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THE PREDICTION OF DUST REMOVAL IN AN OIL-BATH AIR CLEANER

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THE PREDICTION OF DUST REMOVAL IN AN OIL-BATH AIR CLEANER

In order to evaluate the applicability of our present knowledge of particle dynamics to the problem of predicting the performance of an air cleaner, several calculations have been made. These calculations predict the efficiency of dust removal as a function of particle size in a Donaldson Tray-Type Cleaner operated at full capacity (model No. A 1411 at 600 cfm). While the computational methods are somewhat approximate they have been applied to various aerosol collection problems in the past with fair accuracy and are sufficiently reliable to serve as a guide for experimental design.

MECHANISM OF DUST REMOVAL

The mechanism of collection, which is the basis of the methods of prediction, is impaction due to the inertia of the particles. The physical situation consists of a moving dust-carrying air stream striking an obstacle and then changing direction to move around the obstacle. The dust particles tend to continue to move toward the obstacle because of their inertia, but the drag force exerted on them by the turning air stream tends to move them around the obstacle. If a particle has sufficient inertia and the drag force is insufficient to move it around the obstacle, it will strike the obstacle and presumably stick to it. Thus large particles are more easily removed from the air stream than small ones. It will be seen later that for any dust-removal system there is a narrow band of particle diameters above which all particles are removed and below which no particles are removed.

Since this study is primarily concerned with oil-bath air cleaners, the Donaldson cleaner was chosen for the subject of the predictions as it is representative of the class. The method of analysis used here is applicable to all cleaners of this type. It should be noted that the function of the oil is assumed to be merely that of cleaning the impingement surfaces and is assumed not to affect the dust-removal process. It has also been assumed that dust concentration does not affect the mechanism of removal. These points remain to be evaluated by experiment.

Will author rewrite when abstract needed

What's the point of it? computation or check on computation?

THE PREDICTION OF DUST REMOVAL
IN AN OIL-BATH AIR CLEANER

(IP Report No. B-5)

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i.e. Computational methods are developed to predict the efficiency of dust removal in the impingement zone of a Donaldson Tray-Type Cleaner operated at full capacity. The results of These computational methods ~~are compared~~ compare favorably with those of ~~Ranzxandxwongxx~~ W. E. Ranz and J. B. Wong, "Impaction of Dust and Smoke Particles on Surface and Body Collectors", Inddstrial and Engineering Chemistry, 44, 1371 (1952). Although both methods are somewhat approximated, it ~~itx~~ is concluded that each is suitable for the prediction of trends and ~~effe~~ the effect of changes in design.

The basis for the computational methods of prediction ~~presented~~ is derived from the mechanism of collection, from which it can be assummed: that if a particle in a moving dust-carrying air stream has sufficient inertia and the drag force ~~xxxxx~~ is insufficient to move it around an obstacle, the particle will strike the obstacle and presumably stick to it.

~~ThexDonaldsonxCleanerxwasxchosenx~~

As it is representative of the class, the Donaldson cleaner was chosen for the subject of predictions. Neither oil nor dust concentration were assummed to affect the mechanism of removal, ~~and~~ Since the oil-bath cleaner is arbitrarily divided into two zones, impingement and packed, the performance of each is predicted separately.

The impingement zone is that in which the entering air impinges upon and is turned by a baffle plate. Assuming the premise that the trajectory of a particle in a turning air stream may be approximated as an angular rather than a curved turn, calculations were made of the distance the particle would travel before being stopped by the air resistance. If this distance was at least equal to the particle's distance from the collector baffle, the particle

WAS assummed to have struck the baffle. Two arbitrarily chosen loci of points were ~~chosen~~ used to represent the places from which particles were thrown ina flow of streamlines and ~~trajectories with a vena~~ particle trajectories with a vena contracta as occurs in flow through an orifice. The equation of motion of a particle thrown into still air was obtained by equating the forces on a particle to its mass times acceleration. Since its effect was small, the force of gravity was neglected. ~~and~~ Evaluations of drag coefficient for turbulent flow, air density, particle density, and poise were ~~substituted~~ introduced into the equation. By rearrangment and integration, equations were ~~are~~ derived by which the values of velocity and penetration distance versus time could be computed. By using these and an evaluation of maximum penetration time at 0.005 seconds, total penetration and fraction of stream impinging were computed.

¶ A plot of percent removal of dust versus particle size in the Donaldson Cleaner yielded the conclusion that all particles larger than 15 microns are removed and no particles smaller than about two microns are removed. By applying this data to the removal of AC Test Dust, ~~it was then predicted that~~ the efficiency of removal was predicted at 48.6 percent for AC Fine Dust and 79.3 percent for AC Coarse Dust. These figures compared favorably with the prediction of impingement-zone efficiency by use of the Ranz and Wong method in which the impingement section is considered as a cylindrical jet followed by a rectangular jet.

The calculation of packed zone performance was carried out to indicate the difference between the packed zone and the impingement zone. By employing the data of Ranz and Wong on cylindrical collector efficiency, it was predicted that the packing would remove approximately 90 percent of AC Fine Dust and 96.5 percent of AC Coarse Dust.

The results showed that the impingement zone will take out the relatively large particles and no great increase in efficiency should be expected; and that the packed zone is capable of removing the small particles and can be modified to attain increased efficiency.

F - Instruments
up to 300 PSIG Designs

Designation	Location	Purchase Cost	Installation	Piping Tubing	Total Direct Co
TI	Direct	20	10		30
PI	Direct	20	10	5	35
TI	Board	20	10	20	50
PI	Board	20	10	20	50
FI	Board	80	40	20	140
LLI	Direct	225	100	50	375
TR	Board	175	90	35	400
PR	Board	250	125	60	435
TR	Board	150	75	75	300
LLIR	Board	275	150	80	505
TRC	Board	750	325	125	1200
PRC	Board	525	250	150	925
PRC	Board	650	325	150	1125
LLIRC	Board	725	300	150	1175

6. Piping:

   1. Carbon steel

$$\frac{\text{Cost of values}}{.64} = \text{Purchase cost of pipe valves \& fitting}$$

$$\frac{\text{Purchase cost}}{.40} = \text{Installed direct cost}$$

   2. SS & alloys .

$$\frac{\text{Cost of valves}}{73} = \text{Purchase cost of pipe valve \& fitting}$$

$$\frac{\text{Purchase cost}}{.45} = \text{Installed cost direct}$$

H. Structural Costs

	Net
1. Substructures	
Excavation - Machine	$ 1.50/cu yd
- Hand	6.00/cu yd
Backfill	1.80/cu yd
Borrowfill	1.00/cu yd
2. Foundation	
Column pedestals & footers	75.00/cu yd
Walls & footers	95.00/cu yd
3. Superstructure concrete	
Unfinished floor slabs 10" - 400 PS ft	.75/sq ft
Unfinished floor slap 6" - 200 PS ft	.45/sq ft
Misc concrete	100.00/cu yd

The oil-bath air cleaner is arbitrarily divided into two zones. One zone is that in which the entering air impinges upon and is turned by a baffle plate. The other zone is that in which the air passes through layers of small-size packing. The impingement zone will be considered first and its performance will be predicted by two different methods: one derived by us and the other by Ranz and Wong[1]. Next, the packed zone will be considered and its performance predicted from the data of Ranz and Wong.

IMPINGEMENT ZONE

The first method of prediction of dust-collection efficiency is derived from the premise that the trajectory of a particle in a turning air stream may be approximated as that which it would follow if the air made an angular rather than a curved turn. The particle is assumed to be thrown into still air at a given velocity and the distance it would then travel before being stopped by the air resistance is calculated. If this distance is at least equal to the particle's distance from the collector baffle, the particle will strike the baffle.

Figure 1 is a sketch of the impingement section and illustrates the streamlines and particle trajectories which are being considered. While the actual path of a particle might be like the curved dashed line from point 1 to point 5, we will assume that it is thrown abruptly from points 1 and 3 and that the distances 1-2 and 3-4 are equal to the particle's penetration into still air if thrown at the same velocity as at 1 or 3. It will be noted that the air streamlines are drawn (free hand) with a vena contracta as occurs in flow through an orifice. This has been observed in a cutaway model demonstrated for us by the Donaldson Company. Lines A-C and B-C are arbitrarily chosen as the loci of points from which the particles are thrown.

In outline, the following are the steps in this method of calculation (which will be described in detail below):

1. Calculate the distance of penetration for particles thrown into still air with initial velocities equal to that at the end of the vertical tube and that at the vertical plane passing through the vena contracta (a perpendicular passing through point C).

2. Compute the penetration distance for particles of various diameters going around the first bend (A-C).

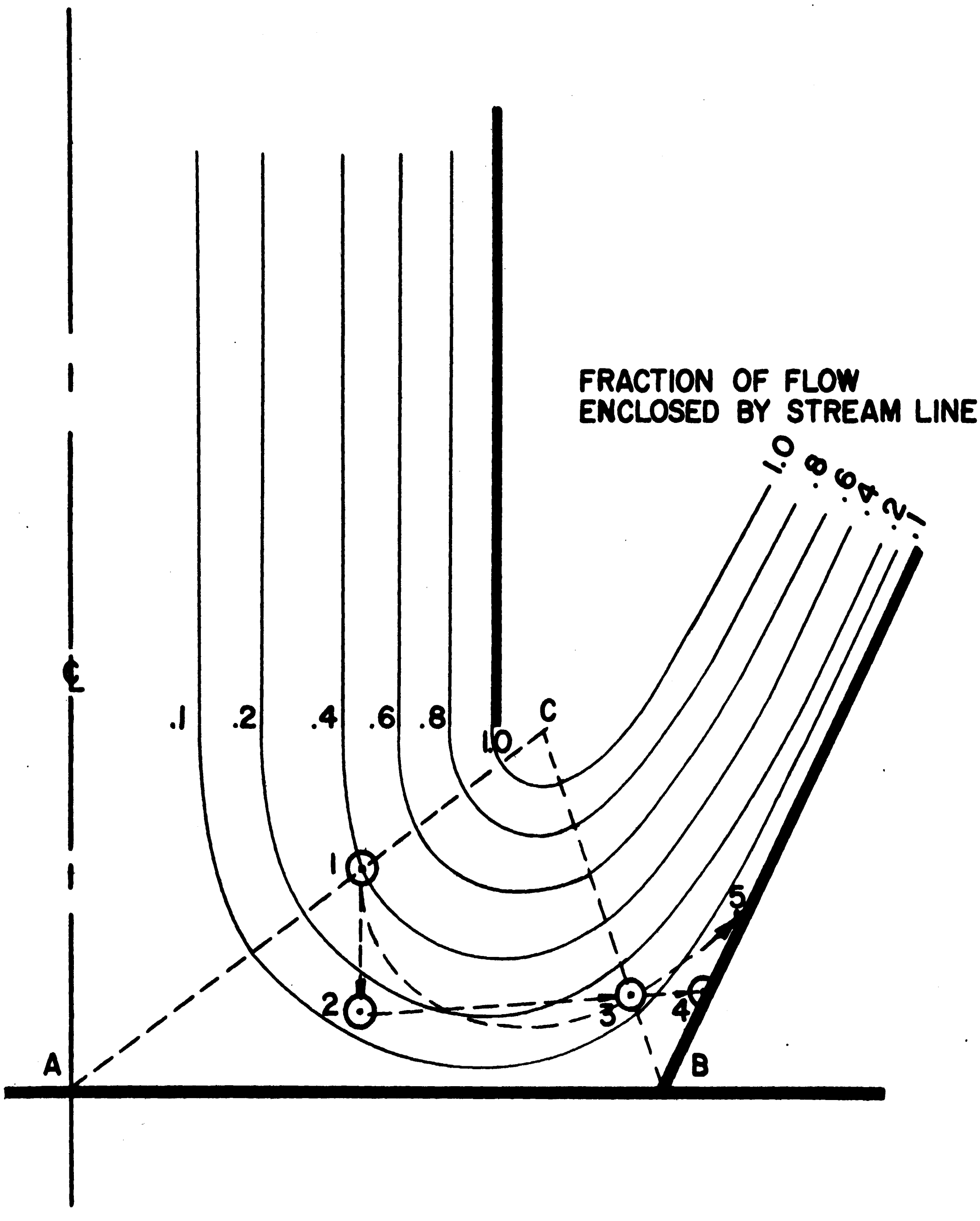


Fig. 1 Air and Dust Flow in the Impingement Section of an Air Cleaner

3. Compute the penetration distance for particles going around the second bend (B-C).

4. Add the penetration on the second bend to R times the penetration on the first bend. R is the ratio of the average distance between streamlines downstream of the second bend to the average distance between streamlines downstream of the first bend. In the example shown in Fig. 1, R is approximately 1/2.

5. Compute the fraction removed for each particle diameter. This is the ratio of total penetration distance to the width of the stream downstream of the second bend. In other words, if particles of a particular diameter will move half the width of the air stream half the original number will strike the baffle while the other half will not go far enough to do so.

Derivation of Equations for Penetration Distance

The equation of motion of a particle thrown into still air is obtained by equating the forces acting on a particle to its mass times acceleration.

$$F = ma = \frac{\pi D_p^2 C \rho u^2}{8} - F_g \qquad (1)$$

where: F = force
m = mass (g)
a = acceleration (cm/sec^2)
D_p = particle diameter (cm)
C = drag coefficient
ρ = air density (g/cm^3)
u = velocity (cm/sec)
F_g = force of gravity

The force of gravity can be neglected since its effect is small, and the drag coefficient for turbulent flow evaluated as:[2]

$$C = \frac{18.5}{Re^{0.6}} \quad (\text{for } 2 < Re < 1000) , \qquad (2)$$

where: Re = Reynolds number = $\frac{D_p u \rho}{u}$,

μ = air viscosity (poise) .

Then:

$$F = \frac{\pi D_p^2 \; 18.5 \; \rho(\mu)^{0.6} \; u^2}{8 \; (D_p u \rho)^{0.6}} = -m \frac{du}{dt} \tag{3}$$

$$m = \text{particle mass} = \frac{\rho_p \pi D_p^3}{6} , \tag{4}$$

where: ρ_p = particle density.

Substitutions into Eq. (3) and evaluation of $\rho = .069$ gm/cc, $\mu = 0.00018$ poise, and $\rho_p = 2.6$ gm/cc yields

$$-\frac{du}{dt} = 5.07 \times 10^3 \frac{(u)^{1.4}}{(D_p)^{1.6}} , \tag{5}$$

where: D_p = particle diameter in microns.

A rearrangment of Eq. (5) gives:

$$\frac{du}{u^{1.4}} = \left[\frac{5.07 \times 10^3}{(D_p)^{1.6}}\right] dt \tag{6}$$

and integration of Eq. (6) gives:

$$u^{-0.4}\Big]_1^2 = \left[\frac{2.03 \times 10^3}{(D_p)^{1.6}}\right] t\Big]_1^2 \tag{7}$$

To find the displacement (penetration), Eq. (7) must be integrated and is first put in the form of an indefinite integral.

$$u^{-0.4} = \left[\frac{2.03 \times 10^3}{(D_p)^{1.6}}\right] t + C_1 \tag{8}$$

at $t = 0$, $u = u_o$, and $C_1 = u_o - 0.4$

$$\left(\frac{dx}{dt}\right)^{-0.4} = At + C_1 , \tag{9}$$

where:

x = distance (cm) ,

$A = \dfrac{2.03 \times 10^3}{(D_p)^{1.6}}$,

u = original velocity (cm/sec).

Then:

$$\frac{dx}{dt} = (At + C_1)^{-2.5} \tag{10}$$

and

$$Adx = (At + C_1)^{-2.5}\ Adt. \tag{11}$$

Integration of Eq. (11) gives:

$$\left[\frac{2.03 \times 10^3}{(D_p)^{1.6}}\right] x = \frac{\left[\left(\frac{2.03 \times 10^3}{(D_p)^{1.6}}\right) t + u_o^{-0.4}\right]^{-1.5}}{-1.5} + \frac{(u_o^{-0.4})^{-1.5}}{1.5} , \tag{12}$$

which is identical with

$$\left[\frac{2.03 \times 10^3}{(D_p)^{1.6}}\right] x = \frac{u_o^{0.6} - u^{0.6}}{1.5} . \tag{13}$$

The equation of motion for laminar flow is developed similarly with the difference being that

$$F = 3\pi\mu u D_p = - ma = \frac{\rho_p \pi D_p^3}{6} \frac{du}{dt} . \tag{14}$$

The end results are:

$$\ln \frac{u}{u_o} = - \left[\frac{1.246 \times 10^5}{D_p^2}\right] t. \tag{15}$$

and

$$\frac{\left[\frac{1.246 \times 10^5}{(D_p)^2}\right]}{u_o} x = 1 - e^{-\left[\frac{1.246 \times 10^5}{D_p^2}\right] t} . \tag{16}$$

By using Eqs. (8) and (12) (or (13)) for Reynolds numbers greater than 2 and Eqs. (15) and (16) for Reynolds numbers smaller than 2, one can compute values of u and x versus t. Figure 2 is a plot of the results of such computations for an initial velocity of 100 feet per second. Figure 3 is a plot of u and x versus time for u_o = 75 feet per second and Fig. 4 is a plot of x versus time for u_o = 50 feet per second.

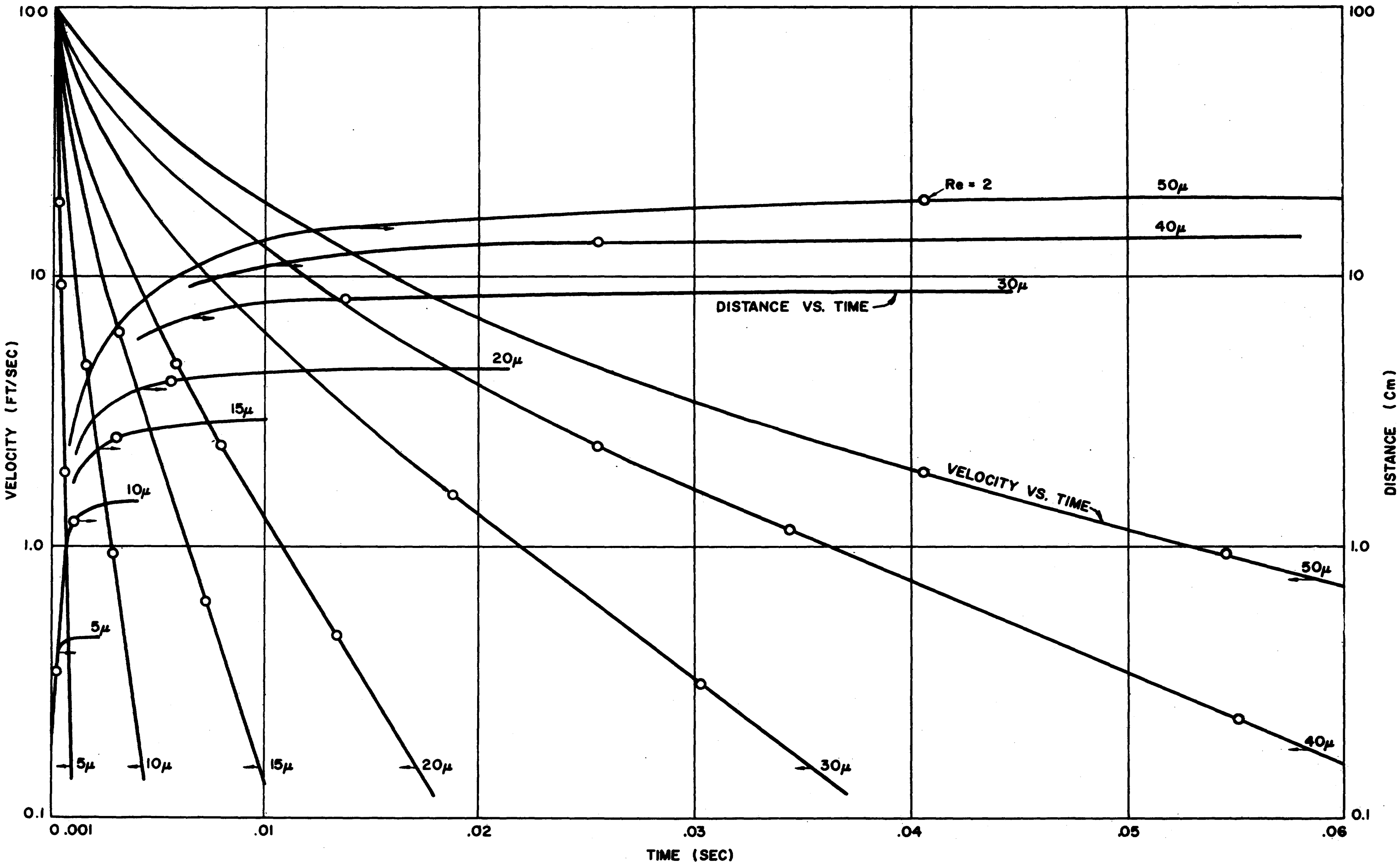


Fig. 2 Velocity and Penetration Distance Versus Time for Particles With an Initial Velocity = 100'/sec

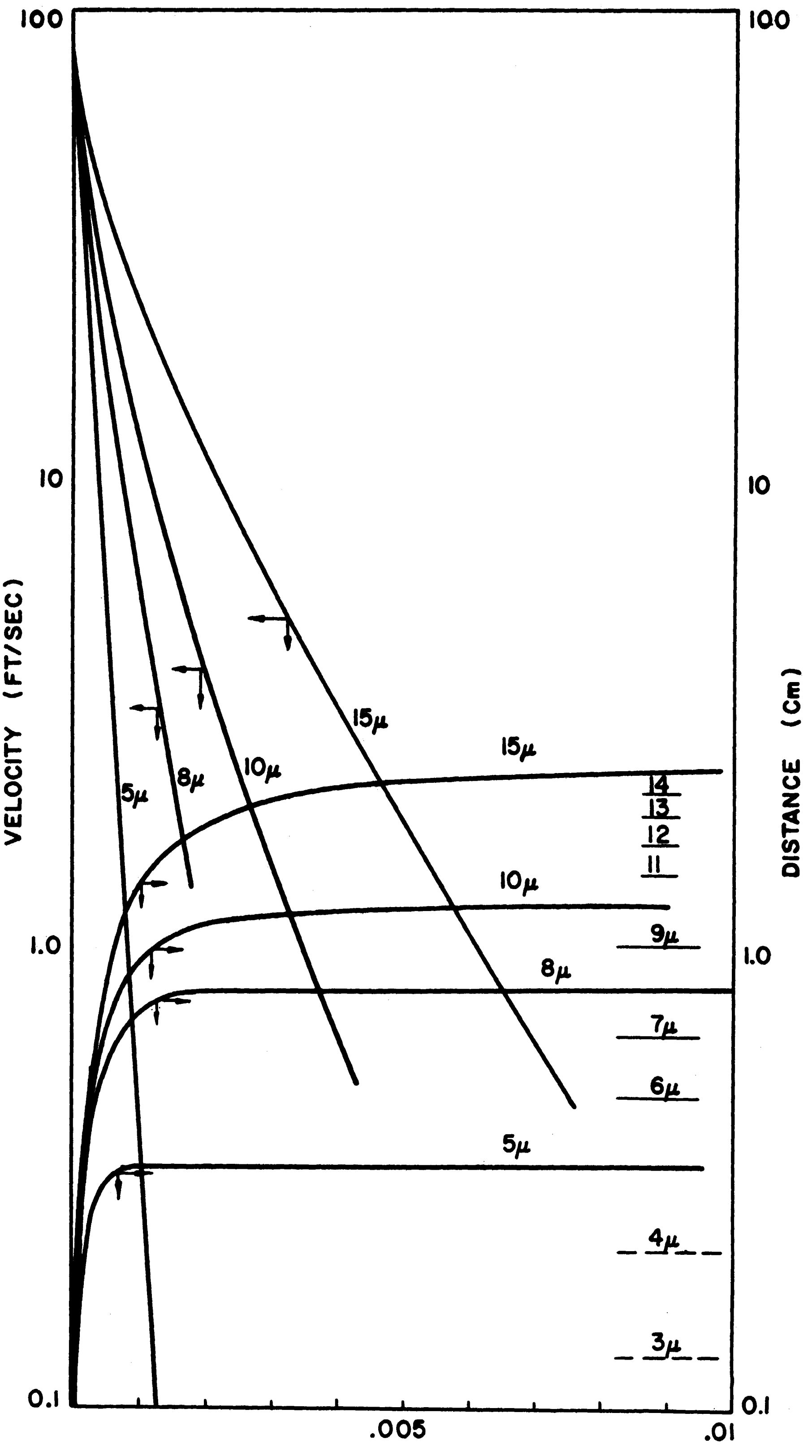
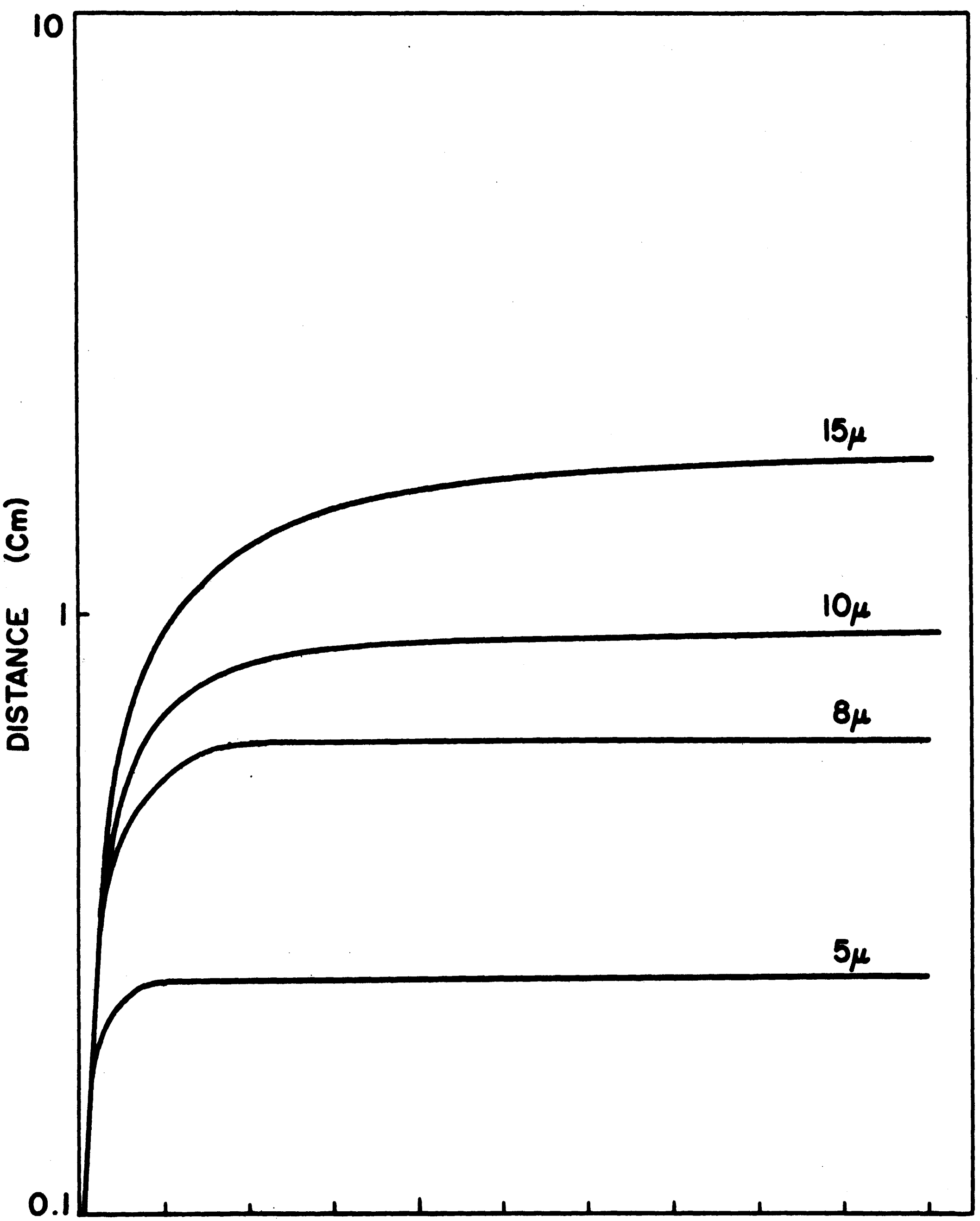


Fig. 3 Velocity and Penetration Distance Versus Time for Particles with an Initial Velocity = 75'/sec



.Fig. 4 Penetration Distance Versus Time for Particles With an Initial Velocity = 50'/sec

Computation of Penetration Distance

The data of Figs. 3 and 4 were used for predicting the performance of an air cleaner with the following dimensions:

(1) Diameter of central tube = 5 in.

(2) Distance from bottom of tube to horizontal portion of baffle = 2 in.

(3) Height of baffle cup = 3 in.

(4) Diameter of baffle cup at bottom = 7 in.

(5) Diameter of baffle cup at 2 in. elevation = 9 in.

Figure 1 is drawn to scale for these dimensions, which are approximately those of a Donaldson Model No. A-1411 Tray Type Cleaner. At the maximum rated capacity of 600 cfm, the velocity through the central tube is about 75 ft/sec and the velocity through the vena contracta at the second bend is about 50 ft/sec. The width of the air stream after the second bend is estimated at 2.5 cm.

A calculation of the time it takes a particle to move along any streamline shows that there is sufficient time for all particles to reach their maximum penetration. Thus the data presented in Table I may be obtained from Figs. 3 and 4 by evaluating maximum penetration at 0.005 seconds, which is about the minimum time for a particle going around the bend.

TABLE I

D_p Particle Diameter (microns)	$x_{max\ 1}$ (cm) Penetration After First Bend	$x_{max\ 2}$ (cm) Penetration After Second Bend *	X_T (cm) Total Penetration $(X_{m2} + \frac{X_{m1}}{2}) = X_T$	Fraction Of Stream Impinging $\frac{X_T}{2.5} = \eta$
15	2.25	(.9) (1.7)	2.66	1.0
14	2.1	(.9) (1.5)	2.4	0.96
10	1.2	(.9) (.9)	1.4	0.56
8	0.8	(.9) (.62)	0.95	0.38
5	0.33	(.9) (.25)	0.39	0.155
3	0.13	(.9) (.11)	0.165	0.066

*Note: Maximum penetration is multiplied by 0.9 to compensate for the slope baffle cup.

The data of Table I were used in drawing Fig. 5, a plot of percent removal of dust versus particle size. It can be seen that all particles larger than 15 microns are removed and no particles smaller than about 2 microns are removed. These data may now be used to predict the efficiency of the impingement section for removing AC Test Dust. The size analysis for AC dust was plotted and then broken down into cumulative weight percent for small fractions. These data were used in Table II which presents the prediction of removal efficiency. The total removal efficiency for AC Fine dust is 48.6 percent, and for AC Coarse dust it is 79.3 percent.

TABLE II

D_p (microns)	Fraction Removed	Weight Percent in AC Fine	Weight Percent of AC Fine Removed	Weight Percent in AC Coarse	Weight Percent of AC Coarse Removed
0-2	0.0	20.0	0.	5.0	0
2-3	0.03	8.0	0.24	2.0	0.06
3-5	0.11	11.0	1.21	5.0	0.55
5-8	0.27	12.0	3.24	7.0	1.9
8-10	0.47	6.0	2.82	5.0	2.35
10-12	0.65	4.0	2.6	3.0	1.95
12-15	0.9	5.0	4.5	5.0	4.5
15	1.0	34.0	34.	68.0	68.0
Total			48.6 %		79.31 %

Prediction of Impingement-Zone Efficiency with the Data of Ranz and Wong

Ranz and Wong have presented experimentally determined data on efficiency of impaction for cylindrical and rectangular aerosol jets impinging on plates, and for cylindrical and spherical collectors in an aerosol stream. Their data are reproduced here in Fig. 6, a plot of efficiency versus the dimensionless inertial parameter ψ, for aerosol jets impinging on infinite flat plates, and Fig. 7, a plot of efficiency versus ψ for variously shaped collectors in an aerosol stream.

The predicted curves shown in these figures were calculated on the basis that potential flow exists. Ranz and Wong assumed simplified boundary conditions for their solution of the differential equations of particle

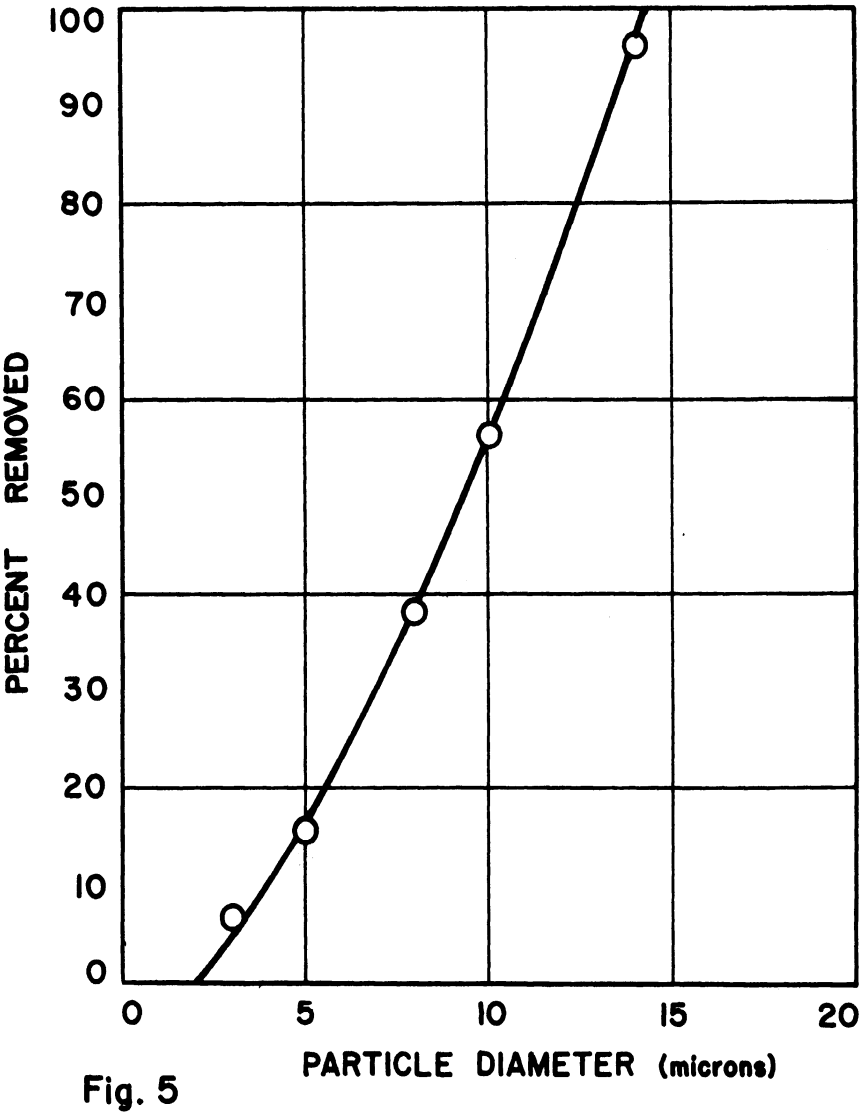


Fig. 5

Dust Removal Versus Particle Diameter Predicted for the Impingement Section motion while Langmuir and Blodgett performed the solutions of their equations on a differential analyser. The basic mechanism behind these calculations is the same as that described in the preceding section. The drag force acting upon a particle is related to its acceleration to define its motion. The methods differ only in the degree of accuracy with which the velocity pattern and its effect on drag are defined.

The efficiency data can be applied to the problem at hand by considering the impingement section to consist of a cylindrical jet followed by a rectangular jet. The dimensions of the rectangular jet are given by the width of the air stream at the vena contracta after the first bend and the circumference of the circle passing through the same position.

In accordance with the above described assumptions we have a system consisting of a cylindrical jet 5 inches in diameter followed by a "rectangular" jet 1.6 inches wide and 15.7 inches long. These dimensions are for the same air cleaner as for the previous calculation of efficiency. The inertial parameter is defined as:

$$\psi = \frac{C\rho_p v_o D_p^2}{18 \mu D_c} , \qquad (17)$$

where:

v_o = average jet velocity,

D_c = width or diameter of jet,

C = drag coefficient = 1.0 for particles larger than 1 micron.

If v_o is 75 ft/sec = 2,280 cm/sec, and D_p is in microns,

$$\psi = 1.44 \times 10^{-3} (D_p)^2 , \qquad (18)$$

$$\sqrt{\psi} = 0.038 (D_p) . \qquad (19)$$

For the rectangular jet, v_o = 50 ft/sec;

$$\sqrt{\psi} = 0.0585 (D_p) .$$

Values of efficiency of collection are taken from Fig. 6 for values of $\sqrt{\psi}$ corresponding to a range of particle diameters. It is found that both the rectangular and the cylindrical jet should take out all particles larger than 15 microns and none smaller than 5 microns. Thus the effect of the rectangular jet is to give greater efficiency only in the range of particle sizes between 5 and 15 microns. This can be seen in Table III which presents the predicted efficiency of removal for each stage and the total of both.

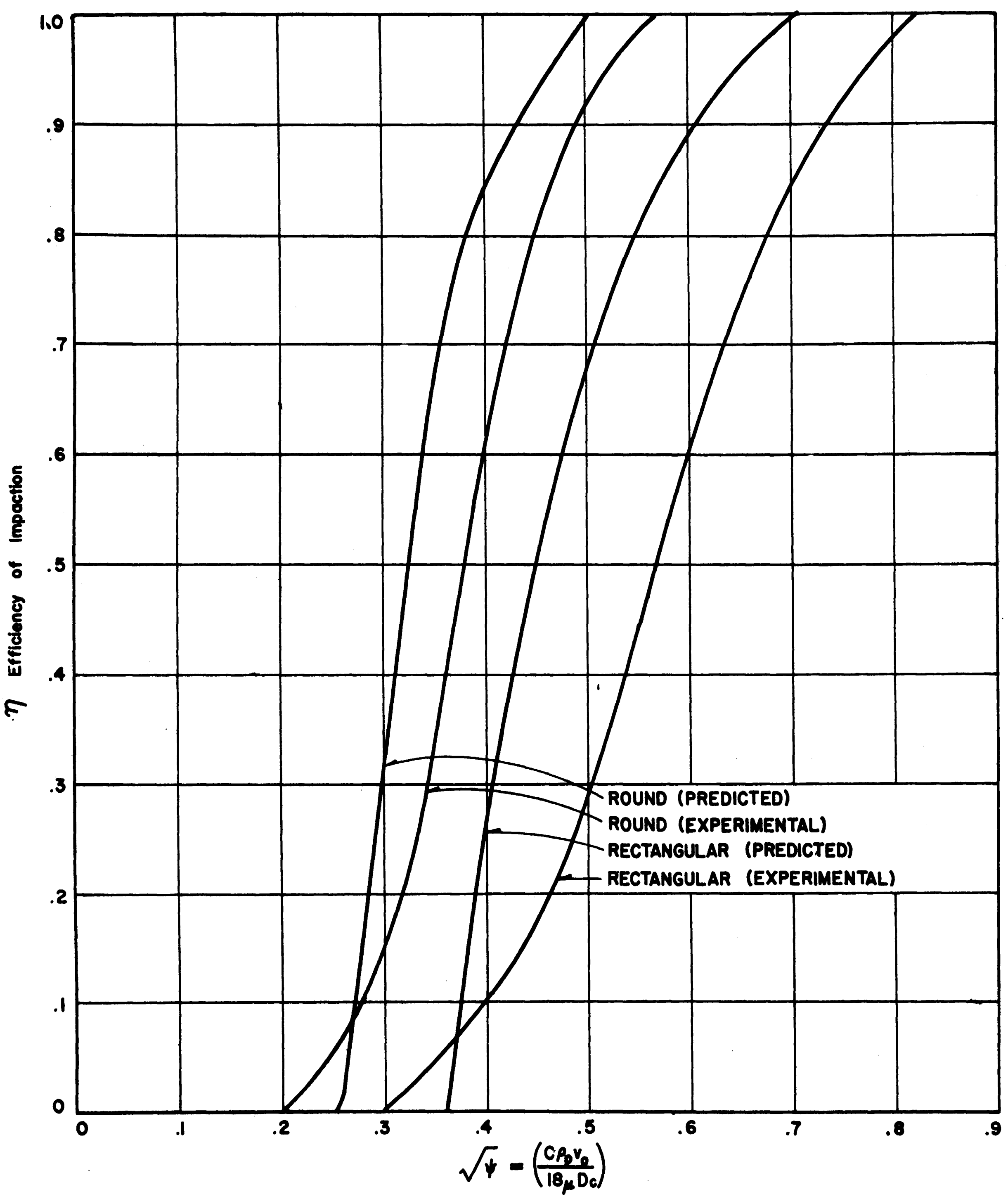


Fig. 6 Predicted and Experimental Impaction Efficiencies for Aerosol Jets

from: Ranz and Wong I&EC 44, 1371

TABLE III

D_p Particle Size (microns)	η_C Efficiency for Cylindrical Jets	η_R Efficiency for Rectangular Jets	η_T Total Efficiency
5	0	0	0
8	0.18	0.26	0.36
10	0.5	0.55	0.75
15	1.0	1.0	1.0

The efficiency of removal of AC dust was computed from the total efficiency and is given in Table IV.

TABLE IV

D_p Particle Size (microns)	η_T Total Efficiency (Average for Range of D_p)	Wt. % in Fraction AC Fine	Wt. % of AC Fine Collected	Wt. % in Fraction AC Coarse	Wt. % of AC Coarse Collected
0-5	0	39.0	0	12	0
5-8	.10	12.0	1.2	7	0.7
8-10	.6	6.0	3.6	5	3.0
10-12	.85	4.0	3.4	3	2.55
12-15	.95	5.0	4.75	5	4.75
15-80	1.0	34.0	34.0	68	68.0
Total			47.0%		79.0 %

The predicted total collection efficiency on AC Fine dust is 47 percent, which agrees with the prediction of 48.6 percent by the other method, and for AC Coarse dust both methods predict 79 percent removal.

PACKED ZONE

The calculation of packed zone performance was carried out more to indicate the difference between the packed zone and the impingement zone than to evaluate the actual packing arrangement in a production air cleaner. The efficiency is predicted for a 1/2-inch-deep bed of 0.025-cm-diameter wires which form 20 layers in the 1/2-inch depth. The air velocity is taken as that which would occur in the annular space of a Donaldson No. A-1411 at 600 cfm or 11 ft/sec. By employing the data of Ranz and Wong on cylindrical collector efficiency we obtain the data presented in Table V.

TABLE V

D_p Particle Diameter (microns)	η Efficiency of Collection	$P = (1-\frac{\eta}{3})$ Penetration per Layer	$P_T = (1-\frac{\eta}{3})^{20}$ Total Penetration for 20 Layers	Efficiency of Removal (percent)
0.76	0	1.0	1.0	0
1.	.05	.983	.71	29.
2.	.27	.91	.15	85.0
3.	.42	.86	.05	95.0
4.	.53	.823	.02	98.0
5.	.64	.786	.0082	99.2
6.	.74	.753	.0034	99.7
8.	.82	.726	.0017	99.8
10.	.87	.71	.0011	99.9

This packing would remove approximately 90 percent of AC Fine dust and 96.5 percent of AC Coarse dust according to the data of Table V.

DISCUSSION OF RESULTS

The most important thing shown by the calculations of efficiency is the relative functions of the two zones of the cleaner. The impingement zone will take out the relatively large particles and no great increase in

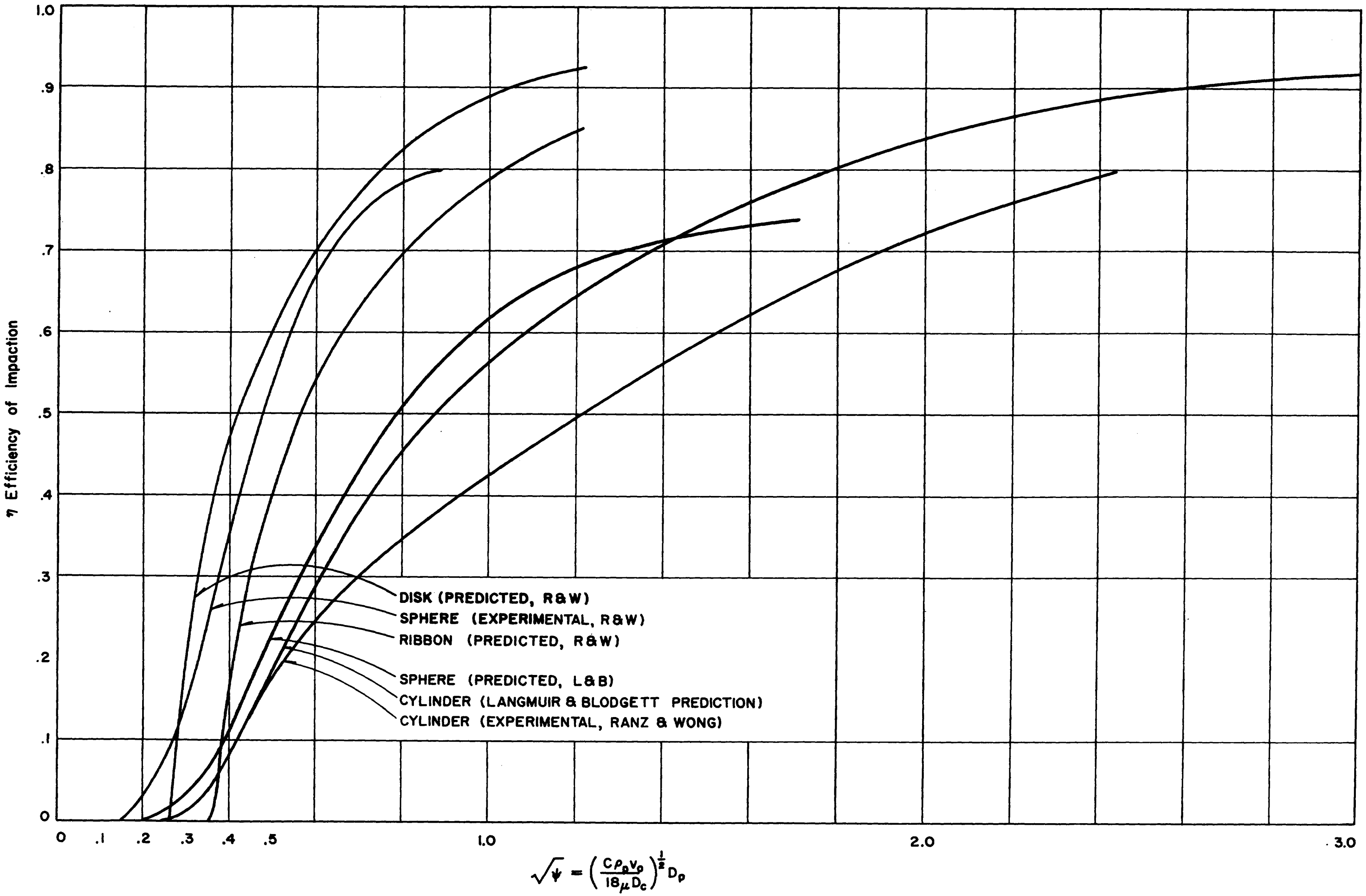


Fig. 7 Predicted and Experimental Impaction Efficiencies for Collectors of Various Shapes from: Ranz & Wong I&EC 44, 1371

its efficiency should be expected. The packed zone is capable of removing the small particles and can be modified in several ways to attain increased efficiency.

While there are several simplifying assumptions in both methods used for prediction of impingment-zone efficiency, these do not detract much from the accuracy since their major effect is to influence the efficiency within a narrow size band (5-1 microns in the cases shown) rather than to shift the limits of the band. The methods certainly are suitable for the prediction of trends and the effect of changes in design.

REFERENCES

1. Ranz, W. E. and Wong, J. B., "Impaction of Dust and Smoke Particles on Surface and Body Collectors," Industrial and Engineering Chemistry, 44, 1371 (1952).

2. Lapple, C. E., "Fluid and Particle Mechanics," University of Delaware (1951).