Final Report

CONTINUOUS TRANSFORMATIONS AND INTEGRAL MANIFOLDS

Lamberto Cesari

ORA Project: 07576

under contract with:

UNITED STATES AIR FORCE
AIR FORCE OFFICE OF SCIENTIFIC RESEARCH
CONTRACT NO. AF-AFOSR-942-65
WASHINGTON, D.C.

administered through:

OFFICE OF RESEARCH ADMINISTRATION ANN ARBOR

November 1968
en
UMR0863
CONTINUOUS TRANSFORMATIONS AND INTEGRAL MANIFOLDS

Final Report

I. RESEARCH PAPERS

During the entire period of research Sept. 1, 1965-Oct. 31, 1968 the following 17 Progress Reports have been issued and distributed. A star denotes that the paper has appeared.


II. SUMMARY OF THE RESEARCH ACCOMPLISHED IN THE FRAME OF THE PROJECT

In previous papers Cesari (Trans. Am. Math. Soc. 124, 1966, 369-412, 413-429) had proved a number of existence theorems for lumped parameter control problems of optimization with bounded or unbounded control space. In particular, Cesari had extended to such problems the Nagumo-Tonelli existence theorem for free problems.

Papers 5, 10, and 17 continue the same line of work for lumped parameter systems. In particular, in 10 the author shows that the consistent use of suitable variants, proposed by Cesari, of Kuratowski's upper semicontinuity condition for variable sets leads to a number of new existence theorems, and moreover to a reinterpretation of parallel results of E. J. McShane and T. Nishiura. The same variants of Kuratowski's upper semicontinuity condition have been later studied by Pavol Brunovsky (SIAM J. Control, 6, 1968, 174-185) and by A. LaSota and C. Olech (Bull. Acad. Polon. Sci. 14, 1966, 615-621). In 17 the authors prove further existence theorem for optimal solutions by a refinement of the same conditions.

Papers 2, 3, 6, 7, 8, 9, 12, 13 concern existence theorems for distributed parameter control problems of optimization, where the solutions are manifolds of suitable dimension. In 2 the author studies distributed parameter systems with compact control space, and proves a Filippov-type existence theorem and other existence theorems by a new argument and consistent use of Sobolev spaces technique. In 6 the author proves existence theorems in situations where the control space is unbounded but $L_p$-compactness arguments with $p>1$ can be used. In 8 the author proves existence theorems, in particular a Nagumo-Tonelli-type theorem, in situations where the control space is unbounded and only $L_1$-integrability can be assured. This is an extremely difficult case. In 8 the author proves also existence theorems with both unbounded domain and unbounded control space. In 9 the author first reformulates the results above for distributed parameter systems with partial differential equations written in the Dieudonné-Rashevskii form. This is a rather general form, which has been proposed, independently and in completely different settings, by J. Dieudonné (e.g. Foundations of Modern Analysis, Academic Press 1960) and by P. K. Rashevski (Geometric Theory of Partial Differential Equations, Moscow 1948). The author then applies the entire technique in 9 to a problem of magnetohydrodynamics (cold plasma) proposed by K. A. Lurie (1963-64) with both unbounded domain and unbounded control space, and the equations written in the Dieudonné-Rashevskii form.

In paper 14 the author gives a theoretical form of the maximum principle for distributed parameter systems with partial differential equations written in the Dieudonné-Rashevskii form. The results seem to have a generality far above that obtained so far in this difficult field. The students B. A.
Suryanarayana and David Cowles are applying these ideas in a number of particular situations in their thesis.

Garth Warner's report No. 1 and J. Breckenridge's thesis concern surface area and continuous parametric manifolds. In 1 the author develops a very general approach to integration. A number of previous integration theories are included in his study, which also applies to theoretical questions of calculus of variations and control. In Breckenridge's thesis the same technique is developed with applications to measure theoretical analysis in surface area and theory of currents.

Reports 3, 15, and 16 concern a method based on functional analysis for existence and error bounds of solutions of boundary value problems in ordinary and partial differential equations. The method was initiated by Cesari in 1939. Report 16 is a survey of papers by different authors in the last ten years within the frame of Cesari's method. In 15 the author gives an existence theorem for solutions of the problem $\Delta u = f(x,y,u)$ in a circle of the $xy$-plane and $u = 0$ on the boundary. Preliminary work on Bessel functions which was needed in this research is contained in C. D. Stocking's thesis. In addition C. D. Stocking gives error bounds for the first Galerkin approximation to the problem above. The error, within certain limitations, is remarkably small.
III. Ph. D. THESSES PREPARED IN THE FRAME OF THE RESEARCH PROJECT

Garth Warner was supported by the present grant in summer 1966 while preparing his thesis for publication.


John Breckenridge is nearing completion of his Ph. D. thesis on measure theoretical problems of Lebesgue area theory. We expect to discuss it during this academic year.

B. A. Suryanarayana is nearing completion of his Ph. D. thesis on optimal control problems with hyperbolic partial differential equations. We expect to discuss it during this academic year.

Charles D. Stocking is nearing completion of his Ph. D. thesis on non-linear boundary value problems in a circle and related questions on Bessel functions. We expect to discuss it during this academic year.

Three other students Tom Angell, Richard Baum, and David Cowles are preparing their Ph.D. theses on theoretical aspects of optimal control theory and the calculus of variations.
IV. LECTURES

Since August 1965 Cesari was invited to give lectures at the following institutions and conferences:


International Summer Center, Bressanone, Italy. An eight lecture summer course, June 10-18, 1966.


University of California, Berkeley, Feb. 3; San Diego (La Jolla, Feb. 7; Los Angeles, Feb. 8, 1967.


Technische Universitaet, Berlin (West), May 5, 1967.


Academy of Science, Department of Mathematics, Prague, May 21-26, 1967.

Tulane University, New Orleans, March 29, 1968.


Northwestern University, Evanston, May 1, 1968.


V. PROGRAM OF LECTURES AT THE UNIVERSITY OF MICHIGAN

In the frame of the research work of the present project the following scientists have been invited to speak at The University of Michigan:

Hubert Halkin, University of California, San Diego (La Jolla).

A. Avez, University of Paris, Paris.

Minoru Urabe, Kyusyu University, Fukuoka, Japan.

Emilio O. Roxin, University of Buenos Aires (now at the Rhode Island State University, Kingston, R. I.).

Stephen A. Williams, California Institute of Technology (now at UCLA).

Hans Lewy, University of California, Berkeley.

Felix Browder, University of Chicago.

Czeslaw Olech, University of Krakow and Brown University.

Pavol Brunovski, University of Bratislava and University of Minnesota.

Peter D. Lax, New York University.

James Serrin, University of Minnesota.