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NOTE ON THE PAIS-PICCIONI EXPERIMENT

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## ABSTRACT

Two experiments to check the Gellmann-Pais<sup>1</sup> "particle mixture" suggestion are analyzed, using a detailed, if crude, phenomenological description. A cloud-chamber experiment of the type proposed by Pais and Piccioni<sup>2</sup> is compared to an experiment using a liquid-xenon bubble chamber.<sup>3</sup> It is found that for likely values of the parameters involved, both experiments are considerably more difficult than envisaged by Pais and Piccioni. For intensity reasons, the bubble-chamber experiment seems preferable. However, unless quite accidentally the relevant cross sections have rather special values, it would seem that experiments of this type would have to wait until accelerators are available capable of producing considerably greater intensities than those in operation at present.

## I. INTRODUCTION

Recently Pais and Piccioni<sup>2</sup> have proposed a cloud-chamber experiment to verify the Gellmann-Pais<sup>1</sup> suggestion that the  $\theta^0$  should be considered as a "particle mixture." In view of the availability of a xenon-filled bubble chamber,<sup>3</sup> it is of some interest to compare the feasibility of a bubble-chamber experiment with the Pais-Piccioni experiment.

Below, a set of phenomenological equations describing a crude model of these experiments is given. Solutions for the two types of experiments are obtained. From these it follows that for likely values of the parameters involved:

- a. Both experiments are considerably more difficult than anticipated by Pais and Piccioni;
- b. Bubble-chamber experiments are somewhat more feasible;
- c. Unless the  $\bar{\theta}^0$  absorption cross section and the  $\theta_1^0, \theta_2^0$  mass difference are miraculously just right, experiments of this sort must wait until greater numbers of  $\theta^0$ 's are available.

## II. PHENOMENOLOGICAL DESCRIPTION

Our model is the following. The wave functions describing  $\theta^0$  and  $\bar{\theta}^0$  particles are to be linear combinations with prescribed phases of functions describing  $\theta_1^0$  and  $\theta_2^0$  particles. Specifically, we have

$$\Psi(\theta^0) = \frac{\Psi_1 + i\Psi_2}{\sqrt{2}} , \quad (1a)$$

$$\Psi(\bar{\theta}^0) = \frac{\Psi_1 - i\Psi_2}{\sqrt{2}} . \quad (1b)$$

The  $\theta_1^0$  is to undergo the familiar two  $\pi$  decay with lifetime  $\tau_1 \sim 1.5 \times 10^{-10}$  seconds. The  $\theta_2^0$  has a completely different set of decay modes with a lifetime  $\tau_2 \gg \tau_1$ . (To obtain detailed numerical results, we will idealize this and put  $\tau_2 = \infty$ .) In passing through matter, the  $\bar{\theta}^0$  can be absorbed while the  $\theta^0$  cannot. (For simplicity we follow Reference 2 and omit consideration of all other processes.) This absorption will be described by an effective lifetime  $\tau$ . Clearly,

$$1/\tau = N\rho v\sigma_a , \quad (2)$$

where  $v$  is the velocity of the  $\bar{\theta}^0$ ,  $\rho$  the density of absorber material,  $N$  the number of absorbing particles per gram, and  $\sigma_a$  the absorption cross section per particle.

Let the wave function describing the state at time  $t$  after a  $\theta^0$  is produced be

$$\Psi(t) = \alpha_1(t) \Psi_1 + \alpha_2(t) \Psi_2 . \quad (3)$$

We have

$$\frac{d\Psi(t)}{dt} = \left[ \frac{d\Psi(t)}{dt} \right]_{ph} + \left[ \frac{d\Psi(t)}{dt} \right]_d + \left[ \frac{d\Psi(t)}{dt} \right]_a . \quad (4)$$

Here

$$\left[ \frac{d\Psi(t)}{dt} \right]_{ph}$$

describes the phase change due to the kinetic energies. Thus, as in Reference 2, we have

$$\left( \frac{d\Psi}{dt} \right)_{ph} = i\omega_1 \alpha_1(t) \Psi_1 - i\omega_2 \alpha_2(t) \Psi_2 , \quad (5)$$

where

$$\omega_i = [c^2p^2 + m_i^2 c^4]^{1/2}/\hbar \quad (i=1,2) . \quad (6)$$

The expression

$$\left[ \frac{d\Psi(t)}{dt} \right]_d$$

is due to the  $\theta^0$  decay. Again, as in Reference 2, we can write it as

$$\left( \frac{d\Psi}{dt} \right)_d = -\frac{1}{2\tau_1} \alpha_1 \Psi_1 - \frac{1}{2\tau_2} \alpha_2 \Psi_2 . \quad (7)$$

The expression

$$\left[ \frac{d\Psi(t)}{dt} \right]_a$$

is due to the absorption of  $\bar{\theta}^0$ 's. Phenomenologically it is

$$\left[ \frac{d\bar{\Psi}(t)}{dt} \right]_a = -\frac{1}{2\tau} [\text{Component of } \bar{\theta}^0 \text{'s in } \bar{\Psi}(t)] \quad (8)$$

Using equations (1) and (3) we obtain

$$\bar{\Psi}(t) = \frac{1}{\sqrt{2}} [\alpha_1(t) - i \alpha_2(t)] \bar{\Psi}(\theta_0) + \frac{[\alpha_1(t) + i \alpha_2(t)]}{\sqrt{2}} \bar{\Psi}(\bar{\theta}^0) \quad (9)$$

Hence,

$$\left[ \frac{d\bar{\Psi}(t)}{dt} \right]_a = \frac{1}{2\tau} \frac{[\alpha_1(t) + i \alpha_2(t)]}{\sqrt{2}} \bar{\Psi}(\bar{\theta}^0) \quad (10a)$$

$$= -\frac{1}{4\tau} [\alpha_1(t) + i \alpha_2(t)] \bar{\Psi}_1 + \frac{i}{4\tau} [\alpha_1(t) + i \alpha_2(t)] \bar{\Psi}_2 \quad (10b)$$

Since

$$\frac{d\bar{\Psi}(t)}{dt} = \frac{d\alpha_1}{dt} \bar{\Psi}_1 + \frac{d\alpha_2}{dt} \bar{\Psi}_2 \quad (11)$$

We obtain, on inserting (5), (7), and (10) into (4) and equating coefficients of  $\bar{\Psi}_1$  and  $\bar{\Psi}_2$ ,

$$\frac{d\alpha_1}{dt} = -\left( \lambda_1^0 + \frac{1}{4\tau} \right) \alpha_1 - \frac{i}{4\tau} \alpha_2 \quad (12)$$

$$\frac{d\alpha_2}{dt} = \frac{i}{4\tau} \alpha_1 - \left( \lambda_2^0 + \frac{1}{4\tau} \right) \alpha_2 \quad (13)$$

where

$$\lambda_i^0 = i\omega_i + \frac{1}{2\tau_i} \quad (i=1,2) \quad .$$

### III. SOLUTIONS

For initial values  $\alpha_1(0)$  and  $\alpha_2(0)$ , the solution of (12) is

$$\alpha(t) = \begin{pmatrix} \alpha_1(t) \\ \alpha_2(t) \end{pmatrix} = \frac{\alpha_1(0) - R \alpha_2(0)}{1 - R^2} e^{-\lambda_1 t} \begin{pmatrix} 1 \\ R \end{pmatrix} + \frac{\alpha_2(0) - R \alpha_1(0)}{1 - R^2} e^{-\lambda_2 t} \begin{pmatrix} R \\ 1 \end{pmatrix}, \quad (14)$$

where

$$\left. \begin{array}{l} \lambda_1 \\ \lambda_2 \end{array} \right\} = \frac{\lambda_1^0 + \lambda_2^0}{2} + \frac{1}{4\tau} + \left\{ \begin{array}{l} + \\ - \end{array} \frac{1}{2} \sqrt{(\lambda_1^0 - \lambda_2^0)^2 + 1/4\tau^2} \right\} \quad (15)$$

and

$$R = -i/2\tau \left[ (\lambda_1^0 - \lambda_2^0) + \sqrt{(\lambda_1^0 - \lambda_2^0)^2 + 1/4\tau^2} \right]^{-1} . \quad (16)$$

The probability of the  $2\pi$  decay at time  $t$  is

$$P_{2\pi}(t) = \frac{|\alpha_1(t)|^2}{\tau_1} . \quad (17)$$

Hence, the ratio of the intensity of decays at time  $t$  after production to those at time zero is

$$\frac{I}{I_0} = \frac{\left| \frac{[\alpha_1(0) - R\alpha_2(0)]}{1 - R^2} e^{-\lambda_1 t} + \frac{[\alpha_2(0) - R\alpha_1(0)]}{1 - R^2} R e^{-\lambda_2 t} \right|^2}{|\alpha_1(0)|^2} . \quad (18)$$

#### IV. VALUES OF THE PARAMETERS

Measuring time in units of  $\tau_1$ , we see that the relative intensities given by (18) depend on the two parameters  $\beta = \tau_1/\tau$  and  $\gamma = \tau_1\Delta\omega$ . Here  $\Delta\omega = \omega_1 - \omega_2$  arises from any difference in the masses of  $\theta_1^0$  and  $\theta_2^0$ . Since such differences presumably would originate from the same interactions which occasion the difference in lifetimes, it seems reasonable that  $\gamma \sim 1$ . To see the effects of differing values of  $\gamma$ , we have computed the results to be expected for  $\gamma = 0, 1$ , and  $2\pi$ .

While the decay lifetime  $\tau_1$  is known to be  $\sim 1.5 \times 10^{-10}$  seconds, comparably accurate information for the  $\theta^0$  absorption lifetime  $\tau$  does not seem to be available at present.

To obtain some idea of  $\tau$ , let us assume each nucleon can absorb  $\theta^0$ 's with a cross section  $\sigma_a$ . Then

$$\tau = \frac{1}{N_0 \rho v \sigma_a} , \quad (19)$$

where  $N_0$  is Avogadro's number and  $\rho$  is the absorber density. Thus,

$$\beta = \frac{\tau_1}{\tau} \sim \left( \frac{\rho}{2.3} \right) \left( \frac{v}{c} \right) \left( \frac{\sigma_a}{10^{-27}} \right) \cdot 6.2 \times 10^{-3} . \quad (20)$$



Assuming  $v \sim c$  and the absorption cross section per nucleon as 1 millibarn, we have

$$\beta(x_e) \approx 6.2 \times 10^{-3} \quad (21a)$$

$$\beta(p_b) \approx 3.0 \times 10^{-2} \quad (21b)$$

With either substance we are certainly in a situation of little absorption.

To obtain an extreme upper limit for  $\beta$ , let us assume  $\sigma_a \sim 5 \times 10^{-26}$  cm<sup>2</sup> (i.e., roughly geometrical per nucleon). Then

$$\beta(x_e) \approx 0.3 \quad (22a)$$

$$\beta(p_b) \approx 1.5 \quad (22b)$$

From (21) and (22) one can conclude that with either xenon or lead we do not have a situation in which  $\beta$  is very large. Upper limits in the two cases are of the order of  $1/3$  and  $1$ , respectively, with somewhat lower values being likely.

#### V. CLOUD-CHAMBER EXPERIMENT

By this we mean an experiment of the type proposed by Pais and Piccioni. A beam of  $\theta^0$ 's produced in a lead plate are to decay in a cloud chamber until all  $\theta_1^0$ 's are removed. The remainder then passes through a lead plate which removes the  $\bar{\theta}^0$  component. Since, effectively,  $\theta_1^0$ 's are produced by removing  $\bar{\theta}^0$ 's, the familiar  $2\pi$  decay can be expected to reappear below the second plate. As indicated in Reference 2, the intensity of decays below the second plate may be expected to be of the order of  $1/4$  of that below the first. However, implicit in the derivation of this result is the assumption that  $\tau(p_b) \ll \tau_1$  (i.e.,  $\beta \gg 1$ ). In the other limit of  $\beta \ll 1$ , qualitatively different results will occur. If  $\tau(p_b) \gg \tau_1$ , most  $\theta_1^0$ 's arising from  $\bar{\theta}^0$  absorption will decay within the lead. The number of decays occurring below the second plate will be very small.

To analyze the situation in detail, we can use equations (12) and (18). On production in the first plate, we have  $\alpha_2(0) = i\alpha_1(0)$ . After passing through a region with  $\tau \sim \infty$  for a time  $T$  long compared to  $\tau_1$ , we will have

$$\alpha_1(T) = 0 \quad (23)$$

and

$$\begin{aligned} \alpha_2(T) &= \alpha_2(0) \\ &= i\alpha_1(0) \quad (24) \end{aligned}$$

Using these as the initial values when the beam enters a second plate, the ratio of  $2\pi$  decays occurring at a time  $t$  later to those occurring just below the first plate is

$$\begin{aligned} \frac{I(t)}{I(0)} &= \frac{|R|^2}{|1-R^2|^2} |e^{-\lambda_2 t} - e^{-\lambda_1 t}|^2 \\ &= \frac{2|R|^2}{|1-R^2|^2} e^{-(1+\beta)t/2\tau_1} [\cosh(\delta_1 t/\tau_1) - \cos(\delta_2 t/\tau_1)] , \end{aligned} \quad (25)$$

where

$$\delta_1 + i\delta_2 = \frac{1}{2} \sqrt{(1 + i2\gamma)^2 + \beta^2} , \quad (26)$$

and

$$R = - \frac{i\beta}{[(1 + i2\gamma) + \sqrt{(1 + i2\gamma)^2 + \beta^2}]} . \quad (27)$$

Let us first consider some limiting cases.

1)  $\beta \gg 1$

Here  $\delta_1 \sim \beta$ ,  $\delta_2 \sim 0$ ,  $R \sim -i$ , and hence,

$$\frac{I(t)}{I(0)} \approx \frac{1}{2} e^{-(1+\beta)t/2\tau_1} \left[ \cosh\left(\frac{\beta t}{2\tau_1}\right) - 1 \right] . \quad (28)$$

The maximum number of  $\theta^0$  decays occurring below the plate is clearly obtained by choosing the thickness of the plate so that the time of traversal  $t_t$  satisfies

$$\tau \ll t_t \ll \tau_1 . \quad (29)$$

In this case we have

$$\frac{I(t_t)}{I(0)} = 1/4 \quad (30)$$

(which is just the Pais-Piccioni prediction).

2)  $\beta \ll 1$

$$\delta_1 \sim \frac{1}{2} , \quad \delta_2 = \gamma , \quad R = - \frac{i\beta}{2(1+i2\gamma)} . \quad (31)$$

$$\frac{I(t)}{I(0)} \approx \frac{\beta^2}{2|1+i2\gamma|^2} e^{-t/2\tau_1} (\cosh t/2\tau_1 - \cos \gamma t/\tau_1) . \quad (32)$$

Now the maximum number of decays occurring below the plate is of the order of magnitude of the coefficient in (32), i.e.,

$$\left[ \frac{I(t)}{I(0)} \right]_{\max} \approx \frac{\beta^2}{2|1+i2\gamma|^2} . \quad (33)$$

Thus, for  $\beta = 0.1$  and choosing the most favorable case of  $\gamma = 0$ , we have

$$\left[ \frac{I(t)}{I(0)} \right]_{\max} \approx .005 , \quad (34)$$

as compared with the value .25 of (30).

3) In Fig. 1 the maximum ratio of the number of decays occurring below the second plate to that below the first is shown as a function of  $\beta$  for  $\gamma = 0, 1, 2\pi$ . (Maximum means having chosen the thickness of the second plate to make this ratio greatest.) It is striking how slowly the value 0.25 is reached as a function of  $\beta$ . For  $\gamma = 2\pi$  and  $\beta = 1.5$  [which we have seen is a probable upper limit for  $\beta(p_b)$ ], we have

$$\left( \frac{I}{I_0} \right)_{\max} = 0.007 .$$

It would appear doubtful whether a recurrence of  $2\pi$  decays with such a low intensity is observable.

## VI. BUBBLE-CHAMBER EXPERIMENT

It should be possible to produce  $\theta^0$ 's directly within a liquid-xenon bubble chamber. Since one would be able to observe  $\theta^0$ 's moving in all directions from the point of production, a considerable increase in the number of  $\theta^0$ 's observed as compared with producing them in a lead plate above a cloud chamber would result. The experiment we have in mind consists of measuring the distance of the  $\theta^0$  decay from the point of production. From a knowledge of the velocity of the  $\theta^0$  the time of decay is known. Equation (18) [with  $\alpha_2(0) = i\alpha_1(0)$ ] shows that the relative number of decays at time  $t$  after production is given by

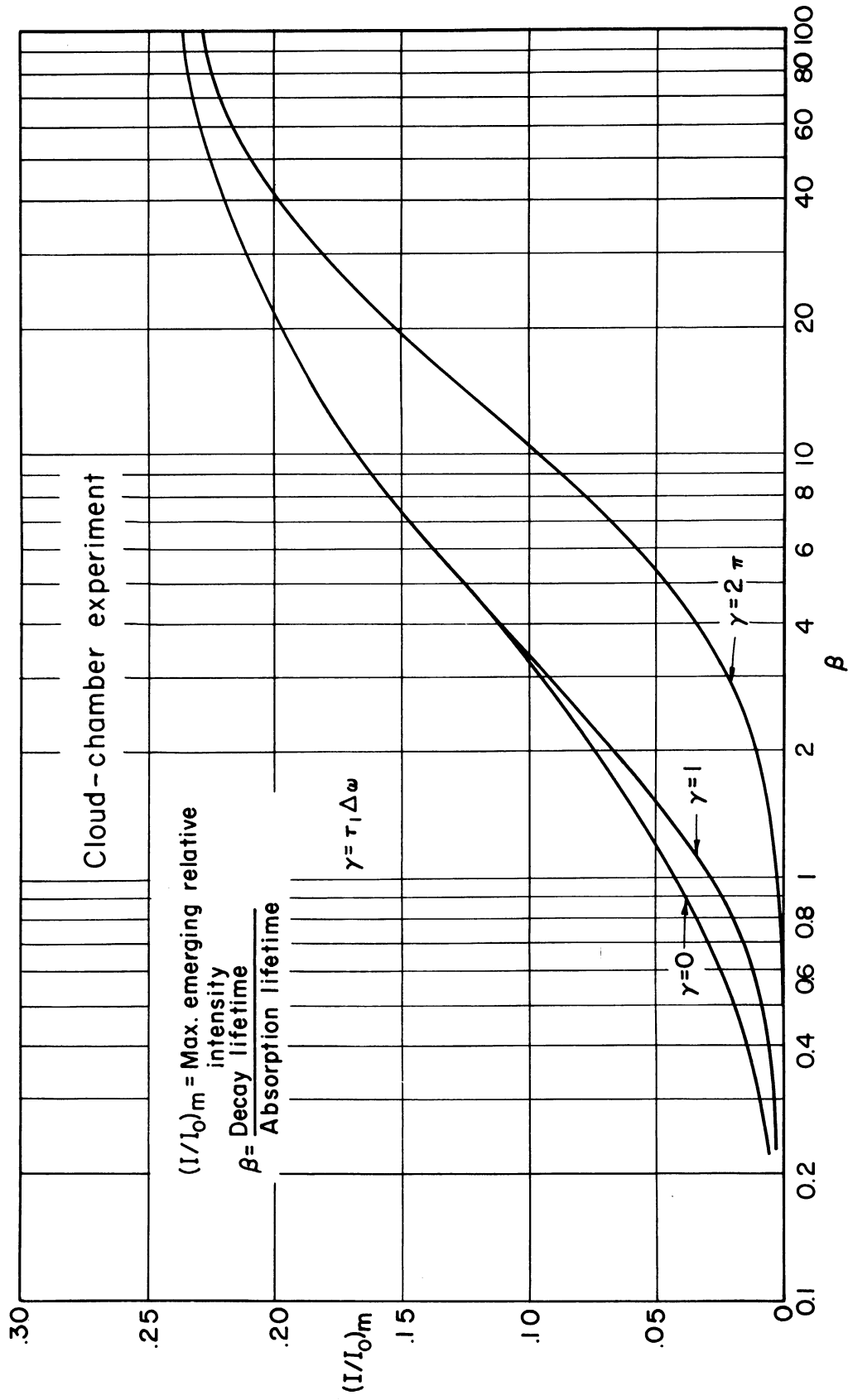


Fig. 1

$$\begin{aligned}
 \frac{I(t)}{I(0)} &= \frac{e^{-t/2\tau_1}}{[(1 - R_1^2 + R_2^2)]^2 + 4R_1^2 R_2^2} \left\{ \right. \\
 &\left. \left\{ \right\} = [(1+R_2)^2 + R_1^2] e^{-(\delta_1 + \beta/2)t/\tau_1} \right. \\
 &+ [(R_1^2 + R_2^2)] [(1 - R_2)^2 + R_1^2] e^{-(\beta/2 - \delta_1)t/\tau_1} \\
 &+ e^{-\beta t/2\tau_1} \left\{ -2[R_1^2(2-R_2) + R_2(1-R_2^2)] \cos \delta_2 t/\tau_1 \right. \\
 &\quad \left. -2R_1(1 - R_1^2 - R_2^2 - 2R_2) \sin \delta_2 t/\tau_1 \right\} .
 \end{aligned} \tag{35}$$

Here  $R = R_1 + iR_2$  is given by (27) and  $\delta_1$  and  $\delta_2$  are given by (26). Now, of course,  $\beta$  refers to the xenon.

1)  $\beta \gg 1$

Equation (35) becomes

$$\frac{I(t)}{I(0)} \approx e^{-t/2\tau_1} . \tag{36}$$

In this limit the  $\Theta^0$ 's would behave as if their lifetime in xenon was double that found in cloud-chamber studies. This would certainly be easy to observe. Unfortunately the estimates of Section IV indicate that we are far from this limit.

2)  $\beta \ll 1$

$$\frac{I(t)}{I(0)} \approx e^{-t/\tau_1} + \frac{\beta^2}{4(1+4\gamma^2)} + \frac{\beta e^{-t/2\tau_1}}{1+4\gamma^2} [\cos(\gamma t/\tau_1) + 2\gamma \sin(\gamma t/\tau_1)] . \tag{37}$$

Thus, only when the number of decays has dropped quite low will there be any deviations from what one could expect for a particle with lifetime  $\tau_1$ .

3) In Figs. 2, 3, and 4 decay curves for various values of  $\beta$  are given, corresponding to  $\gamma = 0, 1, \text{ and } 2\pi$ , respectively.

From Fig. 2 ( $\gamma=0$ ) we see that for  $\beta$  small ( $\beta \lesssim 1$ ) significant deviations from  $\beta = 0$  (no  $\bar{\Theta}^0$  absorption) occur only when the number of decays is quite small. Thus, for  $\beta = 1/3$  there are twice as many decays as for  $\beta = 0$  only when the latter is approximately 1/70 of its initial value.

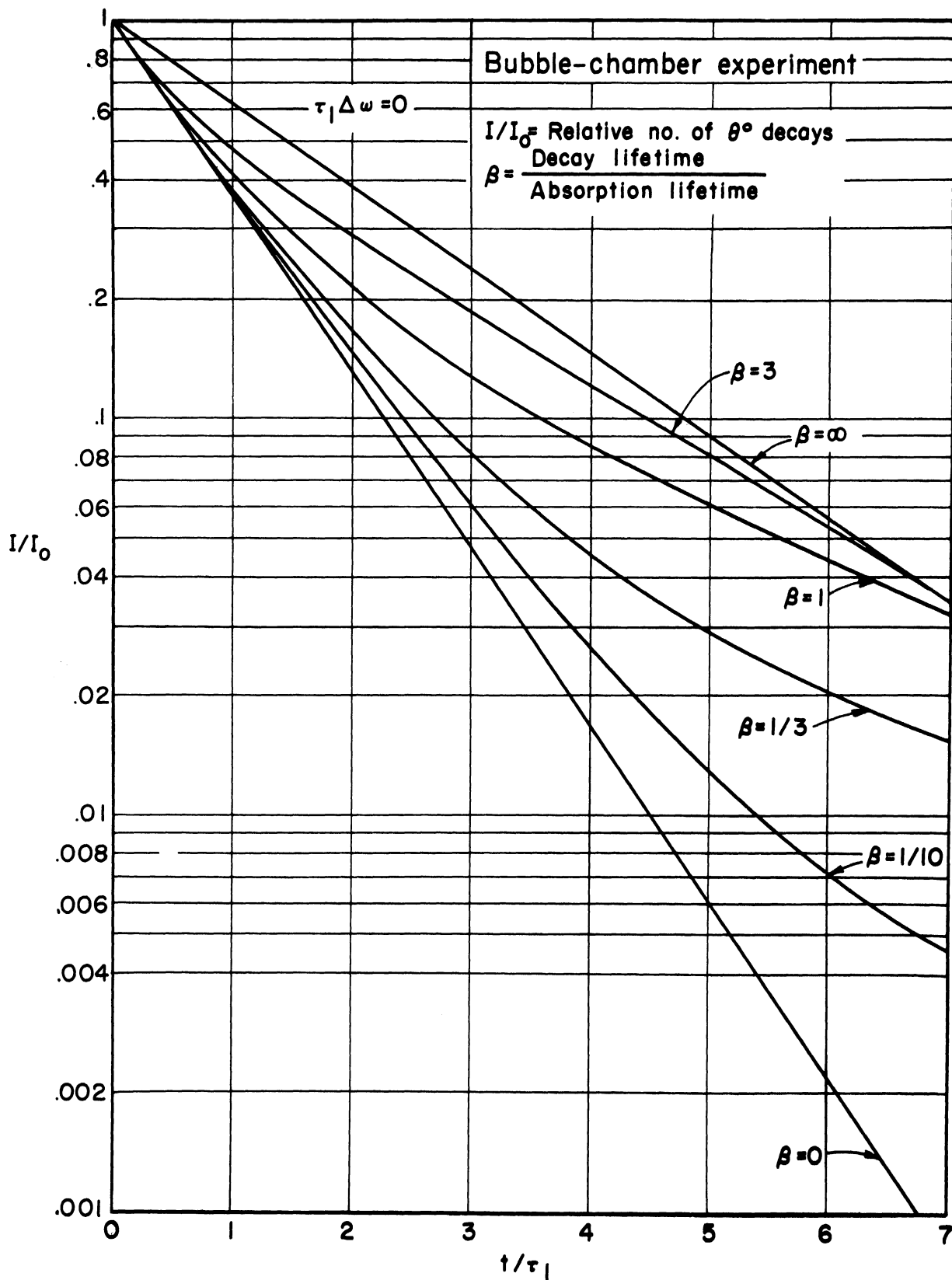


Fig. 2

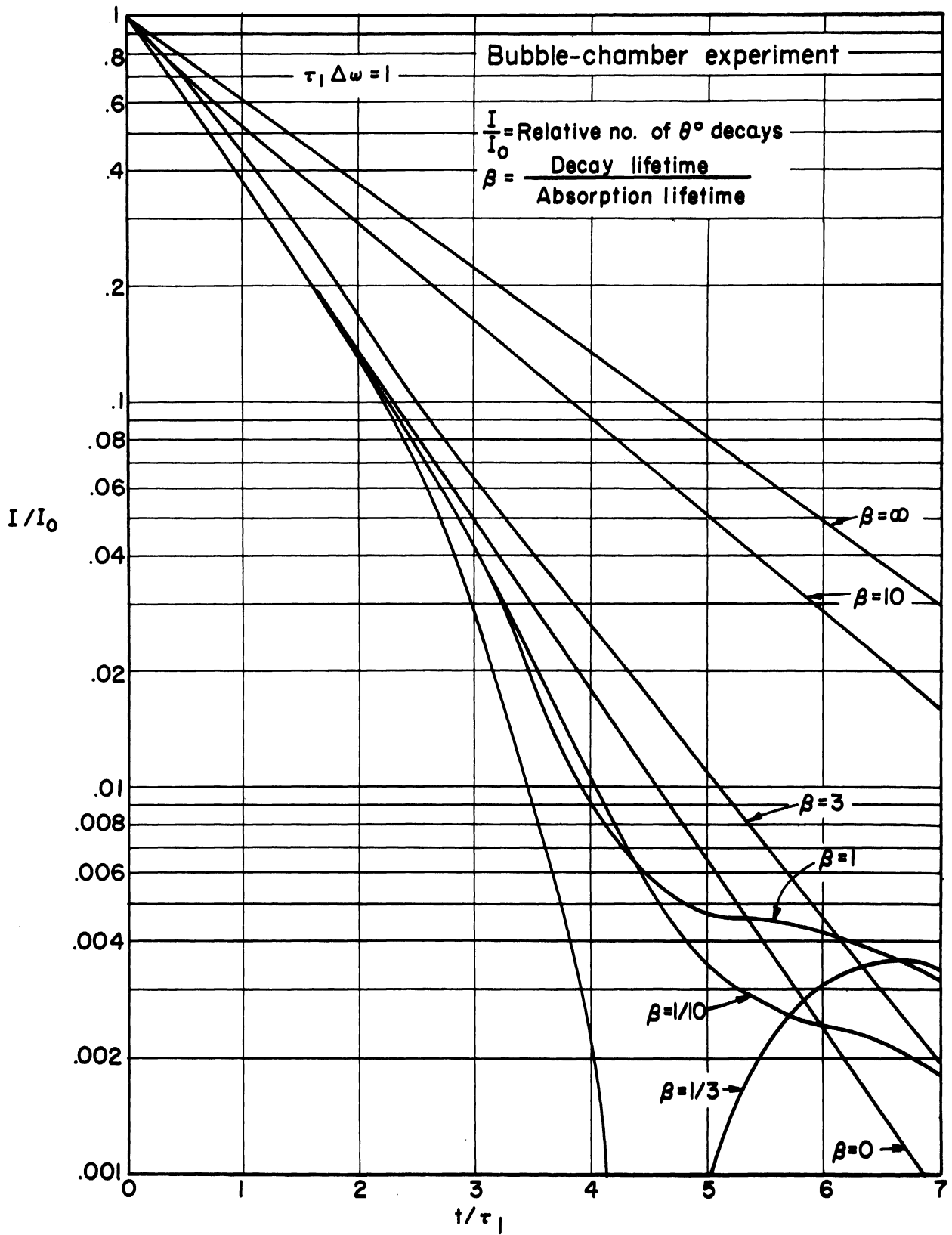


Fig. 3

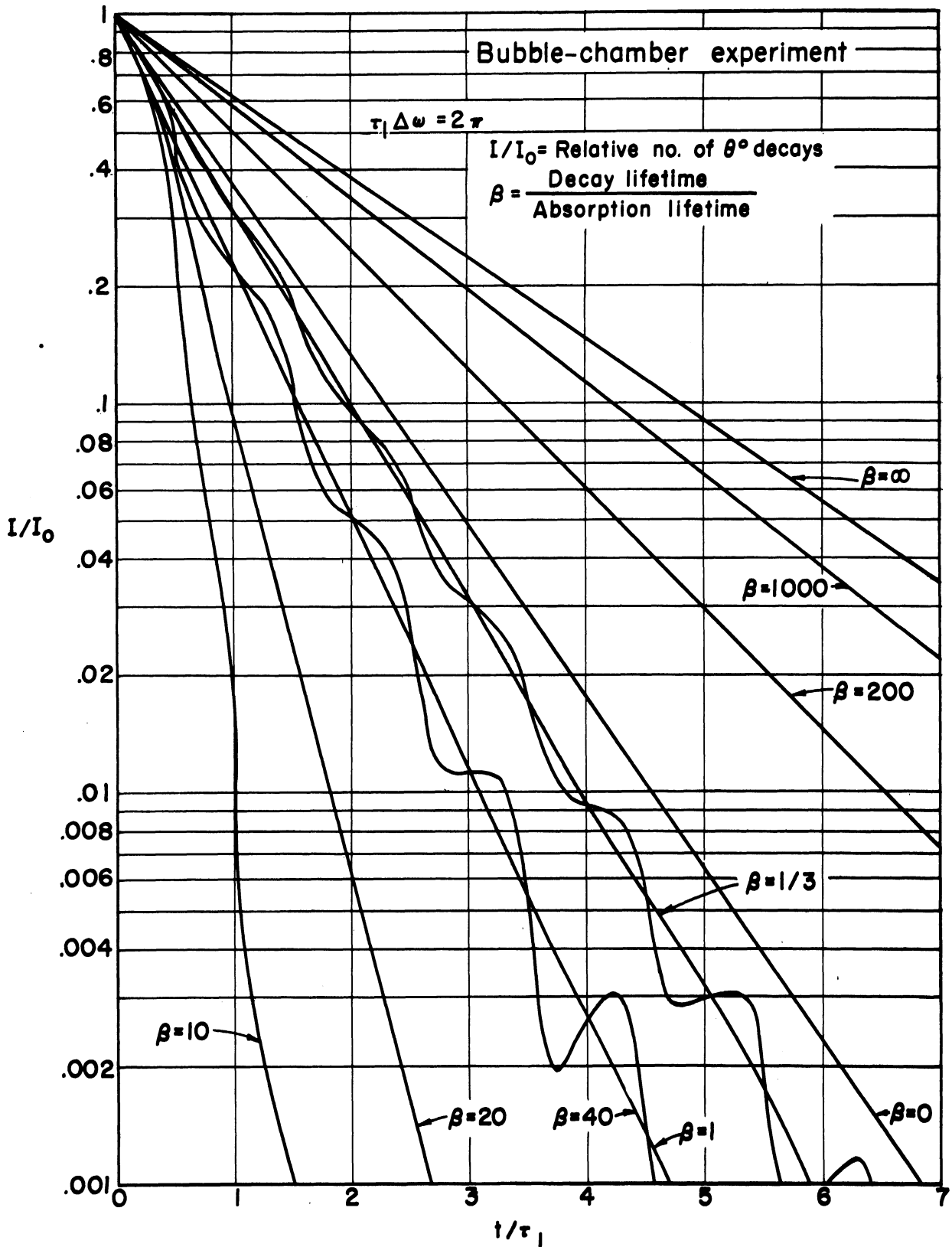


Fig. 4



Figure 3 ( $\gamma=1$ ) leads to similar conclusions, except for one salient feature. For  $\beta \sim 1/3$  there is a tremendous decrease in the number of  $2\pi$  decays expected in the region where, for no absorption of  $\bar{\theta}^0$ 's, we would expect 1% of the decays to occur. While the observation of this effect would be difficult, it is hard to imagine that a decrease by a factor of 100 could not be seen. It should be pointed out that this phenomenon only occurs for a region of values  $\gamma \sim 1$ ,  $\beta \sim 1/3$ . However, the discussion in Section IV by no means rules out this possibility.

The curves of Fig. 4 for  $\gamma=2\pi$  are quite remarkable. Increasing  $\beta$  from zero to infinity does not change the curves smoothly from the straight line describing the decay  $e^{-t/\tau_1}$  to that for  $e^{-t/2\tau_1}$ . As  $\beta$  increases from zero, the decay curves develop characteristic oscillations [due to the sine and cosine terms in (35)]. Since these oscillations probably would be averaged out in an experiment, we have also given curves for this average. These oscillations are most pronounced for  $\beta = 1$ . The decay occurs increasingly rapidly with increasing  $\beta$  until  $\beta \approx 12.5$ . In this region the decay curves look as if the  $\theta^0$  lifetime is considerably shorter than  $1.5 \times 10^{-10}$  sec. Increasing  $\beta$  further the decay curves swing back up, becoming essentially straight. Two phenomena should be observed.

a. Extremely large values of  $\beta$  are required before the curve for  $\beta = \infty$  is even approximately realized. Thus,  $\beta = 200$  is clearly far from this limit.

b. The decay curves for  $12.5 \lesssim \beta \lesssim 60$  coincide with the averaged curves for  $12.5 < \beta < \infty$ . For example, the curve for  $\beta = 40$  falls, within the accuracy with which one can read the figure, exactly on the averaged curve for  $\beta = 1$ . Thus, if one did see an effect of the kind indicated—namely, an apparent increased decay rate—one could only conclude that there are two possible absorption lifetimes  $\tau$  compatible with the results.

## VII. CONCLUSION

We have seen that, in general, for reasonable values of the parameters the "mixture property" of Gellmann and Pais will affect only a small portion of the  $\theta^0$ 's—in either a bubble chamber or a cloud chamber. Under such circumstances the possibility of observing more  $\theta^0$  decays makes the bubble chamber preferable for experiments of this type. At a time when  $\theta^0$ 's are available in sufficient abundance that experiments on 1% of them are feasible, this check on the Gellmann-Pais hypothesis could well be done and would be of great interest.

One proviso to the above general conclusion holds. If, accidentally,  $\tau \sim 1$ ,  $\beta \sim 1/3$  for xenon, the bubble-chamber experiment would be practical even now.

It is a pleasure to thank Professors D. A. Glaser and G. E. Uhlenbeck for helpful discussions.

VIII. REFERENCES

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