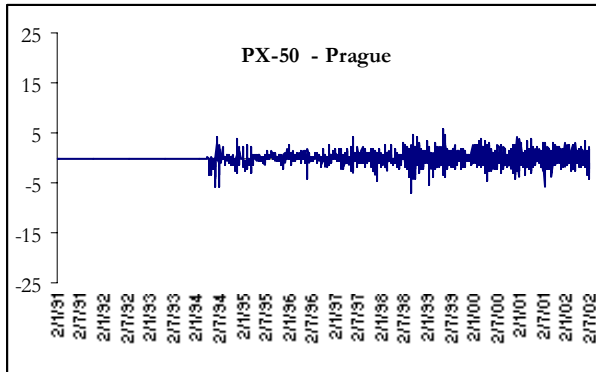
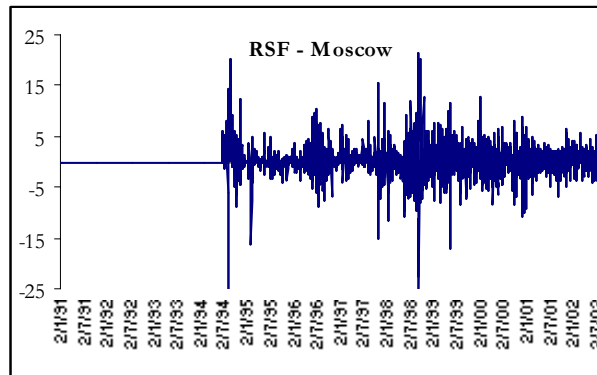
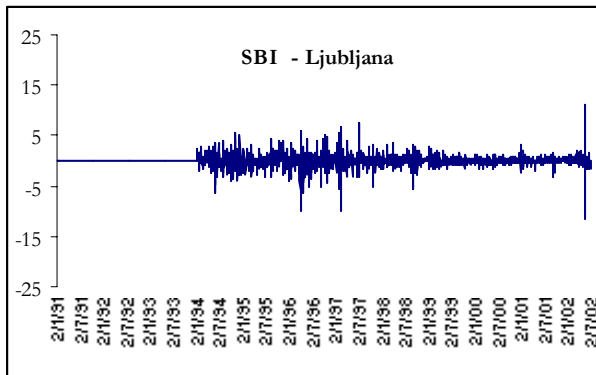
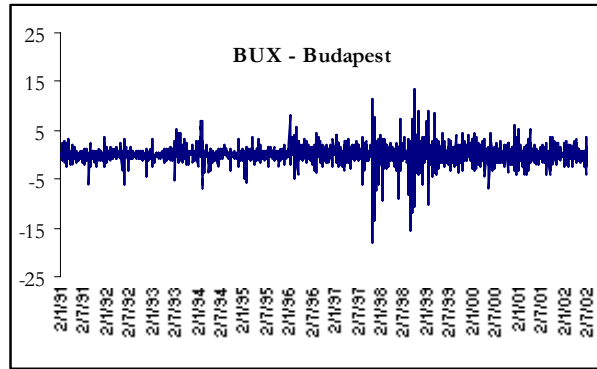
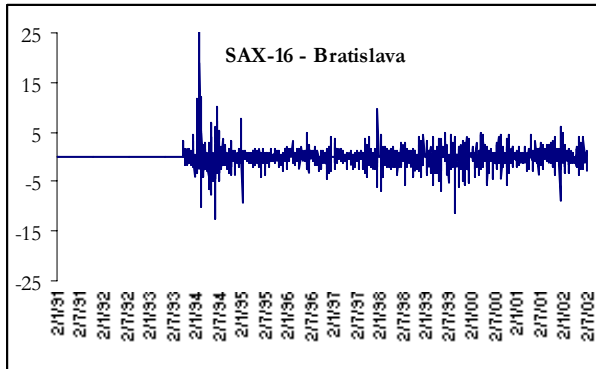
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# Data Appendix

## Return Series of Stock Market Indices, G-7



## Return Series of Stock Market Indices, CEE Economies



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