

THE UNIVERSITY OF MICHIGAN RESEARCH INSTITUTE
ANN ARBOR

THREE METHODS OF NONLINEAR PROCESSING FOR DIRECTION FINDING

Technical Report No. 85

Electronic Defense Group
Department of Electrical Engineering

(Pieter bear!)
By: Pieter G. Cath

Approved by:



A. B. Macnee

Project 2262

TASK ORDER NO. EDG-10
CONTRACT NO. DA-36-039 sc-63203
SIGNAL CORPS, DEPARTMENT OF THE ARMY
DEPARTMENT OF ARMY PROJECT NO. 3-99-04-042

August, 1958

enjm

UMK0872

TABLE OF CONTENTS

	Page
LIST OF ILLUSTRATIONS	iii
ABSTRACT	iv
1. INTRODUCTION	1
2. LINEAR ADDITIVE ARRAYS	2
3. T.A.P. SYSTEMS	8
3.1 T.A.P. System With Two Antenna Elements	8
3.2 T.A.P. System With Four Antenna Elements	11
4. ENHANCED PHASE SYSTEM	15
5. A CORRELATION SYSTEM	21
6. PRACTICAL CONSIDERATIONS	24
7. NOISE CONSIDERATIONS	25
8. CONCLUSION	28
APPENDIX	29
REFERENCES	34
DISTRIBUTION LIST	35

LIST OF ILLUSTRATIONS

	Page
Figure 1 A Linear-Array Direction-Finding System	4
Figure 2 An Array of Antennae	4
Figure 3 Lobe Pattern of an Eight Element Linear Array	6
Figure 4 Two Element T.A.P. System	9
Figure 5 Two Element T.A.P. Antenna Array	9
Figure 6 Response of T.A.P. and Linear Array to Two Signals	12
Figure 7 T.A.P. System With Four Antenna Elements	13
Figure 8 Antenna Configuration	14
Figure 9 Comparison of T.A.P. System With an Eight Element Linear Array for Two Incoming Waves	16
Figure 10 Enhanced-Phase System	18
Figure 11 Comparison of Enhanced Phase and Linear Array	22
Figure 12 A Correlation System	23
Figure 13 Two Antennae With Wavefront	30

ABSTRACT

Three systems employing non-linear signal processing to reduce the physical size of a directive antenna array are compared with a linear additive array for the purpose of direction finding. The three systems are: (1) a time-averaged product system; (2) an enhanced phase system; and (3) a correlation system.

Although the non-linear systems (1, 2 and 3) produce exactly the same lobe pattern as the (equivalent) linear array with a single incoming radio wave, systems 1 and 2 fail to separate two waves of slightly different frequencies arriving from different directions, which can be resolved by the equivalent linear array.

The effect of random noise on these systems is considered, but these considerations do not lead to general conclusions suitable for comparison of these systems with the linear additive array system.

It is concluded that narrow lobe patterns can be obtained for single arriving radio waves, with antenna arrays that are greatly reduced in size, at the cost, however, of a large increase in system complexity.

THREE METHODS OF NONLINEAR PROCESSING FOR DIRECTION FINDING

1. INTRODUCTION

Linear additive arrays have not been widely used for direction finding in the h-f band because of their prohibitive size. To obtain a narrow lobe pattern, it is necessary that the overall length of the antenna system be equal to several wavelengths.

Because, on the other hand, the directive properties of linear arrays are very attractive, methods have been proposed to reduce the size of the antenna arrays without loss of directivity. This can be done by means of non-linear processing. These methods have in common that the signals from a number of antenna elements, with certain relative spacings, are processed and combined in such a (non-linear) fashion that the total effect simulates a linear additive array of much larger dimensions.

Three such methods of non-linear processing are discussed in this report, and it is shown that some of these have the serious limitation of not being able to separate two waves arriving from different directions, even though these waves can be separated by the equivalent linear array. By equivalent linear array is meant the linear array that has the same lobe pattern as is obtained by the processing system with a single incoming wave. In other words, the resolution obtained by those processing systems is less than what one might expect from their directivity pattern for single signals.

Finally in this report, an attempt is made to outline the factors that must be taken into consideration when the effect of noise upon the processing system is calculated. No general conclusions are reached in this

section on noise.

It is assumed throughout the report that there is no interaction between antenna elements in any of the systems discussed.

2. LINEAR ADDITIVE ARRAYS

A linear additive array is composed of a number of identical antenna elements whose outputs are added linearly. The resulting sum is a signal of radio frequency that can be detected subsequently by a radio receiver, which is denoted in this report by the filter F_{ω}^1 . The direction of arrival of a radio wave can be found by rotating the array until its output reaches a maximum.

For the purpose of this discussion, a direction finding system will be studied that consists of a rotatable linear array of antennae, a radio receiver to select a certain frequency band, and a linear detector together with some kind of amplitude display, e.g. a meter display. Such a system is shown in Figure 1.

To serve as a basis for our comparison, the response of this linear array system to one and to two arriving wavefronts will be studied.

Consider that a plane electromagnetic wave is arriving at a linear array of antennae. The antennae elements are situated at A and A', B and B', etc. and the elements are equally spaced along a straight line as indicated in Figure 2. The distance between successive elements is "d". The point "O" is the geometrical center of the array and will serve as a phase reference for our calculations.

¹The word filter in this report means in general any device that selects a narrow frequency band. The filter, therefore, can incorporate local oscillators and mixer stages. When a number of filters are used in one system, the local oscillators, of course, can be common to all. Note that phase relationships are conserved with this general definition of filters.

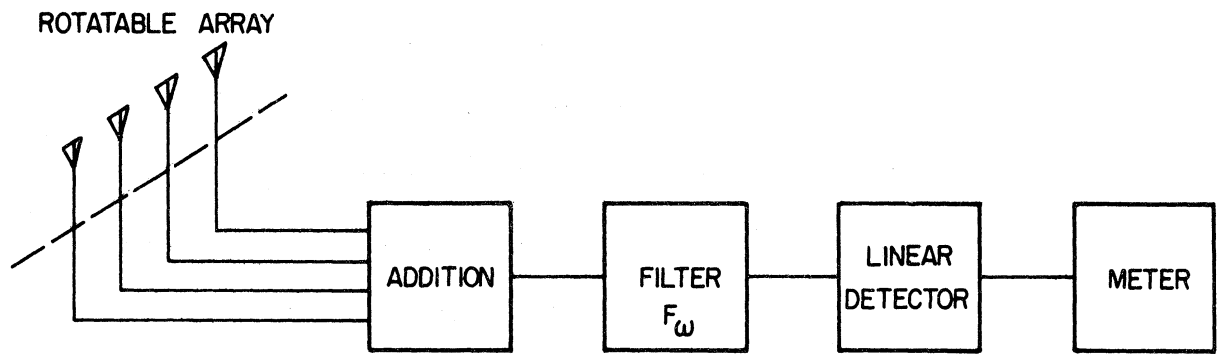


FIG.1 A LINEAR-ARRAY DIRECTION-FINDING SYSTEM

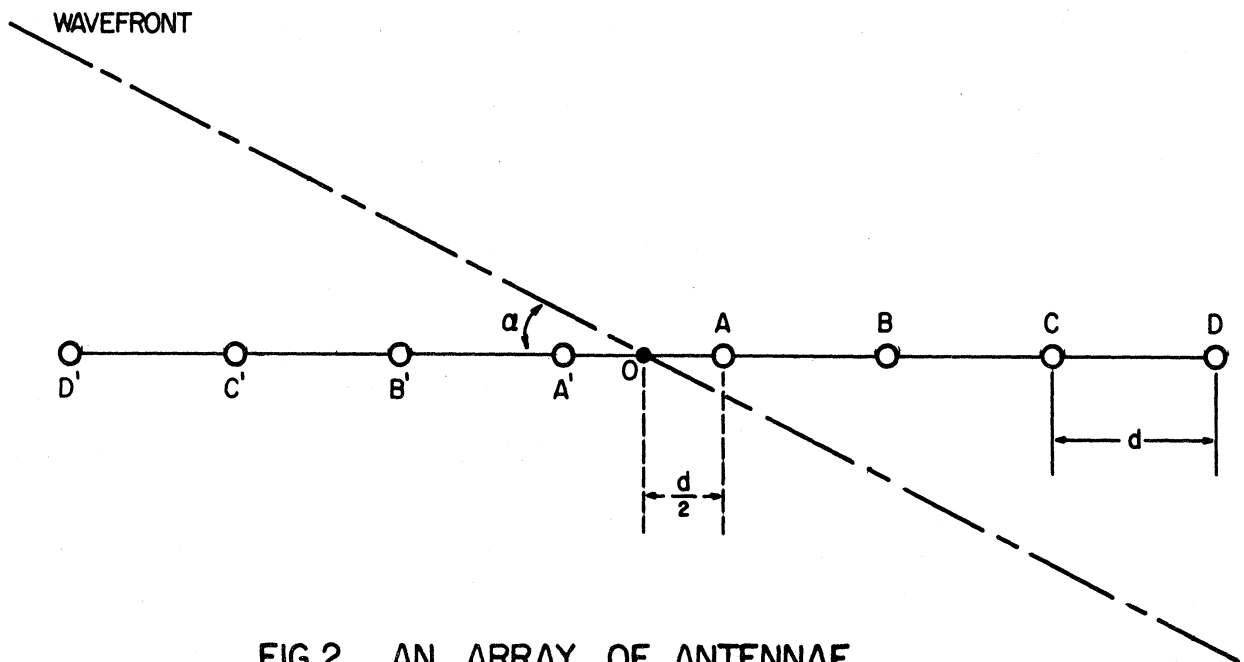


FIG.2 AN ARRAY OF ANTENNAE

The arriving wave has a wavelength (λ) corresponding to a frequency¹
 $\omega = \frac{2\pi c}{\lambda}$ and α is the angle between the wavefront and the line AA' on which the
 antennae are located. (Figure 2) The antenna signals will not differ in mag-
 nitude, only in phase.

If the signal from an antenna placed at point "O" is equal to
 $V_0 = a \cos \omega t$, then the signals from antennae A and A' will be respectively:

$$V_A = a \cos (\omega t + u) \text{ and } V_{A'} = a \cos (\omega t - u)$$

where $2u$ is the total phase difference between the two signals due to the
 angle of arrival (α) of the wave front. This phase difference (u) is there-
 fore, a function of the angle α :

$$u = \frac{2\pi d}{\lambda} \sin \alpha = \frac{\pi d}{\lambda} \sin \alpha \quad (1)$$

When the voltages from antennae A and A' are added the following result is
 obtained:

$$V_A + V_{A'} = a \cos (\omega t + u) + a \cos (\omega t - u) = 2a \cos u \cdot \cos \omega t$$

Similarly:

$$V_B + V_{B'} = a \cos (\omega t + 3u) + a \cos (\omega t - 3u) = 2a \cos 3u \cdot \cos \omega t$$

When the four antenna voltages are added together the result is:

$$V_A + V_{A'} + V_B + V_{B'} = 2a (\cos u + \cos 3u) \cos \omega t.$$

This is a voltage of frequency ω having an amplitude equal to $2a (\cos u + \cos 3u)$.
 After amplitude detection a voltage of magnitude $2a (\cos u + \cos 3u)$ is obtained.
 Since the direction finder is not concerned with the absolute amplitude, the
 factor $2a$ is disregarded and we shall say that the output of a D-F system with
 a four-element linear array is equal to:

$$\cos u + \cos 3u \dots\dots\dots (2)$$

¹The word "frequency" will be used in this report both for frequency (f)
 and for angular frequency ($\omega = 2\pi f$).

The output of a linear array consisting of eight antenna elements is derived in the same manner. With the same assumptions this output after amplitude detection will be:

$$\cos u + \cos 3u + \cos 5u + \cos 7u \dots \dots \dots (3)$$

This output is shown in Figure 3, for a value of $d = \frac{\lambda}{2}$. This figure then represents the rectified output of a linear additive array while this array is rotated, for instance, to find the direction of arrival of an incoming wave.¹

The output of the linear array system to two radio waves arriving simultaneously from different directions is derived in a similar fashion.

Let the directions of arrival be (α) and $(\alpha + \beta)$ so that β is the angular separation between the two wavefronts. The voltages generated in antennae A and A' will then be:

$$V_A = a \cos (\omega t + u) + b \cos (\omega' t + u')$$

and

$$V_{A'} = a \cos (\omega t - u) + b \cos (\omega' t - u')$$

where a and b represent the amplitudes, ω and ω' the frequencies of the two signals and

$$u = \frac{2\pi d}{\lambda} \sin \alpha = \frac{\pi d}{\lambda} \sin \alpha \text{ and } u' = \frac{\pi d}{\lambda} (\alpha + \beta) \dots (4)$$

The frequencies ω and ω' are assumed to lie inside the passband of the filter. (see Figure 1) Following exactly the same procedure as before, we now consider the sum of these two voltages:

¹When a linear detector is used as indicated in Figure 1, the meter will indicate the absolute value of the function shown in Figure 3. In the other systems that will be discussed below, the meter does indicate negative values. Because this fact does not basically alter the comparisons that will be made, we shall continue plotting the response function of the linear array rather than its absolute value.

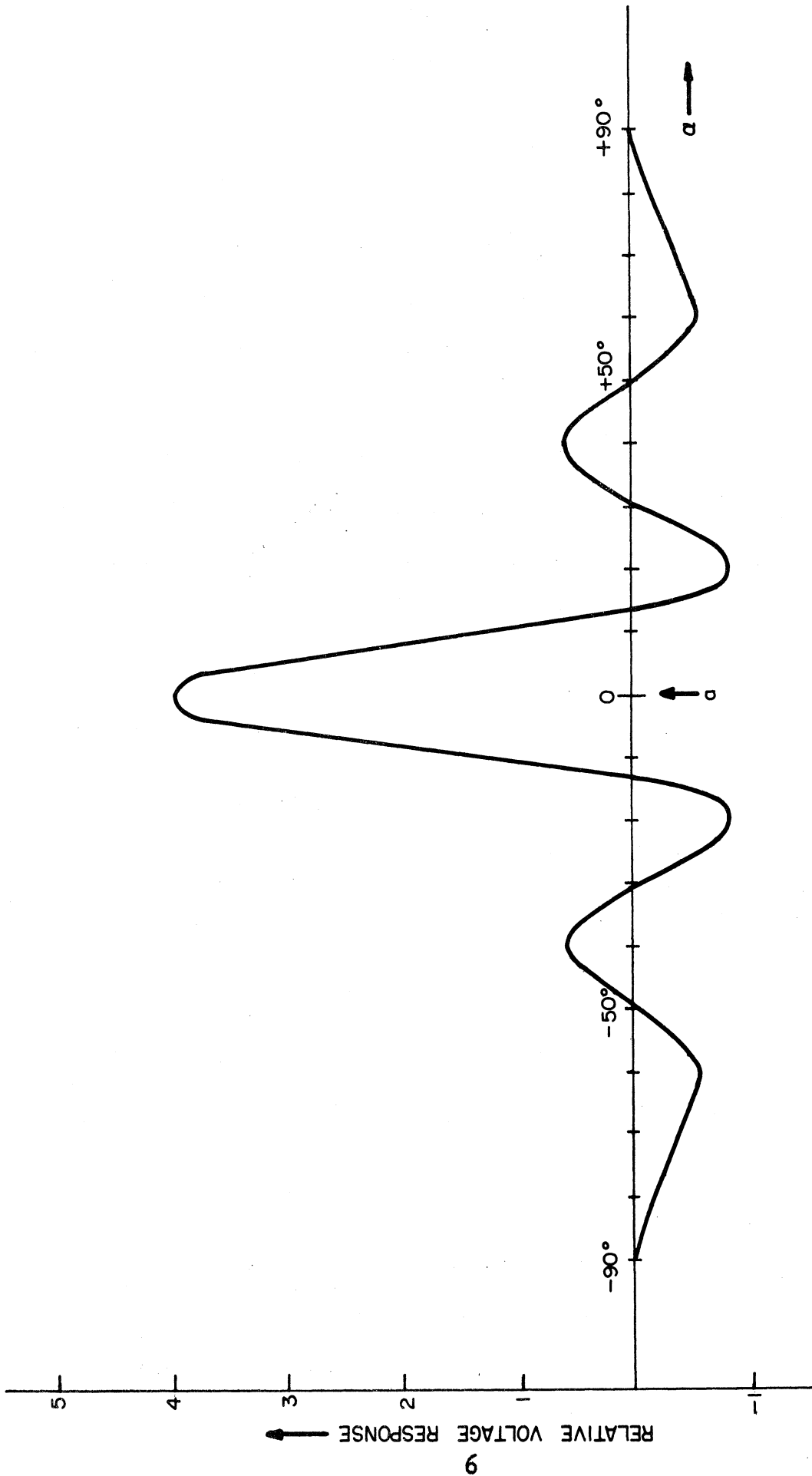


FIG.3 LOBE PATTERN OF AN EIGHT ELEMENT LINEAR ARRAY

$$V_A + V_{A'} = 2a \cos u \cdot \cos \omega t + 2b \cos u' \cdot \cos \omega' t$$

A similar expression is found for the sum of the voltages V_B and $V_{B'}$, the difference being that $3u$ and $3u'$ replace u and u' in the above formula.

To find the response of the system to these two waves one additional assumption will be made, namely that the time averaging inherent to the process of amplitude detection and meter display has a time constant that is larger than the reciprocal of the difference in frequency ($\frac{\omega - \omega'}{2\pi}$) of the two received signals.¹ In that case the detected output of a D-F system with a four-element linear array will be:

$$2a (\cos u + \cos 3u) + 2b (\cos u' + \cos 3u')$$

and when the two signals have equal strength and if we take into account the fact that we are interested in relative amplitudes only, this output becomes:

$$\cos u + \cos u' + \cos 3u + \cos 3u' \dots \dots \dots (5)$$

The response of an eight element linear array to two signals of equal strength will be after amplitude detection:

$$\cos u + \cos u' + \cos 3u + \cos 3u' + \cos 5u + \cos 5u' + \cos 7u + \cos 7u' \dots \dots (6)$$

The response of a four element array to two waves of equal amplitude is shown as the dotted line in Figure 6 (parameters chosen: $\beta = 53.5^\circ$; $d = \lambda/2$) and that of an eight-element linear array is shown as the dotted line in Figure 9 (parameters: $\beta = 26.2^\circ$; $d = \lambda/2$) These plots show that the linear array is capable of separating and finding the approximate directions of arrival of the two incoming waves with these parameters chosen.

¹This assumption will also be made in the rest of this report.

3. T.A.P. SYSTEMS

A time-averaged-product system (T.A.P.) (see Ref. 1)¹ consists of a number of antenna elements, arranged in a certain configuration, together with the necessary processing equipment. Many different configurations are possible for the antenna pattern as well as for the processing equipment. In this section, we shall discuss the response of two different T.A.P. systems to one and two arriving radio waves.

3.1 T.A.P. System With Two Antenna Elements

A T.A.P. system having two antenna elements can be made to simulate a linear array of any dimension, at least when only one signal is received. Assume that a four-element linear additive array with a distance d between antenna elements is to be simulated (Figure 2). This is realized by the T.A.P. system shown in Figure 4. This system consists of two elements together with the processing equipment. Because the response of the T.A.P. system is to be equal to the response of a linear additive array with spacing d , the distance D between the two antenna elements in the T.A.P. system is chosen such that $D = \frac{1}{2} d$ (Figure 5). The two antenna voltages V_1 and V_2 as a result of a plane wave arriving from a direction α , will be:

$$V_1 = a \cos(\omega t) \text{ and } V_2 = a \cos(\omega t + u)$$

where a and ω are the amplitude and the frequency of the arriving wave and

$u = \frac{2\pi D}{\lambda} \sin \alpha$. Note that because the distance D was chosen to be equal to $D = \frac{1}{2} d$, this quantity $u = \frac{\pi d}{\lambda} \sin \alpha$ is exactly equal to the quantity u that was used in the calculations for the linear array (equation 1).

In the T.A.P. system the two antenna signals are multiplied together and the time average of this product is taken. The result of this manipulation

¹The references are listed in the back of this report.

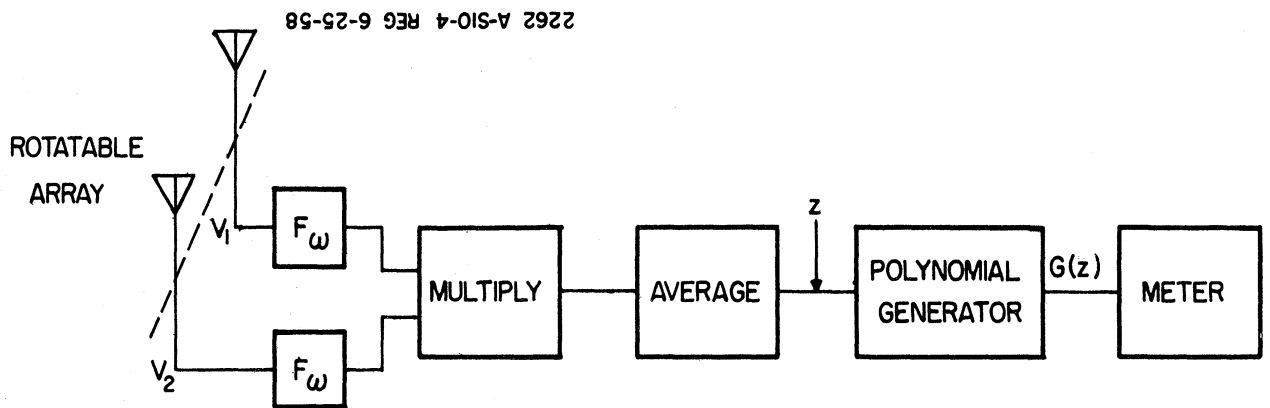


FIG. 4. TWO ELEMENT TAP SYSTEM

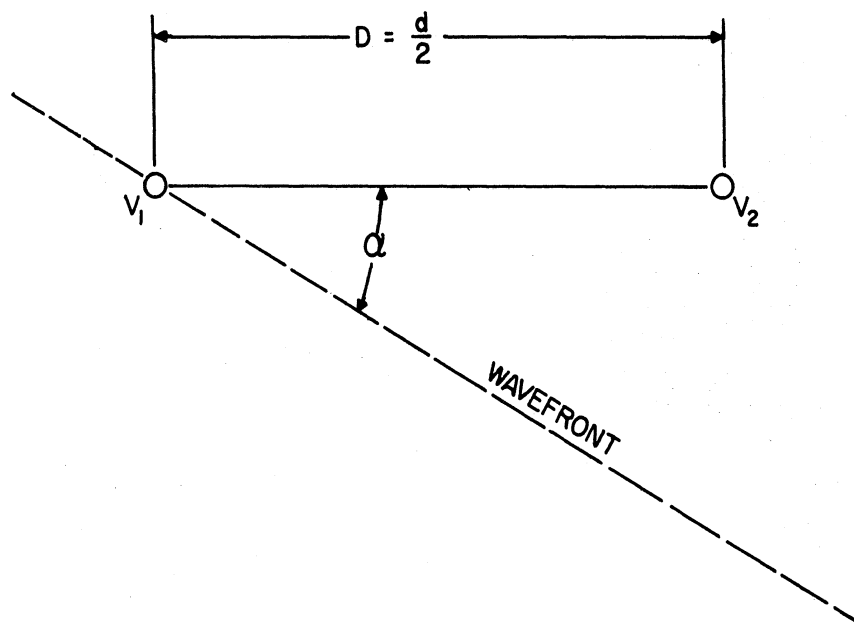


FIG.5 TWO ELEMENT TAP. ANTENNA ARRAY

is a quantity z:

$$z = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T V_1 \cdot V_2 dt = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T a^2 \cos \omega t \cdot \cos (\omega t + u) dt = \frac{a^2}{2} \cos u \quad (7)$$

In this particular case, one is not permitted to neglect the absolute amplitude, because when the polynomial is formed below, the values z are going to be raised to different powers. So, for this one case, the division by $a^2/2$ must actually be performed by the equipment prior to generating the polynomial. In all other systems discussed in this report, neglecting the absolute amplitude is permitted in the sense that we are interested in relative amplitudes only. We shall assume in the following that the division by $a^2/2$ has been performed so that $z = \cos u$.

From this quantity z the following polynomial G(z) is generated:

$$G(z) = 4 z^3 - 2 z = 4 \cos^3 u - 2 \cos u = \cos u + \cos 3u \dots \dots \dots (8)$$

When the meter display indicates the value of this polynomial G(z), its deflection will be exactly the same as that of the meter in Figure 1. (equation 2) In other words, the output of this T.A.P. system is exactly equal to the output of the linear additive array system shown in Figure 1. The output of an eight element linear array can be simulated by forming the polynomial:

$$G'(z) = 64 z^7 - 96 z^5 + 40 z^3 - 4 z$$

because

$$64 \cos^7 u - 96 \cos^5 u + 40 \cos^3 u - 4 \cos u = \cos 7u + \cos 5u + \cos 3u + \cos u \quad (9) \quad \text{(compare eq. 3)}$$

How does the T.A.P. system respond when two radio waves are arriving simultaneously? Let the frequencies of the two waves be ω and ω' , the directions of arrival α and $\alpha + \beta$, and let their amplitudes be equal to a and b respectively.

Then the two antenna voltages V_1 and V_2 will be:

$$V_1 = a \cos (\omega t) + b \cos (\omega' t) \quad \text{and} \quad V_2 = a \cos (\omega t + u) + b \cos (\omega' t + u')$$

where $u = \frac{\pi d}{\lambda} \sin \alpha$ and $u' = \frac{\pi d}{\lambda} \sin (\alpha + \beta)$ as in the two signal case that was discussed for the linear array. (equation 1 and 4)

When the time averaged product of V_1 and V_2 is taken, a quantity z is obtained which will be equal to:

$$z = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T V_1 \cdot V_2 dt = \frac{a^2}{2} \cos u + \frac{b^2}{2} \cos u' \quad (10)$$

As before, we assume that the two arriving signals have equal amplitudes and that we can neglect the absolute magnitude of z because the division by $a^2/2$ has been carried out.

When the polynomial $G(z)$ is formed with this value of z , the output becomes

$$G(z) = 4 z^3 - 2 z = 4 (\cos u + \cos u')^3 - 2 (\cos u + \cos u') \quad (11)$$

This differs from the response of the linear array system to the same incoming waves.

The response of the two element T.A.P. system (equation 11) is plotted in Figure 6 (full line). In this figure the response of the four element array (equation 5) is also shown for comparison (broken line). The angular separation (β) between the two wave fronts was chosen so that the linear array could easily distinguish between the two. As is apparent from Figure 6, the T.A.P. system fails to separate the two directions, even though the responses were exactly the same for a single wave. The parameters chosen were $\beta = 53.5^\circ$ and $D = \lambda/4$.

3.2 T.A.P. System With Four Antenna Elements

A similar conclusion is reached when other types of T.A.P. systems are considered. A T.A.P. system with four antenna elements and a different type of processing will now be studied. By means of the antenna configuration shown in Figure 8, whose overall length is $4 D = 2 d$, an eight element linear array is simulated which has an overall length $7 d$. A notable reduction in size! This system is shown in Figure 7. $V_1, V_2, V_3,$ and V_4 are the signals from the four antennae and three time average products (z_1, z_2, z_3) are formed as indicated in Figure 7. These three quantities are multiplied together to

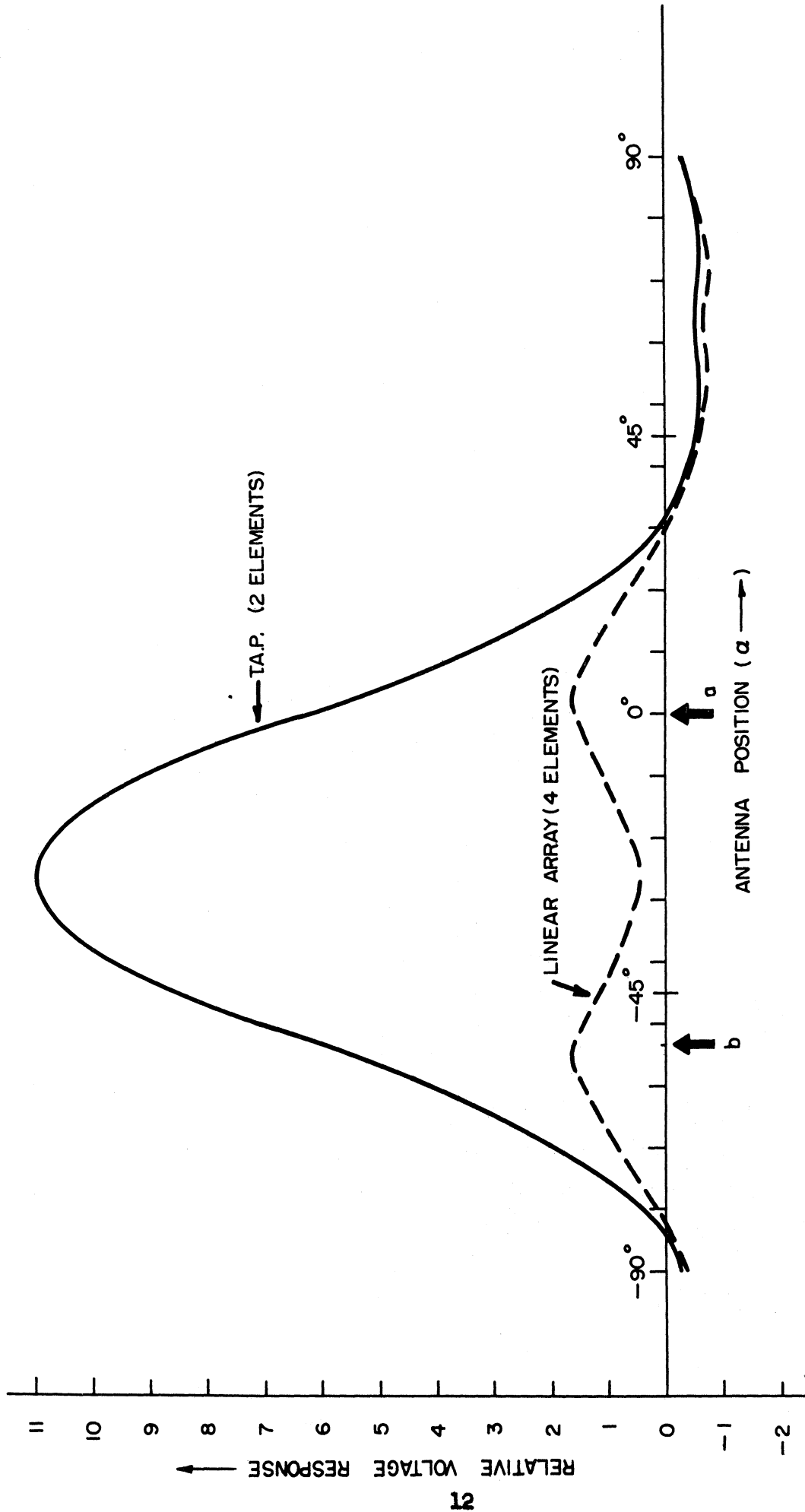


FIG. 6 RESPONSE OF T.A.P. AND LINEAR ARRAY TO TWO SIGNALS

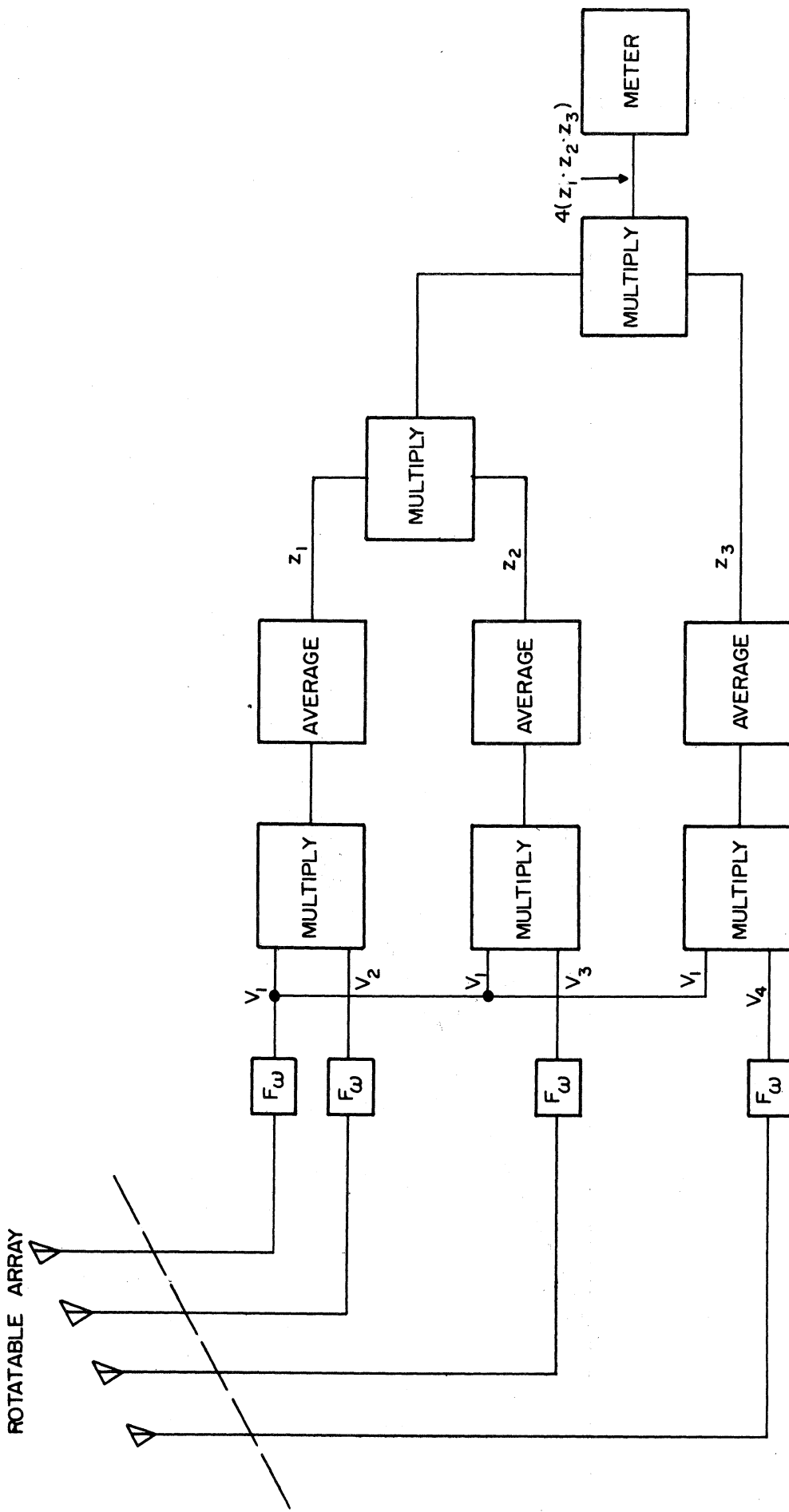


FIG.7 TAP SYSTEM WITH FOUR ANTENNA ELEMENTS

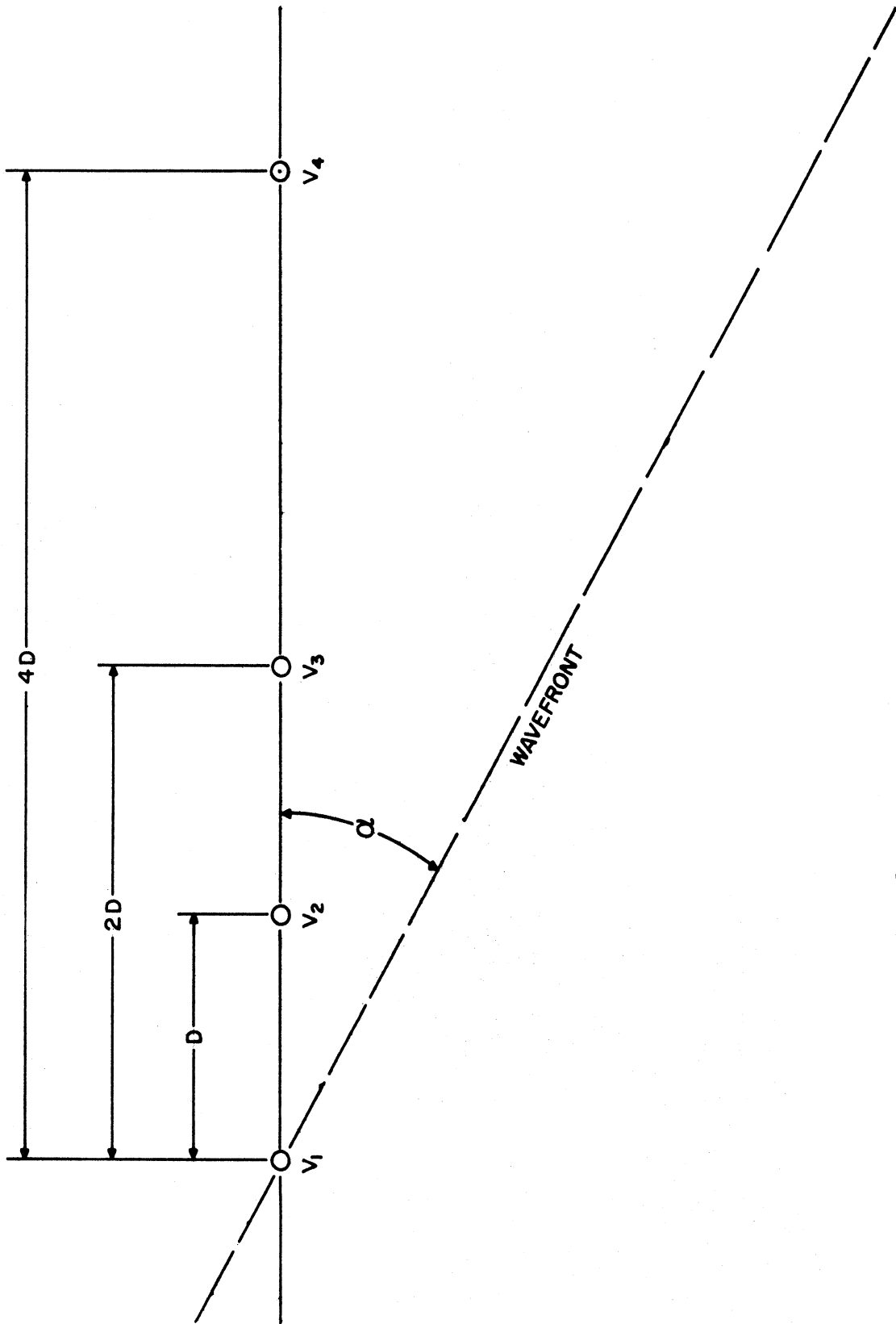


FIG. 8. ANTENNA CONFIGURATION

produce the output of the T.A.P. system. The antenna configuration is shown in Figure 8, and when a plane wave arrives from an angular direction α , the four antenna voltages will be:

$$V_1 = a \cos \omega t; V_2 = a \cos (\omega t + u); V_3 = a \cos (\omega t + 2u); V_4 = a \cos (\omega t + 4u)$$

where again $u = \frac{2\pi D}{\lambda} \cos \alpha = \frac{\pi d}{\lambda} \cos \alpha$ when $D = \frac{1}{2} \lambda$. The three time averaged products now are:

$$z_1 = \cos u; z_2 = \cos 2u; z_3 = \cos 4u$$

As before the absolute amplitudes are neglected. Four times the product of $z_1, z_2,$ and z_3 is generated and the following is obtained:

$$4 z_1 \cdot z_2 \cdot z_3 = 4 \cos u \cdot \cos 2u \cdot \cos 4u = \cos 7u + \cos 5u + \cos 3u + \cos u \quad (12)$$

and this is, of course, exactly equal to the output of an eight element linear array. This output is shown in Figure 3 for a value $D = \lambda/4$.

Again we consider the two signal case, and the response of the T.A.P. system to two arriving waves of equal amplitude and an angular separation of $\beta = 26.2^\circ$ is calculated. For the three time averaged products the following values are found:

$$z_1 = \cos u + \cos u'; z_2 = \cos 2u + \cos 2u'; z_3 = \cos 4u + \cos 4u' \quad (\text{compare eq.10})$$

Now the product $4 z_1 \cdot z_2 \cdot z_3$ is no longer equal to the output of the linear array under the same conditions. The two responses are compared in Figure 9, and again it can be seen that the T.A.P. system does not have the same resolution as does the linear array to which it is equivalent in the single signal case.

4. ENHANCED PHASE SYSTEM

The enhanced phase system is another type of non-linear processing proposed for an antenna array of reduced size. (see Ref. 2) It also is capable of simulating a linear array in the single signal case. It is easily shown,

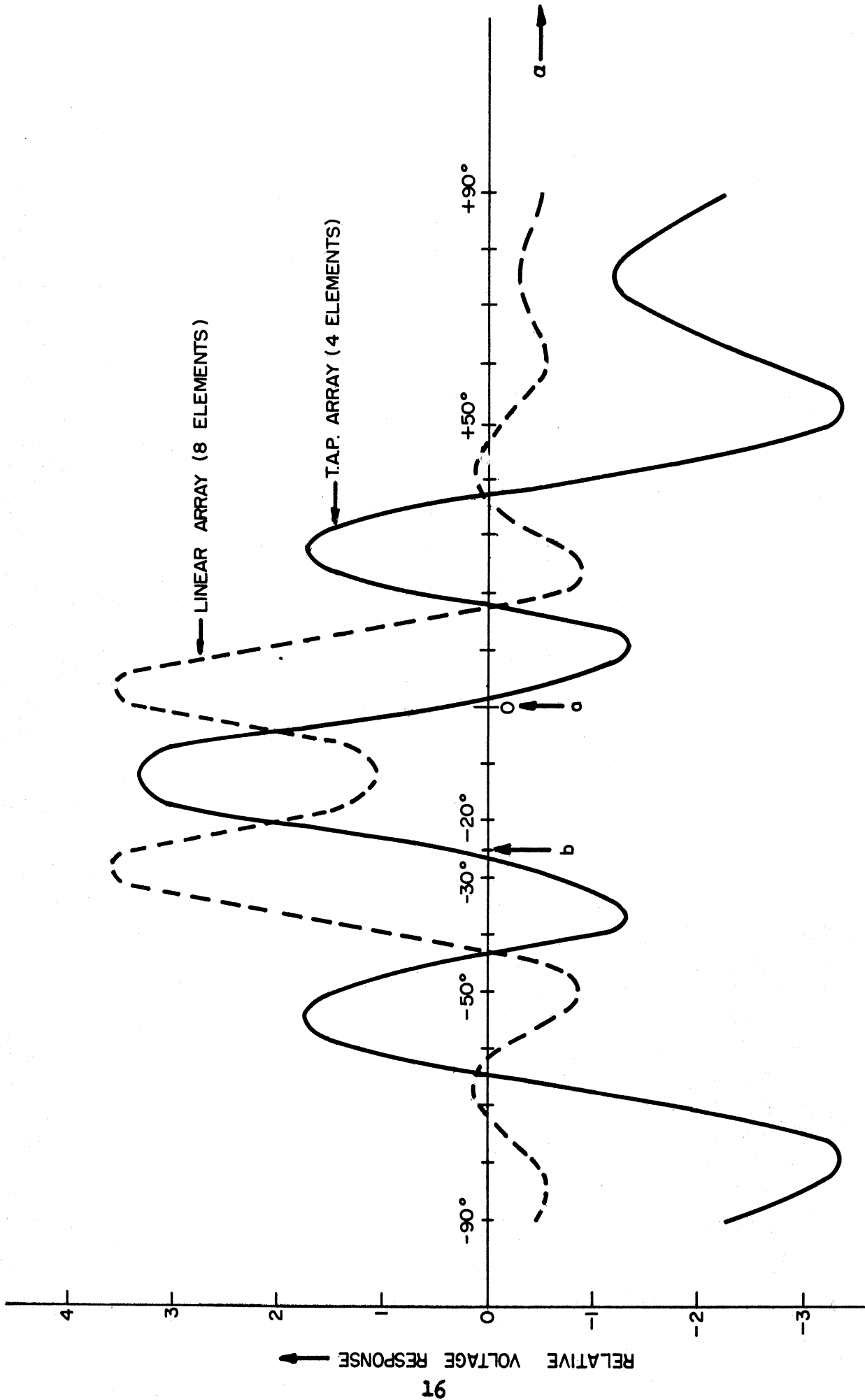


FIG. 9 COMPARISON OF T.A.P. SYSTEM WITH AN 8 ELEMENT LINEAR ARRAY FOR TWO INCOMING WAVES

however, that the system does not separate two waves arriving from different directions, that are easily resolved by the linear array.

The enhanced phase system has the same number of antenna elements as the linear array that is being simulated, only the spacing between antenna elements and, therefore, also the overall size of the array is reduced by a factor $1/n$ where n is an integer. The antenna voltages will have the same amplitude and the same frequency, but generally not the same phase. Passing the antenna signals through a frequency multiplier will at the same time result in signals whose phase differences have been multiplied by the same amount. Subsequently adding these signals will, then, result in a simulation of a linear additive array which is n times as large as the enhanced phase array, where n is the amount by which the frequency was multiplied in the process. Let us examine this closer now with the aid of Figure 10.

As was stated in the introduction, it is assumed that there is no interaction between antenna elements. Figure 10 shows the complete system where the frequency of the antenna signals is multiplied by a factor n in the frequency multipliers. (see also footnote on page 2) A frequency multiplier consists of a non-linear device whose output contains a large number of frequency components. The desired frequency is selected by a filter. Let $\cos(\omega t + y)$ be the input to the frequency multiplier. Then the output of the frequency multiplier will contain a component of frequency $n\omega$ only if there is a non-linearity of the n^{th} order in the frequency multiplier which produces a term

$$c_n \cos^n(\omega t + y) = \frac{c_n}{2^{n-1}} \cos(n\omega t + ny) + \text{terms of other frequencies.}$$

The filter associated with the frequency multiplier rejects all frequencies other than $n\omega$.

Consider an eight element antenna array the spacing of which is D (compare Figure 2). The respective antenna voltages will then be:

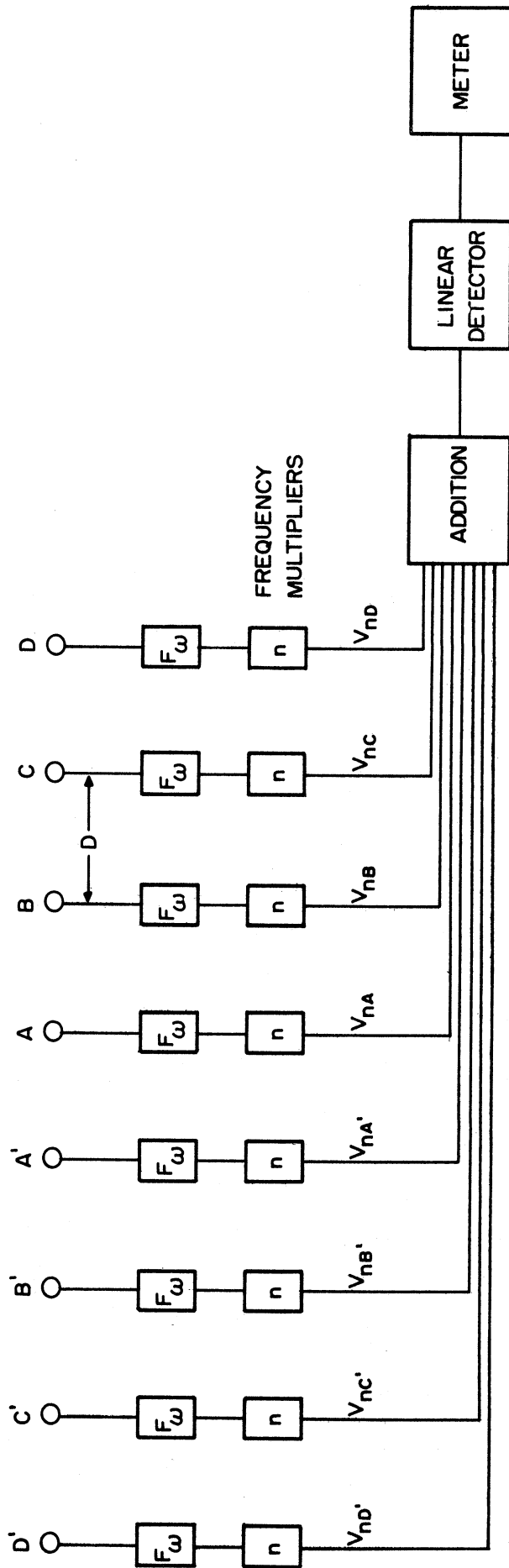


FIG.10 ENHANCED-PHASE SYSTEM

$$V_A = a \cos (\omega t + y); V_{A'} = a \cos (\omega t - y); V_B = a \cos (\omega t + 3y)$$

$$V_{B'} = a \cos (\omega t - 3y), \text{ etc.} \quad y = \frac{2\pi D}{\lambda} \sin \alpha$$

These voltages will produce the following outputs after frequency multiplication:

$$V_{nA} = \frac{a^n \cdot c_n}{2^{n-1}} \cos (n\omega t + ny); \quad V_{nA'} = \frac{a^n \cdot c_n}{2^{n-1}} \cos (n\omega t - ny)$$

Adding the outputs of all the frequency multipliers and disregarding the absolute amplitude results in:

$$V_{nA} + V_{nA'} + V_{nB} + V_{nB'} + V_{nC} + V_{nC'} + V_{nD} + V_{nD'} =$$

$$\cos (n\omega t) \cdot \left(\cos ny + \cos 3ny + \cos 5ny + \cos 7ny \right)$$

After amplitude detection the output of the system becomes:

$$(\cos ny + \cos 3ny + \cos 5ny + \cos 7ny)$$

This is equal to the output of the linear array system (equation 3) when $ny = u$. This condition is fulfilled when D is chosen such that $D = d/n$ where d is the spacing in the linear array. This shows that the size of the antenna array has been reduced by a factor n .

When the frequency is multiplied by a factor $n = 3$, an eight element antenna array with a spacing $D = \lambda/2 \cdot 3 = \lambda/6 = d/3$ will simulate a linear additive array of eight elements and with a spacing $d = \lambda/2$, provided, of course, that only one signal is arriving. This output has already been plotted in Figure 3.

It is not immediately obvious what the response of the enhanced phase system will be to two simultaneously arriving waves. The method by which this can be calculated for the enhanced phase system will now be outlined and the result given.

Consider the system mentioned above with $n = 3$, $D = d/3 = \lambda/6$, with an array of eight antennae. Two wavefronts are received whose amplitudes are equal ($=a$) and whose frequencies are ω and ω' . The antenna voltages V_A and $V_{A'}$ will be:

$$V_A = a \cos (\omega t + y) + a \cos (\omega' t + y'); \quad V_{A'} = a \cos (\omega t - y) + a \cos (\omega' t - y')$$

These signals are passed through frequency multipliers. To produce frequencies 3ω , a non-linearity of the form $c_3 x^3$ is necessary. The voltages V_A and $V_{A'}$ are subjected to this non-linearity. When the resulting voltage is passed through a band-pass filter tuned to a frequency 3ω , components having the following frequencies will be passed: 3ω , $2\omega + \omega'$, $\omega + 2\omega'$ and $3\omega'$. These are the only frequencies that are approximately equal to 3ω . All other frequency components that are produced by the non-linearity, are removed by the filter. The filter outputs will, therefore, be:

$$V_{3A} = \frac{c_3 \cdot a^3}{4} \left[\cos (3\omega t + 3y) + 3 \cos (2\omega t + \omega' t + 2y + y') \right. \\ \left. + 3 \cos (\omega t + 2\omega' t + y + 2y') + \cos (3\omega' t + 3y') \right]$$

and for $V_{3A'}$, a similar expression is found in which all quantities y and y' in the above expression have the opposite sign. When these two voltages are subsequently added, the sum will be:

$$V_{3A} + V_{3A'} = \frac{c_3 \cdot a^3}{2} \left[\cos 3\omega t \cdot \cos 3y + 3 \cos (2\omega + \omega')t \cdot \cos (2y + y') \right. \\ \left. + 3 \cos (\omega + 2\omega')t \cdot \cos (y + 2y') + \cos 3\omega' t \cdot \cos 3y' \right]$$

As in the case of the linear array, this signal is amplitude detected and indicated by a meter. Also only relative amplitudes are of interest. Then the above term gives the following contribution to the output of the enhanced phase system:

$$\cos 3y + 3 \cos (2y + y') + 3 \cos (y + 2y') + \cos 3y'$$

When the contributions from the three other antenna pairs are added to this the total output of the enhanced phase system becomes:

$$\begin{aligned} \text{Output} = & \cos 3y & + 3 \cos (2y + y') & + 3 \cos (y + 2y') & + \cos 3y' \\ & + \cos 9y & + 3 \cos (6y + 3y') & + 3 \cos (3y + 6y') & + \cos 9y' \\ & + \cos 15y & + 3 \cos (10y + 5y') & + 3 \cos (5y + 10y') & + \cos 15y' \\ & + \cos 21y & + 3 \cos (14y + 7y') & + 3 \cos (7y + 14y') & + \cos 21y' \end{aligned} \quad (13)$$

This response of the enhanced phase system to two incoming waves is plotted in

Figure 11. And again we see that the system is unable to separate two incoming waves that are easily resolved by the linear array.

5. A CORRELATION SYSTEM

One is inclined to draw the conclusion that whatever type of processing is used to reduce the size of a linear array, (see Ref. 3) this always results in a loss of resolving power. That this is not the case is shown in the following example.

The system now under study consists of a number of antennae together with the necessary processing equipment, as did the systems previously considered.

The antennae elements are positioned at the points O, A, B, C, and D as defined for the linear additive array (Figure 2). The antenna voltages will be:

$$V_0 = a \cos \omega t; V_A = a \cos (\omega t + u); V_B = a \cos (\omega t + 3u), \text{ etc.}$$

where $u = \frac{\pi d}{\lambda} \sin \alpha$ as defined for the linear array (equation 1). The time averaged product of V_0 with each of the other antenna voltages is now taken and the resulting quantities z_1, z_2, z_3, z_4 are then added. As was shown in the discussion of the T.A.P. arrays, the time averaged product of V_0 and V_A will be:

$$\overline{V_0 V_A} = z_1 = \frac{a^2}{2} \cos u$$

Similarly:

$$z_2 = \frac{a^2}{2} \cos 3u; z_3 = \frac{a^2}{2} \cos 5u; z_4 = \frac{a^2}{2} \cos 7u$$

and the sum of these quantities is:

$$z_1 + z_2 + z_3 + z_4 = \frac{a^2}{2} (\cos u + \cos 3u + \cos 5u + \cos 7u)$$

which is exactly equal to the output of an eight element linear array if the factor $\frac{a^2}{2}$ is disregarded. For further clarification, the complete system is shown in Figure 12. It can easily be seen that the overall length of the

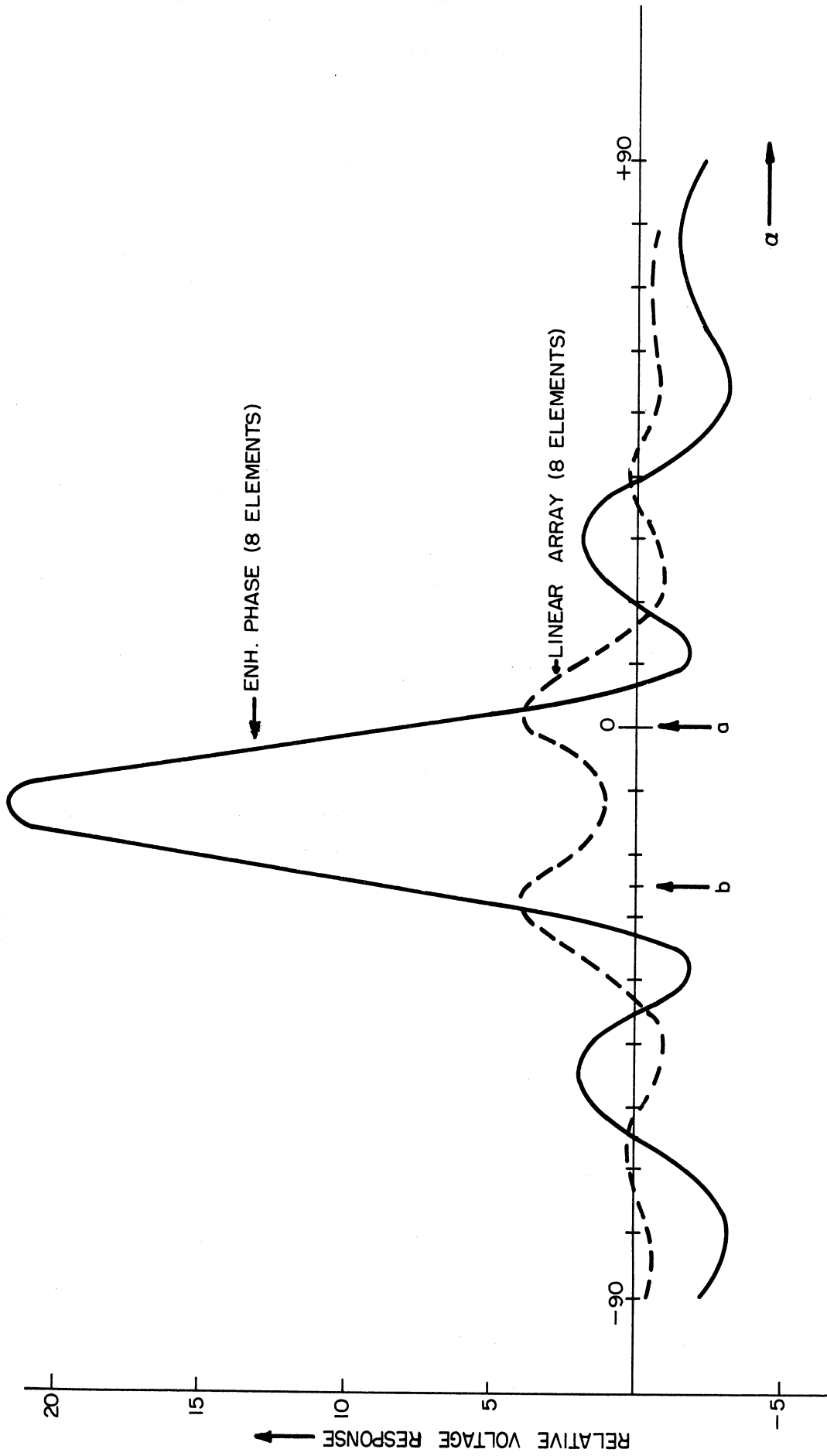


FIG. II COMPARISON OF ENHANCED PHASE AND LINEAR ARRAY

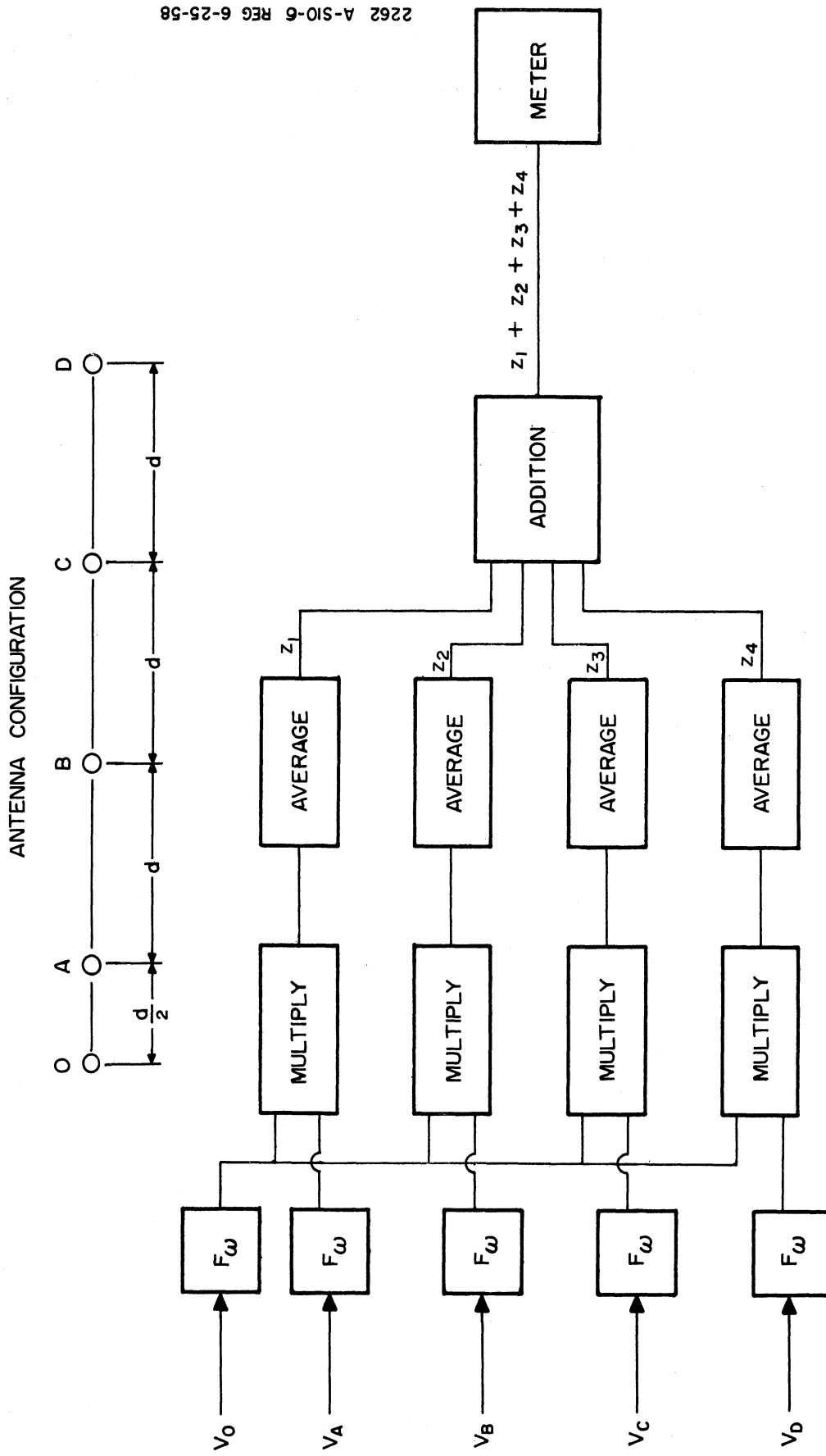


FIG. 12. A CORRELATION SYSTEM

antenna array in this system is half that of the linear array (compare Figure 2).

When two waves of equal amplitude are arriving, the voltages on the antennae will be as follows:

$$V_0 = a (\cos \omega t + \cos \omega' t)$$

$$V_A = a \left[\cos (\omega t + u) + \cos (\omega' t + u') \right] \quad \text{etc.}$$

The time averaged products of these voltages are:

$$\begin{aligned} z_1 &= \frac{a^2}{2} (\cos u + \cos u') & z_3 &= \frac{a^2}{2} (\cos 5u + \cos 5u') \\ z_2 &= \frac{a^2}{2} (\cos 3u + \cos 3u') & z_4 &= \frac{a^2}{2} (\cos 7u + \cos 7u') \end{aligned}$$

The sum of these quantities is:

$$\begin{aligned} z_1 + z_2 + z_3 + z_4 &= \cos u + \cos u' + \cos 3u + \cos 3u' + \cos 5u + \cos 5u' + \cos 7u \\ &\quad + \cos 7u' \end{aligned} \quad (14)$$

where the factor $\frac{a^2}{2}$ has been neglected because only relative amplitudes are important.

This output is exactly the same as the output of the linear array system (equation 6) with two incoming waves and this correlation system, therefore, has the same resolving power as does the linear array even though its antenna array is only half as long.

In general it can be said that a linear array consisting of $2n$ elements can be reduced to half its length by using the correlation method employing $(n + 1)$ elements.

6. PRACTICAL CONSIDERATIONS

As we have seen, different methods of non-linear processing are possible to obtain narrow lobe patterns for narrow aperture antenna arrays. How useful are these methods for the purpose of radio direction finding?

In the first place, one should bear in mind that even though a

narrow lobe shape is obtained, the resolution is not what one would expect. This was explained in detail in the previous sections of this report.

Secondly, one must remember that the reduction of the antenna array size has to be paid for by a tremendous increase in complexity of the equipment. Not only does the necessary non-linear processing have to be instrumented, but in addition it is necessary to have matched bandfilters between the antennae and the processing equipment. (see also footnote on page 2) These filters are denoted by F_{ω} in Figures 4, 7, 10 and 12. The bandfilters are necessary to prevent radio signals at different frequencies from producing intermodulation products in the non-linear device that would fall in the desired frequency range. Such narrow band-pass filters with identical phase response are difficult to build and keep in alignment. These filters, therefore, increase the complexity of the systems studied considerably.

7. NOISE CONSIDERATIONS

The object of this section is to consider the effect of random noise. Although no general conclusions are reached, this section discusses the system variables that affect the output noise.

Sections 2, 3, 4, and 5 of the report are devoted to studying the effect of one interference source on the performance of the T.A.P. and enhanced phase systems. We can now ask what happens when a large number of interference sources are present. If the way in which these sources are distributed is known, their effect can be calculated separately for each possible distribution.

This situation can be carried to the extreme; instead of a limited number of interference sources, an infinite number of uncorrelated noise sources are considered. These noise sources are located at a very large distance and are distributed uniformly over an angle of 360° . In addition, they all radiate

the same average power. The noise field generated by this distribution of noise sources will be called an isotropic noise field.

Consider the behavior of a linear array system (see Figure 1) which is placed in an isotropic noise field. We can then define the output noise as the fluctuation of the indicating device that shows the output amplitude. The output noise will depend both on the predetection bandwidth of the receiver and on the integration time or bandwidth of the post-detection filter and the indicating device. By making this integration time long, the output noise can be made arbitrarily small, but the time necessary to take a bearing will be increased.

The same general remarks apply to the T.A.P. systems. Here too, practical considerations limit the maximum feasible integration time. The shorter this time, the larger the fluctuations of the output signal. The contribution of the noise to the output signal in this case, however, cannot be made arbitrarily small because the partial correlation between the noise voltages generated in the antennae, contributes a non fluctuating DC term to the output voltage, even when the time average is taken over an infinitely long period of time. This DC component in the output due to the noise does not, however, affect the accuracy with which the bearing can be determined, because it is independent of the orientation of the antenna array, because the noise field was assumed isotropic. It is only the fluctuating noise components in the output, that influence the bearing-accuracy.

The output fluctuations of the Linear Array System and of the T.A.P. system can be calculated once all time constants or bandwidths involved are specified. As we pointed out above, the choice of these time constants depends on the speed with which the array is to be rotated and the bearing be found. Another consideration that enters into the calculation of the output noise is the correla-

tion (c_i) that exists between the noise voltages produced by two antenna elements which are spaced a distance d apart. This correlation coefficient can be calculated for noise with a rectangular frequency spectrum centered around a frequency ω , corresponding to a wavelength (λ), provided that the relative bandwidth $w < 0.01$ and $d < 5\lambda$. With these assumptions the correlation c_i between the noise voltages generated in two antenna elements spaced a distance d apart is given to within $\frac{1}{2}$ % by:

$$c_i = J_0 \left(\frac{2\pi d}{\lambda} \right)$$

where J_0 is the Bessel function of order zero. A derivation of this expression is given in the appendix.

No calculation of the noise outputs of any of these systems are included in this report. It does not appear that the results obtained from such calculations would be worth the effort required at the present time. This follows because the non-linear systems are not presently used for radio direction finding and because the performance of direction finding systems does not appear to be limited by an isotropic noise field. Propagation effects due to the system environment and ionospheric reflection seem to be more important limitations at present.

Once the noise output has been determined the question can be asked as to how this noise affects the estimated value of the bearing. This again will depend on the period of time an operator spends taking his readings and also on the procedure he follows in estimating the bearing from these readings. If the operator just tries to position the antenna array to maximize the output signal, the variance of his bearings may be larger than in the case when he takes the average of two bearings on either side of the maximum, where the outputs are equal and on a part of the antenna pattern where the signal decreases rapidly with change in angle of orientation (α). This is a method often used

to find a maximum that is not sharply defined, as for instance, the resonant point of a tuned circuit. To study these effects and to determine the optimum manner to take a bearing is beyond the scope of this paper.

8. CONCLUSION

Three methods of reducing the size of a linear additive antenna array have been studied. It has been shown that they all are capable of producing a directivity pattern identical to that of the linear array when only one wave is received.

Two of these methods, the T.A.P. and the enhanced phase systems, do not have the resolution that the equivalent linear array has, when two waves arrive simultaneously. The third, a correlation system, does have the same resolution but it only allows the size of the array to be reduced by a factor of two.

The non-linear processing methods have another drawback when used in radio direction finding; because the antenna signals have to be filtered before being processed, a number of narrow band filters balanced in phase, equal to the number of antenna elements, is required. They increase the complexity of the systems and make maintenance difficult.

When these systems are placed in an isotropic noise field, the noise interference can be made arbitrarily small by allowing large averaging times. When finite periods of time are used, the residual noise can be calculated for each system, provided the relevant time constants are known. The effect of noise on the estimate of the bearing also depends on the manner in which the reading is taken.

APPENDIX

In this appendix the derivation will be given of the formula

$c_i = J_0 \left(\frac{2\pi \cdot d}{\lambda} \right)$ for the correlation between the two noise voltages V_1 and V_2 generated in two antennae spaced a distance d apart.

The following assumptions are made:

1. The two antennae are placed in an isotropic noise field. This is the noise field generated by an infinite number of uncorrelated noise sources at a very large distance, each generating the same average noise power. These noise sources are distributed evenly over 360° .
2. Because normally a narrow band receiver is used behind the antennae, it will only be necessary to consider a small "rectangular" frequency band of noise of bandwidth $w_\omega = \omega_2 - \omega_1$ centered around the frequency ω_0 . The center frequency ω_0 corresponds to a wavelength $\lambda = \frac{2\pi c}{\omega_0}$.
3. It is assumed that the relative bandwidth $w_r = \frac{w_\omega}{\omega_0}$ is small, for instance, $w_r = 0.01$.
4. The distance d between the two antenna elements is not larger than 5λ .
5. The antenna elements are identical and do not have any mutual coupling.
6. The rms noise voltages generated in the two antennae are equal for reasons of symmetry.

To derive the formula two other cases have to be considered first. This is done in sections A and B. The expression for c_i is derived in section C.

A. Instead of two antennae placed in an isotropic noise field, where they would receive noise arriving from all directions, we first consider the case that the two antennae receive the noise from one noise source only. The noise source is a narrow band noise source according to assumption 2. The noise source is situated at a large distance in the direction α so that the wave received by the two antennae is essentially plane. (Figure 13) It is obvious from Figure 13

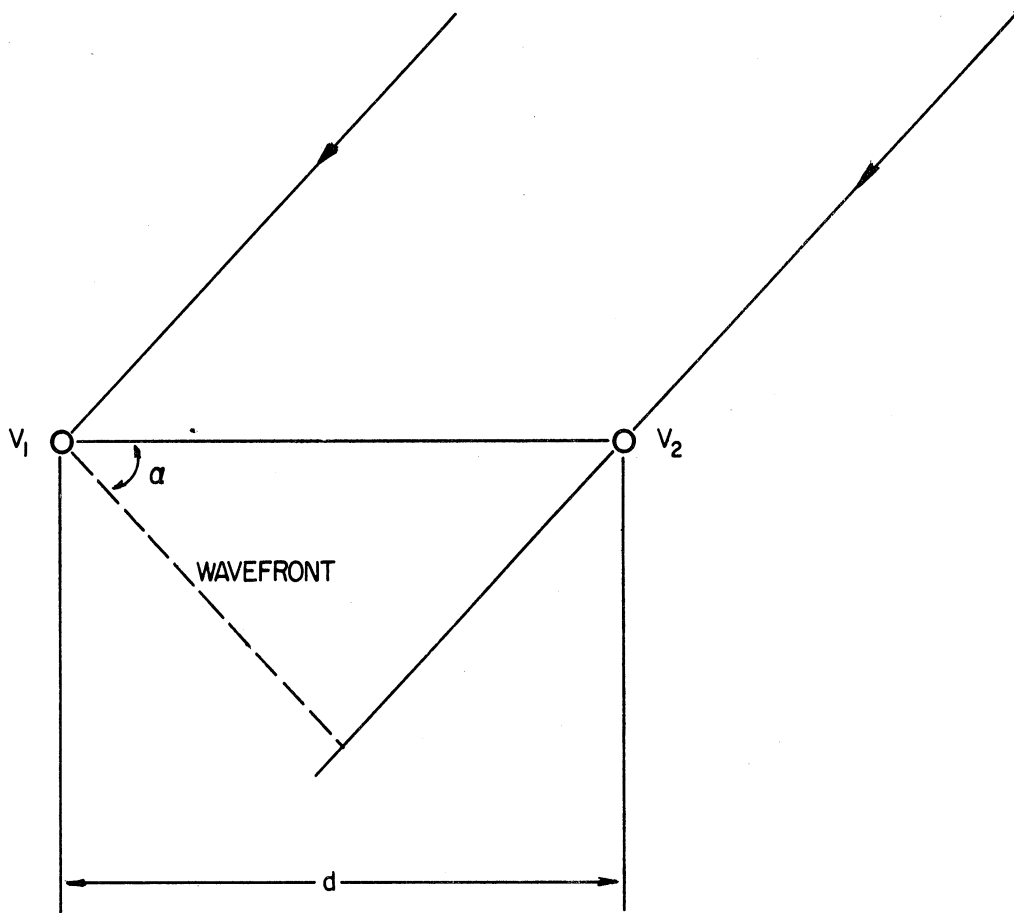


FIG. 13

TWO ANTENNAE WITH WAVEFRONT

that the noise that is received by antenna 2, is exactly the same as that received by antenna 1 a moment τ later. The period τ is a function of the angle of arrival α :

$$\tau = \frac{d \sin \alpha}{c} \quad (c = \text{velocity of propagation})$$

The correlation $c(\tau)$ between two noise voltages that are identical except for a time delay τ , is a function of the bandwidth w_ω and the center frequency ω_0 , and is given by the autocorrelation function for a "rectangular" noise band (assumption 2):

$$c(\tau) = \cos \omega_0 \tau \frac{\sin (w_\omega \tau / 2)}{w_\omega \tau / 2} \quad (\text{Ref. 4})$$

With $d = n\lambda$; (assumption 4: $n < 5$) $\tau = \frac{d \sin \alpha}{c} = \frac{2\pi d \sin \alpha}{\omega_0 \lambda} = \frac{2\pi n \sin \alpha}{\omega_0}$

and $w_\omega = w_r \cdot \omega_0 = 0.01 \omega_0$, (assumption 3), we find that an error of less than 1/2% is made, for any value of α , if we set the following quantity equal to one:

$$\frac{\sin (w_\omega \tau / 2)}{w_\omega \tau / 2} = \frac{\sin (0.01 \pi n \sin \alpha)}{0.01 \pi n \sin \alpha} = 1$$

With this substitution we find the following expression for the correlation coefficient

$$c(\tau) = \cos (\omega_0 \tau)$$

or

$$c(\alpha) = \cos \left(\frac{2\pi d}{\lambda} \sin \alpha \right)$$

This then is the correlation between the noise voltages V_1 and V_2 when the noise received is coming from a single noise source in the direction α .

B. Next we consider the correlation between the antenna voltages V_1 and V_2 when two noise waves of equal average noise power are arriving from two different directions α_1 and α_2 . The two noise waves, of course, are generated by two noise sources with equal bandwidth (assumption 2). The two noise sources

are denoted by N_1 and N_2 , and they are uncorrelated (assumption 1). N_1 generates the voltage n_{11} in antenna 1 and the voltage n_{12} in antenna 2, where n_{11} and n_{12} stand for instantaneous voltages. We know that the average noise powers $\overline{n_{11}^2}$ and $\overline{n_{12}^2}$ are equal (assumption 6) and also that the correlation between n_{11} and n_{12} is equal to (Ref. 4)

$$c_1 = \frac{\overline{n_{11} \cdot n_{12}}}{\sqrt{\overline{n_{11}^2} \cdot \overline{n_{12}^2}}} = \frac{\overline{n_{11} \cdot n_{12}}}{\overline{n_{11}^2}} = \cos \left(\frac{2\pi d}{\lambda} \sin \alpha_1 \right)$$

Similarly we define the voltages induced in antenna 2 by the noise source N_2 as n_{21} and n_{22} respectively. The correlation between them is c_2 . Because N_1 and N_2 are uncorrelated noise sources of equal average power, we can write down the following equalities:

$$\overline{n_{11}^2} = \overline{n_{12}^2} = \overline{n_{21}^2} = \overline{n_{22}^2} \quad \text{and}$$

$$\overline{n_{11} \cdot n_{21}} = \overline{n_{12} \cdot n_{21}} = \overline{n_{12} \cdot n_{22}} = \overline{n_{11} \cdot n_{22}} = 0$$

We are now in a position to calculate the correlation (c_0) between the two antenna voltages $V_1 = n_{11} + n_{21}$ and $V_2 = n_{12} + n_{22}$, and we find

$$c_0 = \frac{\overline{V_1 \cdot V_2}}{\sqrt{\overline{V_1^2} \cdot \overline{V_2^2}}} = \frac{\overline{(n_{11} + n_{21}) \cdot (n_{12} + n_{22})}}{\overline{(n_{11} + n_{21})^2}}$$

$$c_0 = \frac{\overline{n_{11} \cdot n_{12}} + \overline{n_{21} \cdot n_{22}} + \overline{n_{11} \cdot n_{22}} + \overline{n_{21} \cdot n_{12}}}{\overline{n_{11}^2} + \overline{n_{21}^2} + 2 \overline{n_{11} \cdot n_{21}}}$$

$$c_0 = \frac{c_1 \cdot \overline{n_{11}^2} + c_2 \cdot \overline{n_{12}^2}}{2 \overline{n_{11}^2}} = \frac{c_1 + c_2}{2}$$

This result indicates that the correlation between the voltages V_1 and V_2 when two noise signals of equal average power are received, is the arithmetical average of the correlation coefficients c_1 and c_2 as defined above.

C. In section B a configuration with two noise sources was considered.

When additional noise sources are added, all uncorrelated and of equal average power, a calculation similar to that of section B shows that the correlation between V_1 and V_2 is equal to the average of the correlation coefficients of the voltages V_1 and V_2 generated by each of the noise sources separately.

When the two antennae are now placed in an isotropic noise field, which was defined as the noise field generated by an infinite number of uncorrelated noise sources, distributed evenly over 360° , (assumption 1) the correlation coefficient c_i of the voltages V_1 and V_2 will be equal to the average of $c(\alpha)$ taken over all directions α .

$$c_i = \frac{1}{2\pi} \int_{-\pi}^{+\pi} c(\alpha) d\alpha = \frac{1}{2\pi} \int_{-\pi}^{+\pi} \cos\left(\frac{2\pi d}{\lambda} \sin \alpha\right) d\alpha$$

Because the integrand is an even function of α , this can be written as:

$$c_i = \frac{1}{\pi} \int_0^{+\pi} \cos\left(\frac{2\pi d}{\lambda} \sin \alpha\right) d\alpha = J_0\left(\frac{2\pi d}{\lambda}\right)$$

where J_0 is the Bessel function of order zero. This proves our formula.

REFERENCES

1. Theory of Time-Averaged-Product Arrays, A. Berman and C. S. Clay. The Journal of the Acoustical Society of America, Vol. 29, Nr 7, July 1957.
2. A Tactical Direction Finding System, Lyman W. Orr. University of Michigan, Electronic Defense Group, Internal Memorandum No. 25. September 30, 1957.
3. Space Frequency Equivalence, W. E. Kock and J. L. Stone, Proceedings of the I.R.E. Vol. 46 No. 2, February 1958, correspondence pages, 499-500.
4. Noise, A. van der Ziel, Ed. Prentice-Hall, Inc., 1954. See pages 317 and 4.

DISTRIBUTION LIST

1 Copy Document Room
Stanford Electronic Laboratories
Stanford University
Stanford, California

1 Copy Commanding General
Army Electronic Proving Ground
Fort Huachuca, Arizona
Attn: Director, Electronic Warfare Department

1 Copy Chief, Research and Development Division
Office of the Chief Signal Officer
Department of the Army
Washington 25, D. C.
Attn: SIGEB

1 Copy Chief, Plans and Operations Division
Office of the Chief Signal Officer
Washington 25, D. C.
Attn: SIGEW

1 Copy Countermeasures Laboratory
Gilfillan Brothers, Inc.
1815 Venice Boulevard
Los Angeles 6, California

1 Copy Commanding Officer
White Sands Signal Corps Agency
White Sands Proving Ground
Las Cruces, New Mexico
Attn: SIGWS-EW

1 Copy Commanding Officer
White Sands Signal Corps Agency
White Sands Proving Ground
Las Cruces, New Mexico
Attn: SIGWS-FC

1 Copy Commanding Officer
Signal Corps Electronics Research Unit
9560th USASRU
Mountain View, California

65 Copies Commanding Officer
 USASRDL
 Fort Monmouth, New Jersey

 FOR - USASRDL Accountable Officer
 Inspect at Destination
 File No. 22824-PH-54-91(1701)

1 Copy H. H. Goode
 University of Michigan Research Institute
 Ann Arbor, Michigan

1 Copy Technical Documents Services
 Willow Run Laboratories
 University of Michigan
 Ypsilanti, Michigan

11 Copies Electronic Defense Group Project File
 University of Michigan
 Ann Arbor, Michigan

1 Copy University of Michigan Research Institute Project File
 Ann Arbor, Michigan

